Explanationist Aid for the Theory of Inductive Logic

A central problem facing a probabilistic approach to the problem of induction is the difficulty of sufficiently constraining prior probabilities so as to yield the conclusion that induction is cogent. The Principle of Indifference, according to which alternatives are equiprobable when one has no grounds for preferring one over another, represents one way of addressing this problem; however, the Principle faces the well-known problem that multiple interpretations of it are possible, leading to incompatible conclusions. In the following, I propose a partial solution to the latter problem, drawing on the notion of explanatory priority. The resulting synthesis of Bayesian and inference-to-best-explanation approaches affords a principled defense of prior probability distributions that support induction.

1 A PROBABILISTIC FORMULATION OF THE PROBLEM OF INDUCTION

The problem of induction is the problem of explaining why it often makes sense to accept conclusions that are supported only by inductive arguments. I take an inductive argument to be a species of non-demonstrative argument in which what is known to be true of a sample from some population is extended to other members of the population not included in the sample. Sometimes induction is represented as proceeding according to the following pattern:

> All observed A's have been B.
> Therefore (probably), all A's are B.

Sometimes, instead, induction is represented as following this pattern:

All observed A's have been B.

Therefore (probably), *the next* A to be observed will be B.

Hereafter, I shall focus mainly on the second sort of inductive inference, partly because it seems more likely that the second sort of induction can be justified than that the first can, though the justification of the second sort of induction is nevertheless nontrivial and philosophically interesting.

The problem of induction is a problem largely because of the perceived force of inductive skepticism, the view that the premises of an inductive argument as such provide *no* epistemic reason for accepting the conclusion of that argument. While a number of influential philosophers have embraced it,[1] the view's counter-intuitiveness—entailing as it does that we presently have no evidence that the Earth revolves around the sun, and that there is no epistemic reason to think that placing my hand in a fire will be painful—seems sufficient reason for seeking a way of avoiding inductive skepticism. In any case, I shall assume hereafter that a non-skeptical resolution of the problem of induction is desirable.

This is not to say that we should aim at defending the rationality of *every* inductive inference. A plausible theory of induction may impose strictures on cogent inductive inferences that rule out many actual or possible inductions. Two candidate strictures that come to mind are that the sample that the inductive premises concern should be large, and that it should be sufficiently varied. Doubtless there are other plausible such conditions. But we shall be satisfied if we can defend the thesis that at least *some* inductive inferences are cogent. Hereafter, when I discuss inductive inferences, I shall have in mind those inductive inferences that are the best candidates for cogent inferences—that is, inductions in which the sample is large and varied; there are no special reasons for doubting the conclusion; the premises and conclusion use ordinary predicates, rather

[1] Hume (1975, pp. 25-39) and Popper (1961, pp. 27-30) endorse inductive skepticism. Peirce (1932, pp. 470-71), Wittgenstein (1981, 5.15), and Keynes (1921, pp. 56-7) endorse what I refer to in the text below as the skeptical probability distribution.

than "grue-like" predicates; and so on. This assumption is fair, since inductive skeptics deny that induction can be justified even in the most favorable of circumstances.

Clearly the conclusion of an inductive argument is not *certain* to be true given that the premises are. Once we acknowledge this, it is natural to turn to a *probabilistic* formulation of the issue: those who accept the cogency of some forms of induction (hereafter, "inductivists") are naturally taken as claiming that the conclusion of an inductive argument is supported by its premises in the sense that the premises render the conclusion *more probable*. Inductive skeptics are naturally read as claiming that the conclusion of an inductive argument is not rendered more probable by its premises. This formulation of the issue requires an epistemic or logical interpretation of probability, rather than a physical interpretation. Hereinafter, I shall assume that such an interpretation is acceptable, addressing myself to the question of to what extent the notion of epistemic probability affords a solution to the problem of induction.

It will be convenient hereafter to discuss inductivism and inductive skepticism in terms of a simple, admittedly artificial example. If we can come to an understanding of this case, we will have a better chance of subsequently generalizing our results:

> **Example 1:** A physical process X has been discovered, the laws governing which are as yet unknown, except that the process must produce exactly one of two outcomes, A or B, on every occasion. No relevant further information is known about X, nor about A or B. We plan an experiment in which X will occur n times, and we will observe on each occasion whether A or B results.

Let A_i = [Outcome A occurs on the ith trial.]

U_i = [Outcome A occurs on all of the first i trials.]

In this case, we wish to consider whether (at least for large values of i), U_i provides probabilistic evidence for A_{i+1}. The following three positions are possible:

Inductivism: $P(A_{i+1} \mid U_i) > P(A_{i+1})$

Skepticism: $P(A_{i+1} \mid U_i) = P(A_{i+1})$

Counter-inductivism: $P(A_{i+1} \mid U_i) < P(A_{i+1})$

Inductivism, inductive skepticism, and counter-inductivism, so defined, are each probabilistically coherent views. Perhaps the easiest way to see the coherence of inductive skepticism (*pace* David Stove[2]) is to consider a model of inductive skepticism, that is, a possible case in which the correct probability distribution would in fact be the one employed by the inductive skeptic: Suppose a fair coin is to be flipped a large number of times. Suppose that the first fifty flips result in heads up. Given this, what is the objective chance that the coin will land heads up on the next flip? Answer: ½, the same as the prior probability of the coin landing heads up on any given trial. What happens during the first fifty flips is independent of what happens on any subsequent flip, since the coin is fair and has no memory of what happened previously. Since this is a possible distribution for the objective chances, it is also a *coherent* distribution for epistemic or subjective probabilities, since the latter are governed by the same axioms. The inductive skeptic's view is that distinct observations are analogous to distinct flips of a coin known to be fair: they are entirely probabilistically independent of each other.

The counter-inductivist distribution, on the other hand, is similar to the probability distribution appropriate to a game of Russian Roulette: the more times you have pulled the trigger (without spinning the barrel again) and not been shot, the more likely it is that you *will* be shot the next time. Despite the occasional human tendency to commit the gambler's fallacy, there may be no one who has advanced a general counter-inductivist probability distribution.[3] The interest of counter-inductivism is purely theoretical—it serves as an example of something that a satisfactory solution to the problem of

[2]Stove (1986, pp. 51-4) argues that inductive skepticism is probabilistically incoherent, given some assumptions regarding non-extreme prior probabilities. My characterization of inductive skepticism in the text enables the skeptic to avoid Stove's argument.

[3]Though Popper and Miller (1983) come close.

induction should avoid.

Our task, then, is to explain why the admittedly coherent probability distribution of the skeptic or the counter-inductivist is rationally inferior to some inductivist probability distribution.

2 A PROBLEM WITH OBJECTIVE BAYESIANISM

2.1 Intuitive Motivation for the Principle of Indifference

Objective Bayesians recognize constraints on initial probability distributions that go beyond the Kolmogorov axioms.[4] Ideally, we might hope that such constraints will uniquely determine the prior probability of every proposition. But even much more modest constraints could suffice to avoid inductive skepticism—as long as we can constrain priors sufficiently that, for example, the drawing of a series of black balls from an urn supports the hypothesis that the next ball drawn will be black, we will have made significant progress on the problem of induction.

