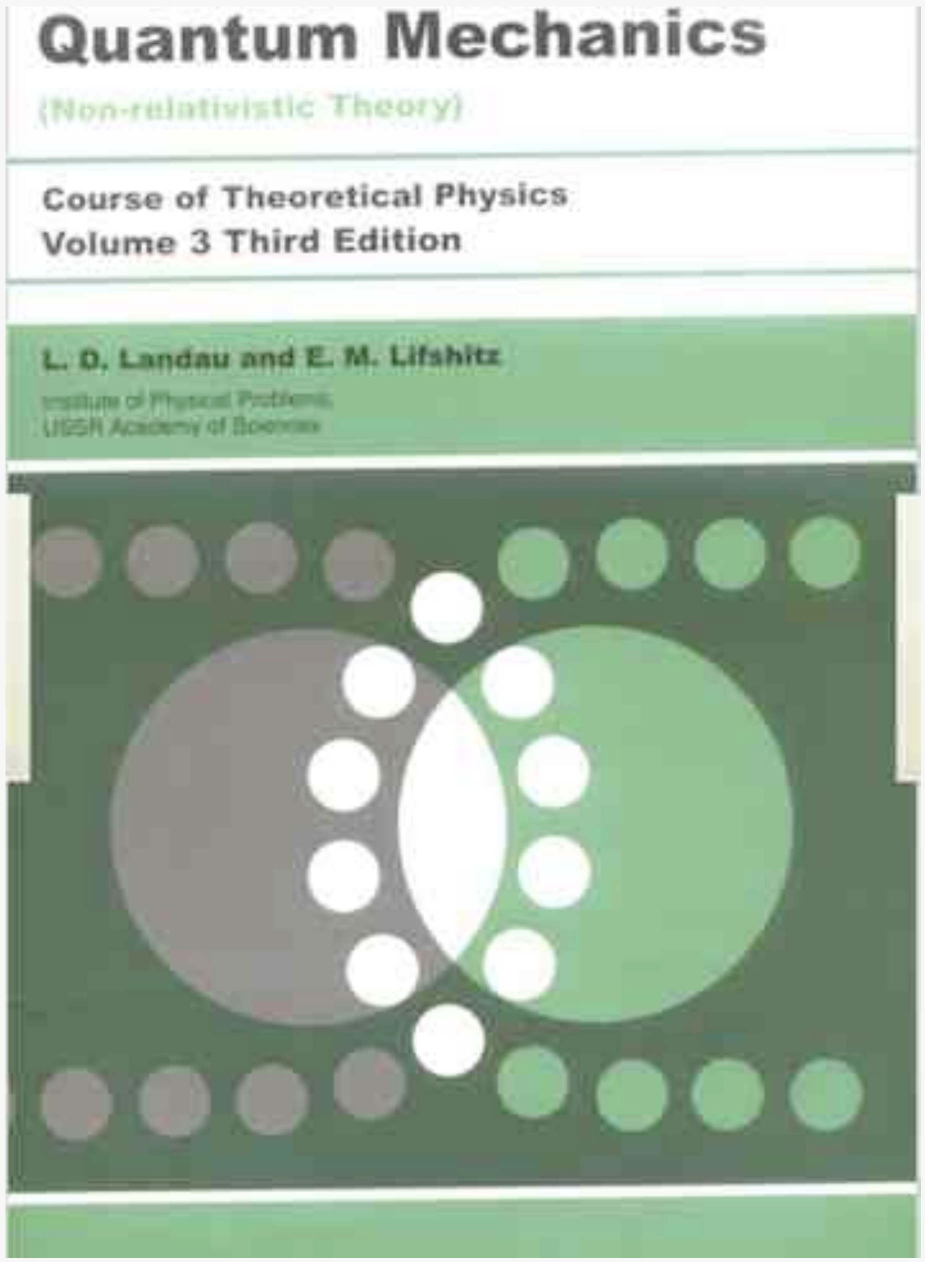
Non-Abelian particles in a two dimensional world

Victor Gurarie

University of Colorado
Boulder

Bosons and fermions

Bosons and fermions



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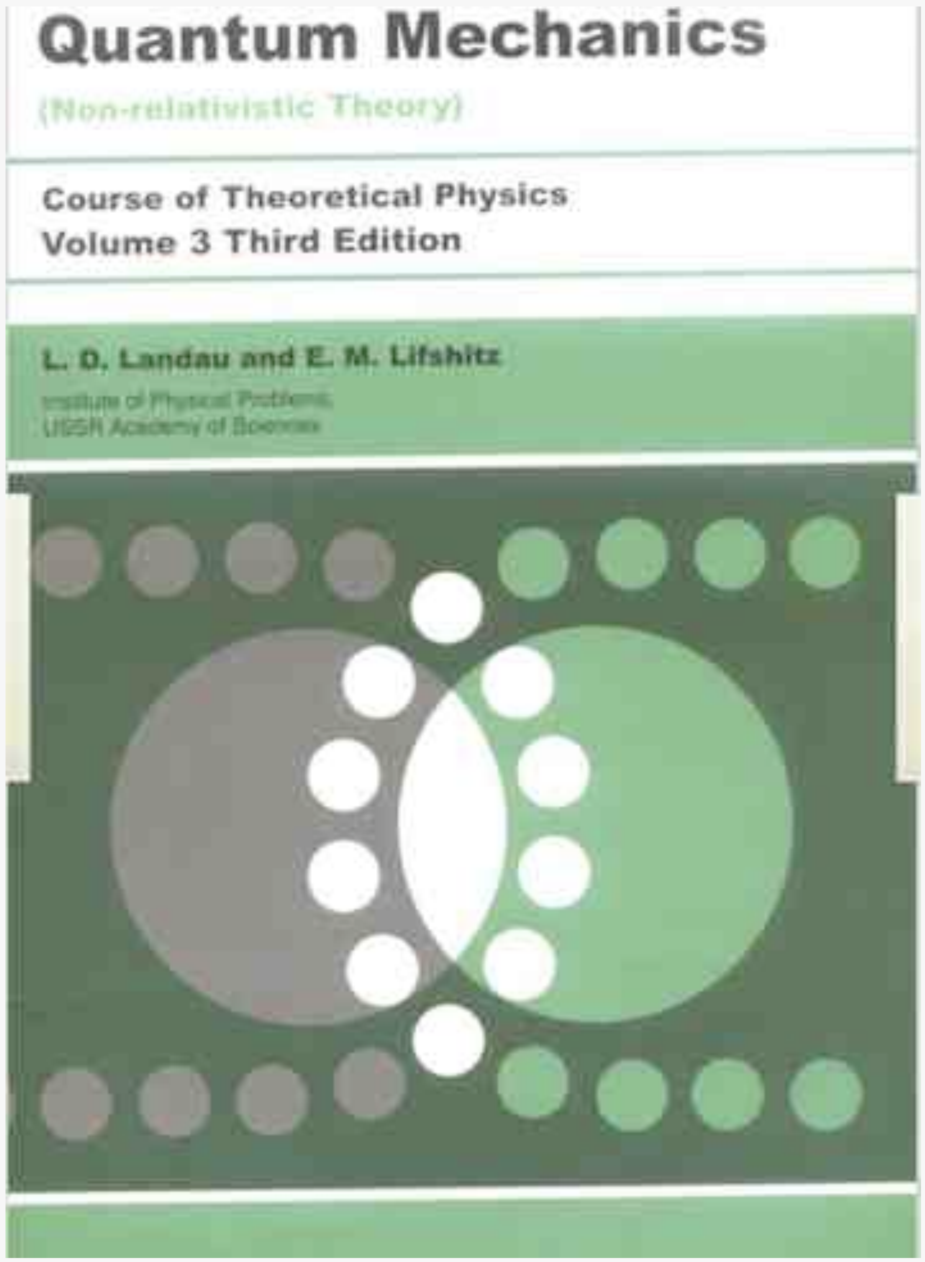

L.D. Landau and E.M. Lifshitz

Bosons and fermions

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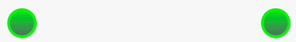
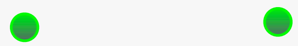


$$\Psi\left(\mathbf{r}_1, \mathbf{r}_2\right)$$

Bosons and fermions

L.D. Landau and E.M. Lifshitz



$$\Psi\left(\mathbf{r}_2,\mathbf{r}_1\right)=e^{i\theta}\Psi\left(\mathbf{r}_1,\mathbf{r}_2\right)$$

Bosons and fermions

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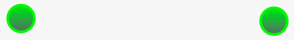

$$\Psi\left(\mathbf{r}_1, \mathbf{r}_2\right) = e^{i\theta}\Psi\left(\mathbf{r}_2, \mathbf{r}_1\right) = e^{2i\theta}\Psi\left(\mathbf{r}_1, \mathbf{r}_2\right)$$

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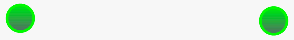

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$$e^{2i\theta}=1$$

Bosons and fermions

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$$e^{2i\theta} = 1$$

$$\theta = 0 \;\rightarrow \Psi\left(\mathbf{r}_1,\mathbf{r}_2\right) = \Psi\left(\mathbf{r}_2,\mathbf{r}_1\right)$$

Bosons

Bosons and fermions

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$$e^{2i\theta} = 1$$

$$\theta = 0 \;\rightarrow \Psi\left(\mathbf{r}_1,\mathbf{r}_2\right) = \Psi\left(\mathbf{r}_2,\mathbf{r}_1\right)$$ Bosons

$$\theta = \pi \;\rightarrow \Psi\left(\mathbf{r}_1,\mathbf{r}_2\right) = -\Psi\left(\mathbf{r}_2,\mathbf{r}_1\right)$$ Fermions

2D world: anyons

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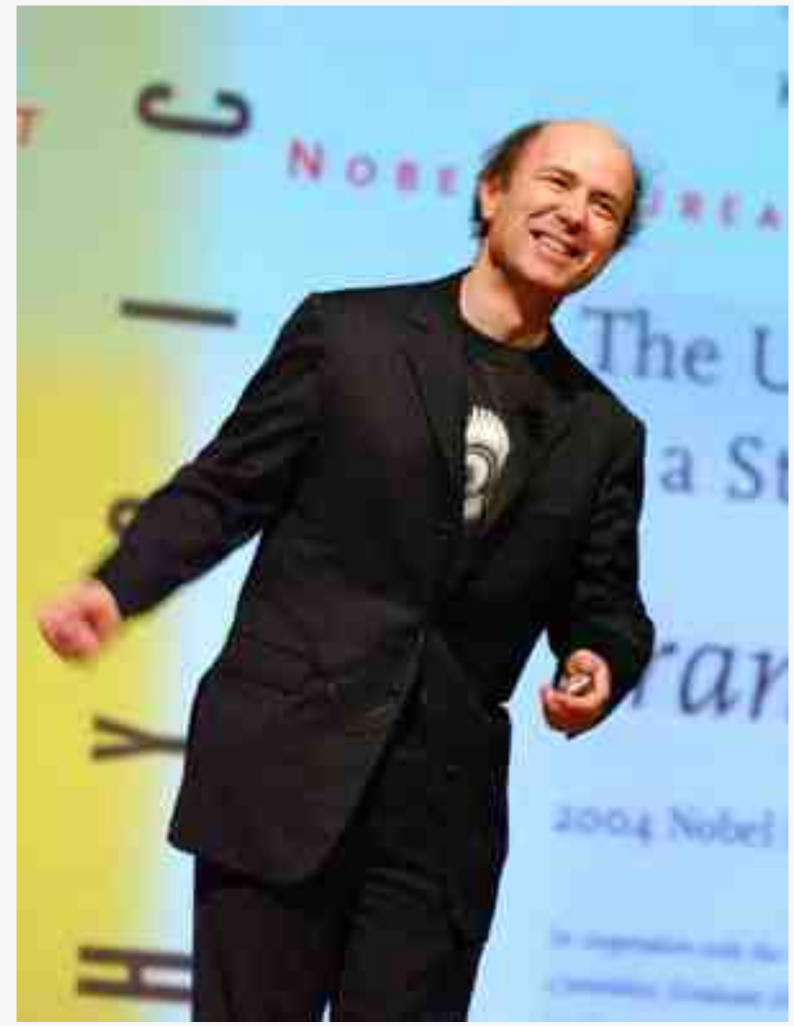
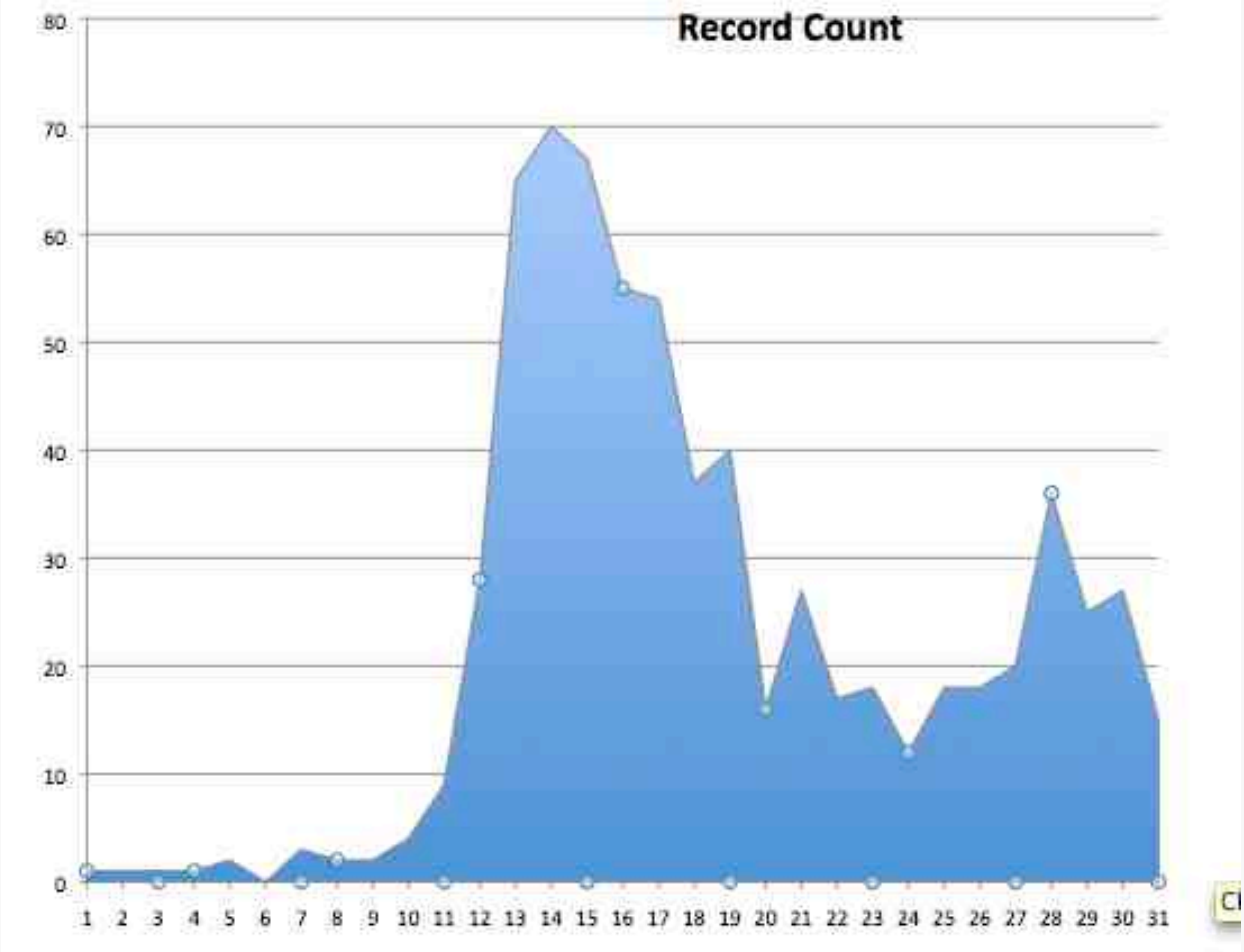
J. M. Leinaas
(with J. Myrheim)
1977



F. Wilczek
1982 and on

2D world: anyons



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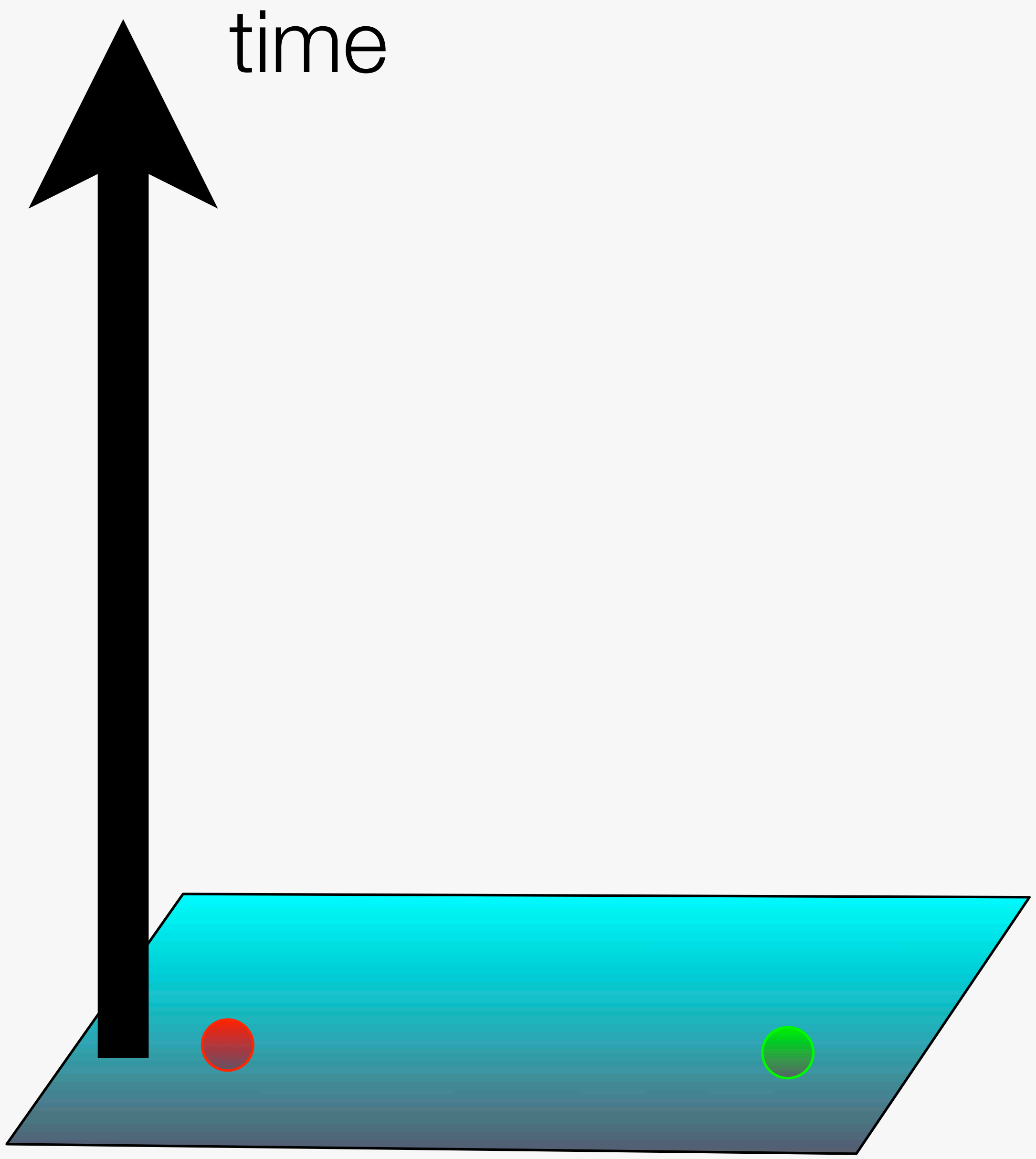

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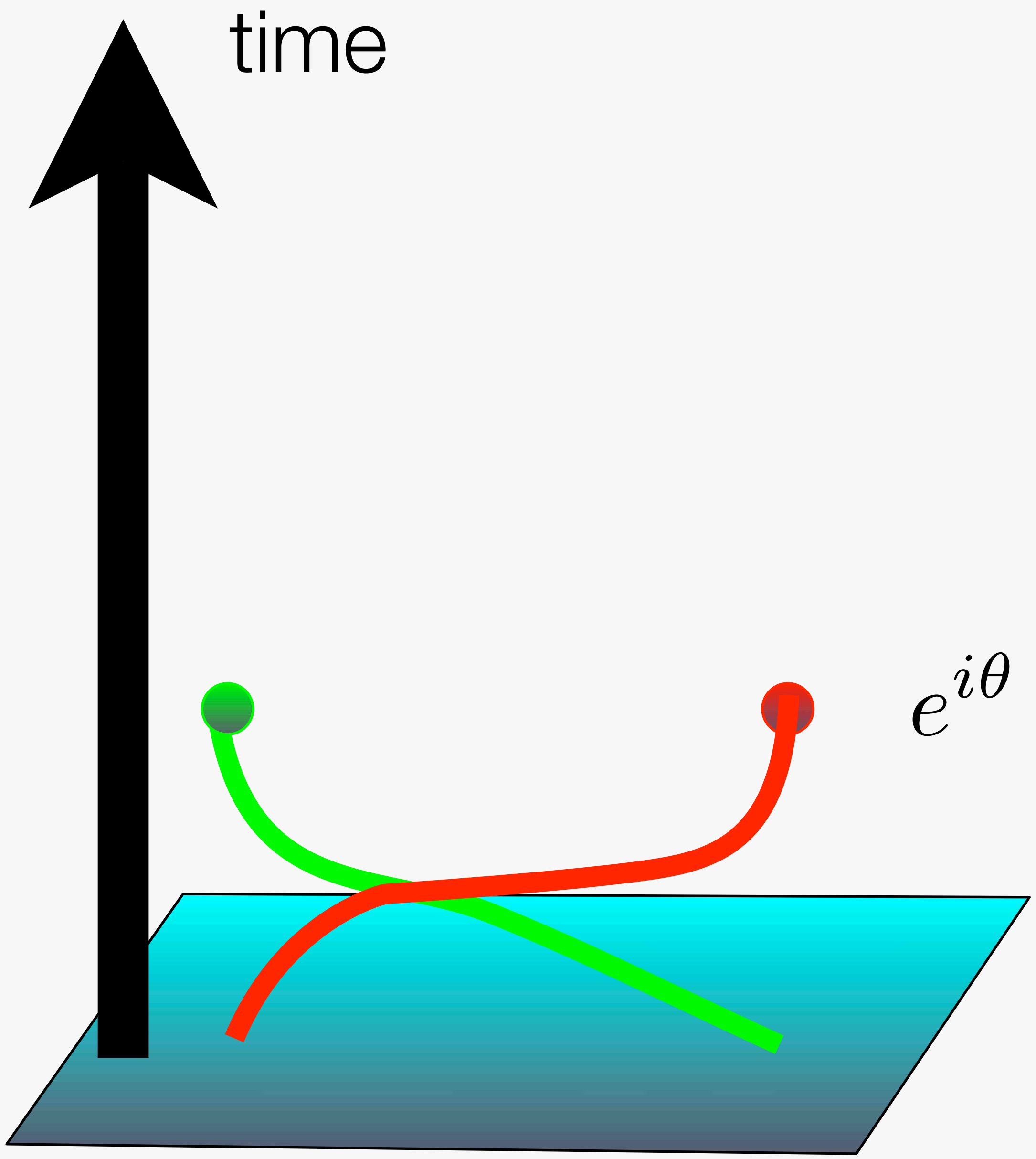

counterclockwise braid

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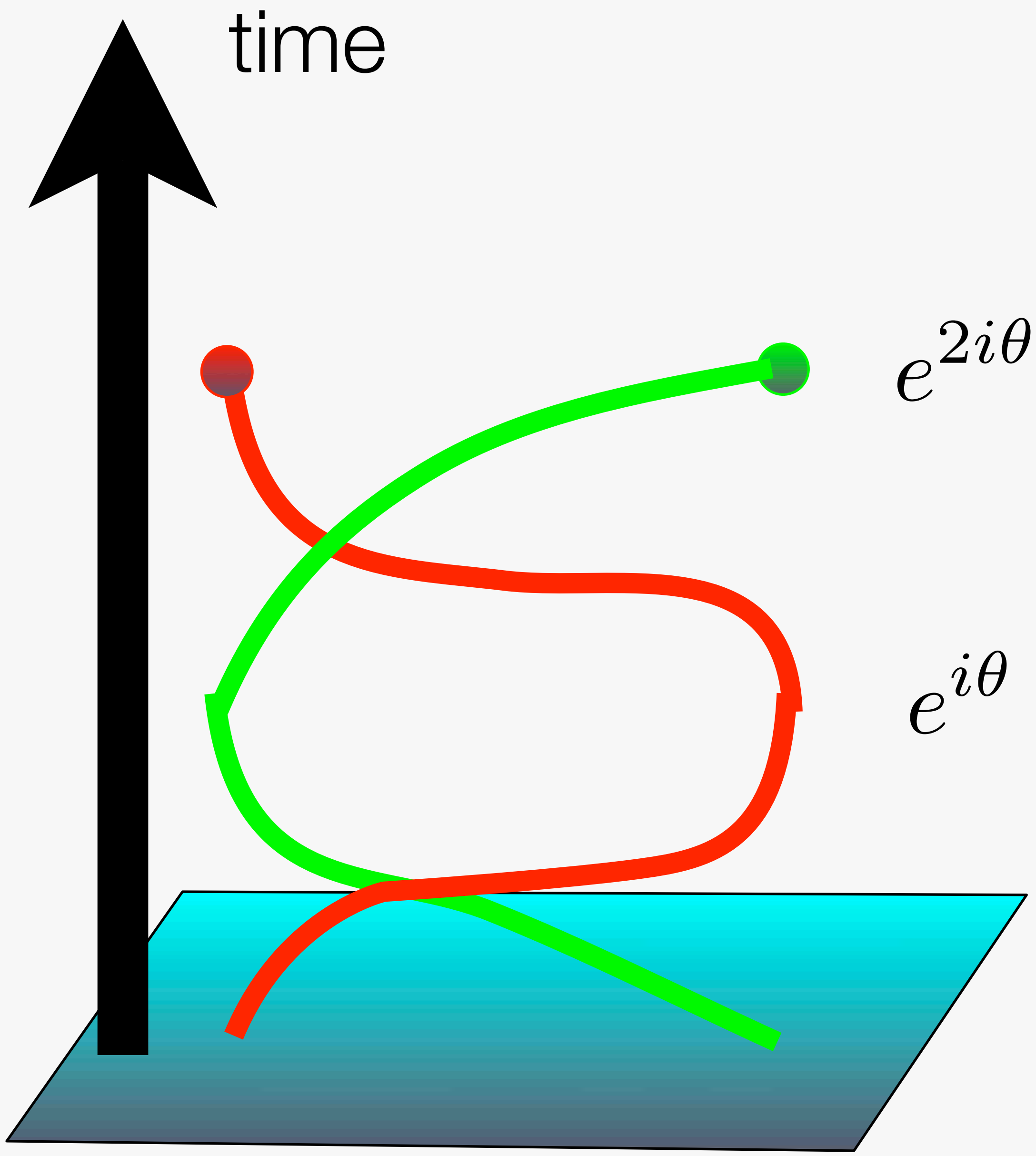