The Principle of Indifference, according to which the probabilities of two alternatives are equal whenever one lacks reason for favoring one over the other, is perhaps the most popular way of constraining prior probabilities. This principle can be motivated by an epistemic or logical interpretation of probability. Suppose that the probability of a proposition (for a given person) is understood as a measure of how much reason one has to believe that proposition, or the degree to which that proposition is supported by one's evidence. Then the Principle of Indifference amounts to the claim that, if one has no reason for preferring one alternative over another, then one has as much reason, or evidence, for the one proposition as for the other. This principle seems close to an analytic truth, though it presupposes the substantive assumption that how

[4]Objective Bayesians introduce constraints beyond the axioms (*i*) that the probability of any proposition must be greater than or equal to zero, (*ii*) that the probability of a tautology must be 1, (*iii*) that $P(A \vee B) = P(A) + P(B)$ whenever A and B are mutually exclusive, and (*iv*) that $P(A \& B) = P(A) \times P(B|A)$. I lack space here to discuss the more popular, subjective variety of Bayesianism (see Howson & Urbach 2006; de Finetti 1974).

much reason one has to believe a proposition may be treated as a quantity. It seems that, if one does *not* have as much reason to believe A as to believe B, then one must have *more* reason to believe one than to believe the other. But this is incompatible with one's having no reason to prefer either alternative. Therefore, if one has no reason to prefer either A or B, then they must have equal epistemic probabilities.

2.2 The Inconsistency Objection

Consider one illustration of the common charge that the Principle of Indifference is inconsistent:

> **Example 2:** Sue has taken a trip of 100 miles in her car. The trip took between 1 and 2 hours, and thus, Sue's average speed was between 50 and 100 miles per hour. Given only this information, what is the probability that the trip took between 1 hour and 1½ hours?[5]

Here is one solution. Using a generalization of the Principle of Indifference, we assign a flat probability density over the range of possible durations of the trip, from 1 hour to 2 hours. Since the interval from 1 hour to 1½ hours is one-half of the total range of possibilities, the probability of the true time falling in that interval is ½.

Here is another solution. Again using a generalization of the Principle of Indifference, we assign a flat probability density over the range of possible average velocities with which Sue may have traveled. Now, the time of Sue's journey was between 1 hour and 1½ hours if and only if her velocity was between 66⅔ mph (= 100 miles/1½ hours) and 100 miles per hour (= 100 miles/1 hour). Since the interval from 66⅔ mph to 100 mph is two-thirds of the total range of possible velocities, the probability of the true velocity falling in that interval is ⅔.

These two answers are inconsistent, yet both seem to be arrived at by equally natural

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[5]Fumerton (1995, p. 215) discusses this example.

applications of the Principle of Indifference. At worst, we might conclude that the Principle of Indifference is inconsistent.[6] At best, we might say that the principle stands in need of clarification: When we wish to deploy the Principle of Indifference, under what way of partitioning the possibilities ought we to assign each possibility an equal prior probability? For cases with a continuous range of alternatives, with respect to what variable ought we to assign a uniform prior probability density?

2.3 An Effort to Contain the Problem

We might seek to limit the impact of the inconsistency objection by arguing that at least in some cases, we have clear intuitions about which of a set of partitions of the space of possibilities is relevant. In those cases, we may deploy the Principle of Indifference. In cases like the above, in which we have no clear intuitions discriminating among some possible ways of characterizing the possibilities, perhaps we are unable to determine which of a set of numbers is the correct probability for a given proposition, or perhaps there is no uniquely correct probability.

Suppose, for example, that I inform you that I have a playing card in my pocket. Suppose you know nothing about me, so that you have no knowledge of what sort of playing cards I might prefer to keep in my pocket, and I refuse to tell you by what physical process the playing card in my pocket was selected. Given this, what is the probability that the card in my pocket is a four of clubs? Here is one solution: the four of clubs is one of 52 possible playing cards. Applying the Principle of Indifference, each of the possible cards has an equal probability of being in my pocket. So the probability that the card in my pocket is the four of clubs is 1/52.

Now, in the style of the Inconsistency Objection, here is another solution. The card in my pocket is either a three, or a four, or something else. Applying the Principle of Indifference, each of these alternatives has an equal probability. So the probability of the card's being a four is ⅓. Now, if it is a four, then it is either a club or not a club.

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[6] Van Fraassen 1989, p. 303; Howson and Urbach 1989, pp. 45-8.

Applying the Principle of Indifference again, each of these alternatives receives ½ probability. So the probability of the card's being the four of clubs is (⅓)(½) = 1/6.

This version of the inconsistency objection is intuitively uncompelling. The reason is that, though we may lack a general account of how possibilities should be partitioned when applying the Principle of Indifference, the partitioning required by the "1/6" solution to the problem does not strike us as equally natural as the partitioning required by the "1/52" solution. Rather, the partitioning used for the "1/52" solution is clearly the more natural. In contrast, Sue's journey (Example 2) presents an intuitively compelling puzzle because the *speed* of Sue's car and the *time* of her journey seem equally natural variables in terms of which to characterize the possibilities. As a result, we might say that in the case of Sue's journey, the answer to the problem is either indeterminate or unknown, but that nevertheless, in the case of the card in my pocket, the problem has a clear, unique answer of 1/52.

Though I have some sympathy with this line of thought, it offers us little help with the problem of induction. For skeptics can defend their position with an application of the Principle of Indifference that seems intuitively natural, or at least not clearly artificial as in the "1/6" solution to the playing card problem. This application of the Principle of Indifference is to assign an equal initial probability to each possible sequence of observations, or to each possible way of distributing properties to individuals. In Example 1, this amounts to assigning to each possible sequence of A and B results the same probability. Since there are 2^i possible ways of distributing A and B among i members of a sequence, the probability of each possible sequence is $(½)^i$. This is not an intuitively strained or artificial way of interpreting the Principle of Indifference. But of course, it amounts to the "fair coin" probability distribution: the outcome of any iteration of process X will be probabilistically independent of the outcomes of any other iterations. $P(A_{i+1}) = ½$, since A_{i+1} is one of the two possible outcomes of the $(i+1)$th iteration of X; $P(U_i) = (½)^i$, since U_i describes exactly one of the 2^i possible sequences of the first i outcomes; and $P(U_i \,\&\, A_{i+1}) = (½)^{i+1}$, since $(U_i \,\&\, A_{i+1})$ describes exactly one of the 2^{i+1} possible sequences of the first $i+1$ outcomes. Applying the axiom of conditional probability, we obtain $P(A_{i+1} \mid U_i) = P(U_i \,\&\, A_{i+1})/P(U_i) = (½)^{i+1}/(½)^i = ½$, the same as the initial probability of A_{i+1}. Hence, inductive skepticism seems to be vindicated.