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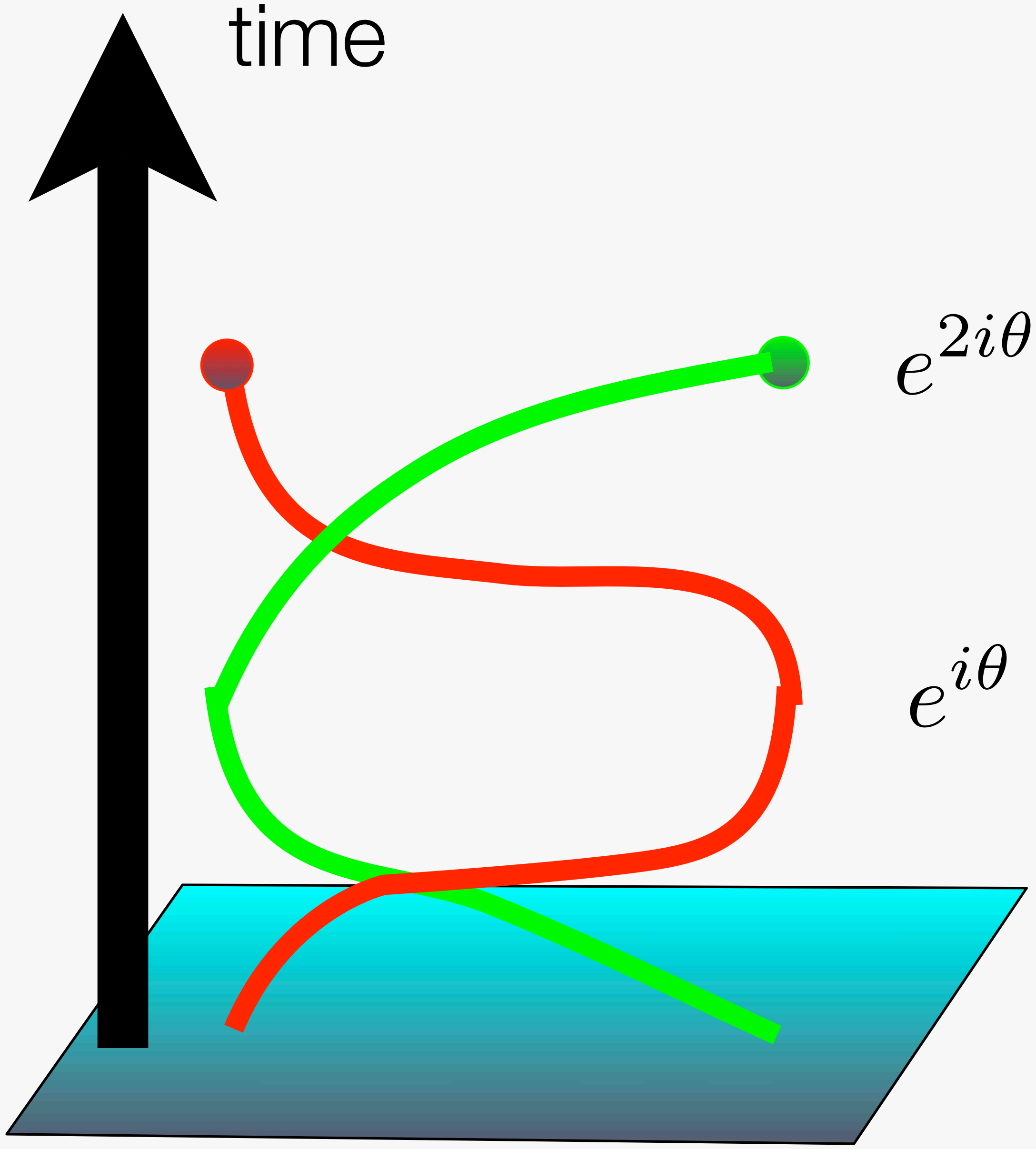

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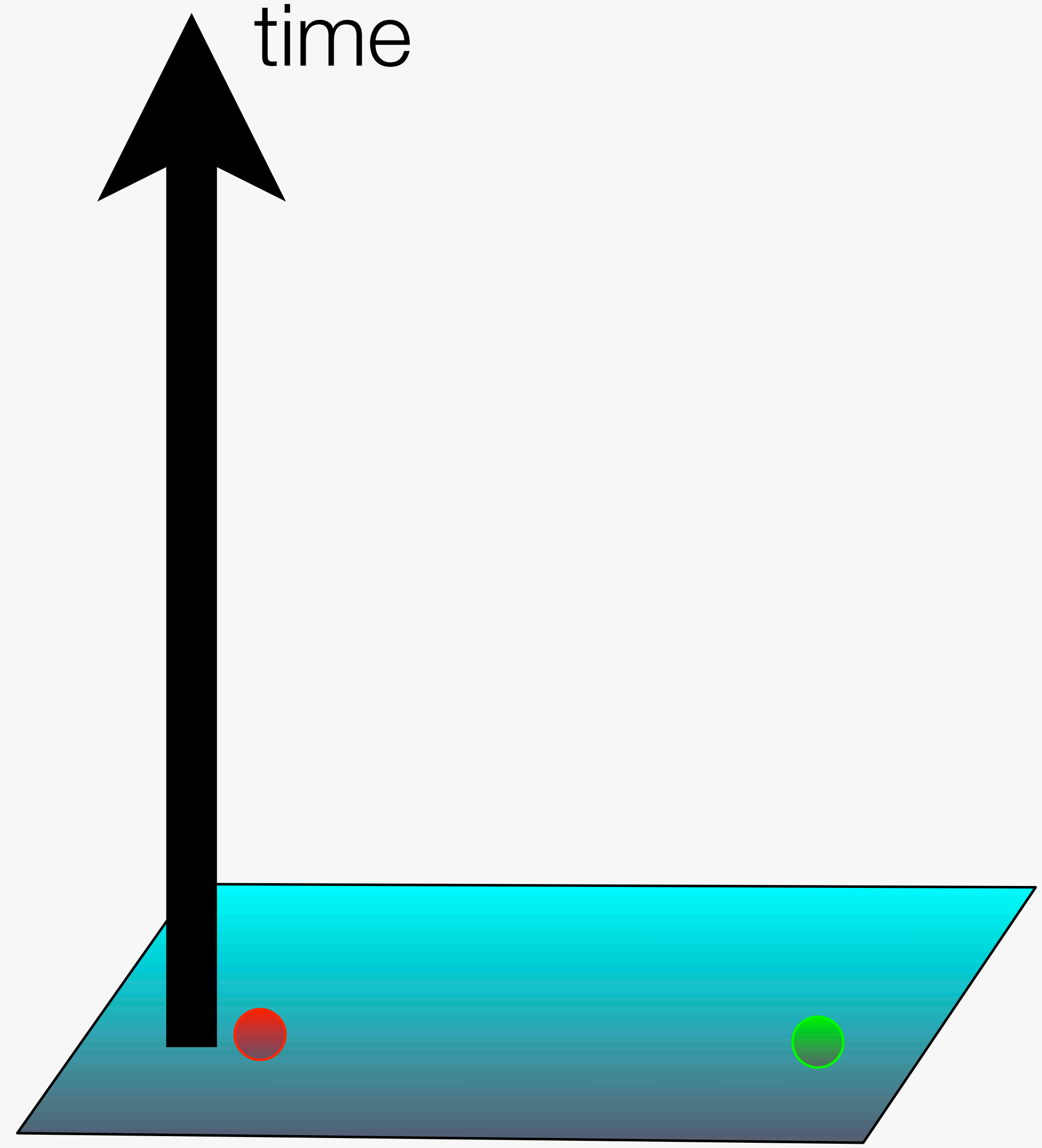

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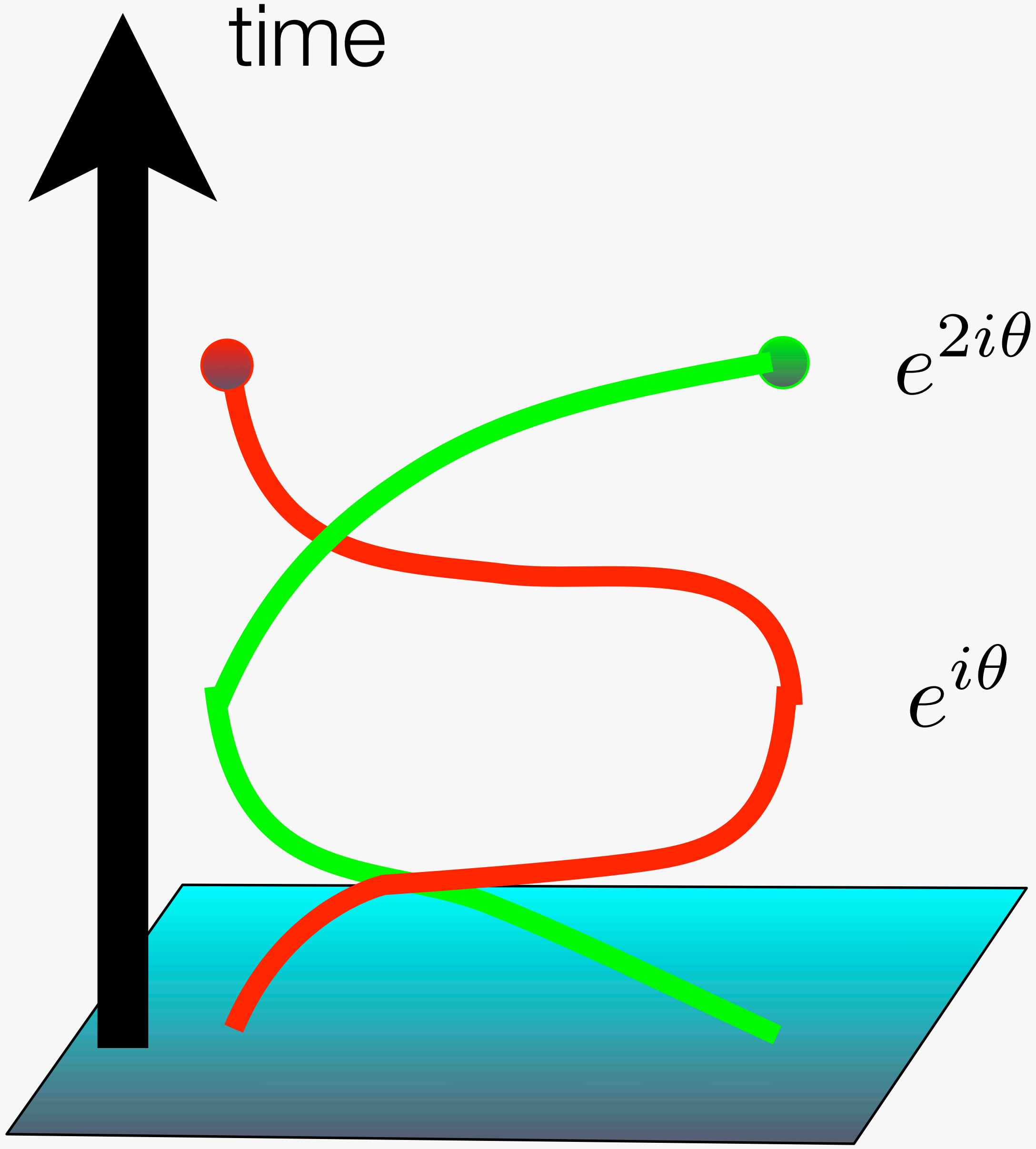

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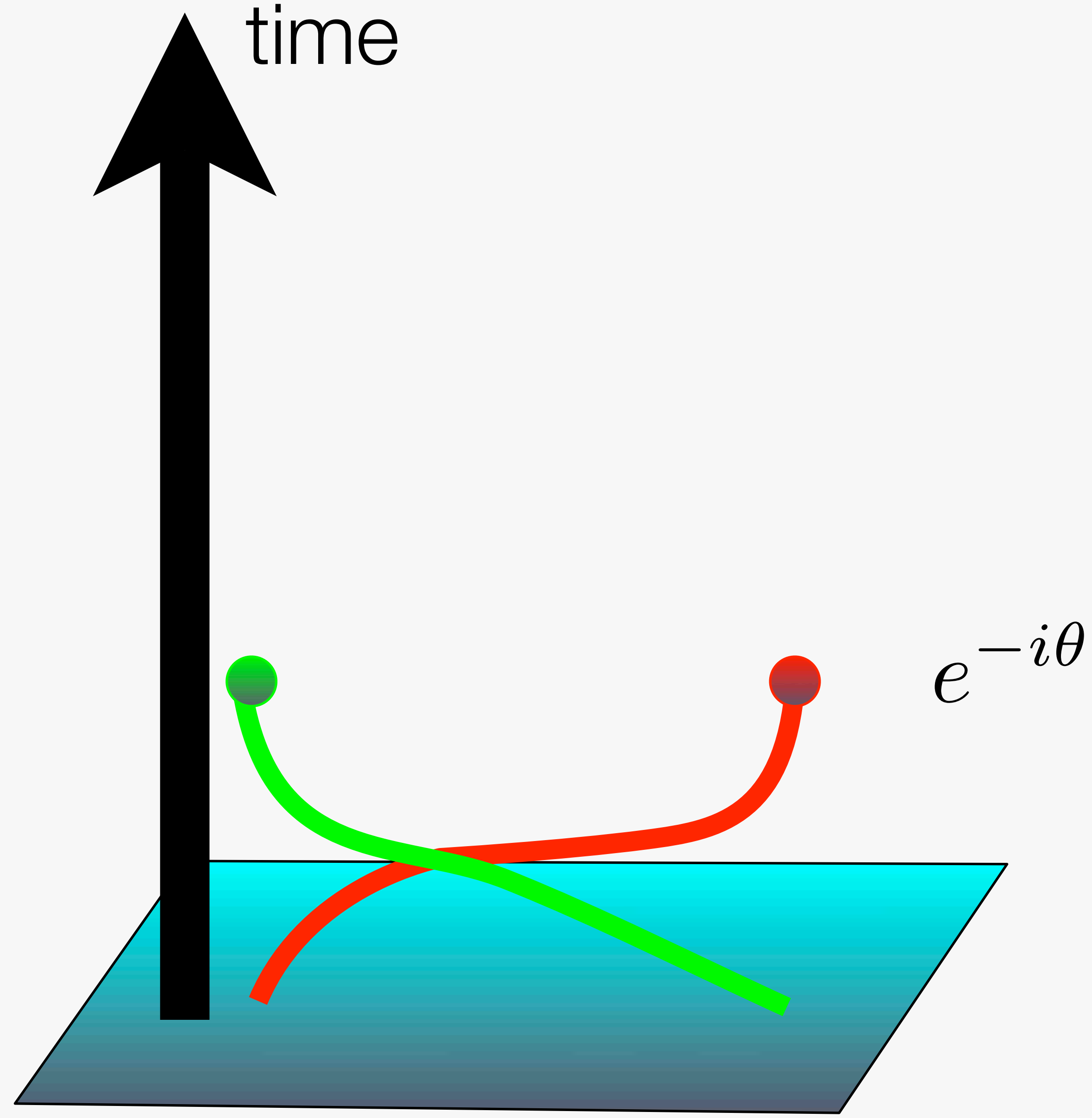

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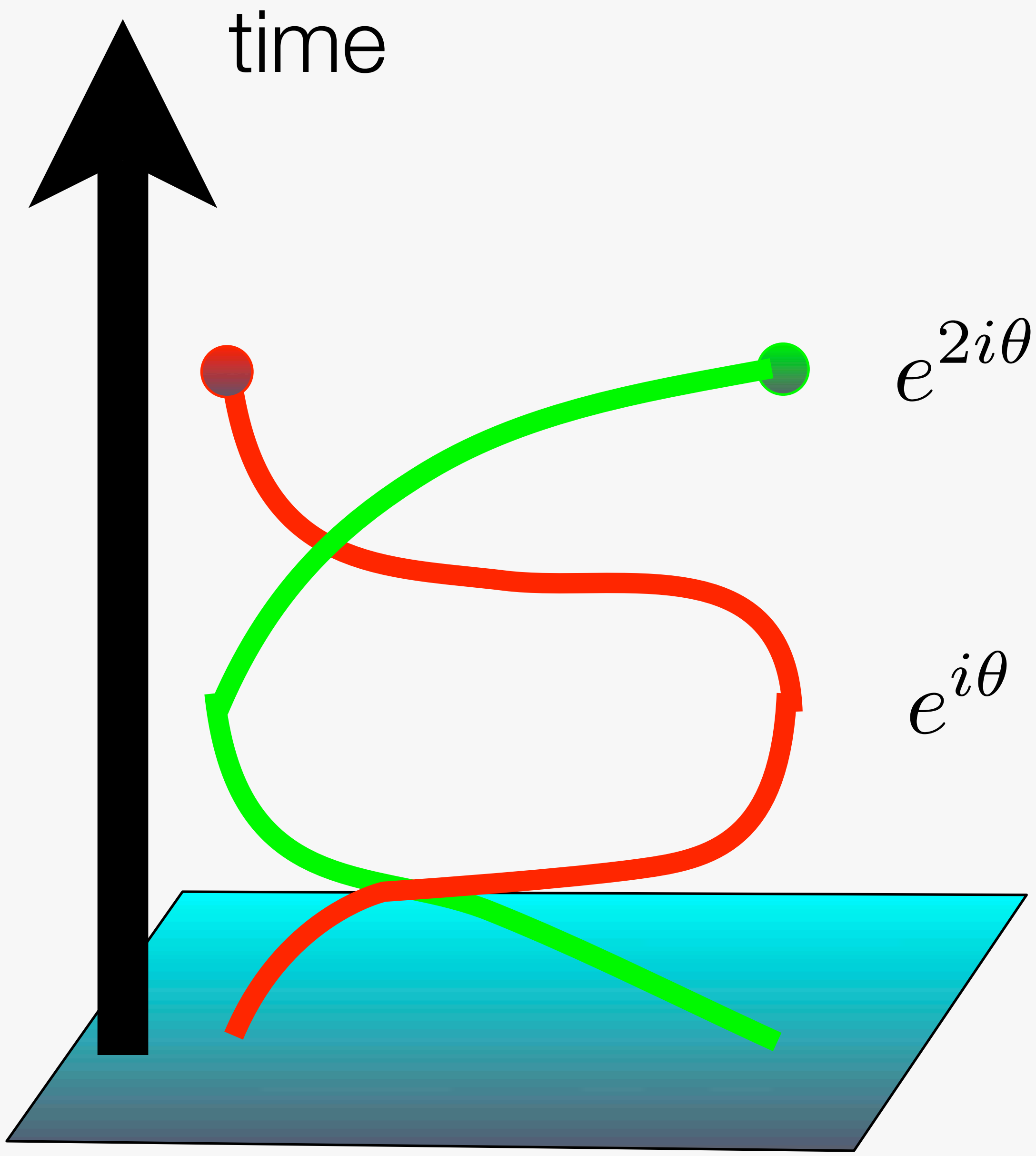

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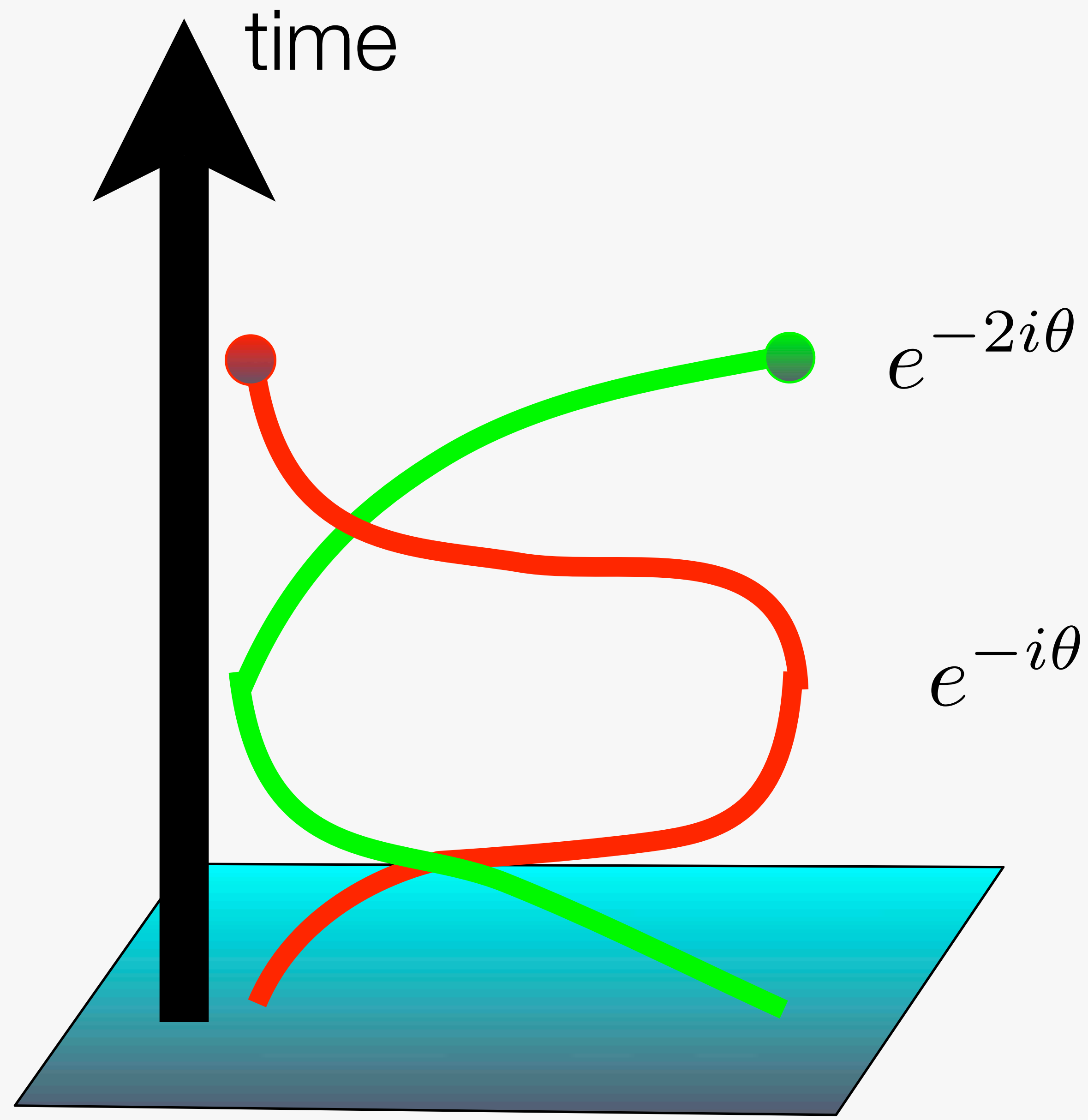

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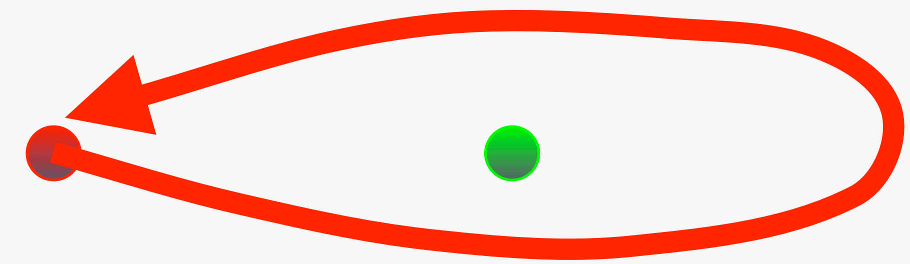
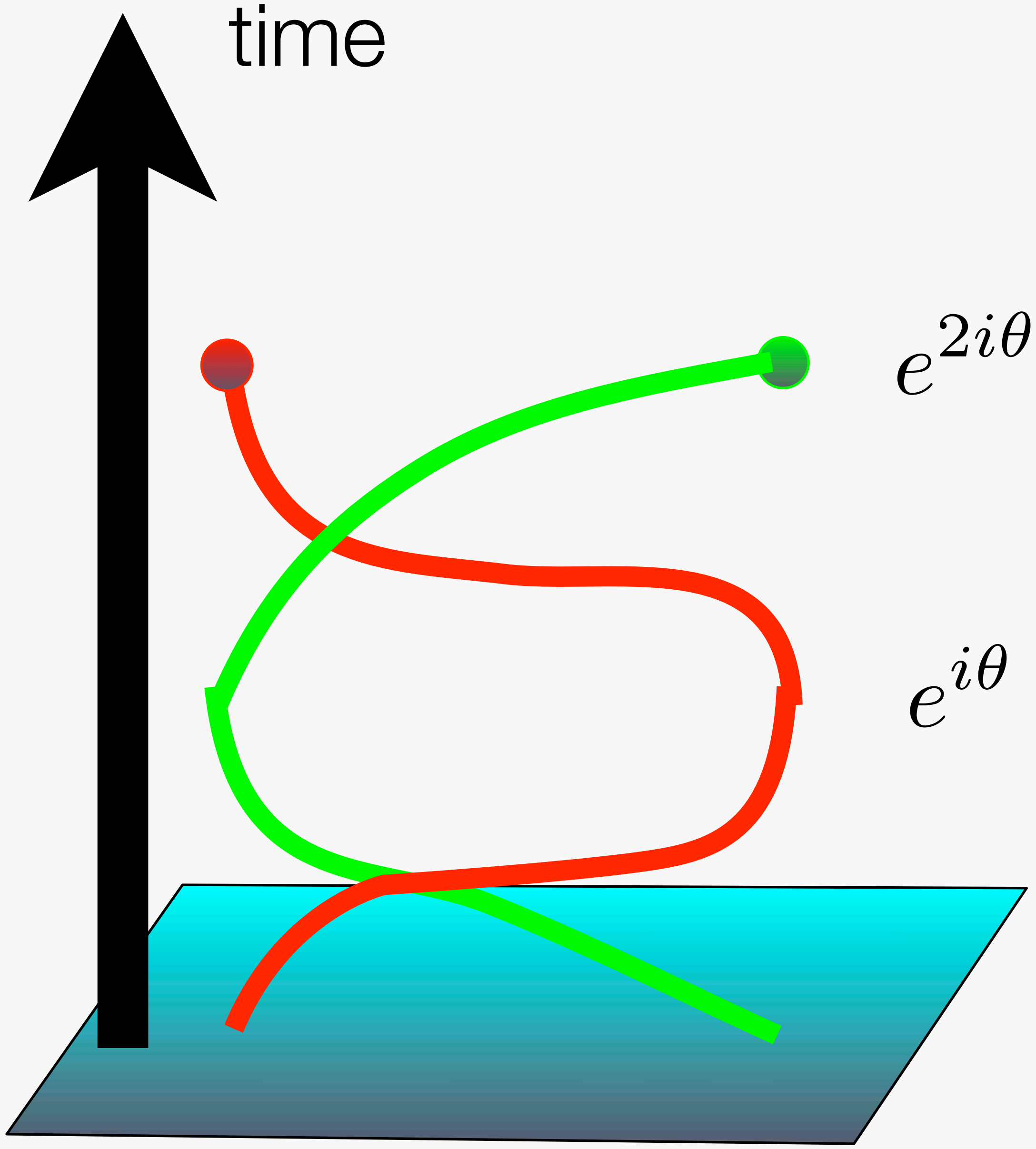
3D:
not possible





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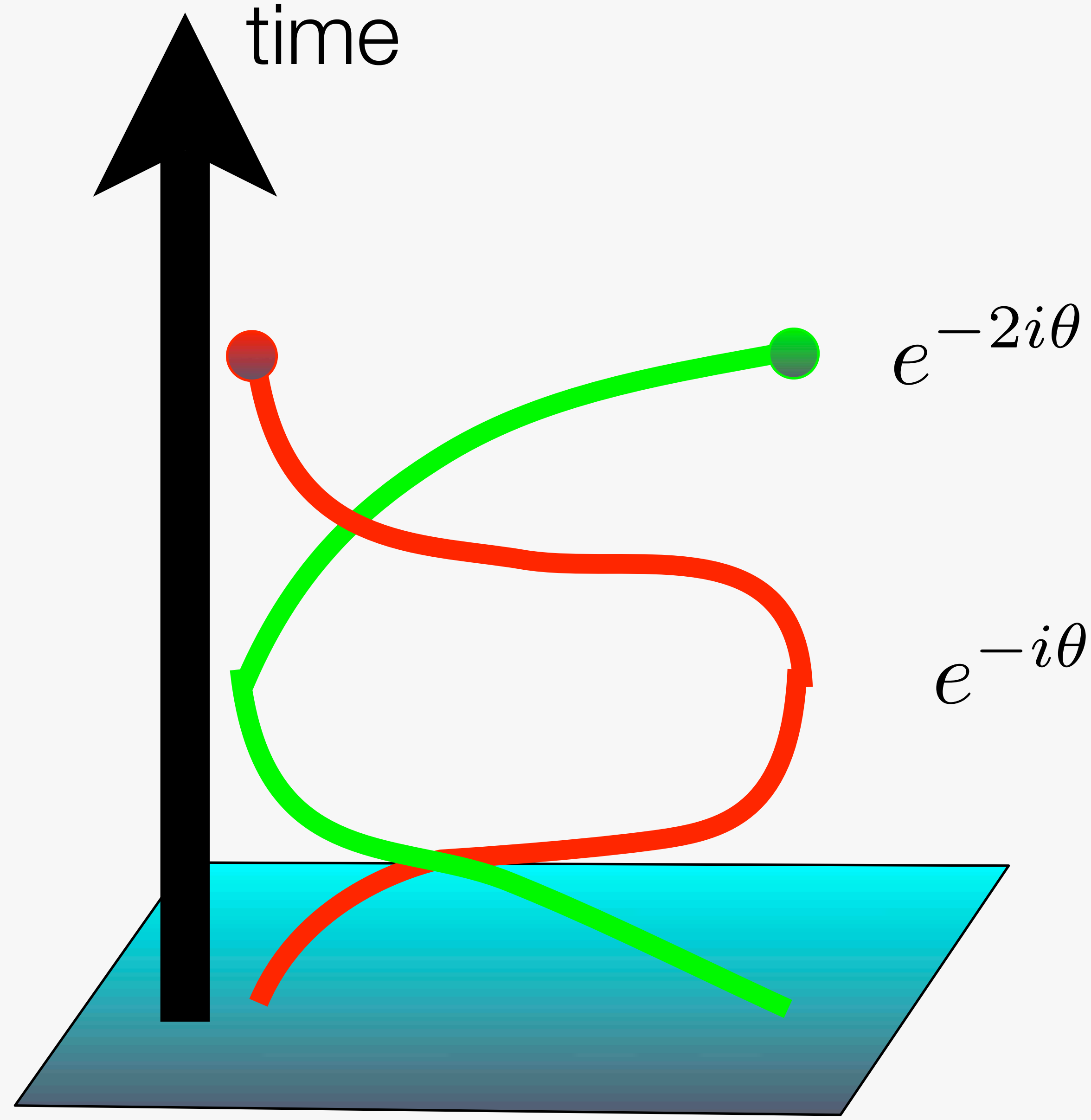

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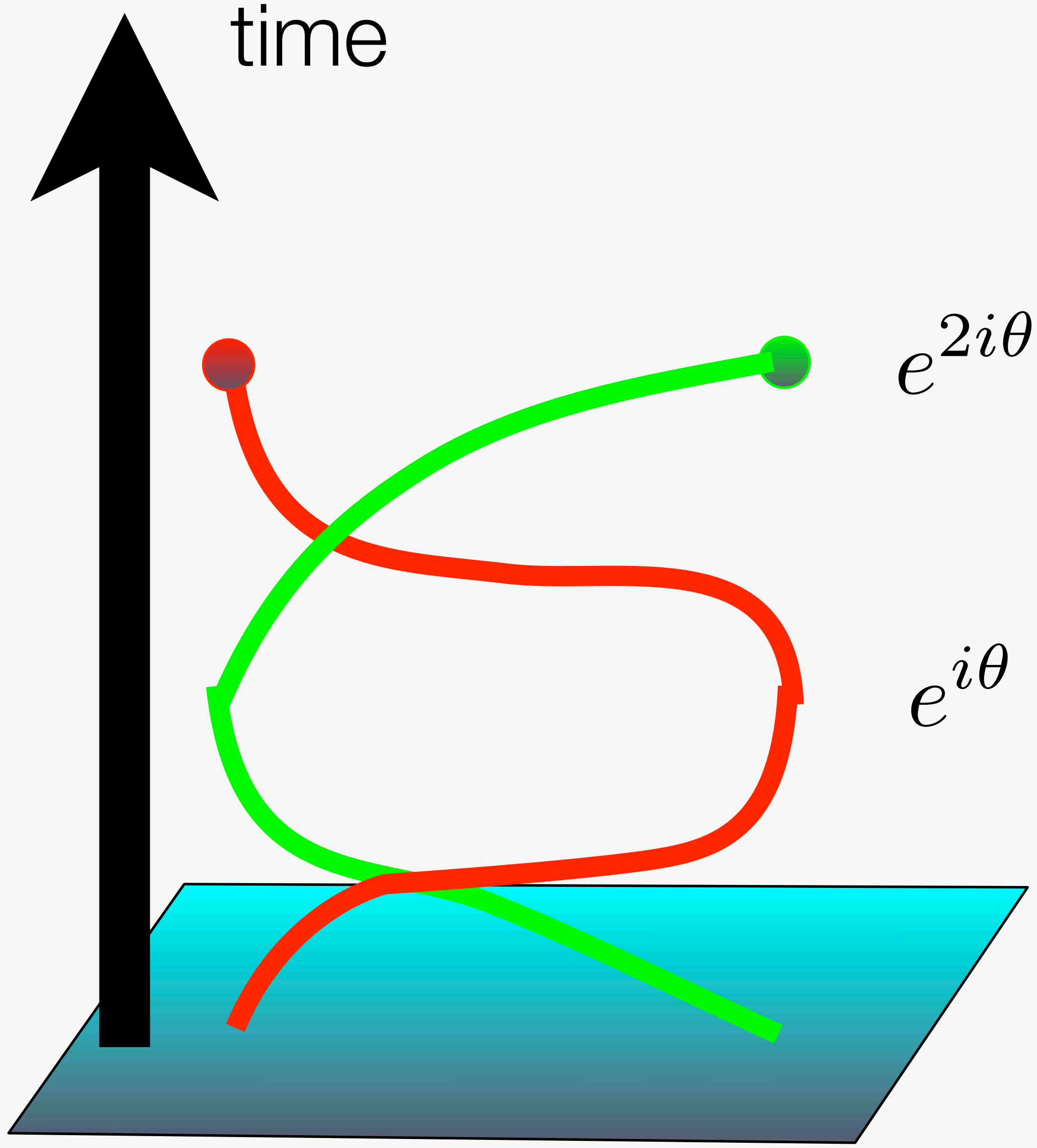
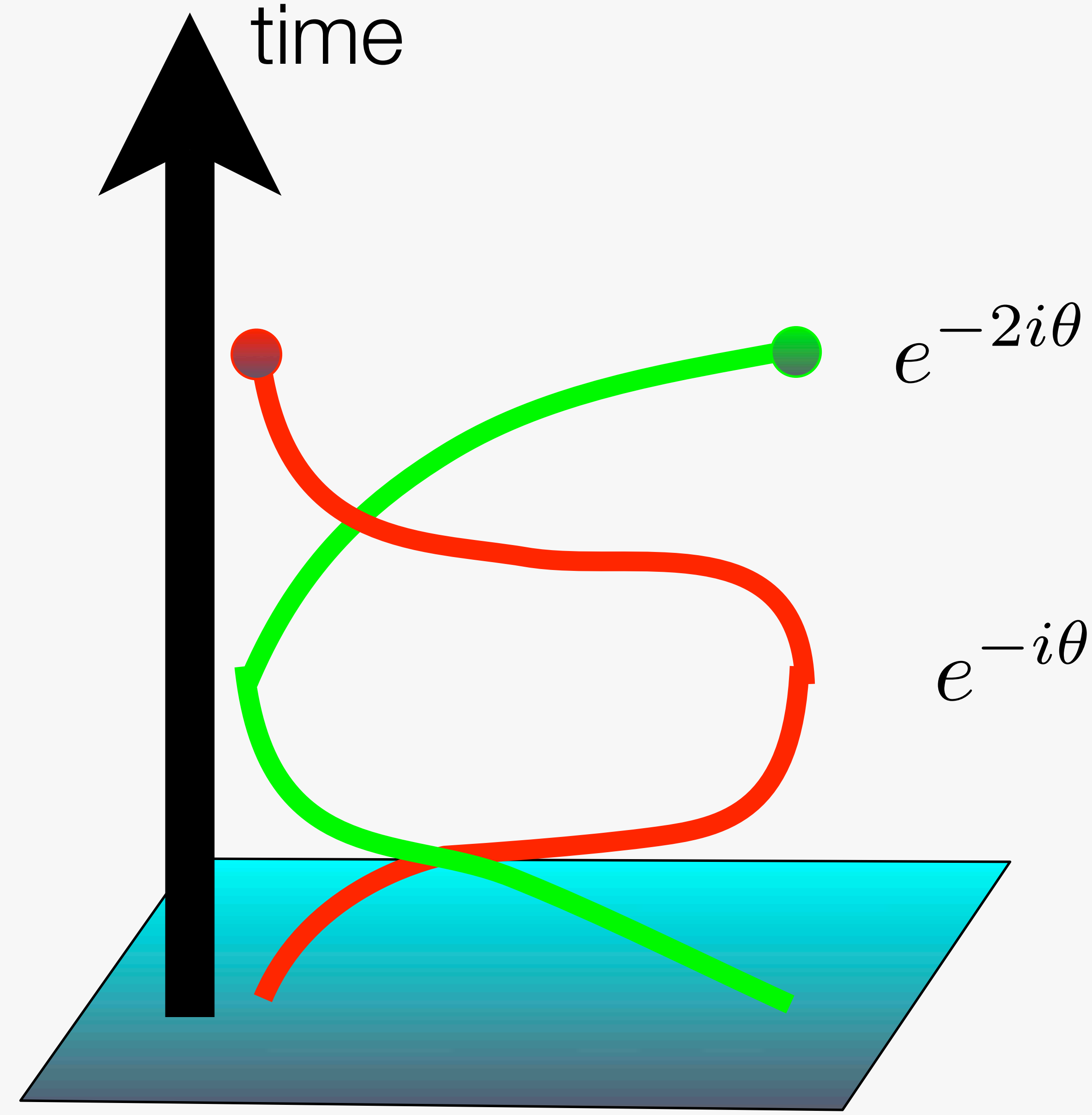


counterclockwise braid



clockwise braid

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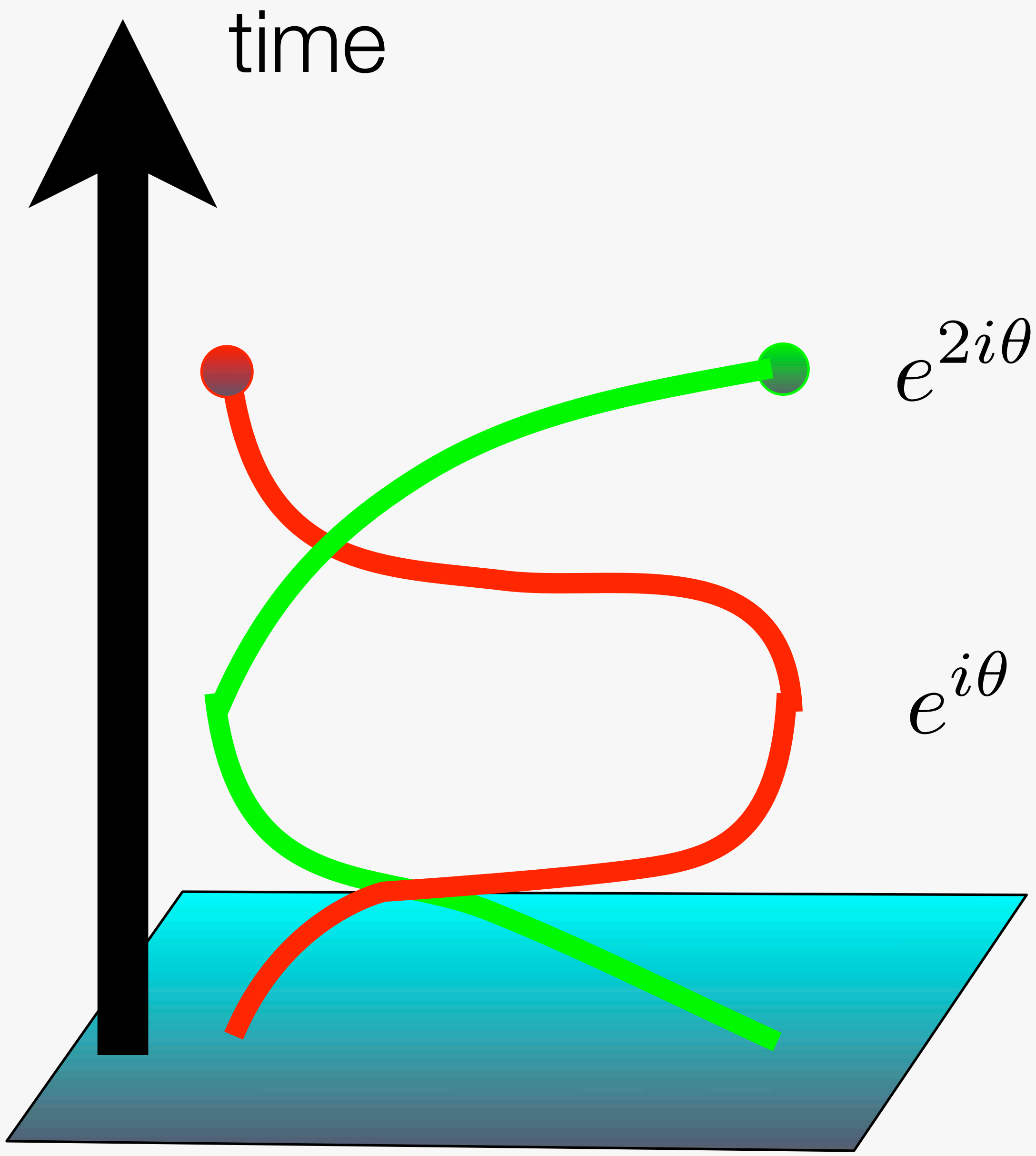
θ arbitrary



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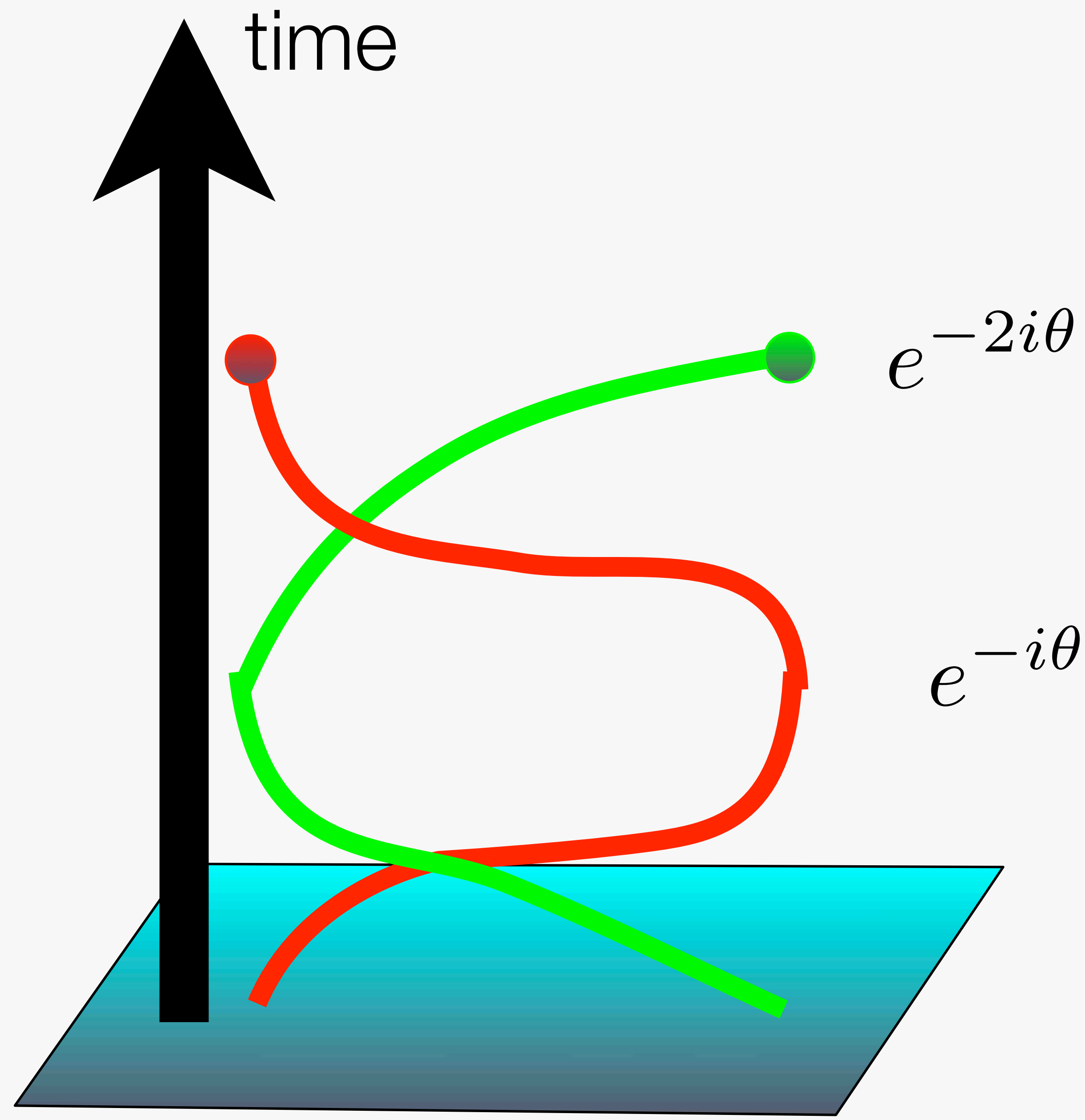

clockwise braid

2D world: “non-Abelions” (particles with non-Abelian statistics)

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N. Read, 1991
(originally with G. Moore)

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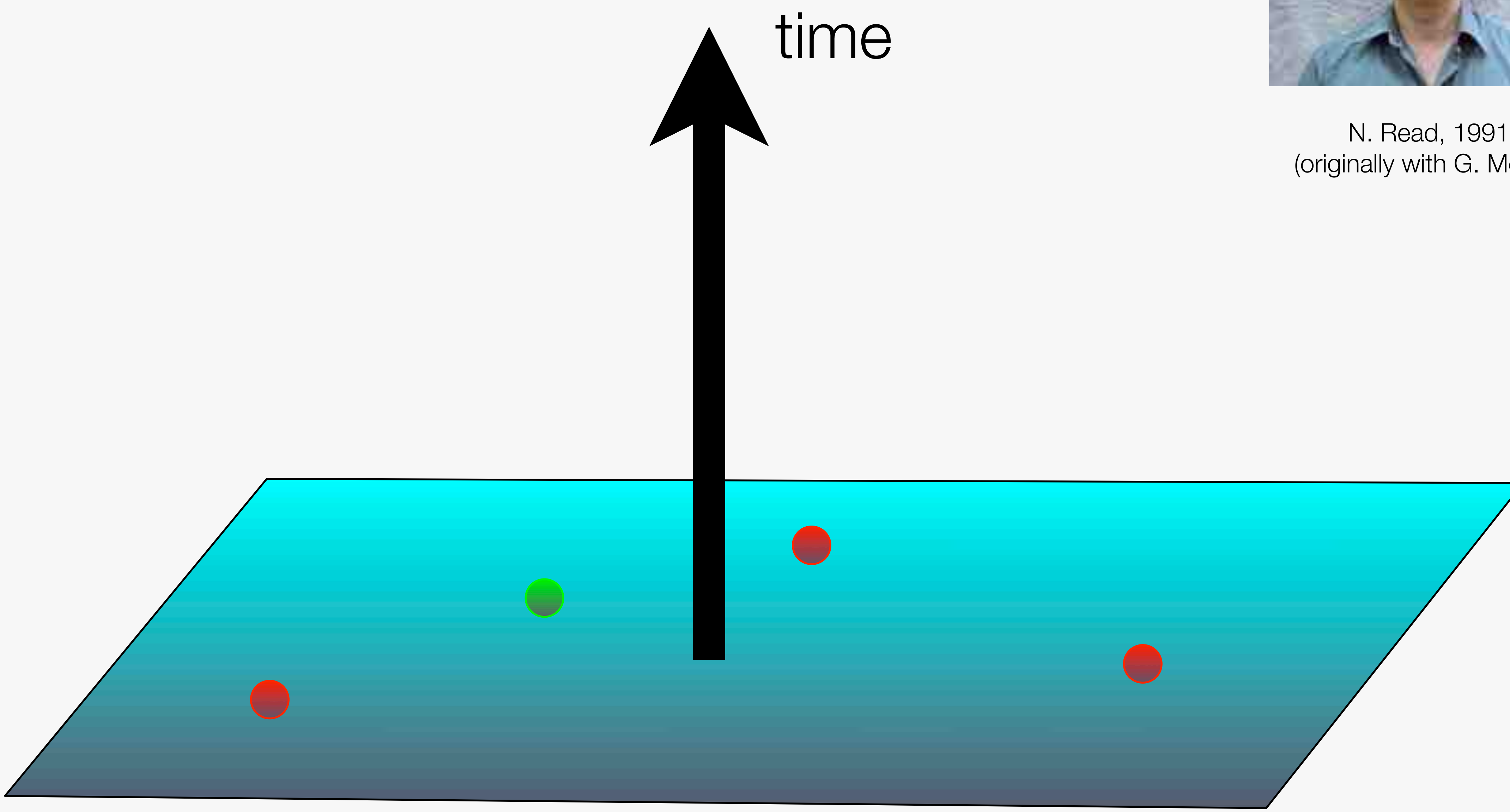

$$\Psi_\alpha(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)$$

$$\alpha = 1, \ldots, n$$

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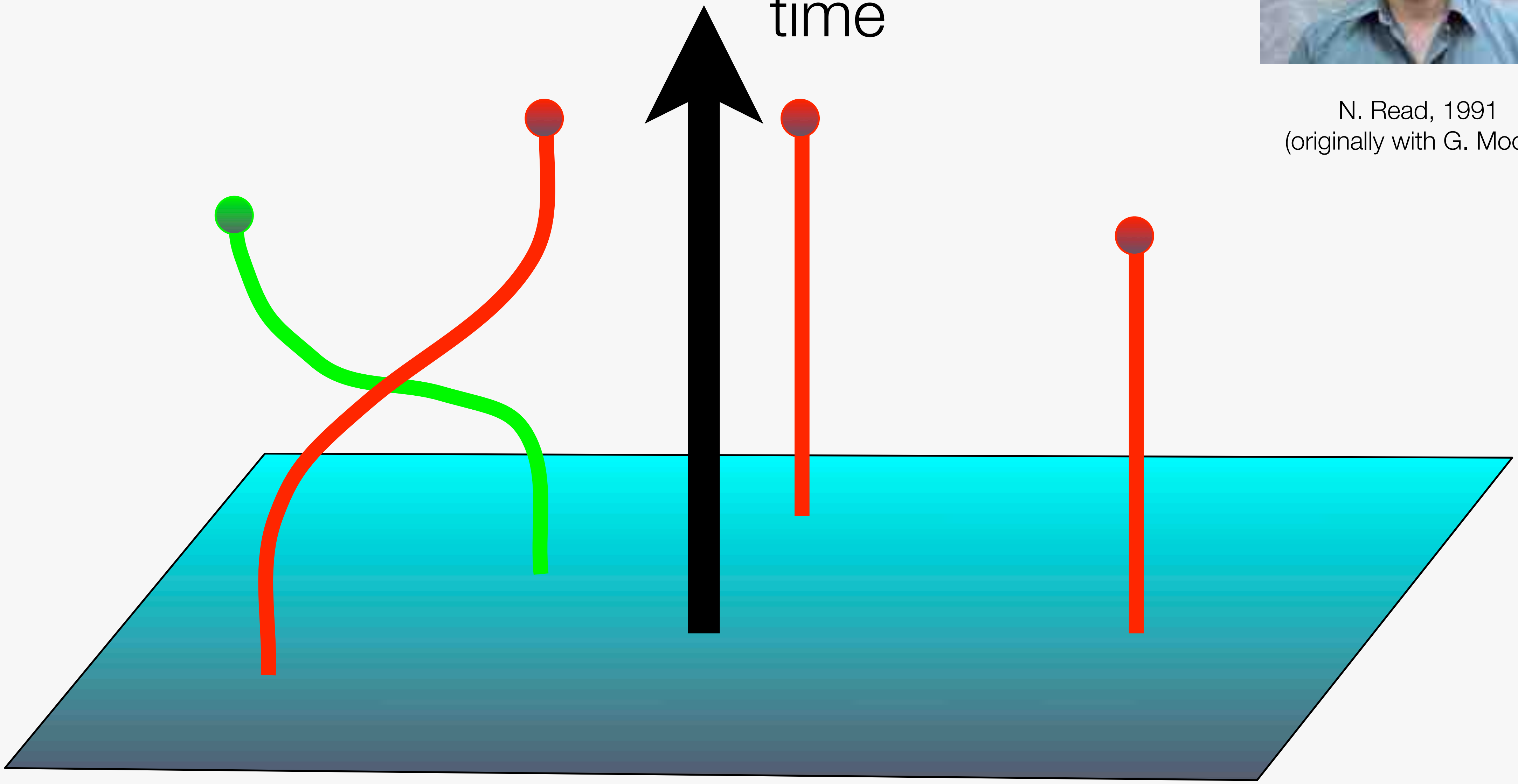

$$\Psi_\alpha(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) \rightarrow \sum_{\beta=1}^{n} U^{(1,2)}_{\alpha,\beta} \Psi_\beta(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = \Psi_\alpha(\mathbf{r}_2, \mathbf{r}_1, \mathbf{r}_3, \mathbf{r}_4)$$