Another seemingly natural interpretation of the Principle of Indifference results in an inductivist probability distribution, which we may call the Laplacean distribution. This interpretation assigns an equal initial probability to each possible *proportion* of A's in the sequence. The proportion of A's in a sequence of *i* instances of process X is either $0/i$ or $1/i$ or . . . or i/i. So each of these possibilities has an initial probability of $1/(i+1)$. This distribution favors induction: after *i* cases of A have been observed, with no B's, the probability of the next observed case being A as well is given by

$$P(A_{i+1} \mid U_i) = \frac{P(A_{i+1} \,\&\, U_i)}{P(U_i)} = \frac{P(U_{i+1})}{P(U_i)} = \frac{1/(i+2)}{1/(i+1)} = \frac{i+1}{i+2}.$$

This is the Rule of Succession invoked by Bayes, Laplace, and others to defend induction.[7]

If we are to wield the Principle of Indifference against inductive skepticism, then, we must supply a rationale for preferring an inductivist prior probability distribution, such as Laplace's distribution, over the inductive skeptic's distribution. It is here that objective Bayesians are most in need of aid. And it is here that explanationism enters our story.

[7]Laplace 1995, pp. 10-11. Bayes (1763, scholium to proposition 9) also employs a Laplacean distribution. The Laplacean distribution is equivalent to Carnap's (1962, pp. 562-77) *m** measure, leading to his recommended confirmation function, *c**. Carnap (1962, p. 567-8) derives a general formula that gives the Rule of Succession as a special case when families of two atomic predicates are considered. Unfortunately, Carnap later took back his support for *c** (1980, pp. 110-19).

3 Some Explanationist Relief for Objective Bayesianism

3.1 Explanation and Explanatory Priority

The explanationist holds that much of our non-demonstrative reasoning is to be understood in terms of inference to the best explanation.[8] Whether and how this approach comports with Bayesianism remains a matter of dispute. Bayes' Theorem seems to provide at least partial support for the explanationist approach: in choosing between candidate explanations h_1 and h_2 for evidence e, one factor that seems relevant is the likelihood ratio $P(e|h_1)/P(e|h_2)$. The greater this is, the better h_1 is as an explanation of e, compared to h_2—other things being equal, the hypothesis that more strongly predicts the evidence is the better explanation. Bayesians will go along with this approach so far.

But there is more to explanation than likelihood ratios reveal. An explanation must do more than induce a higher probability for the explanandum than the explanandum's initial probability. For instance, typically $P(e|a \,\&\, e) > P(e)$, yet $(a \,\&\, e)$ does not count as an *explanation* of e. Importantly, the explanans must be in some sense *prior* to (or more basic than, or more fundamental than) the explanandum. $(a \,\&\, e)$ violates this criterion for explaining e. Henceforth, I shall refer to this crucial relation that an explanatory fact must bear to its explanandum as "explanatory priority." Following are examples of some kinds of explanatory priority:

1. *Causal priority:* If A (partly) causes B, then the occurrence of A is "prior" to that of B in the order of explanation, meaning that A's occurrence is a candidate to figure in an explanation of B's occurrence, whereas B's occurrence is not fit to serve in an explanation of A's.
2. *Temporal priority:* If A is a fact about events or states that are temporally prior to (exist before) the events or states that B is about, then A is explanatorily prior to B. (A

[8] Harman 1965; Foster 1982-3; Niiniluoto 1999; Lipton 2004.

may still, of course, fail to satisfy some other requirement for explaining B.)[9] For these purposes, an eternal or timeless fact may also be treated as prior to facts about what happens at particular times.[10]

3. *The part-whole relation:* The existence, arrangement, and intrinsic features of the parts of an object are explanatorily prior to the existence and features of the whole.
4. *The in-virtue-of relation:* If B holds in virtue of A's holding, then A is explanatorily prior to B. The determinable-determinate relation may be a species of the in-virtue-of relation: if *d* is a determinate of *D*, then an object that has *d* will also have *D in virtue of* its having *d*. So a thing's having *d* will be explanatorily prior to its having *D*.[11]
5. *Supervenience:* At least some forms of supervenience are also instances of explanatory priority. For instance, the object on which I am seated is a chair in virtue of its parts having certain microphysical properties and relations, the properties and relations on which its chairhood supervenes. So the instantiation of those properties and relations is explanatorily prior to this object's being a chair.

I treat explanatory priority as a relation between facts or propositions, since I take facts or propositions to be the sort of things that explain and are explained. (Suitable rephrasing, however, could accommodate the view that events may explain or be explained.) I shall not attempt to fully analyze the concept of explanation. I assume, however, that A's being a *good explanation* of B has at least these two important necessary conditions: (*i*) that A should be explanatorily prior to B, and (*ii*) that B should be more

[9]Consider a case in which a kind of event, C, regularly causes A followed by B, where A and B are not causally related to each other. In such a case, the occurrence of A might well raise the probability of B's occurring due solely to its raising the probability that C has occurred. In my view, A would also be explanatorily prior to B. Yet A would not explain B. This shows that some further relation between A and B is required for explanation beyond explanatory priority and the probabilistic relation.

[10]Principles (1) and (2) can come into conflict if backwards causation is possible. In such a case, I believe that principle (1), that causal priority implies explanatory priority, would take precedence; however, I hold that backwards causation is not possible.

[11]I thank Christian Lee for this example of explanatory priority.

probable given A than otherwise ($P(B \mid A) > P(B)$), understanding probability in a logical or epistemic sense.[12] In some cases, such as the case of causal explanations, it may seem as though the relevant sort of probability in condition (*ii*) is physical probability. However, provided that the explanans A includes a description of the relevant causal laws or other facts that determine the physical probabilities, the relation between the epistemic probabilities $P_e(B \mid A)$ and $P_e(B)$ will mirror the relation between the physical probabilities $P_p(B \mid A)$ and $P_p(B)$. And it is reasonable to hold that A must include such a specification of the causal laws to be a candidate for the full explanation of B.

3.2 Explanatory Priority and the Assignment of Priors

In the literature on Bayesianism and inference to the best explanation, some have suggested that explanationism should be incorporated into a Bayesian framework through prior probabilities, roughly by one's assigning higher probabilities to propositions that are felt to be explanatory.[13] Van Fraassen has suggested, instead, that the explanationist would give a bonus to the *posterior* probability of a hypothesis that is judged as the best explanation of the evidence, thereby violating Bayesian

[12]The notion of a "good" explanation is partly epistemic; it is close to that of a *satisfying* explanation. Notably, A may be in fact the *correct* explanation of B without A's being a very *good* explanation of B—consider a case in which the actual causal history of B involves a highly complex and improbable sequence of coincidences. A description of that causal history would correctly (truthfully) explain B, yet it would not be satisfying as an explanation. Meanwhile, a more simple and elegant hypothesis, better supported by our available evidence, might offer a more satisfying explanation of B and yet be false. This shows that inference to the best explanation is a fallible form of inference.

As Michael Tooley has pointed out (p.c.), there may even be cases in which A correctly explains B despite A's lowering the probability of B: suppose there are probabilistic laws of nature, that A has a 50% chance of probabilistically causing B, but that, due to A's interfering with other potential causes of B, the occurrence of A actually lowers the probability of B's occurrence overall. On a given occasion, A together with a description of the relevant probabilistic laws might correctly explain B. Nevertheless, this would not be a "good" explanation of B.

[13]Niiniluoto 1999, p. S448; Okasha 2000, pp. 702-4; Lipton 2004, pp. 115-16.

conditionalization.[14] Each of these proposals seems artificial. Both have the flavor of *ad hoc* modifications to Bayesianism designed to humor explanationists.

Explanatory priority may affect the assignment of prior probabilities in a different way: it may feature in a partial solution to the problem of the interpretation of the Principle of Indifference, so that, rather than humoring explanationists, Bayesians may receive crucial aid from explanationists on a central problem for their view. The way in which considerations of explanatory priority may modify (or clarify) the Principle of Indifference is this: in applying the Principle of Indifference, one ought to assign equal probabilities (or a uniform probability density) at *the most explanatorily basic level*. I call this the Explanatory Priority Proviso to the Principle of Indifference. Suppose, that is, that we have two partitions of the space of possibilities, one that divides the possibilities into mutually exclusive, jointly exhaustive alternatives $h_1, ..., h_n, ...$, and another that divides the possibilities into mutually exclusive, jointly exhaustive alternatives $j_1, ..., j_n, ...$[15] Suppose further that each of the h_i is explanatorily prior to each of the j_i. Then the former partition should be preferred to the latter for purposes of applying the Principle of Indifference. For the case of continuous ranges of possibilities, suppose we have two variables, v_1 and v_2, each of whose values exhaust the possibilities. But suppose that v_1's having the value that it does is explanatorily prior to v_2's having its value. Then v_1 should be preferred to v_2 for purposes of applying the Principle of Indifference.

Let us begin with some examples designed both to clarify how this interpretation may be applied and to exhibit its plausibility.

Example 3: You are informed that a certain lamp is either on or off, and also that a single marble was recently drawn from a bag containing only red, blue, and/or green marbles. If a red marble was drawn from the bag, then the person drawing the

[14]Van Fraassen 1989, pp. 138, 160-9.

[15]Exclusiveness and exhaustiveness should be understood probabilistically, i.e., we may call h_1 and h_2 mutually exclusive iff $P(h_1 \& h_2) = 0$ (even if h_1 does not logically contradict h_2). Similarly, the h_i are jointly exhaustive iff $P(h_1 \vee h_2 \vee ...) = 1$.

marble made sure the lamp would be on (turning it on if necessary). If either a blue or a green marble was drawn, then he made sure the lamp would be off. Given just this information, what is the probability that the lamp is on? What is the probability that a red marble was drawn?

Solution #1: The lamp is either on or off. Applying the Principle of Indifference, each of these alternatives has probability ½. The lamp is on if and only if a red marble was drawn from the bag. So the probability that a red marble was drawn is also ½, while the probability of a blue marble is ¼, and the probability of a green marble is ¼.

Solution #2: The marble drawn from the bag was red, blue, or green. Applying the Principle of Indifference, each of these alternatives has probability ⅓. The lamp is on if and only if a red marble was drawn from the bag. So the probability that the lamp is on is ⅓.

Solution 2 is the intuitively correct one. This is explained by the Explanatory Priority Proviso. The drawing of the marble is causally and temporally prior to the lamp's current state, so the possible results of the marble-drawing are explanatorily prior to the possible states of the lamp. Therefore, the Principle of Indifference is to be applied to the possible marble-drawing results. Solution 1 is incorrect, because the lamp's state is determined by the prior results of the drawing; we must therefore first assign probabilities to the possible results of the drawing, and determine the probability of the lamp's being on from that probability distribution.

Example 4: You are informed that a conscious brain has recently been artificially created. (This supposition is meant to neutralize your background knowledge of the sorts of states that brains are typically in.) The brain has been put in one of the 4 million possible states recognized by modern brain science. Assume that mental states supervene on physical states, and that 100,000 of the 4 million possible brain states realize overall painful mental states, 50,000 realize pleasurable mental states, and the remainder realize hedonically neutral mental states (or states that are between pleasure and pain). What is the probability, on this information, that the brain is in pain?

Solution #1: The brain is either in a painful state, in a pleasurable state, or in a hedonically neutral state. Applying the Principle of Indifference, each of these alternatives has a probability of ⅓.

Solution #2: Each of the possible brain states is equally probable. Since 100,000 of those states realize pain, the probability that the brain is in pain is 100,000/4,000,000 = .025.

Again, Solution 1 is intuitively wrong. One should not assign ⅓ probability to the brain's being in pain, because the brain's hedonic state is determined by its (explanatorily prior) physical state, and only .025 of the possible physical states give rise to pain.

Now that we have a sense of the plausibility of the Explanatory Priority Proviso, let us apply it to the problematic case discussed in section 2.2:

Example 2: Sue has traveled a distance of 100 miles in between 1 hour and 2 hours. Her average velocity was between 50 mph and 100 mph. What is the probability that her trip lasted between 1 and 1.5 hours and thus that her average velocity was between 66.7 mph and 100 mph?

Solution: The length of time that Sue's journey took is causally explained (given a fixed distance) by the speed at which she was driving, not vice versa. Therefore, we assign a uniform probability density over the possible average velocities of Sue's trip. Since the measure of the interval [66.7, 100] is ⅔ of the measure of [50, 100], the probability of Sue's velocity falling in the former interval is ⅔.

Note that here we do not apply a uniform probability density to the possible durations of Sue's trip on the grounds that "velocity" is defined in terms of distance and time. The sort of priority invoked in the Explanatory Priority Proviso is metaphysical rather than conceptual. What matters is the fact that velocity is metaphysically prior to duration in this example, because the velocity causally determines the time it will take to go 100 miles—not the ostensible fact that the *concept* of velocity is dependent on the *concept* of duration. One reason for preferring a reliance on metaphysical priority rather than conceptual priority is that conceptual priority may differ between different subjects. Suppose one individual formed the concept of duration first, and then formed the concept of velocity by defining velocity as distance traveled per unit time, while another individual formed the concept of velocity (or rate of change) first, and only later formed the concept of duration.[16] It seems that these beings might nonetheless have the same information relevant to assigning probabilities in Example 2, and thus that our theory should not require them to endorse different answers to the problem.

One might doubt that conceptual priority relations can differ between subjects in this way. However, another argument against relying on conceptual priority is that doing so may result in intuitively wrong answers in cases like Example 4. Suppose that the mental concepts used in Example 4 ("pain," "pleasure") are psychologically basic, since they are formed on the basis of direct introspection. But suppose that the concepts used for identifying the four million different brain states are largely theoretical and require complex definitions. Intuitively, this makes no difference to the correct solution to Example 4.

3.3 In Defense of Laplace

The Explanatory Priority Proviso does not resolve every puzzle regarding the interpretation of the Principle of Indifference. In some cases, we may have two ways of characterizing the possibilities, neither of which is intuitively more natural than the other, and neither of which classifies the alternatives in terms of explanatorily prior propositions. In such cases, perhaps the relevant probabilities are indeterminate, or perhaps some other principle is required to assess the relevant probabilities.

Nevertheless, the Explanatory Priority Proviso makes important progress towards

[16] Piaget (1969, chapter 2) claims that the latter is in fact the situation with human children.

solving the problem of induction, as it helps to resolve the dispute between the skeptical interpretation of the Principle of Indifference and the inductivist interpretation discussed in section 2.3 above. Return to our original example:

> **Example 1:** Process X is to be repeated *n* times, producing either A or B on each occasion. Where A_i is the proposition that outcome A occurs on the *i*th trial and U_i is the proposition that A occurs on all of the first *i* trials, what is $P(A_{i+1} \mid U_i)$?