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(1,2) - permuting particles 1 and 2

2D world: "non-Abelions" (particles with non-Abelian statistics)

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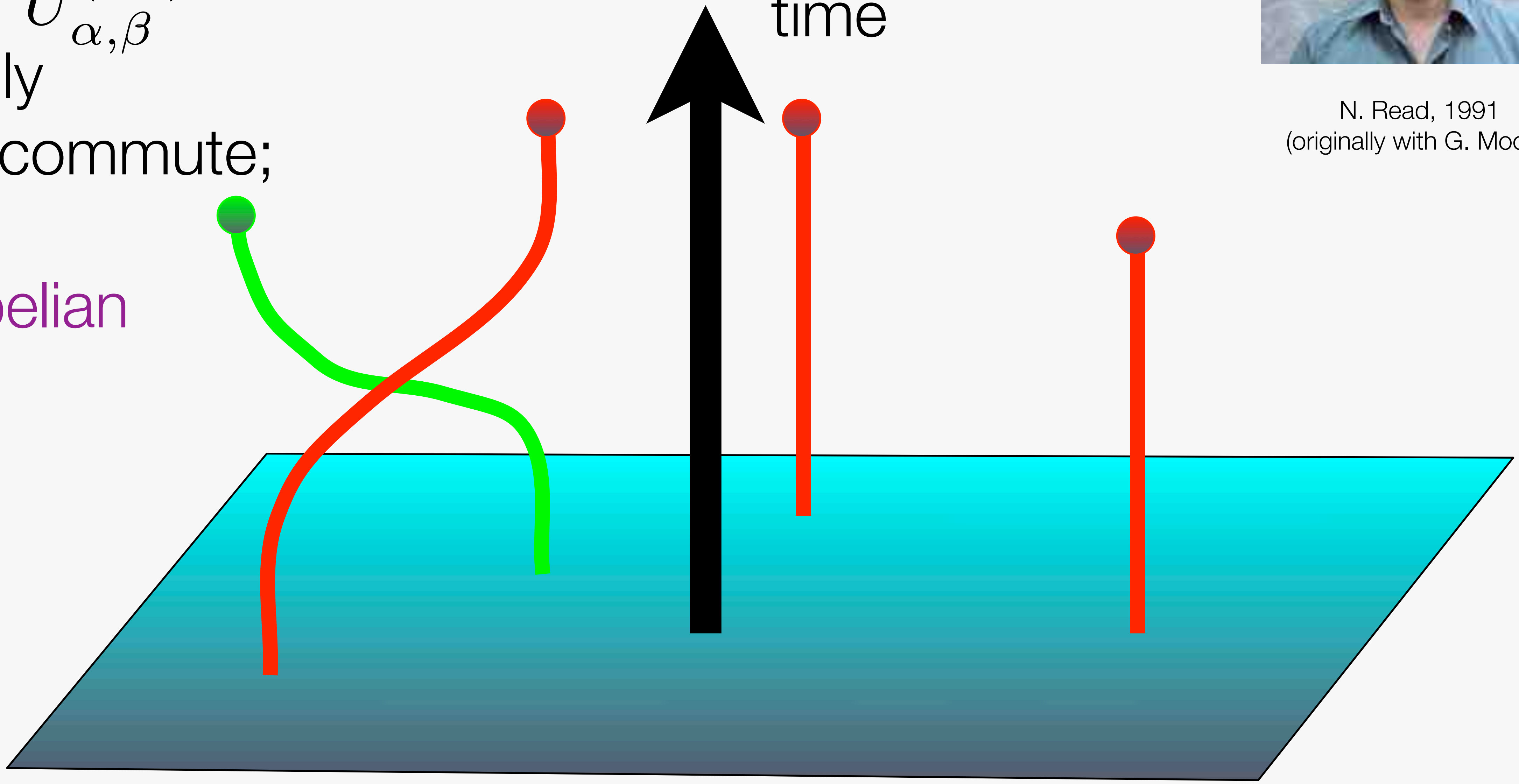
(originally with G. Moore)

Matrices
$U^{(1,2)}_{\alpha,\beta}$, $U^{(1,3)}_{\alpha,\beta}$
generally
do not commute;
hence
non-Abelian



$$\Psi_\alpha(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) \to \sum_{\beta=1}^{n} U^{(1,2)}_{\alpha,\beta}\,\Psi_\beta(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = \Psi_\alpha\left(\mathbf{r}_2,\mathbf{r}_1,\mathbf{r}_3,\mathbf{r}_4\right)$$

$$\alpha = 1,\ldots,n$$

(1,2) - permuting particles 1 and 2

What are anyons good for?

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A. Kitaev, 1997

Don’t know about anyons, but non-Abelions are good for the “topologically protected quantum computing”!

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Quantum bit - qubit

$$\begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix}$$

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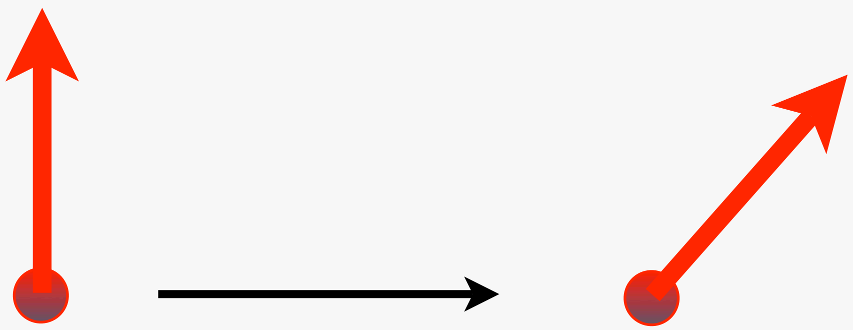

$$\begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} \rightarrow U \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = \sum_{\beta=\uparrow,\downarrow} U_{\alpha,\beta} \psi_\beta$$

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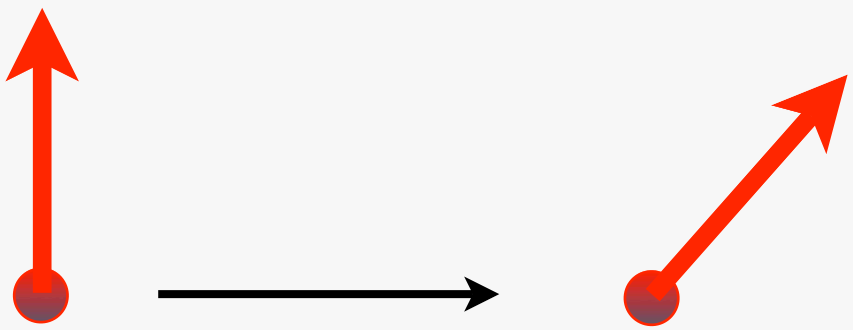

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Decoherence - the enemy of quantum computing

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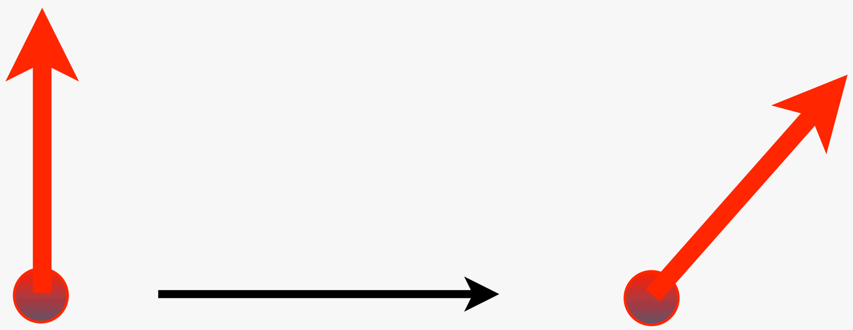

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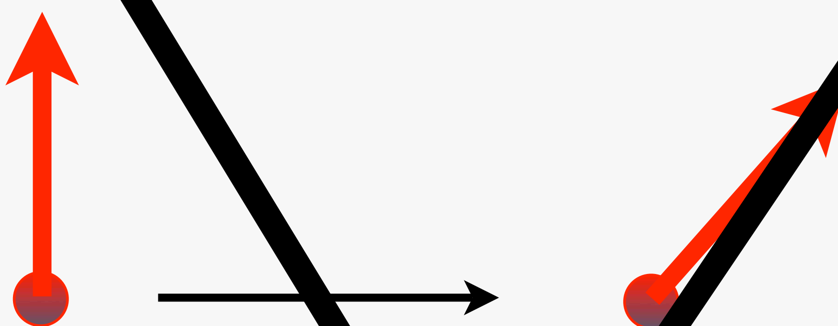
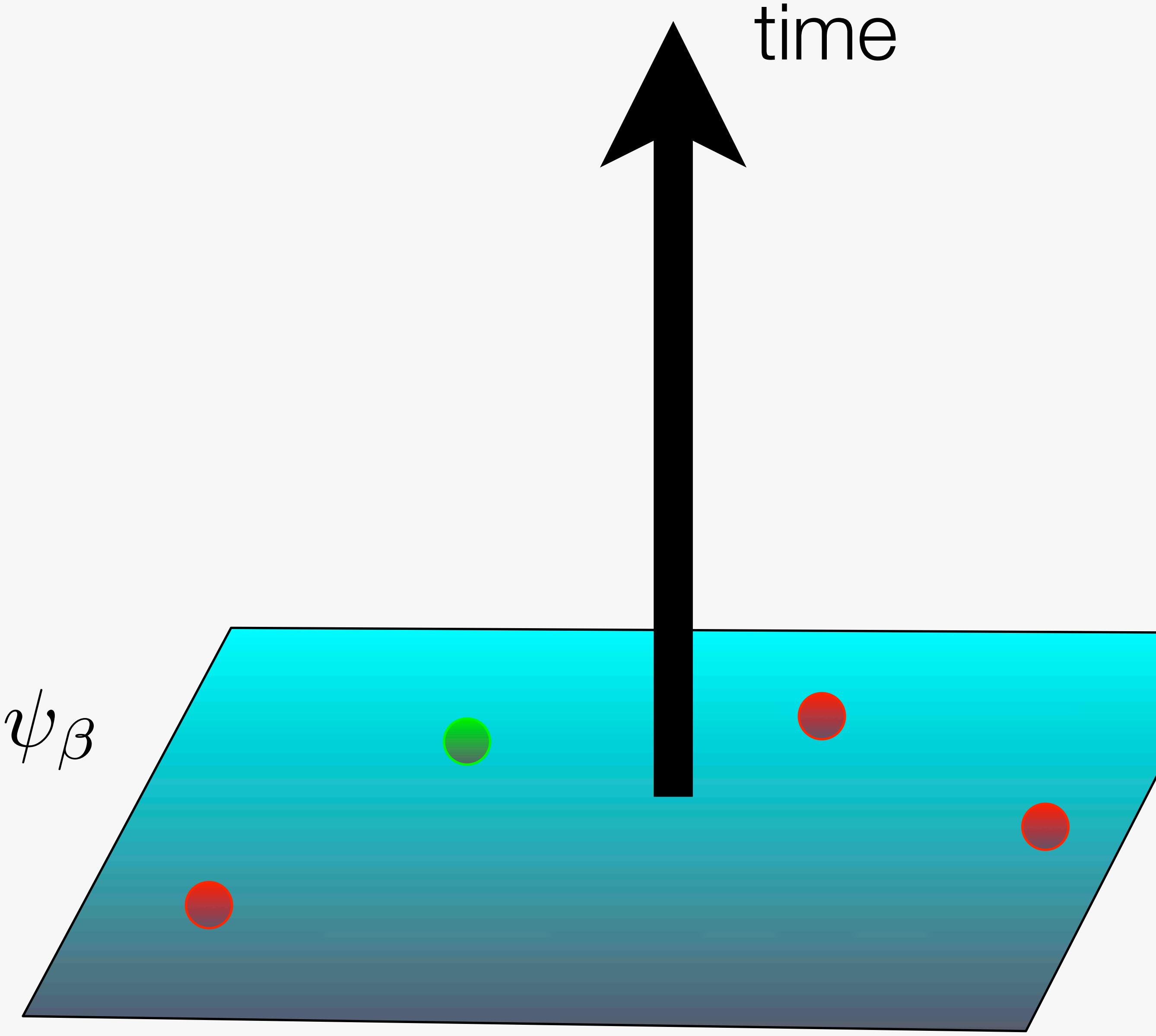


$$\begin{pmatrix}\psi_\uparrow \\ \psi_\downarrow\end{pmatrix} \to U \begin{pmatrix}\psi_\uparrow \\ \psi_\downarrow\end{pmatrix} = \sum_{\beta=\uparrow,\downarrow} U_{\alpha,\beta}\psi_\beta$$

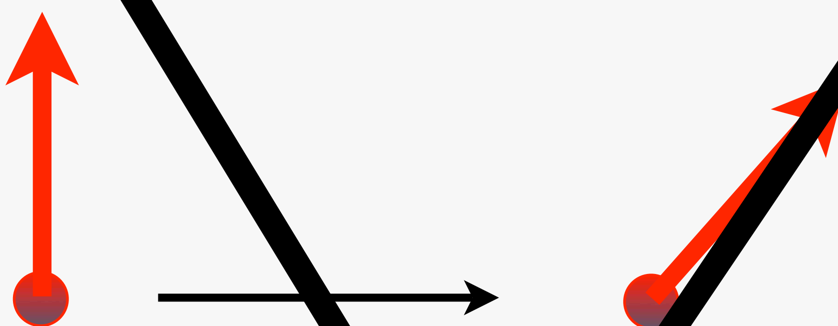
Decoherence - the enemy of quantum computing



What are anyons good for?

random noise is powerless and decoherence is absent!

Quantum bit - qubit



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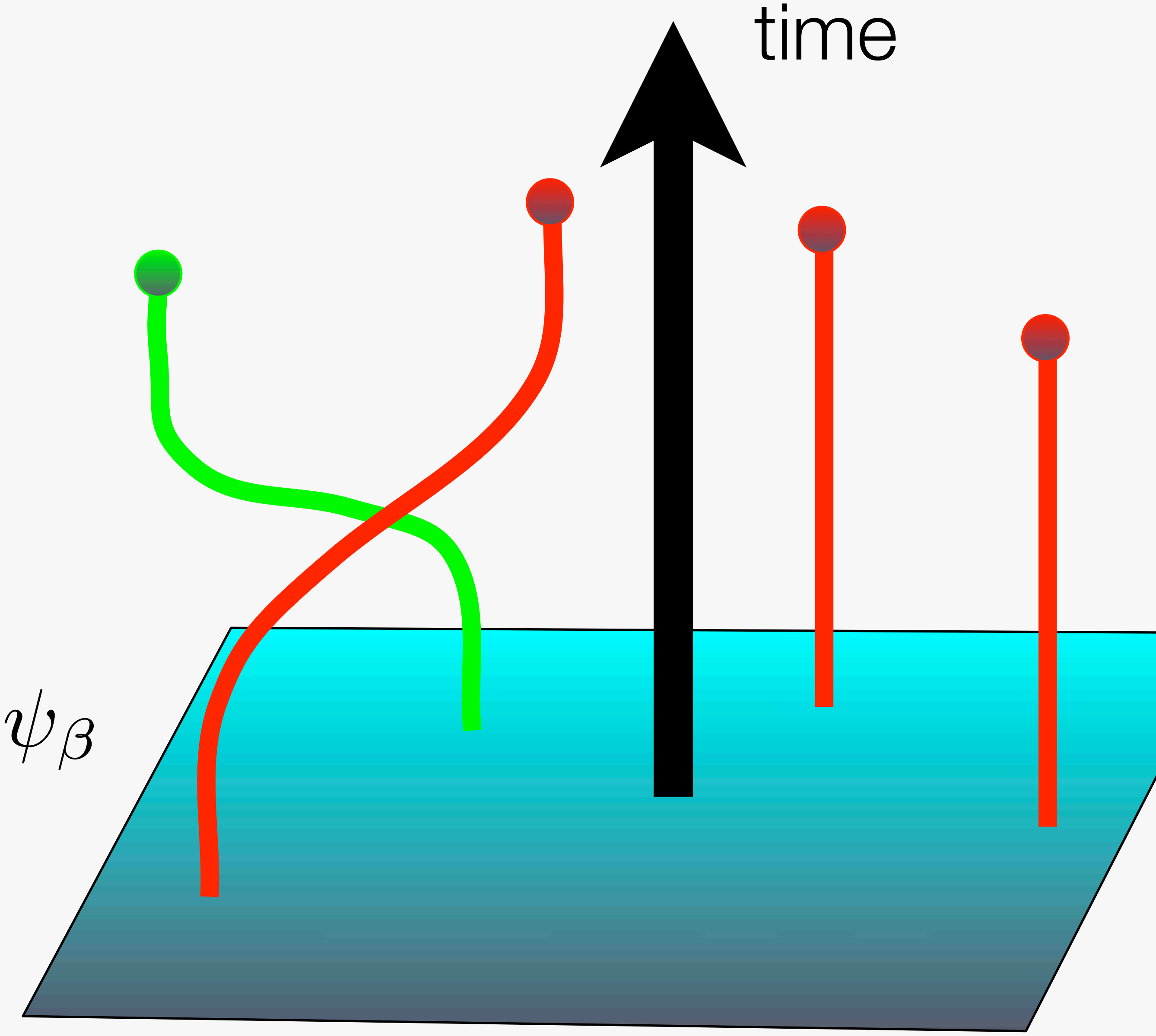

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Who is interested in topological quantum computing?

One proponent is familiar to all of us...

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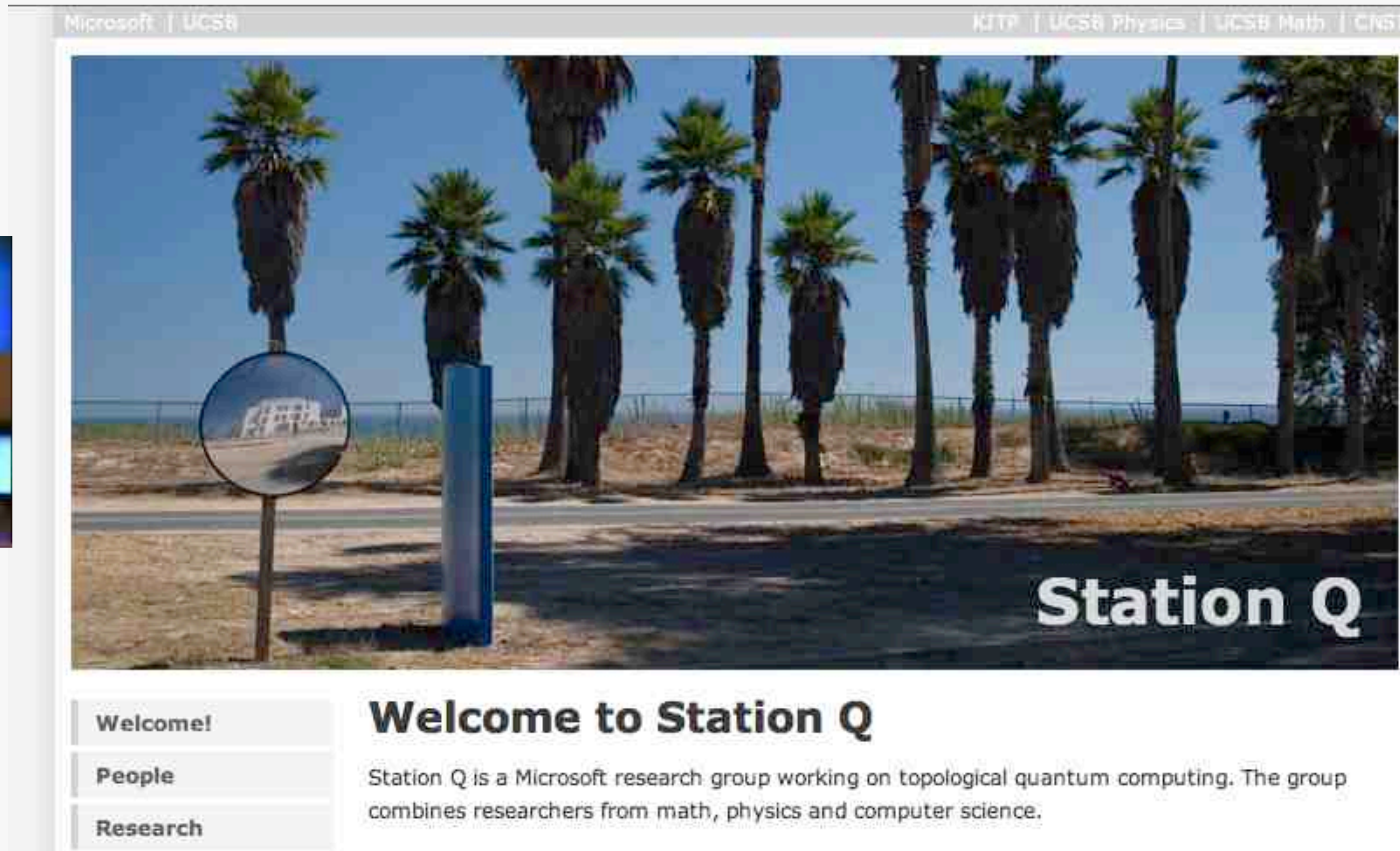

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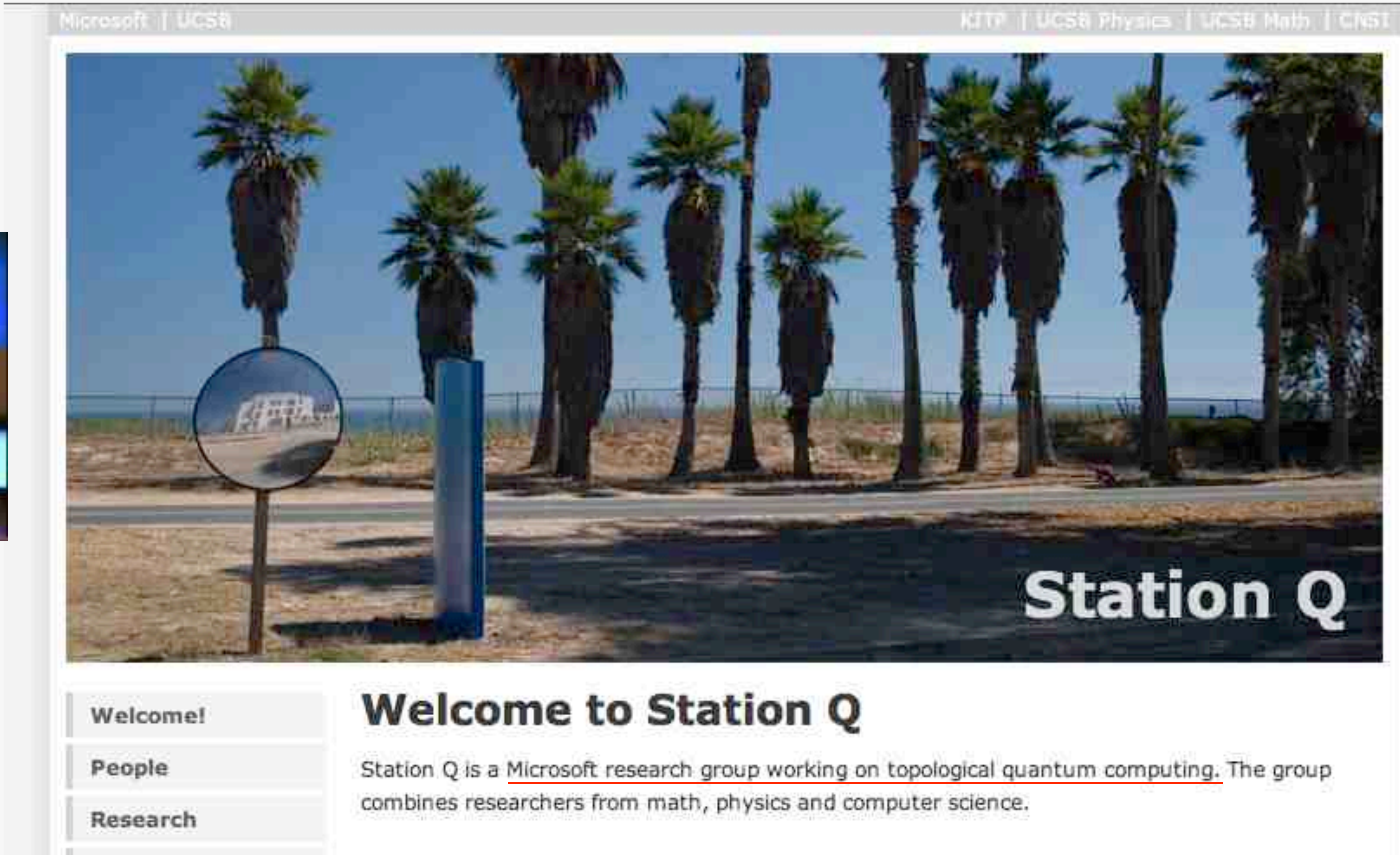

Where can we find anyons?

They were actually found in the studies of fractional quantum Hall effect!

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Nobel Prize 1998

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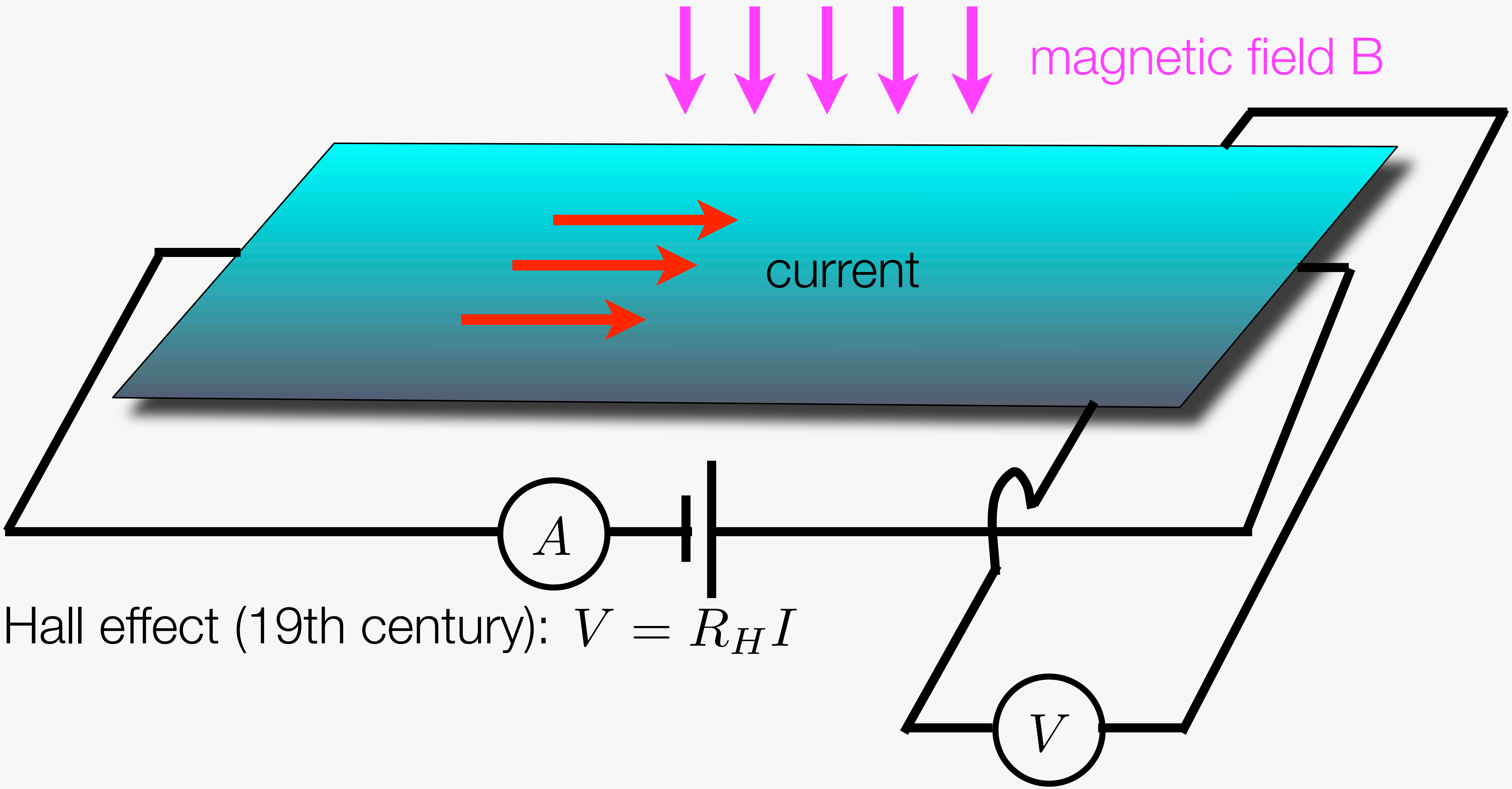
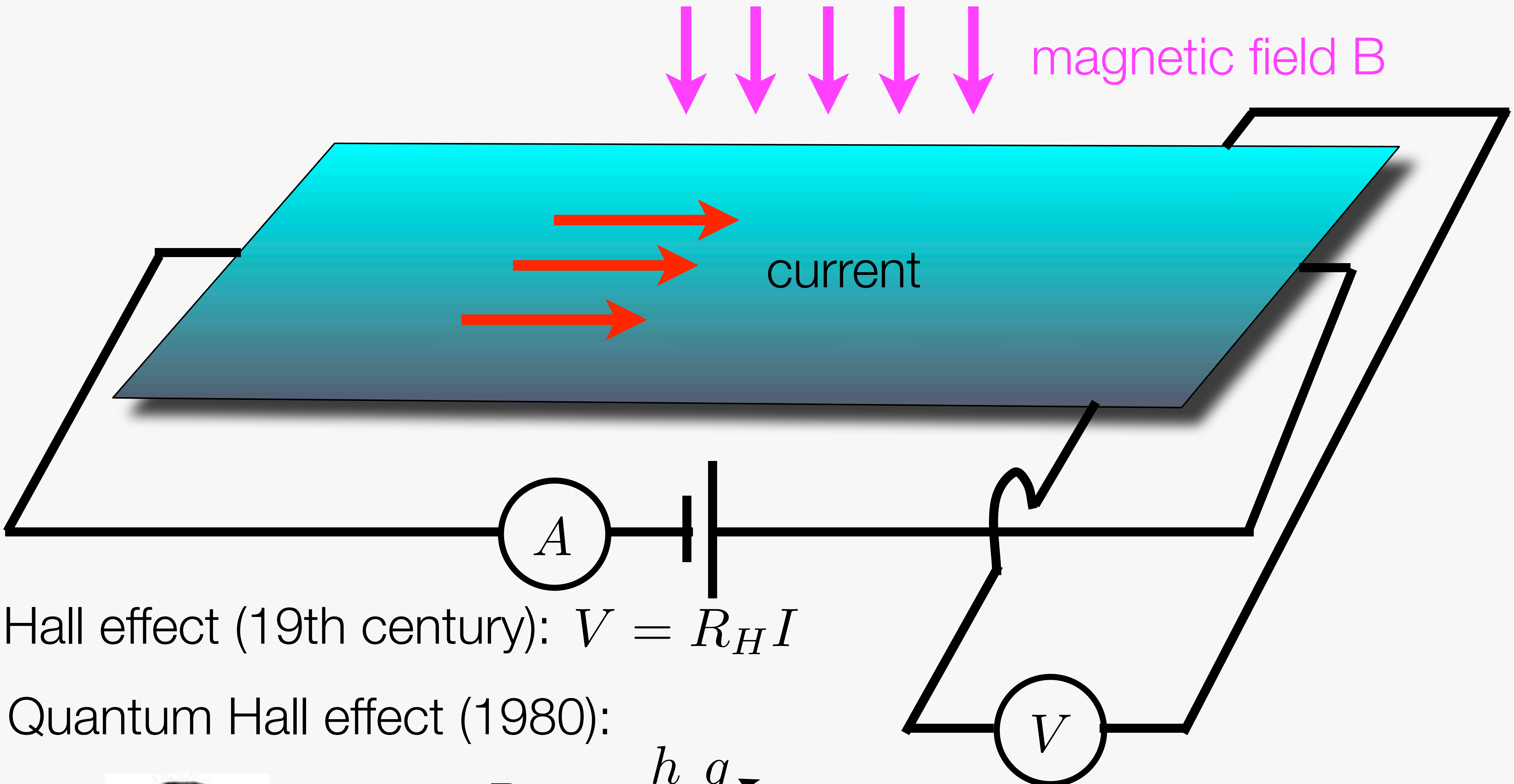
Dirty little secret: those "fractionally charged excitations" are actually anyons!

Quantum Hall Effect



Hall effect (19th century): $V = R_H I$

Quantum Hall Effect



Hall effect (19th century): $V = R_H I$

Quantum Hall effect (1980):

$$R_H = \frac{h}{e^2}\frac{q}{p}$$ integers

Klaus von Klitzing

Nobel Prize 1985: for the discovery of QHE

Fractional Quantum Hall effect

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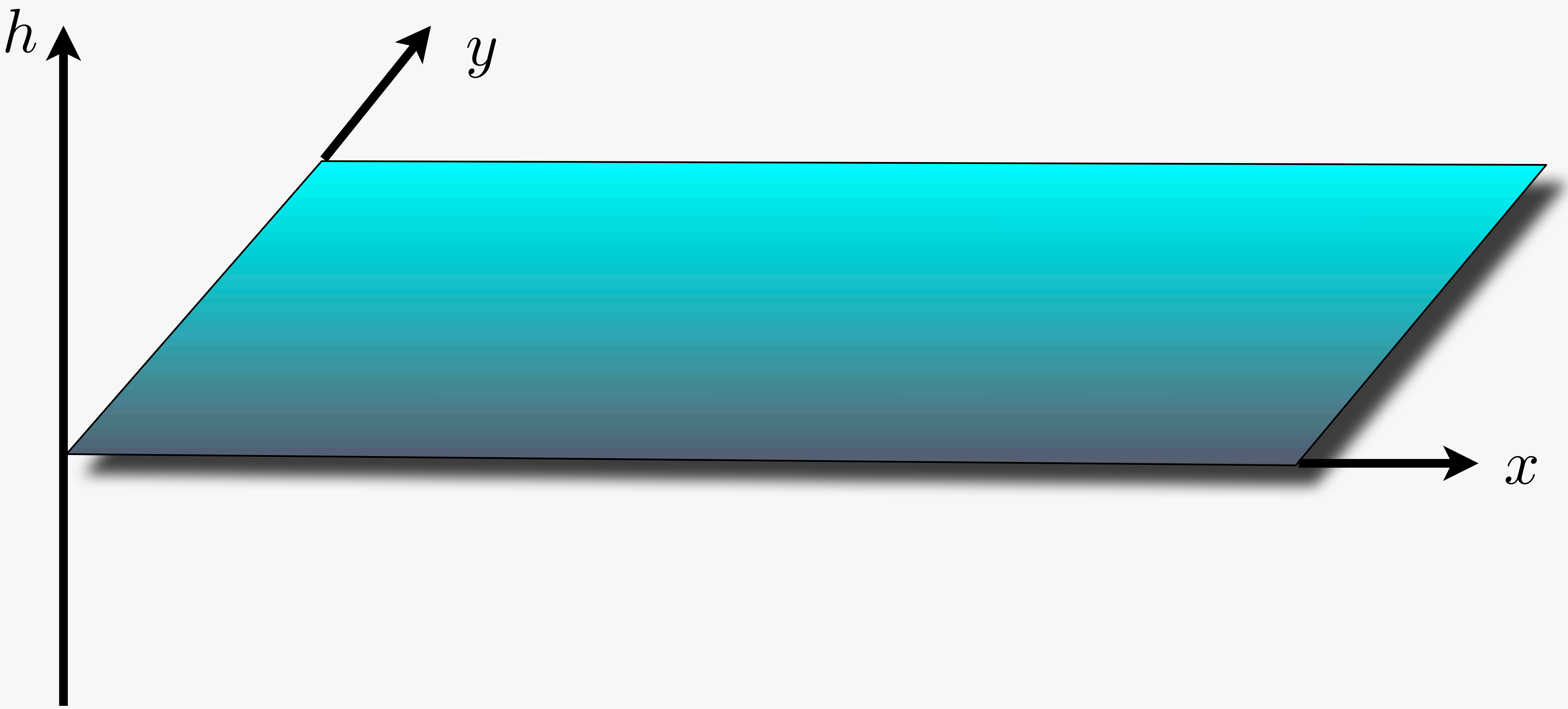

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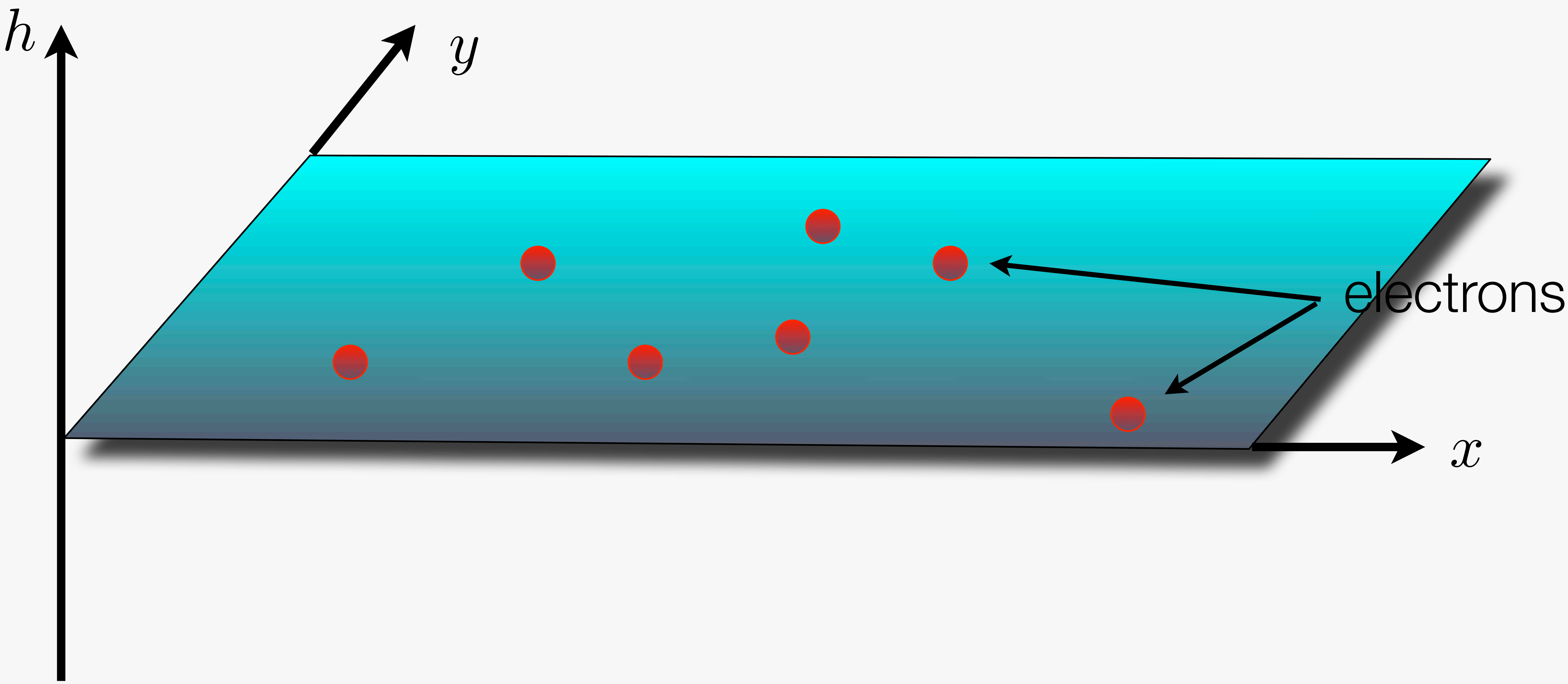

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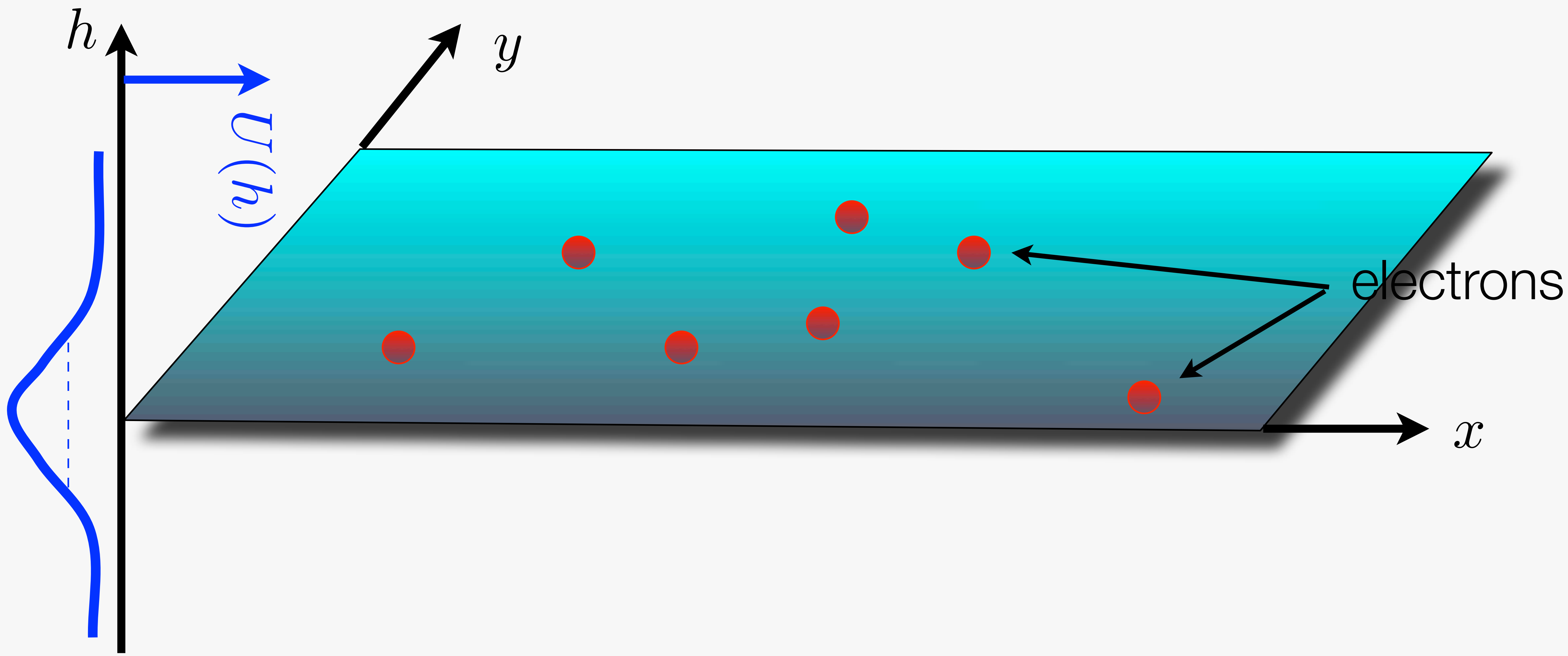

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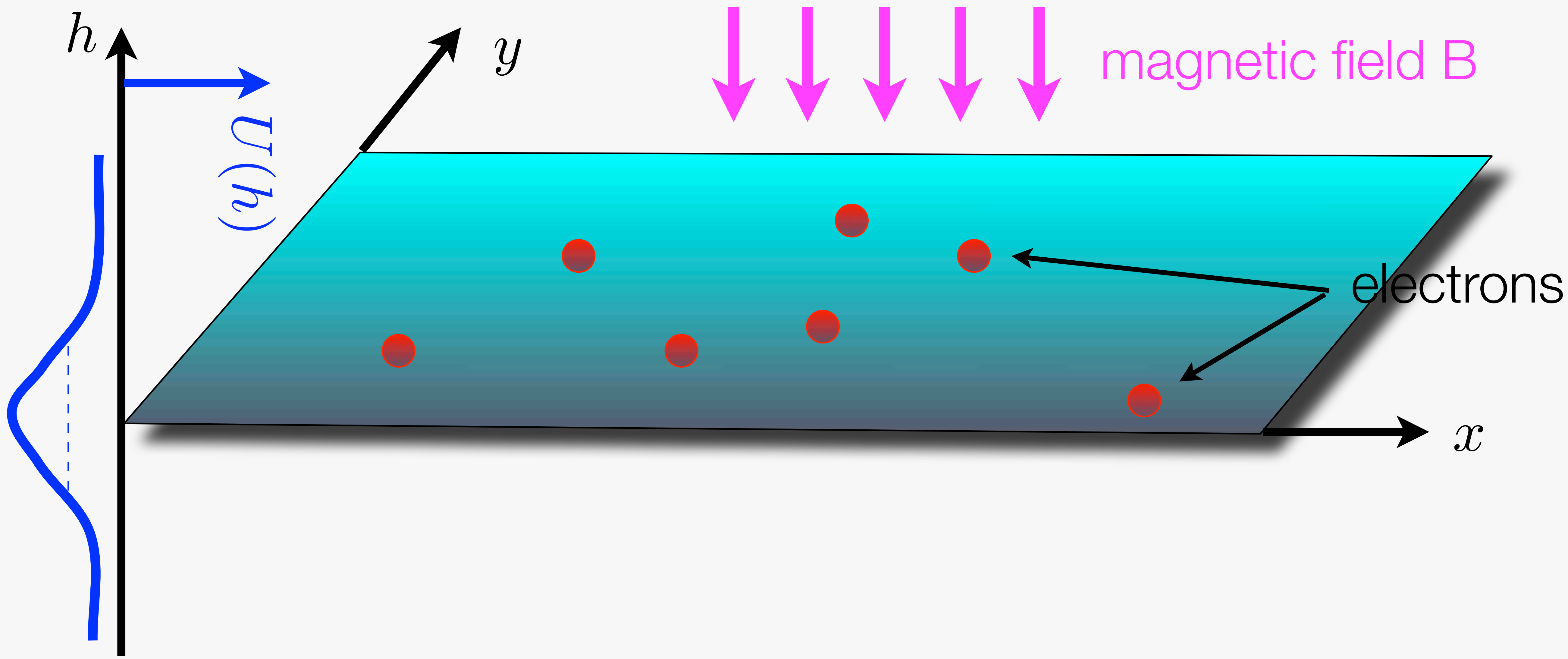

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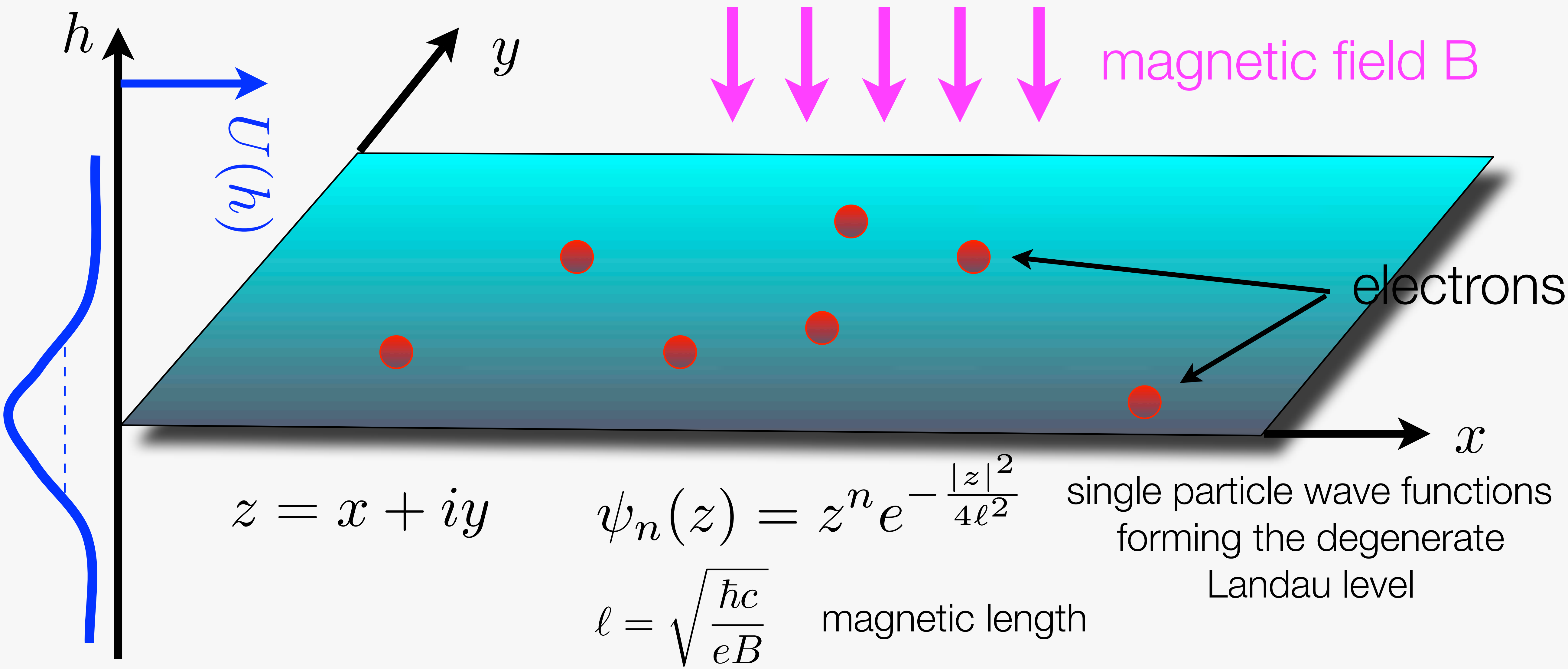