Solution: The physical process in question has some *physical* probability, or objective chance, of producing A on any given occasion. This objective chance is explanatorily prior to the individual outcomes or sequences of outcomes. Therefore, we assign a uniform (epistemic) probability density over the possible values of this objective chance, rather than over the possible sequences of outcomes.[17] Thus, we assign

$$\begin{aligned} \rho(c) &= 1, \text{ for } 0 \le c \le 1 \\ &= 0 \text{ otherwise.} \end{aligned} \tag{1}$$

where c is the objective chance and $\rho(c)$ is the probability density function for c. To find $P(A_{i+1} \mid U_i)$, we invoke the axiom of conditional probability:

$$P(A_{i+1} \mid U_i) = \frac{P(A_{i+1} \,\&\, U_i)}{P(U_i)} = \frac{P(U_{i+1})}{P(U_i)} \tag{2}$$

To determine the quantities on the right hand side of equation (2), we use the probability density given in equation (1).

[17]Bayes (1763, scholium) takes this approach.

$$\mathrm{P}(U_i) = \int_0^1 \rho(c) \times \mathrm{P}(U_i|C{=}c)\,dc = \int_0^1 \mathrm{P}(U_i|C{=}c)\,dc$$
$$\mathrm{P}(U_{i+1}) = \int_0^1 \rho(c) \times \mathrm{P}(U_{i+1}|C{=}c)\,dc = \int_0^1 \mathrm{P}(U_{i+1}|C{=}c)\,dc \tag{3}$$

where "$C = c$" denotes the proposition that the objective chance of outcome A is c. The probability that the first i instances of the experiment will result in outcome A, given that the objective chance on each instance is c, is c^i (invoking a version of the Principal Principle[18]). Thus, we have:

$$\mathrm{P}(U_i) = \int_0^1 \mathrm{P}(U_i|C{=}c)\,dc = \int_0^1 c^i\,dc = \left.\frac{c^{i+1}}{i+1}\right]_0^1 = \frac{1^{i+1}}{i+1} - \frac{0^{i+1}}{i+1} = \frac{1}{i+1}$$
$$\mathrm{P}(U_{i+1}) = \int_0^1 \mathrm{P}(U_{i+1}|C{=}c)\,dc = \frac{1}{i+2} \tag{4}$$

Substituting equations (4) into equation (2), we arrive at the Rule of Succession:[19]

[18]Lewis 1986.

[19]In Carnap's system, this result holds only for the case in which A is one of two possible outcomes of X, each of which has a "logical width" of 1 (Carnap 1962, p. 568). In my derivation, the latter assumption, regarding "logical width," is replaced with the assumption that one has no relevant information about outcomes A and B beyond that they are the two possible outcomes of X.

For the more general case in which A is one of n mutually exclusive and jointly exhaustive outcomes, and j of the first i instances of X resulted in outcome A, the appropriate deployment of the Principle of Indifference is to assign a uniform probability density over the interior of the (n-1)-dimensional simplex defined by $c_1+\ldots+c_n = 1$, $0 \le c_i \le 1$, where the c_i are the objective, single-case chances of each of the n outcomes. This procedure leads in effect to Carnap's rule that $\mathrm{P}(A_{i+1} \mid j \text{ of the first } i \text{ instances result in } A) = \dfrac{j+1}{i+n}$.

$$\mathrm{P}(A_{i+1} \mid U_i) = \frac{\mathrm{P}(U_{i+1})}{\mathrm{P}(U_i)} = \frac{1/i+2}{1/i+1} = \frac{i+1}{i+2} \quad \textbf{(5)}$$

Here, the Rule of Succession is justified, not by an arbitrary decision to privilege the classification of possibilities in terms of the possible *proportions* of A and B results over the classification in terms of the possible *sequences* of A and B results, but by the fact that the objective chances are explanatorily prior to the sequences, and thus that the Principle of Indifference must be applied at the level of objective chances. With the Rule of Succession, we have a reasonably strong form of inductivism: if we observe 98 A's in a row, we have a 99% probability that the next case will also be an A.

3.4 The Metaphysics of the Explanationist Defense: Causation and Laws

This defense of Laplace's probability function and the Rule of Succession is available only on certain metaphysical assumptions, which may explain why neither Hume nor Carnap followed this route. To employ the preceding defense, one must accept the existence of objective chances, and one must accept that objective chances are explanatorily prior to particular events. Under what metaphysical conditions would objective chances be explanatorily prior to particular events? Suppose that the objective chance of outcome A is determined by the laws of nature and general, standing background conditions. Suppose further that laws of nature are conceived as eternal or timeless facts that in some sense "govern" what happens in the world. Then the laws will be explanatorily prior to particular events. The standing background conditions as well will typically be explanatorily prior to the particular events whose objective chances we are concerned with, due to their temporal and causal priority. This makes it plausible that the resulting objective chances are explanatorily prior to particular events.

Consider an alternative metaphysical view. Suppose that, as Hume would have it, causation is nothing but constant conjunction, so that whether a type of event, A, causes a type of event B is determined by whether, in general, events of kind A are followed by events of kind B.[20] This view seems to entail a reversal of the explanatory priority relation that we normally take to obtain. On the common sense view of causation, one ought to say: Events of type A are generally followed by events of type B, because A causes B. On the Humean view, one ought to say: A causes B, because events of type A are generally followed by events of type B. I take it that the "because" in each case signals (at least) an explanatory relation. On the Humean view, we first have facts about what particular events occur at what times and places. Facts about causation then supervene on, and are nothing over and above, those particular facts. On this view, then, causal priority ought *not* to be taken as implying explanatory priority. To say that A's cause B's is just to say that A-type events are always followed by B-type events. The mere existence of such a contingent pattern in the phenomena is not explanatorily prior to the particular occurrences of B's.[21] Rather, just as facts about the parts of some whole are explanatorily prior to facts about the whole, the facts about which sorts of particular events occur at what times are explanatorily prior to propositions describing the patterns in the particular events.

A similar observation holds for a broadly Humean view about laws of nature. If laws of nature are taken as facts that in some sense govern, or determine, the way particular events unfold, then the laws of nature are explanatorily prior to particular occurrences. But if laws of nature are mere summaries of patterns in the particular events, as in David Lewis' view,[22] then the facts about particular events are explanatorily prior to the laws.

For this reason, given the Explanatory Priority Proviso, Humean views of causation and laws induce a different sort of probability distribution from non-Humean, "realist" views. On a Humean view, the appropriate application of the Principle of Indifference is to assign equal probabilities to the possible sequences of particular events, resulting in

[20] Hume 1975, p. 76.

[21] Compare Dretske's (1977, p. 262) argument that mere regularities do not explain their instances.

[22] Lewis 1994.

the inductive skeptic's probability distribution.[23] On a realist view, on the other hand, causal and nomological facts, including facts about objective chances, are explanatorily prior to facts about sequences of particular events, and the resulting interpretation of the Principle of Indifference, as we have seen, yields an inductivist probability distribution. This is one reason for preferring non-Humean theories of causation and laws, since they yield the intuitively correct sort of probability distributions.