$z = x + iy$

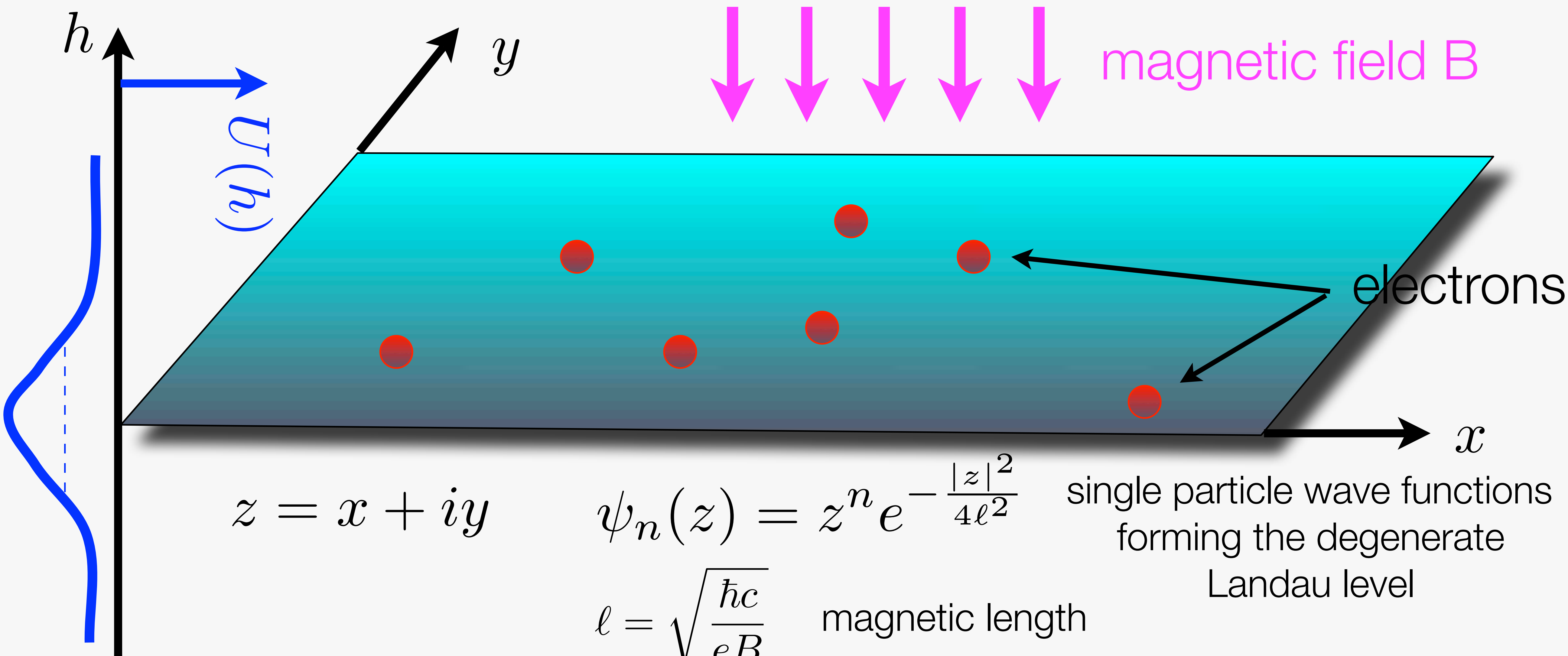
$$\psi_n(z) = z^n e^{-\frac{|z|^2}{4\ell^2}}$$

single particle wave functions forming the degenerate Landau level

$$\ell = \sqrt{\frac{\hbar c}{eB}}$$

magnetic length

Fractional Quantum Hall effect



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Arbitrary many-body wave function

(antisymmetrized product of single particle wave functions)

$$\psi(z_1, z_2, \ldots) = A(z_1, z_2, \ldots)\, e^{-\frac{1}{4\ell^2}\sum_k |z_k|^2}$$

electrons' coordinates

arbitrary antisymmetric polynomials

Anyons in fractional quantum Hall effect

Laughlin's insight: simplest possible ground state

$$\psi_0(z_1, z_2, \ldots) = \prod_{l<m} (z_l - z_m)^3 \, e^{-\frac{1}{4\ell^2} \sum_k |z_k|^2}$$

simplest possible excited state

$$\psi_\eta(z_1, z_2, \ldots) = \prod_k (\eta - z_k) \prod_{l<m} (z_l - z_m)^3 \, e^{-\frac{1}{4\ell^2} \sum_k |z_k|^2}$$

Arbitrary many-body wave function

(antisymmetrized product of single particle wave functions)

$$\psi(z_1, z_2, \ldots) = A(z_1, z_2, \ldots) \, e^{-\frac{1}{4\ell^2} \sum_k |z_k|^2}$$

electrons' coordinates

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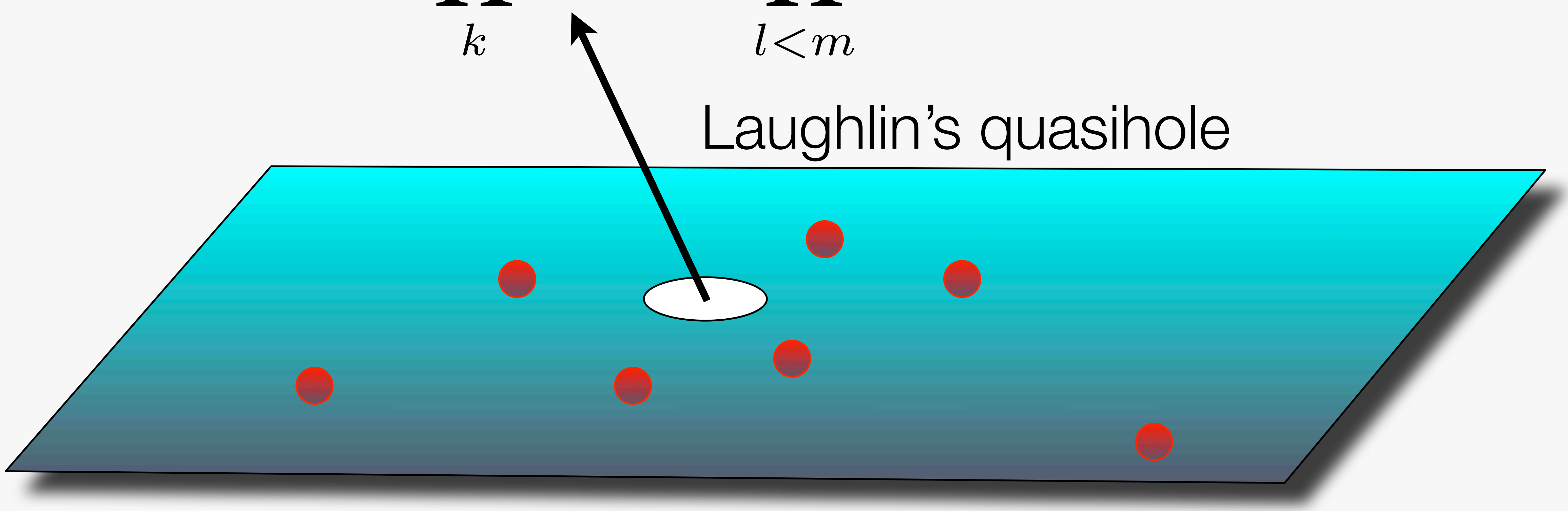

Anyons in fractional quantum Hall effect

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The normalization integral is the partition function of a 2D plasma!

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2D plasma: definitions

Two chargers interact logarithmically $U_{12}(r) = -e_1 e_2 \ln(r)$

The partition function is $Z = \int \prod_k d^2 r_k \; e^{-\frac{1}{T} \sum_{j \neq l} U_{jl}(r_{jl})}$

Non-Abelions in fractional quantum Hall effect

$$\psi(z_1, z_2, \dots) = A(z_1, z_2, \dots)\, e^{-\frac{1}{4\ell^2}\sum_k |z_k|^2}$$

Conjecture: these are the correlation functions of a two dimensional scale invariant quantum field theory (in other words, of a statistical mechanical system at a point of a second order phase transition), and Laughlin's guess is but a particular case of that, corresponding to a free field theory.

N. Read and G. Moore, 1991

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Moore and Read: let's take the simplest two-dimensional critical model: 2D Ising model!

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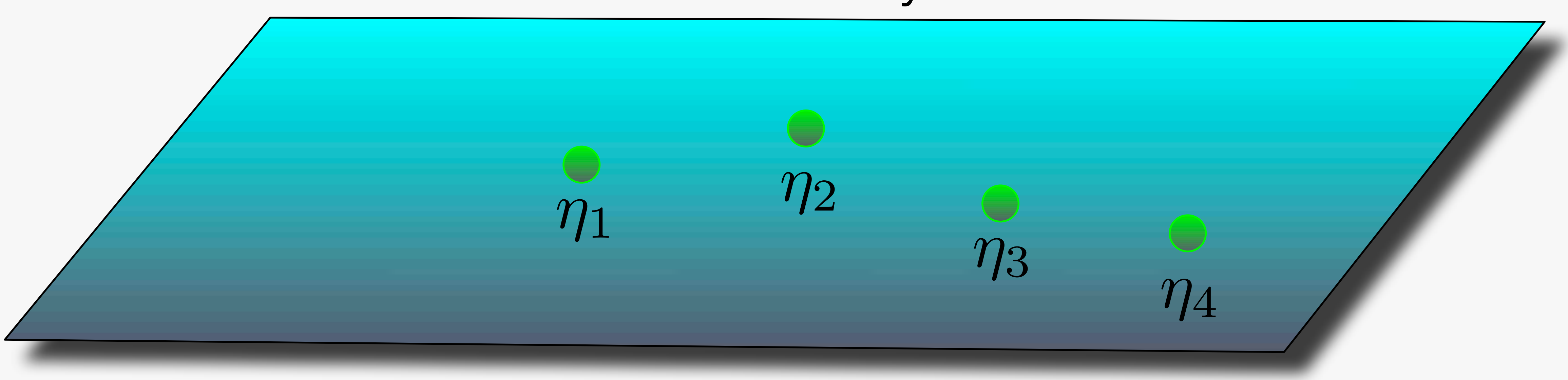
Polynomials



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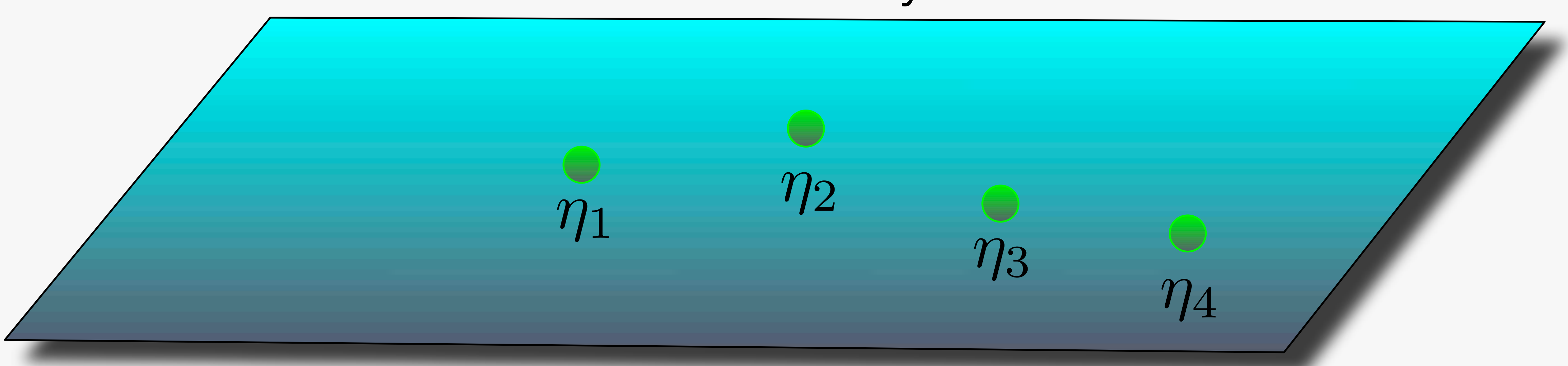

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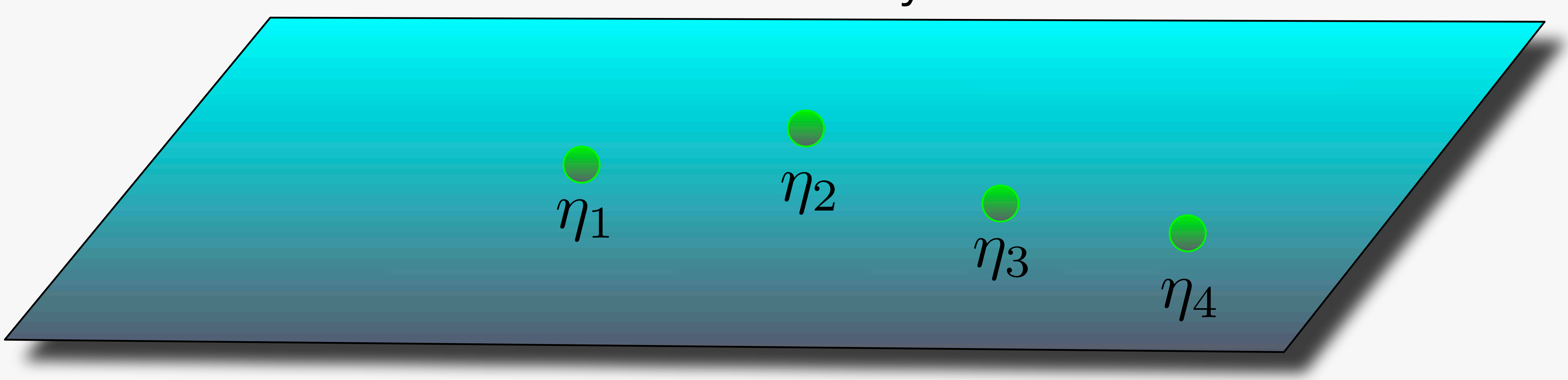

Why this construction works

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Conjectured degenerate wave functions

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where did these come from?

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where did these come from?

Need to prove that $\int \prod_k d^2 z_k\ \psi^*_\alpha\, \psi_\beta = \delta_{\alpha,\beta}$

Proven by VG, C. Nayak, 1997 and 2009

Status of the Non-Abelions in FQHE

- Overwhelming numerical evidence that the non-Abelian quantum Hall states exist as well as firm experimental evidence that they have been observed (states were observed which, as is firmly believed, must have particles with non-Abelian statistics).

- However, nobody was able to probe the non-Abelian statistics experimentally. They see the fractional charge consistent with statistics, but not the statistics itself.

Vol 452 | 17 April 2008 | **doi:10.1038/nature06855** nature

ARTICLES

Observation of a quarter of an electron charge at the $\nu = 5/2$ quantum Hall state

M. Dolev[1], M. Heiblum[1], V. Umansky[1], Ady Stern[1] & D. Mahalu[1]

Question: can we look for the non-Abelian particles elsewhere?

Topological states of matter

X.-G. Wen

Topological states of matter

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Topological states of matter:
2D states of matter with fractional and/or non-Abelian excitations

Topological states of matter

Examples realized or potentially realizable in nature:

1. Fractional Quantum Hall Effect. It's observed and is surely topological. Attempts to observe its non-Abelian particles were so far not successful. More work is ongoing.

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Proposal to realize such superconductor using cold atoms, VG, A. Andreev, and L. Radzihovsky (2004-05).

3. Chiral spin liquids. Long sought after topological state of quantum magnets.

Proposal to realize it using cold atoms,
M. Hermele, VG, Ana-Maria Rey (2009).

Superconductivity

Kamerlingh Onnes
1911

Nobel Prize 1913

Superconductors:
conduct electricity without any resistance;
expel magnetic fields (Meissner effect), levitate in a mag field;
are Bose-condensates of pairs of electrons, “Cooper pairs”;
form when electrons experience attraction;

Excitations in a superconductor

N N Bogoliubov

Bogoliubov quasiparticles

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Bogoliubov quasiparticles

Quasiparticle
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$$\hat{\gamma}_n = \int d\mathbf{r}\ \left[u_n(\mathbf{r})\,\hat{a}(\mathbf{r}) + v_n(\mathbf{r})\,\hat{a}^\dagger(\mathbf{r})\right]$$

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Not a creation operator of anything...

Excitations in a superconductor

N N Bogoliubov

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Not a creation operator of anything...

$\hat{c} = \hat{\gamma}_1 + i\hat{\gamma}_2$ These are legitimate creation and annihilation operators

$\hat{c}^\dagger = \hat{\gamma}_1 - i\hat{\gamma}_2$ Each of these γ are half of the electron! (an anyon, isn't it??)

Excitations in a 2D $p_x + i\, p_y$ superconductor

This is a 2D superconductor where Cooper pairs of electrons spin about their center of mass with angular momentum 1(p-wave) and with $\ell_z = 1$ ($p_x + i\, p_y$).

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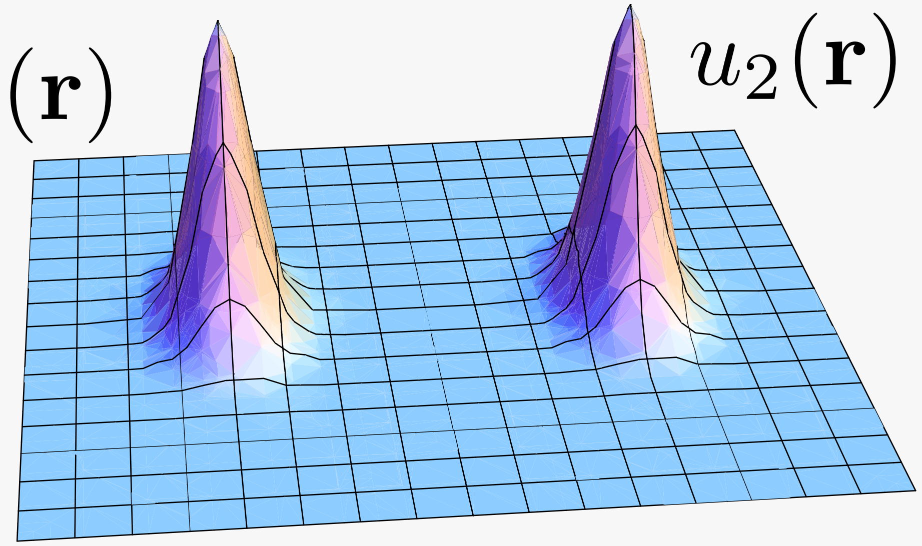

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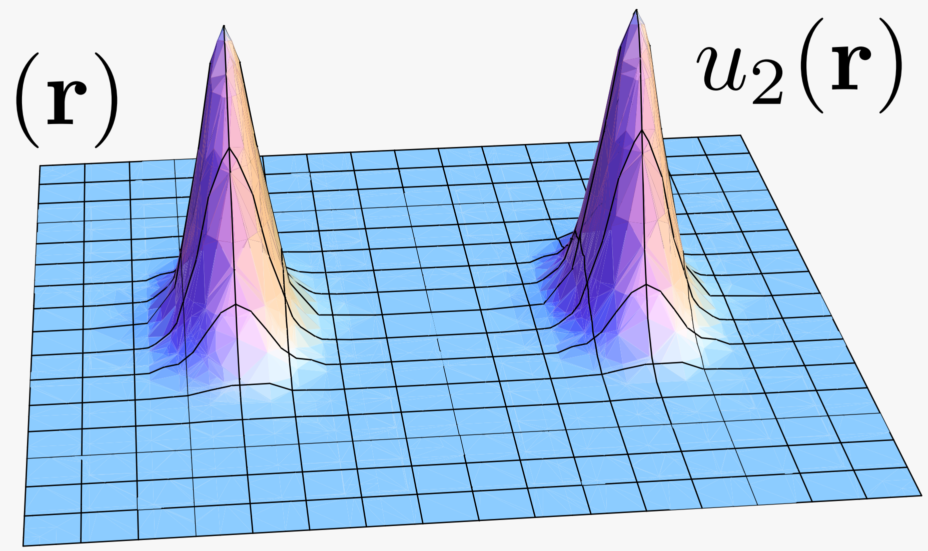

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Where can we find such a superconductor?

Cold atoms to the rescue?

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The drawbacks of cold atoms:

- Not all interactions can be modeled. Atoms are neutral, so magnetic fields are hard to emulate. Coulomb or other long range interactions are hard as well

- Cold atom systems often tend to be unstable, especially those with interesting interactions

Superconductivity with cold atoms

Take a bunch of fermionic atoms (common examples ^{40}K or ^{6}Li), "turn on" attractive interactions between them, cool them down, and they form a superconductor!

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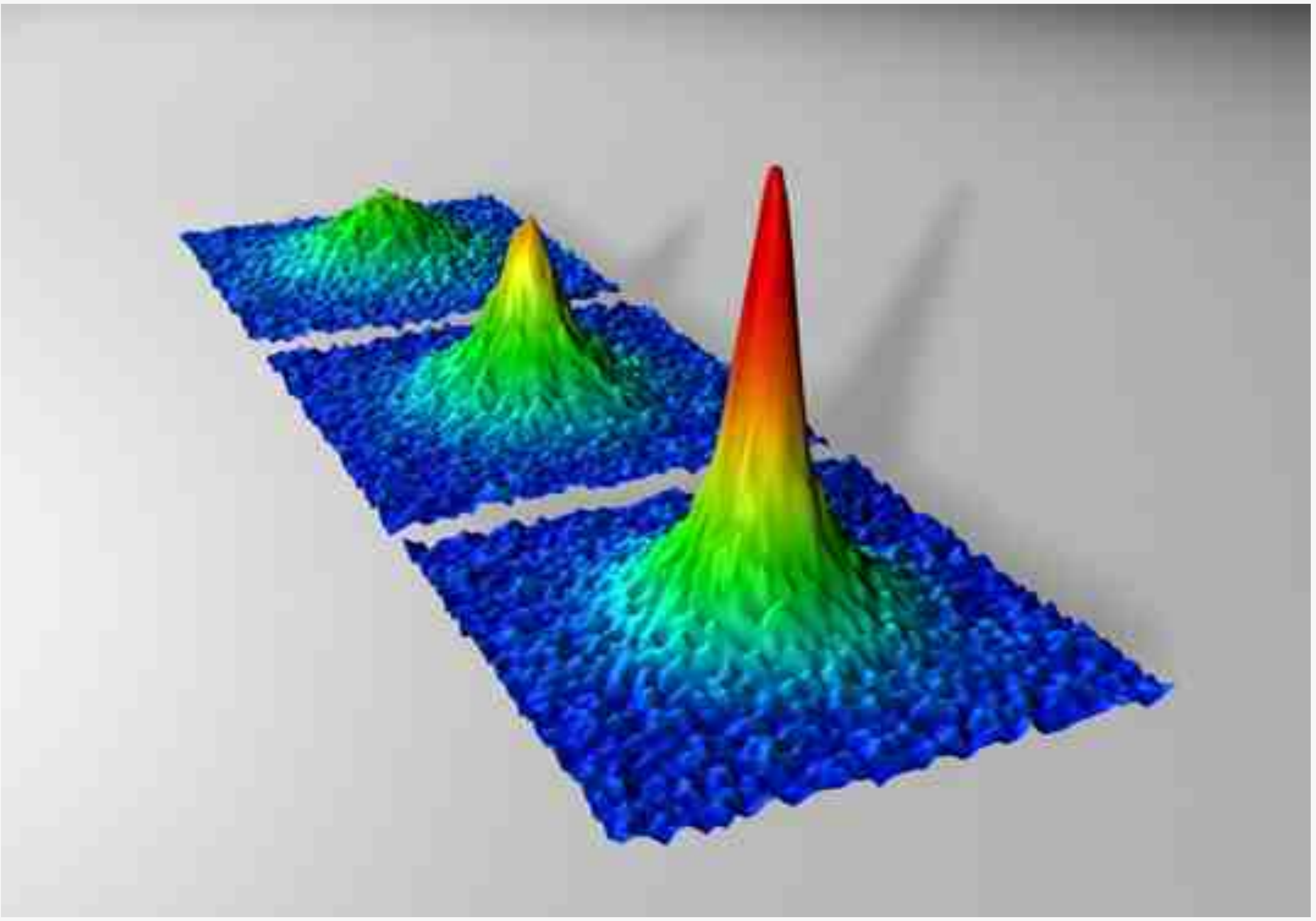