3.5 Inference to the Best Explanation?

In what sense, if any, does the approach here advanced involve inference to the best explanation? Initially, it might appear that, while the approach makes use of the notion of explanatory priority, no actual *inference to the best explanation* is required to arrive at inductive conclusions. Rather, it appears that one arrives at an inductive conclusion by simply conditionalizing on some set of evidence, starting from an inductivist prior probability distribution. Considerations of explanatory priority feature in the motivation for that prior distribution, but even that does not obviously involve one in making an inference to the best explanation—at no point in the reasoning given in defense of the Laplacean distribution did we need to make the claim that some hypothesis was *the best explanation* for anything, as opposed to the mere claim that some hypotheses are *explanatorily prior* to others. And it seems that, once we have the appropriate prior distribution, at no later stage need we make the claim that some hypothesis is the best explanation for anything, either.

Though I am not greatly concerned with whether my approach involves genuine inference to the best explanation, it seems to me that it at least involves something very much like inference to the best explanation. On my approach, one begins by considering a set of alternatives that are explanatorily prior to the data and so are in that minimal

[23]This explains and vindicates the intuition, often pressed by advocates of inference to the best explanation (Dretske 1977, p. 267; Foster 1982-3, pp. 91-2; Armstrong 1983, pp. 52-9), that non-realist views about causation or laws engender inductive skepticism. Among such advocates, Tooley (1987, p. 135) comes closest to justifying the intuition in a probabilistic framework.

sense *potential* explanations of the data. Which of these alternatives has the greatest posterior probability will be determined by the initial plausibility of each alternative together with the degree to which it predicts the evidence, $P(e|h)$. The notion of an initially plausible, explanatorily prior hypothesis that confers a high probability on the evidence is at least something close to that of *a good explanation* of the evidence.

In our above example used to derive the Rule of Succession, the essential reason why A_{i+1} receives a high probability conditional on U_i (for large i) is that the truth of U_i confers a high posterior probability on hypotheses placing the objective chance of outcome A near the top of its range of possible values. The initially flat density distribution over C becomes skewed towards the top end. An explanationist might plausibly say: the *best explanation* for the evidence U_i is that c is close to 1. This is the best explanation, because this hypothesis (a) is explanatorily prior to the data, and (b) confers a much higher likelihood on the data than the alternative explanatorily prior hypotheses (such as that c is close to ½ or that c is close to 0). We infer that this hypothesis is probably correct—which is to say, we raise our degree of belief in it—whereupon we must also raise our degree of belief that outcome A will occur in the future. It seems to me that the inductive prediction is supported both by an inference to the best explanation and by good Bayesian reasoning.

4 Problems and Objections

The Explanationist-Bayesian approach raises a number of issues and problems that require further analysis. Here, I can offer only brief sketches of how a defender of the approach might seek to address just three of these problems.

4.1 Unknown Explanatory Possibilities

The Explanatory Priority Proviso calls for the Principle of Indifference to be applied to the alternatives at the most explanatorily basic level. But in some cases, we do not know what the most explanatorily basic level is. Indeed, sometimes empirical investigation reveals new explanatory possibilities of which we were previously unaware. This is particularly to be expected if, as I have suggested, both causal priority and the part-whole relation imply explanatory priority. Suppose, for example, that we seek to explain the behavior of some chemical substance. In the light of atomic theory, hypotheses about the properties and arrangement of the atoms of which that substance is composed are among the explanatorily prior alternatives. We would thus want to begin by assigning probabilities to those alternatives in a suitably neutral manner. Later investigation may reveal, however, that atoms are composed of subatomic particles. We would thus want to assign probabilities in a neutral manner to alternative hypotheses about the *subatomic* particles, rather than to the alternative hypotheses about atoms.

The case of unknown explanatory possibilities raises a number of issues. One issue is familiar to Bayesians for other reasons: the Explanatory Priority Proviso appears to impose an unrealistic demand on epistemic agents. Given that we are often unaware of the explanatorily most basic alternatives, we cannot follow the directive to assign equal probabilities to these alternatives. This is analogous to a problem sometimes raised for Bayesians: given that mortal humans are unable to identify all the necessary truths, it is unrealistic to require that a rational person assign probability 1 to every proposition that is *in fact* necessary.

Perhaps the most natural way to deal with the problem is to say that one rationally ought to apply the Principle of Indifference to the alternatives at the most explanatorily basic level *that one is aware of*. This naturally suggests the view that, when one learns of new potentially explanatory alternatives, one will need to revise one's degrees of belief by a process other than conditionalization, a process designed to adjust one's degrees of belief to what they would have been, had one known of the new potentially explanatory alternatives earlier and had one then assigned each of them equal probabilities. Though Bayesians may be uncomfortable here, presumably this is the same *sort* of response as one would want to make to the problem of unknown necessary truths: when one discovers a new necessary truth—say, by proving a new theorem—one should revise one's degrees of belief (leaving aside the issue of uncertainty as to the soundness of the proof) by assigning probability 1 to that newly discovered truth. This is not a process of conditionalization, but rather, one might say, a process of correcting for one's earlier cognitive limitation.

A second problem raised by the possibility of unknown explanatory alternatives is that of how one should deal with situations in which the existence of a certain explanatory level itself is in dispute. For instance, suppose that, atomic theory having already been accepted, we face a dispute over whether subatomic particles ought to be introduced into our theory of matter. Those who accept the existence of subatomic particles, it seems, will assign prior probabilities in one way, while those who remain with the older theory will assign probabilities in another way, and perhaps individuals with some entirely different theory will assign probabilities in yet a third way. How can we assign probabilities so as to respect the Explanatory Priority Proviso, without begging questions concerning what potentially explanatory entities exist?

In some cases, we may be able to resolve this sort of problem by moving to a more abstract level of description at which it will be possible to agree on what the potentially explanatory alternatives are. For instance, it might be argued that the question of *whether atoms have parts*—or more generally, what sort of thing is the most basic constituent of matter—is prior to that of what the characteristics of those parts might be. Thus, we might apply the Principle of Indifference first at the level of the competing theories as to what the most fundamental constituents of matter are (including the theory on which matter is infinitely divisible). Each of these theories may then specify what the most explanatorily fundamental alternatives are *given* the truth of the theory.

4.2 The Probability of Deterministic Laws

The Laplacean probability function recommended in section 3.3 lends support to Karl Popper's claim that the initial probability of any universal deterministic law is zero.[24] For

[24]Popper (1961, pp. 363-8) claims that the initial probability of any universal law applying to an infinite population is 0. Carnap (1980, p. 145) recognizes as a problem that his system of inductive logic generates this result for all values of λ.

the hypothesis of a deterministic law—say, a law requiring A always to result from process X—is equivalent to the hypothesis that the objective chance of A's resulting from X is 1. That is one possible value of the objective chance, out of a continuous infinity of possibilities, so the prior probability of the objective chance taking on *exactly* that value is zero. Now, Bayes' Theorem tells us, for any hypothesis *h* and evidence *e*:

$$P(h \mid e) = \frac{P(h) \times P(e \mid h)}{P(e)}$$

If P(*h*) = 0, then P(*h* | *e*) = 0, for any possible evidence *e* that itself has a non-zero prior. (And if P(*e*) = 0, then P(*h* | *e*) is undefined.) Therefore, a hypothesis with zero initial probability can never be confirmed. And so it seems that deterministic laws can never be confirmed. On this view, if A were observed to result from process X a large number of times with no exceptions, we would have evidence only for the claim that the objective chance of A is *very high*—for instance, we might confirm that it is greater than .999—but not that it is exactly 1.

This conclusion strikes me as incorrect. While I would not wish to entirely rule out the hypothesis that the objective chance of A is some very high number less than one, I think that with a sufficient number of positive instances of A, in a sufficiently wide variety of circumstances, with no known exceptions, scientists would reasonably conclude that a deterministic law was operating.

It therefore seems to me that the Laplacean probability distribution requires modification. In particular, it seems to me incorrect to equate a deterministic law with a law that sets the objective chance of some outcome of a process to either 1 or 0.[25] To see why, consider the following descriptions of two allegedly possible worlds:

In World 1, there is a deterministic law that process X must produce outcome A.

[25]I owe the following argument in the text to Michael Tooley (p.c.), though he may not endorse it.

> Process X occurs an infinite number of times in World 1, and on exactly five of those occasions, A fails to result.
>
> In World 2, there is a law that the probability of outcome A resulting from process X is 1. Process X occurs an infinite number of times in World 2, and on exactly five of those occasions, A fails to result.

World 1 is obviously logically impossible. World 2, however, is logically possible. In standard probability theory, that an event has probability zero is logically compatible with that event's actually occurring. (If an infinitely sharp dart is thrown so as to hit a random location on a dartboard, each geometric point on the board has zero probability of being hit, since it constitutes a measure-zero proportion of the board; yet it is guaranteed that some point will be hit.) Likewise, A's failing to occur in some circumstance is logically compatible with its having a probability 1 of occurring in that circumstance. Indeed, A's failure to occur on five out of infinitely many trials is not even *improbable* relative to the supposition that A has probability 1 of occurring on each occasion. After all, the relative frequency with which A fails to occur in World 2 matches the objective chance of A's failing to occur (assuming that we treat $5/\infty$ as 0), just as we would expect. It is thus difficult to see what objection one could have to the possibility of World 2.

Now, if World 1 is logically impossible while World 2 is logically possible, then a deterministic law to the effect that X must produce A must be something different from a law to the effect that X has probability 1 of producing A. So deterministic laws are something qualitatively distinct from probabilistic laws; they are not merely a special case of probabilistic laws. Once we recognize this, we need to modify our view of the space of possibilities in Example 1. Earlier, we held that the possibilities with regard to the laws governing process X can be adequately characterized by assigning a value between 0 and 1 inclusive to the objective chance of A's eventuating. But having recognized a qualitative distinction between deterministic and indeterministic laws, we should now reason as follows. There are two fundamental alternatives: either the outcome of X is causally determined, or it is undetermined. Each of these possibilities has a prior probability of ½. If determinism holds, then either A is necessary or A is impossible, so each of these alternatives has a prior probability of ¼. Finally, we assign a uniform probability density over all the possible indeterministic values of c, that is: $\rho(c) = ½$, for $0 \le c \le 1$.

This new proposed probability distribution is even more friendly to induction than Laplace's. In place of the Rule of Succession, it leads to the stronger inductivist conclusion:[26]

$$\mathrm{P}(A_{i+1} \mid U_i) = \frac{\mathrm{P}(U_{i+1})}{\mathrm{P}(U_i)} = \frac{i^2 + 5i + 4}{i^2 + 5i + 6}$$

And it allows the deterministic hypothesis that A is causally necessary to be confirmed, in accordance with the formula:[27]

$$\mathrm{P}(\text{A is necessary} \mid U_i) = \frac{i+1}{i+3}$$

For example, the observation of 97 A's in a row confers a 98% probability on the universal law that all outcomes are A. However, this probability distribution also gives the scarcely believable verdict that, after observing *just one* instance of A, one should have 83% confidence that the next iteration of X will produce outcome A as well.

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[26]$\mathrm{P}(U_i)$ here is calculated by the formula, $\mathrm{P}(U_i) = \mathrm{P}(U_i \mid \text{A is necessary})\cdot\mathrm{P}(\text{A is necessary}) + \mathrm{P}(U_i \mid \text{A is impossible})\cdot\mathrm{P}(\text{A is impossible}) + \int_0^1 \rho(c)\cdot \mathrm{P}(U_i \mid C = c)dc$, where P(A is necessary) = ¼, $\mathrm{P}(U_i \mid \text{A is necessary}) = 1$, P(A is impossible) = ¼, $\mathrm{P}(U_i \mid \text{A is impossible}) = 0$, $\rho(c) = ½$, and $\mathrm{P}(U_i \mid C{=}c) = c^i$.

[27]This formula is derived starting from the equation, $\mathrm{P}(\text{A is necessary} \mid U_i) = \frac{\mathrm{P}(\text{A is necessary}) \times \mathrm{P}(U_i \mid \text{A is necessary})}{\mathrm{P}(U_i)}$, where P(A is necessary) is ¼ and $\mathrm{P}(U_i \mid \text{A is necessary})$ is 1. $\mathrm{P}(U_i)$ is calculated as in the previous note.

We might try to correct the probability distribution in one or more ways. First, the supposition that, if determinism is true, then the objective chance of A is either 1 or 0 is mistaken: the truth of determinism means that *some* causally sufficient conditions, either for outcome A or for outcome B, are present in each iteration of process X. This does not entail that the conditions defining process X themselves contain such sufficient conditions; thus, the objective chance of outcome A, relative to the reference class of all instances of process X, may be non-extreme. (Analogously, though the outcomes of coin flips may well be determined in the actual world, it is not the case that the objective chance of a coin coming up heads on any flip is 1, nor is it 0; it is ½.) One would therefore need to assign probabilities, perhaps through another application of the Principle of Indifference, to the alternative hypotheses as to *which* possible conditions determine the outcome of X, and to the possible distributions of the potentially causally relevant initial conditions among instances of X, in order to determine the probability that $c = 1$ given determinism.

Second, we might think that, rather than just two fundamental alternatives—determinism and indeterminism—we should consider three fundamental alternatives: (*i*) the outcome of X is governed by deterministic laws, (*ii*) the outcome is governed by indeterministic laws, and (*iii*) the outcome is governed by no laws (which perhaps implies that the physical probability of A on each occasion is .5).

Each of these suggested corrections would preserve the possibility of confirmation for the universal deterministic law that all outcomes of X are A, while reducing the ease with which inductive conclusions are reached (that is, increasing the amount of evidence needed to reach a given level of probability, either for A_{i+1} or for the hypothesis that A is necessary).

4.3 Scruples Concerning A Priori Probability

Some philosophers would object to the notion of the *a priori* probability of a proposition that I have relied upon throughout. They would say that to assign probabilities to propositions, one must always have at least some empirical evidence. In the absence of any relevant evidence, one should say that the probability of a hypothesis is simply *unknown* or *indeterminate*.