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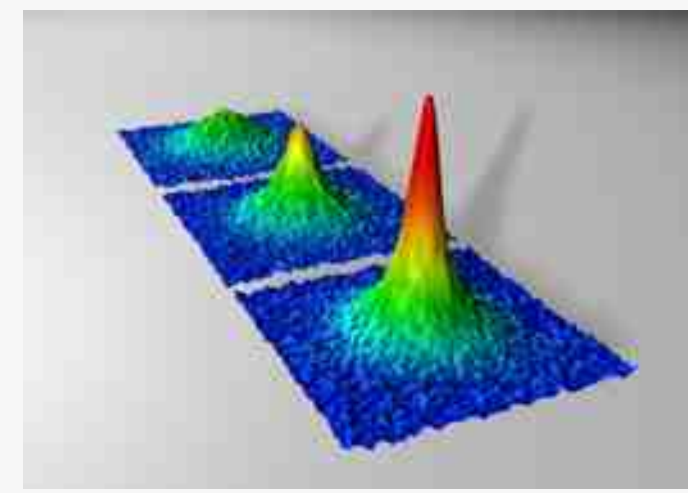

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^{40}K, $\boldsymbol{F_z=-9/2}$ 

^{40}K, $\boldsymbol{F_z=-7/2}$ 

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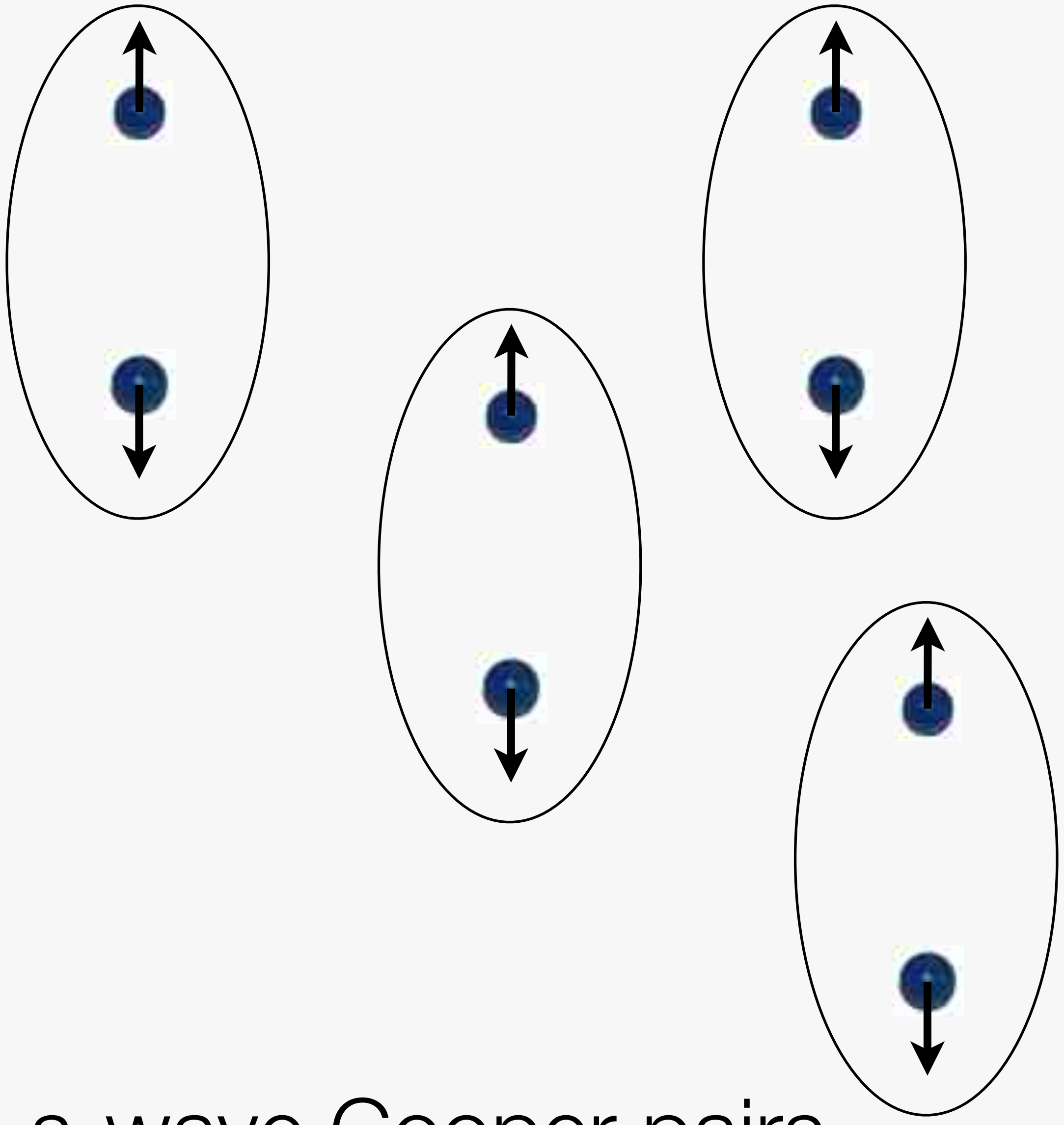
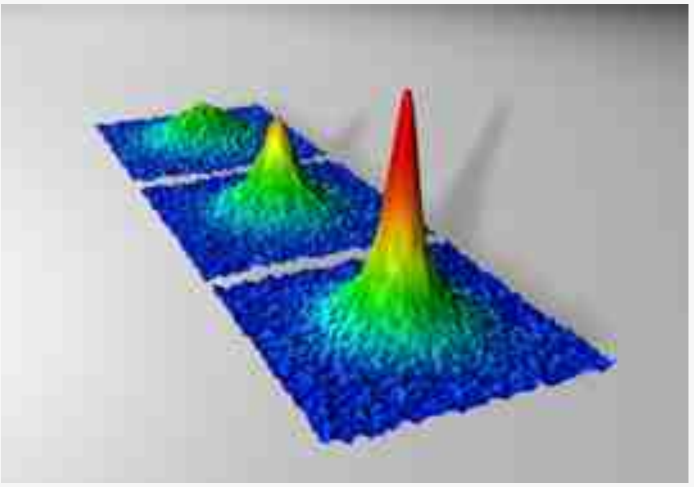


D. Jin, M. Greiner, C. Regal, '03-04



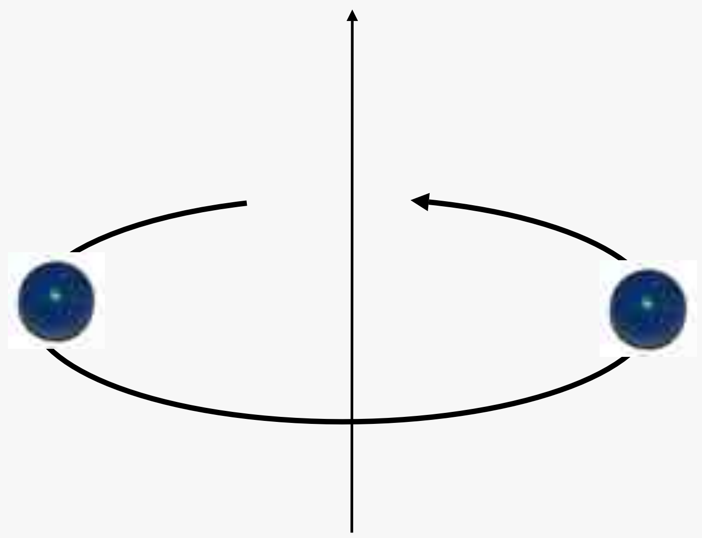
s-wave Cooper pairs

p-wave superconductors with cold atoms

VG, A. Andreev, L. Radzihovsky, 04-05

p-wave superconductors with cold atoms

Observation 1: identical fermionic atoms form Cooper pairs with odd angular momentum. For example, L=1.



Atoms in the same state - identical

VG, A. Andreev, L. Radzihovsky, 04-05

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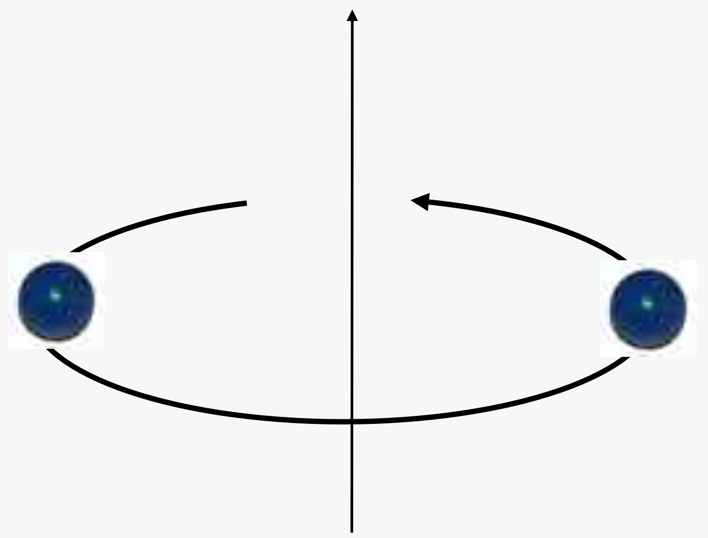

Atoms in the same state - identical

Observation 2: it is energetically favorable for the Cooper pairs to have $\ell_z = 1$ (to verify this requires a many-body calculation)

VG, A. Andreev, L. Radzihovsky, 04-05

p-wave superconductors with cold atoms

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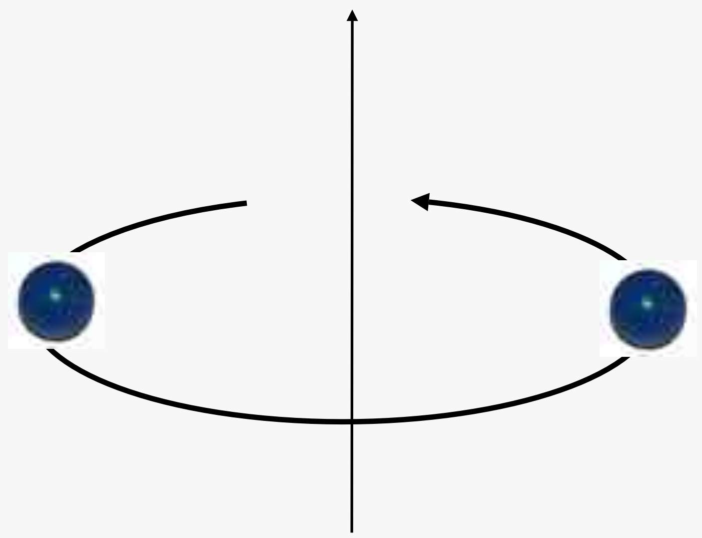


Atoms in the same state - identical

Observation 2: it is energetically favorable for the Cooper pairs to have $\ell_z = 1$ (to verify this requires a many-body calculation)

Observation 3: take identical fermionic atoms, cool them down, confine them to 2D, turn on attractive interactions, and you will get a 2D $p_x + i\, p_y$ superconductor

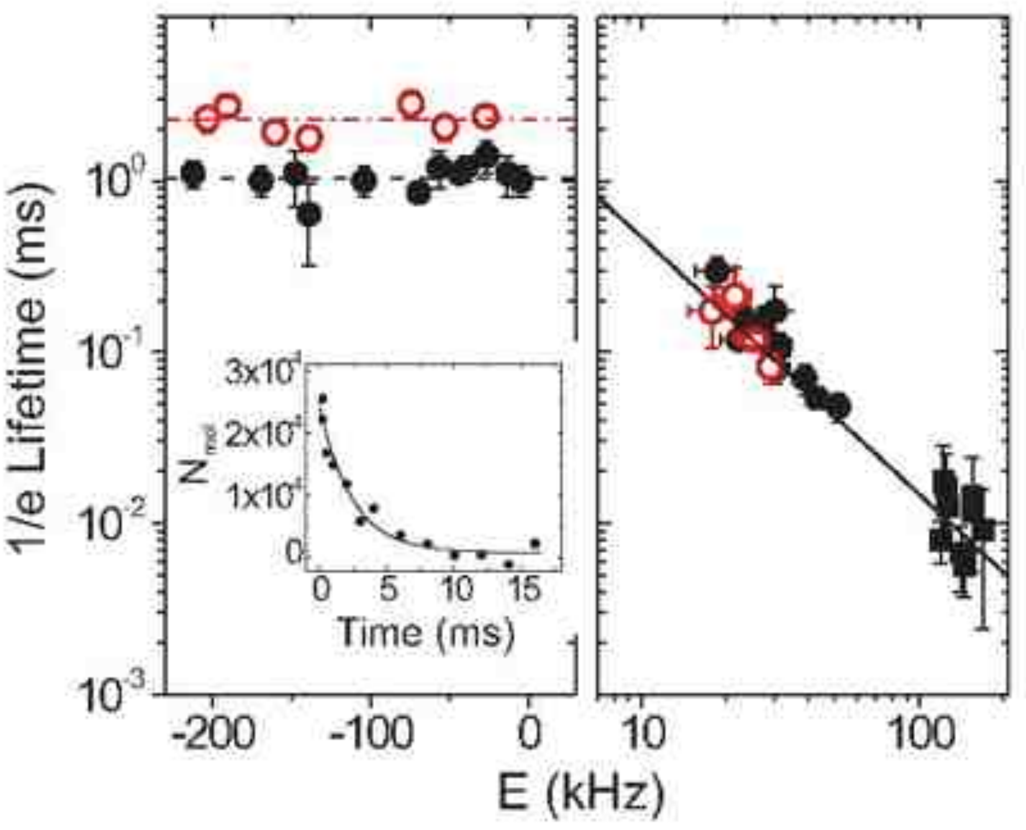
VG, A. Andreev, L. Radzihovsky, 04-05

Experiments

PRL **98,** 200403 (2007) PHYSICAL REVIEW LETTERS week ending 18 MAY 2007

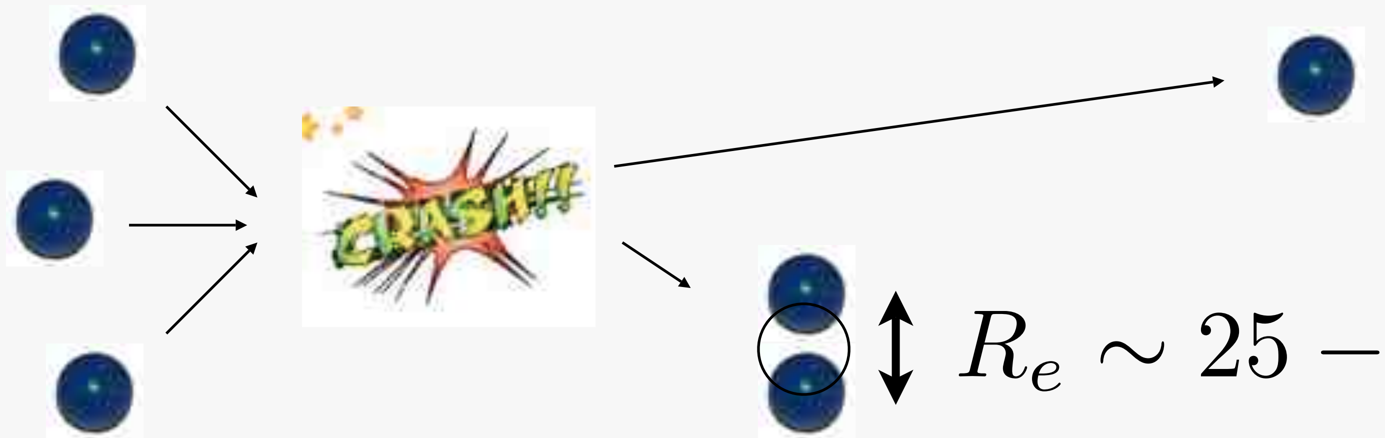
p-Wave Feshbach Molecules

J. P. Gaebler,* J. T. Stewart, J. L. Bohn, and D. S. Jin
JILA, Quantum Physics Division, National Institute of Standards and Technology and Department of Physics, University of Colorado, Boulder, Colorado 80309-0440, USA
(Received 2 March 2007; published 16 May 2007)



Bottom line:
the molecules are unstable,
with $\tau \sim 2ms$

Origin of instability: 3 body recombination



$$R_e \sim 25 - 50\, a.u.$$

R_e is the so-called van der Waals length (the typical interaction range)

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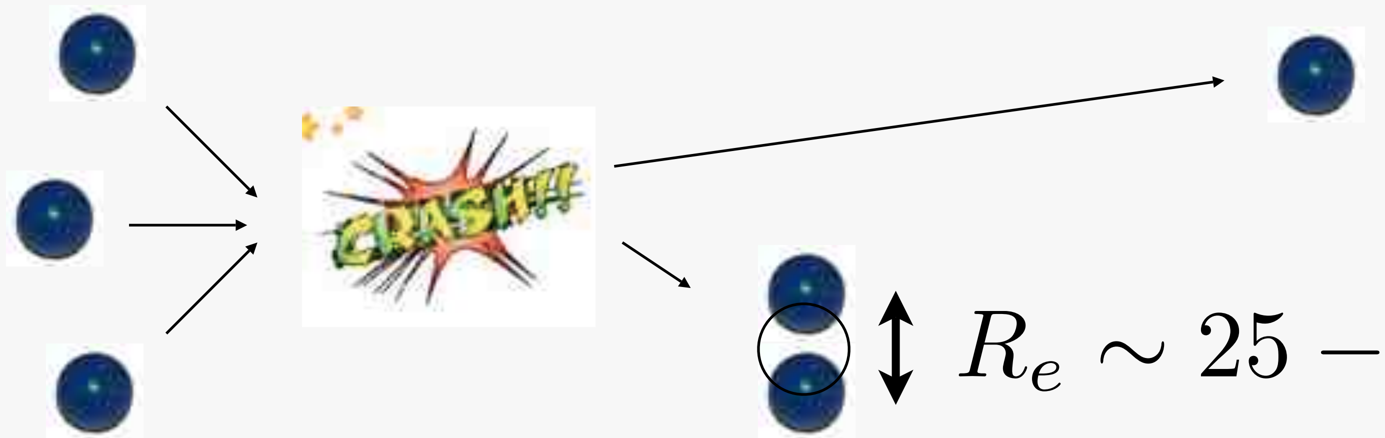

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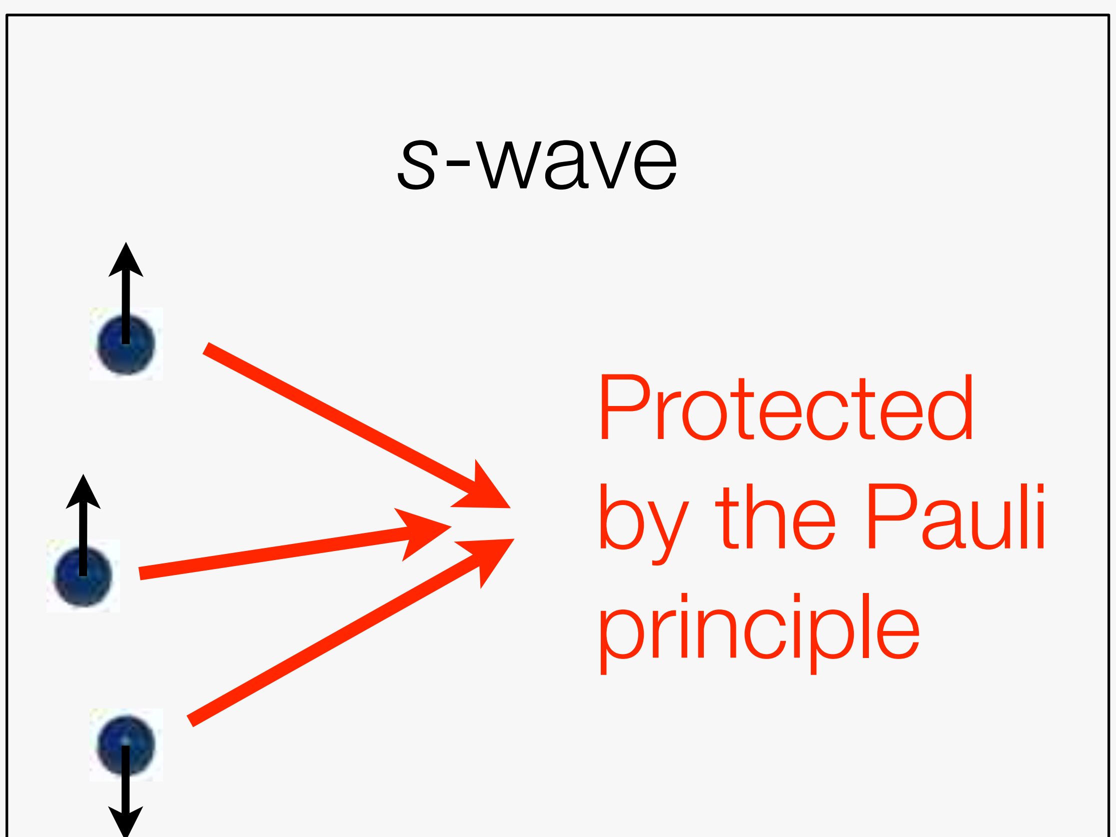

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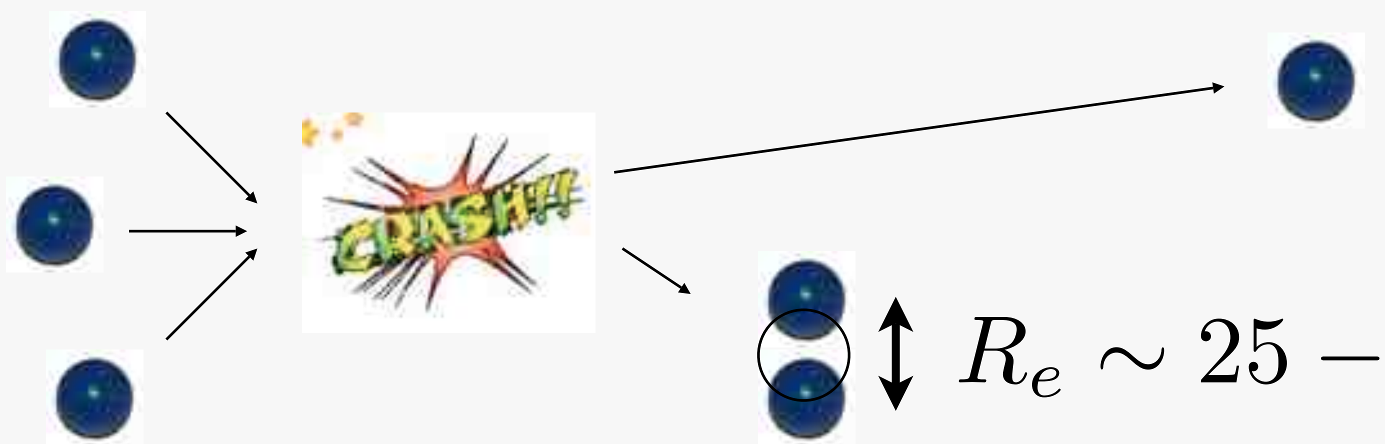

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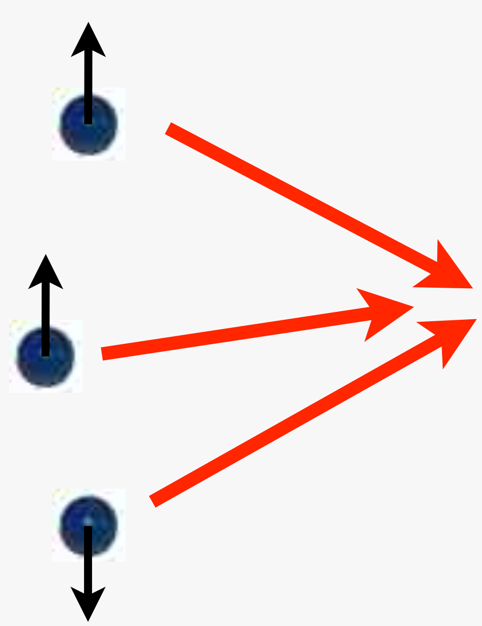
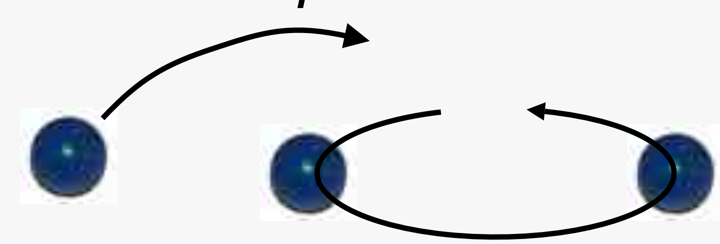


Protected by the Pauli principle

p-wave



By having opposite angular momenta, *p*-wave fermions beat the Pauli principle

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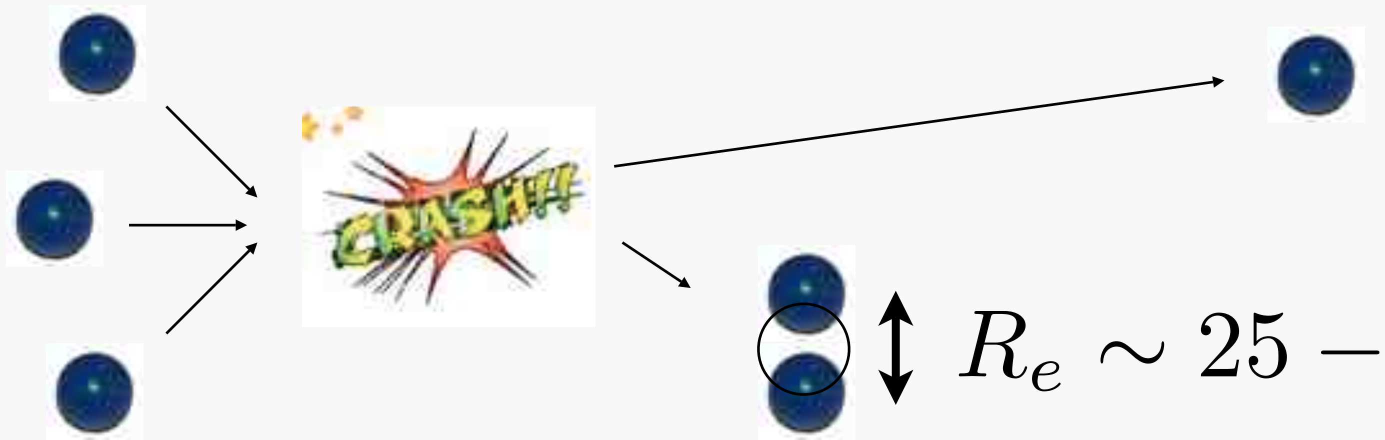

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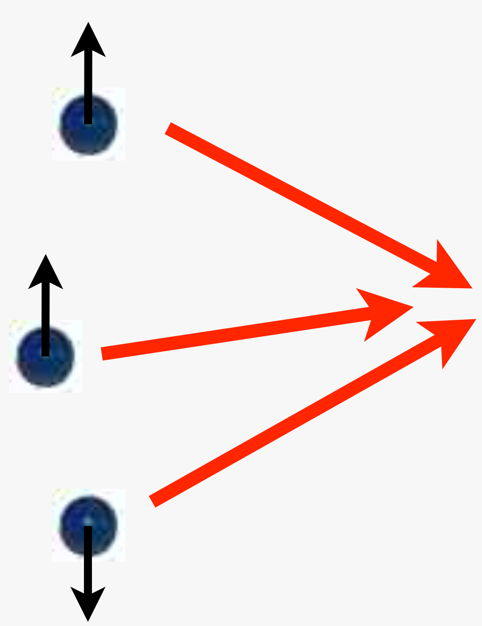

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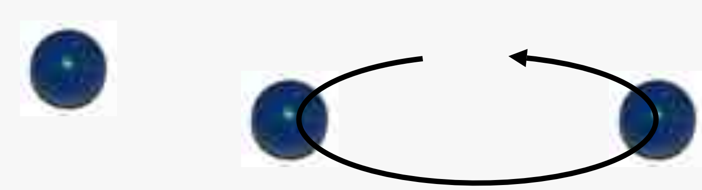

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Lifetime calculations

Interatomic distance

atomic mass

$$\text{Lifetime} = \frac{mr^2}{\hbar}\frac{r}{R_e} \sim 20\text{ms}$$

van der Waals length

Probably, their life is too short!

J. Levinsen, N. Cooper, VG, 07-08

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J. Levinsen, N. Cooper, VG, 07-08

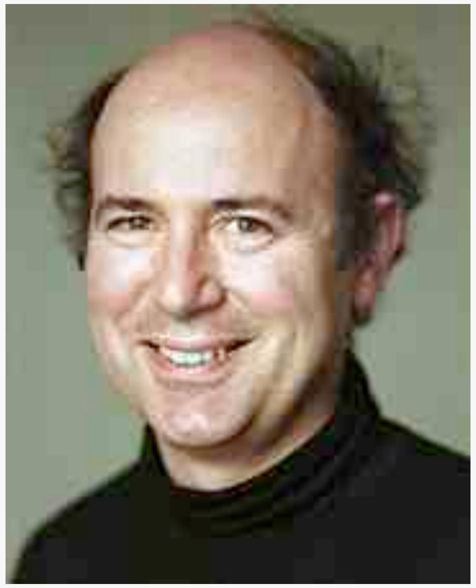
Optical lattices may provide a way to overcome short lifetimes...

P. Zoller et al, 09

topological magnets

topological magnets

X.-G. Wen



F. Wilczek

A. Zee

1989

topological magnets

X.-G. Wen F. Wilczek A. Zee

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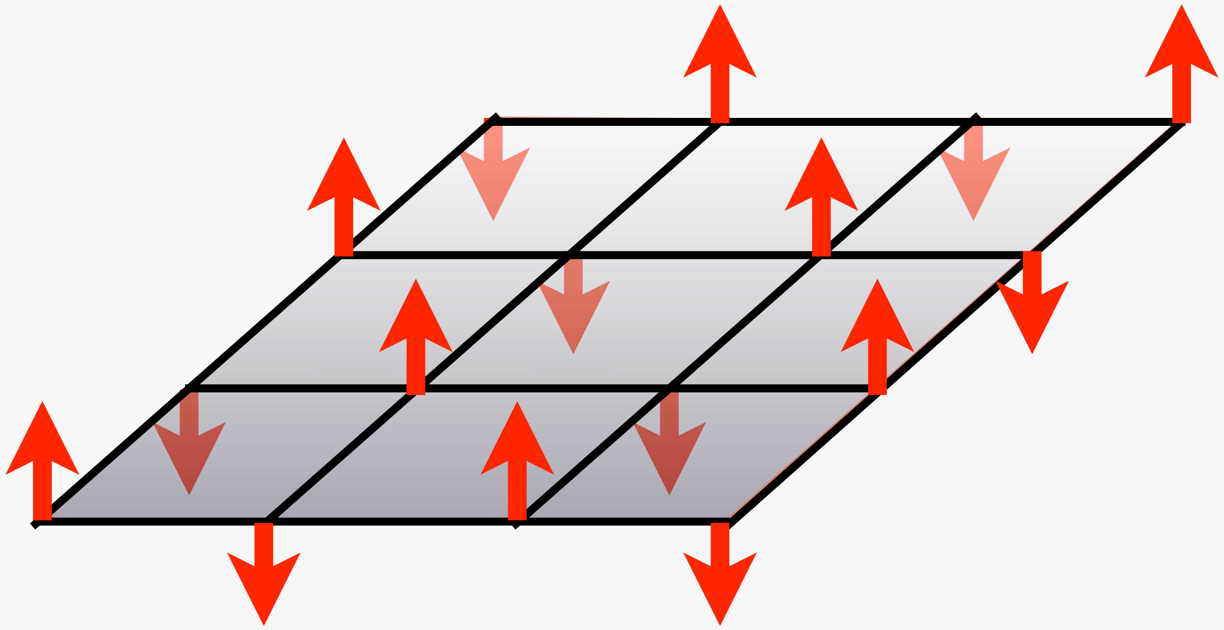
Heisenberg antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

Nearest neighbors



Néel state

topological magnets

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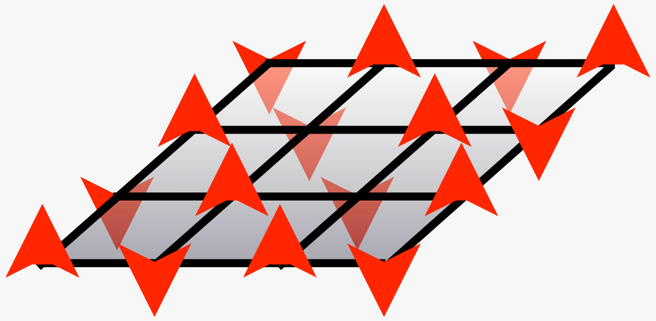

Néel state

Chiral spin liquid (CSL)

Think of spin as attached to particles

$f_{i\uparrow}^{\dagger}, f_{i\uparrow}; f_{i\downarrow}^{\dagger}, f_{i\downarrow}$

spin-up spin-down

$$H = J \sum_{<ij>,\alpha,\beta=\uparrow,\downarrow} f_{i,\alpha}^{\dagger} f_{i,\beta} f_{j,\beta}^{\dagger} f_{j,\alpha}$$

topological magnets


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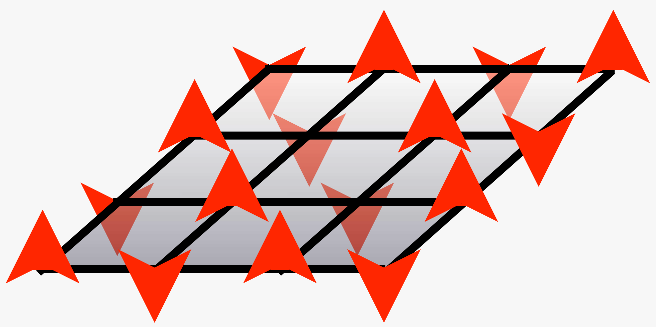

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t_{ij}

What if $\sum_\alpha \left\langle f^\dagger_{i,\alpha} f_{j,\alpha} \right\rangle = t_{ij}$ $\quad H = J \sum_{<ij>,\beta} t_{ij} f^\dagger_{i,\beta} f_{j,\beta} + \ldots$

“tight-binding Hamiltonian”

topological magnets


X.-G. Wen


F. Wilczek


A. Zee

1989

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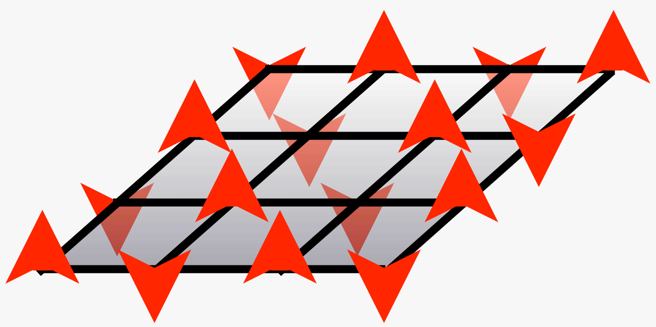

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"tight-binding Hamiltonian"

But what if t_{ij} correspond to a constant magnetic field?

This is CSL (or a topological magnet), by analogy with QHE

topological magnets

X.-G. Wen

F. Wilczek

A. Zee

1989

20 years and 552 citations later, nobody could still point out the Hamiltonian for which this scenario would work.

A proposal to generalize spin from SU(2) to to SU(N)

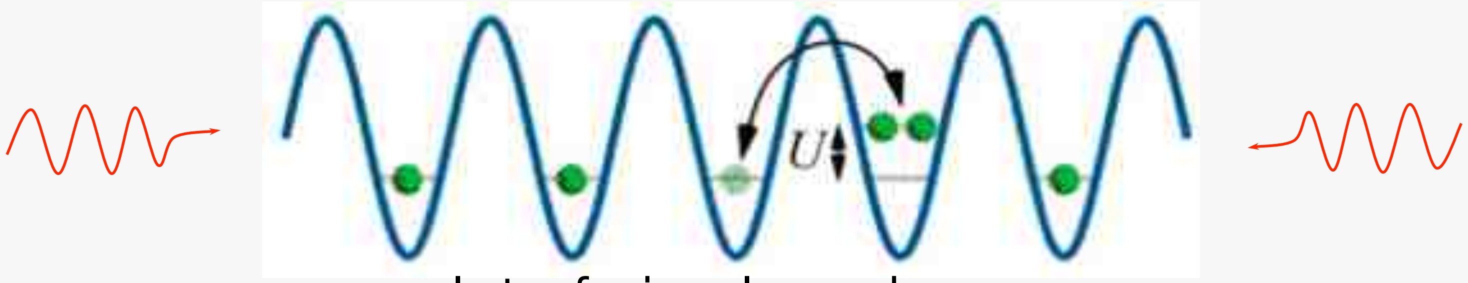
Generalize the usual spin to SU(N) spin by using alkaline-earth atoms. Their nuclear spin does not interact and behaves like an electron spin, only larger.

The spin ***I*** can be as large as *9/2* (for ^{87}Sr).
Then N=***2I+1*** is as large as 10.

A.-M. Rey (2009)

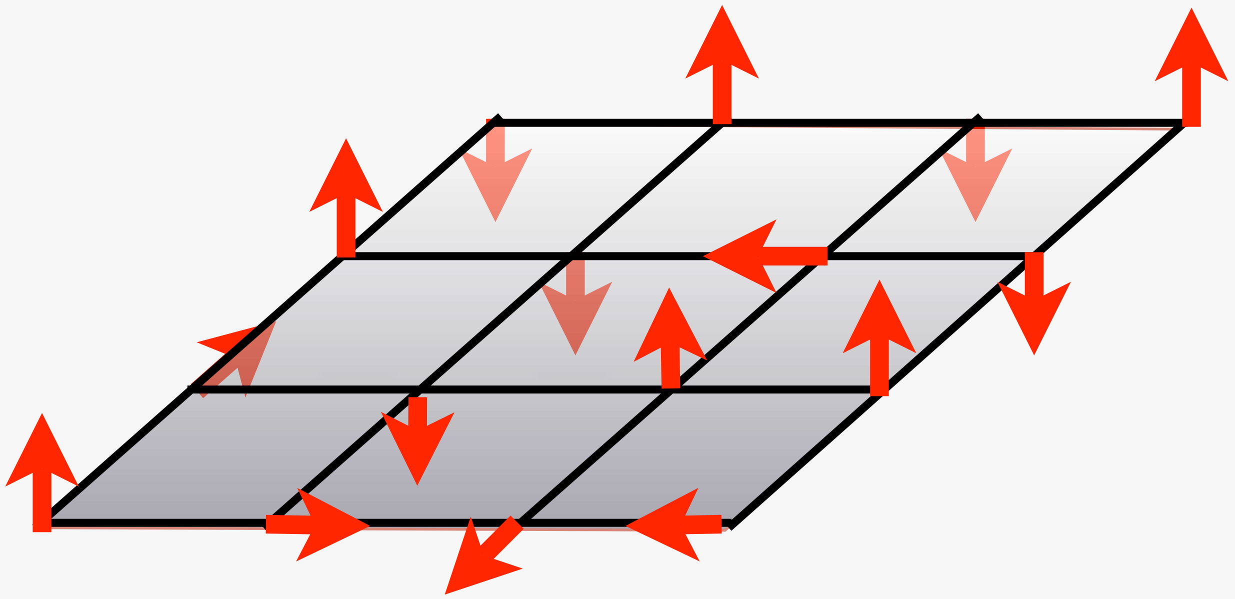
A. Gorshkov, M. Hermele, VG, C. Xu, P. Julienne, J. Ye, P. Zoller, E. Demler, M. Lukin and A.M. Rey (2009)

SU(N) antiferromagnets in optical lattices



Interfering laser beams

SU(N) antiferromagnets in optical lattices



^{87}Sr atoms

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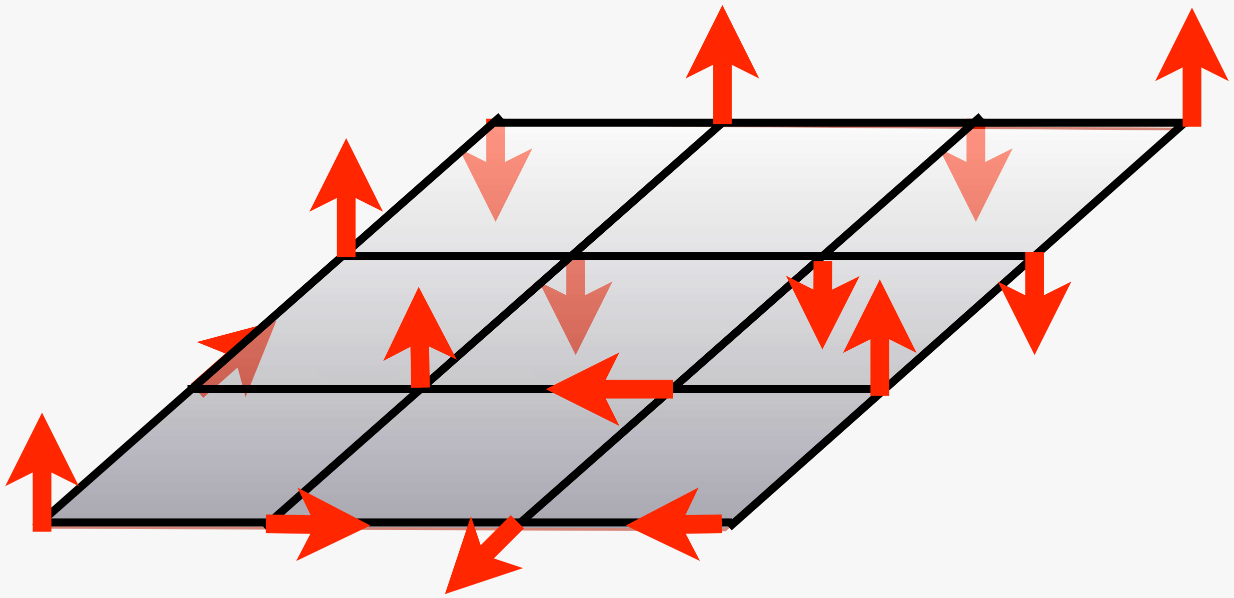
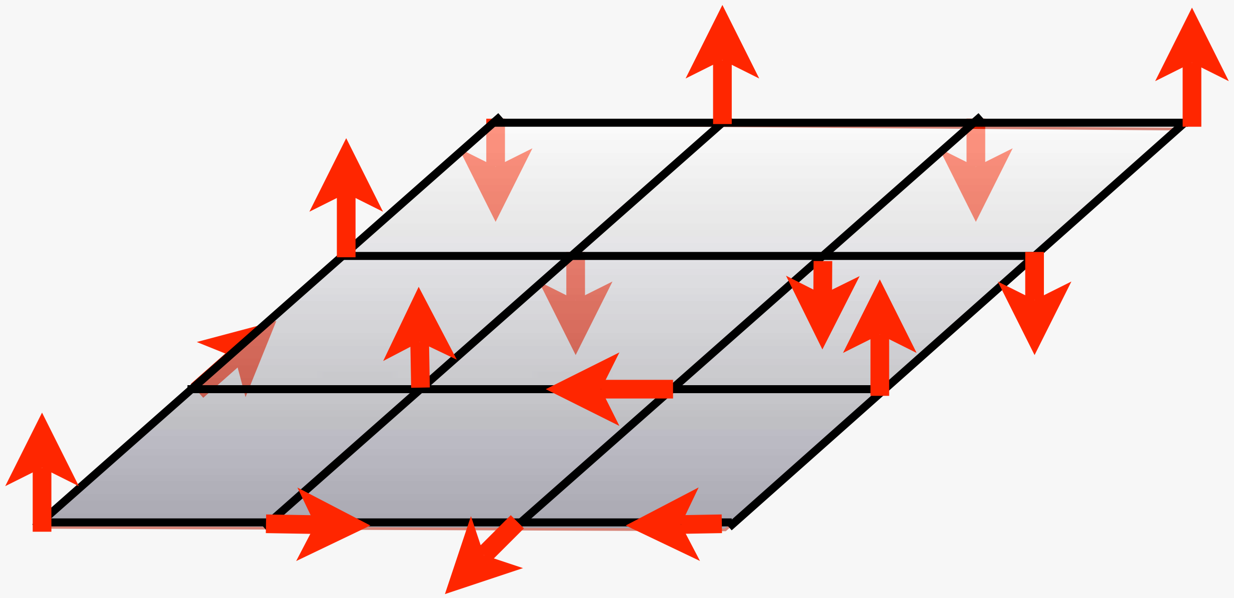


^{87}Sr atoms

$$H = J \sum_{<ij>,\ \alpha,\beta=1,\ldots,N} f_{i,\alpha}^\dagger f_{i,\beta} f_{j,\beta}^\dagger f_{j,\alpha}$$

Atom exchange leads to antiferromagnetic interactions (for nuclear spin).

SU(N) antiferromagnets in optical lattices



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Such SU(N) spins have a hard time ordering: too many directions nearby spins can point to while still being "opposite" to each other (minimize $\vec{S}_i \cdot \vec{S}_j$)

M. Hermele (2009)

Topological SU(N) antiferromagnet

It turns out, for N≥5, the ground state is a chiral spin liquid (that is, a topological magnet), exactly of the type proposed by Wen, Wilczek and Zee.

M. Hermele, VG, A.-M. Rey, (2009)

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M. Hermele, VG, A.-M. Rey, (2009)

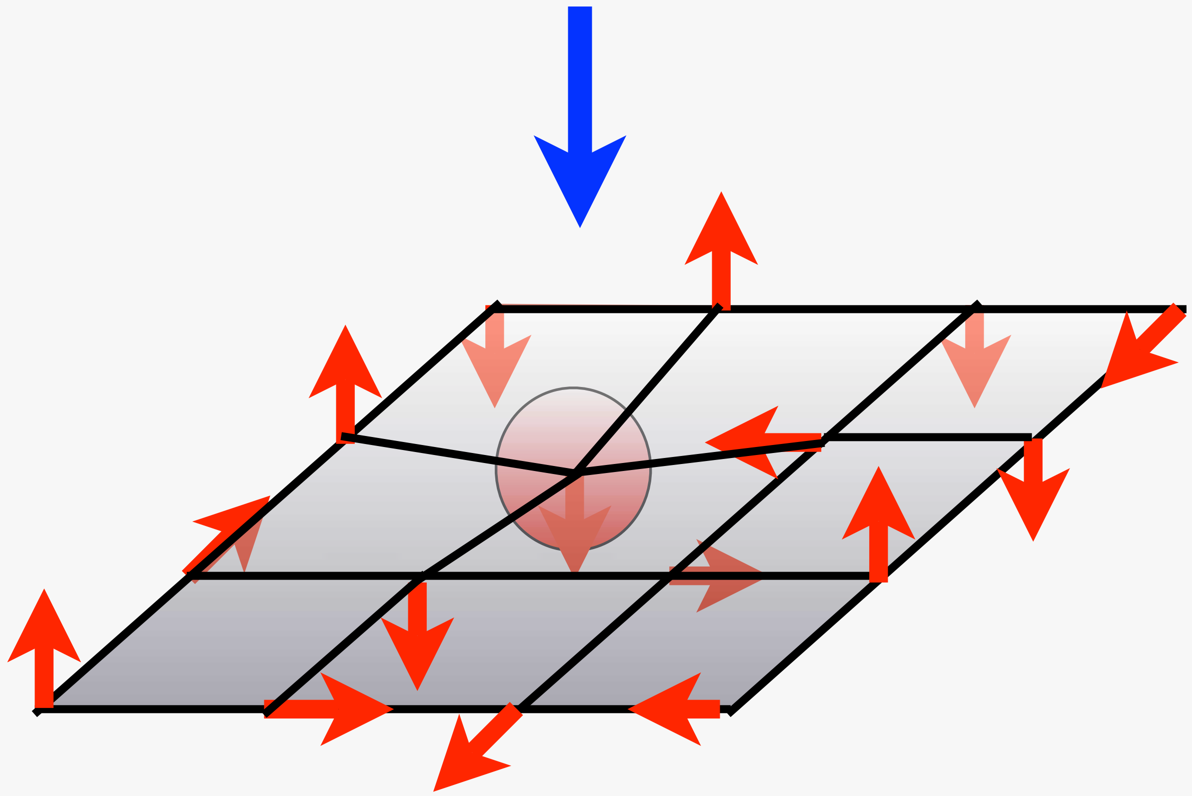
To show that, we employed the large N techniques:

$$H = J \sum_{i,\alpha} t_{ij} \left(f^\dagger_{i,\alpha} f_{j,\alpha} + hc \right) + \frac{N}{J} \sum_{<ij>} |t_{ij}|^2$$

$$S = N \mathrm{Tr} \log \left[\mathcal{S}_{ij} \right] + \frac{N}{J} \sum_{<ij>} |t_{ij}|^2$$

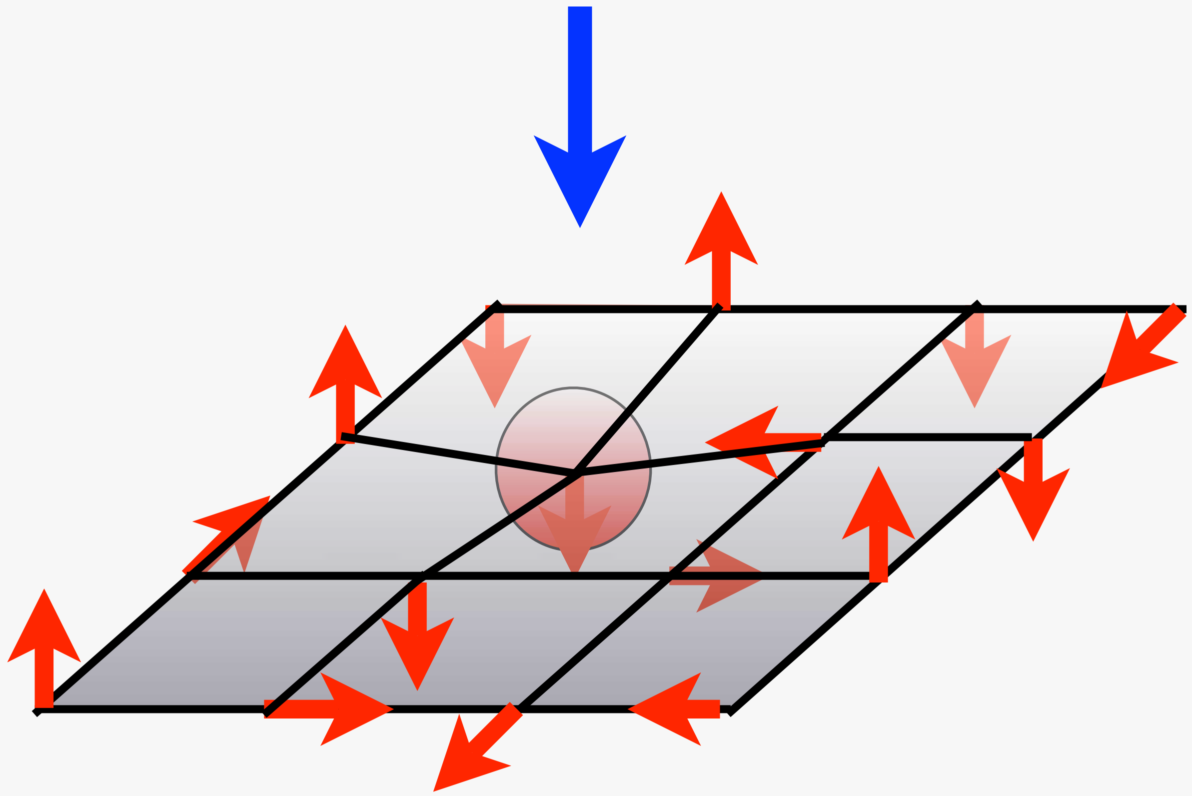
+ saddle point in *t*

Anyons and non-Abelions



Lowering the potential at one site localizes a fractional or non-Abelian particle at that site.

Anyons and non-Abelions



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Experimental detection? Too soon to tell...

Conclusions and outlook

Non-Abelian particles:

- definitely exist, but have not yet been seen
- would be very exciting to find, both for fundamental and applied reasons
- have excellent prospects of being found
- are a wonderful playground for a theorist

The end.