It is important first to clarify this idea. If the sort of probability one has in mind is physical probability, then it is very plausible to claim that one must have some empirical evidence to assess the probability of an event or proposition. On the other hand, if one has in mind *epistemic* probability, it is not obvious why one's ignorance of empirical facts should in principle interfere with one's assignment of probabilities. For an epistemic probability assessment is simply an assessment of how much reason one currently has for believing some proposition. It is unclear why one's ability to say how much reason one presently has for believing something should require empirical information beyond the knowledge of what one's current mental state is. A shortage of empirical information may prevent one from describing the physical facts about some process—but it should not prevent one from describing the state of one's information itself. According to the motivation for the Principle of Indifference that I suggested in section 2.1, in assigning a uniform probability distribution over some set of alternatives, one is simply claiming that one has as much reason to believe any of the alternatives as to believe any of the others. That, of course, is compatible with the claim that one has exactly *no* reason to believe any of the alternatives. Indeed, the latter claim would seem if anything to *support* the former, rather than undermining it as the present objection supposes.

Those who press the present line of objection, then, either reject the notion of epistemic probability, or claim that, despite what I have said, one cannot assess how much reason one has for believing a proposition without specific empirical evidence bearing on that proposition.[28]

Now it seems to me that the latter position is more problematic than is commonly recognized. Consider two fundamental principles that are true of conditional probabilities on the standard conception. First: if, in the light of some evidence *e* (where *e* may include relevant background knowledge), *h* has a certain probability of being true, then that

[28]Achinstein (1995) defends the latter view.

probability is P(*h*|*e*). I have stated this principle in such a way as to leave open the possibility that *h* may fail to have to have any probability relative to some sets of evidence. Perhaps, as an empiricist would wish to maintain, when *e* contains no evidence relevant to *h*, then *h* has no probability relative to *e*. Nevertheless, most empiricists would accept that on some occasions, a proposition has a probability in the light of our evidence; for example, in the light of my current knowledge, the probability that the next coin I flip will come up heads is .5. I assume then that, where *e* includes all of my current actual evidence relevant to coin flips, P(the next coin I flip will come up heads|*e*) = .5.

Second, according to the axiom usually taken to define conditional probabilities, for any hypothesis *h* and evidence *e*, P(*h*|*e*) = P(*h*&*e*)/P(*e*). On the standard view of conditional probability, if P(*h*|*e*) exists, then the terms on the right hand side of that equation must have values. If either P(*h*&*e*) or P(*e*) is indeterminate, inscrutable, undefined, or the like, then it seems to follow that P(*h*|*e*) is also indeterminate, inscrutable, undefined, or the like.[29] This is true regardless of what *h* and *e* are. It is true even if *e* is the richest set of empirical information you care to consider. Thus, if it is in general impossible to assign the unconditional probability of any contingent proposition—perhaps because such probabilities are always indeterminate or inscrutable—then it seems that it is also impossible to assign the probability of any contingent proposition in the light of any evidence whatsoever. No amount of data collection, in other words, will help us overcome the initial inscrutability, because the assessment of that data—what it supports, and how strongly—is relative to an initial

[29]Hájek (2003) poses powerful objections to this standard view, which I lack space to discuss here. To give some of the flavor of the objections, let Q be a logically possible proposition with zero initial probability. Intuitively, it seems that P(Q|Q) = 1, even though P(Q&Q)/P(Q) = 0/0 is undefined.

Hájek takes conditional probability as primitive, arguing that conditional probabilities may exist when the relevant unconditional probabilities do not. Nonetheless, his view provides no way of determining the value of P(*h*|*e*) in the present context, so opponents of *a priori* probability should still regard P(*h*|*e*) as indeterminate or inscrutable.

probability distribution.[30]

I think this point is often overlooked because it is imagined that one can assign probabilities of events simply by looking at the observed relative frequencies in a large number of trials—we naively, and quite wrongly, are inclined to think that the need or relevance of a prior probability distribution will be overcome if we only take a sufficiently large sample.[31] One way to see the wrongness of this is simply to recall some of the prior probability distributions we have discussed above. The skeptical distribution, the Laplacean distribution, and the distributions discussed in section 4.2 above, all differ on what the probability of A_{i+1} is, and they continue to disagree no matter how much data comes in. That is, they differ on the value of $P(A_{i+1} \mid U_i)$ for every i.[32]

Now, if the opponent of *a priori* probabilities were to propose some specific rule for assigning probabilities in the light of a given amount of evidence, we could look at the results this rule delivers to see which (if any) prior probability distributions it agreed with. (If the rule does not agree with the outcome of conditionalization starting from *any* prior probability distribution, then the rule is probabilistically incoherent.) For instance, suppose one proposes that, for all $i \leq 1000$, $P(A_{i+1} \mid U_i)$ is indeterminate, but thereafter, $P(A_{i+1} \mid U_i) = (i+1)/(i+2)$. Then one is agreeing with the results of the Laplacean

[30]This argument applies to the claim that initial probabilities are *entirely* indeterminate. If one accepts that the initial probabilities of propositions are confined to non-extreme ranges, though perhaps lacking perfectly precise values, and one posits certain nice properties of acceptable probability distributions, then one will typically find posterior probabilities confined to narrower ranges (as the convergence theorems of Savage and Hawthorne show). But I take it that principled opponents of *a priori* probability would see no reason to accept these assumptions.

[31]One version of this mistake is to endorse "the straight rule," the idea that the probability of a given outcome's occurring on the next of a series of trials should be reckoned equal to the relative frequency with which that outcome has occurred in the previous trials. See Carnap (1980, pp. 85-6) for brief but effective criticisms of this rule.

[32]The convergence theorems often invoked by Bayesians (Savage 1954, pp. 46-50; Hawthorne 1993; 1994) require ruling out at the start some coherent probability distributions, including the skeptical distribution. In addition, they fail to guarantee convergence for any given, finite amount of data. For any finite set of data, and any desired degree of *divergence*, there exist prior probability distributions, satisfying the stipulations of the convergence theorems, such that the desired amount of divergence will exist after conditionalizing on the given data, starting from those prior probabilities.

distribution, though restricting one's agreement to cases where $i > 1000$. It is hard to see how one could have a coherent rationale for such a view. For if one initially objects to the alleged arbitrariness of selecting Laplace's distribution *a priori* over other equally coherent probability distributions, then shouldn't one equally object to the arbitrariness of selecting the Rule of Succession for cases where $i > 1000$ over other, equally coherent rules, such as those that would agree with the results of some of the alternative, equally coherent prior probability distributions just mentioned? The same objection would apply whatever rule one adopts—any scruples about privileging one prior probability distribution over other, equally coherent distributions ought to apply equally to privileging one rule for assigning posterior probabilities over other, equally coherent rules. Any rationale for considering one rule to be uniquely correct will likely constitute a rationale for considering the prior probability distributions that agree with that rule to be objectively preferable to all other prior probability distributions.

The initial motivation for the scruples about *a priori* probability is clear enough: devising a comprehensive and precise set of general principles delimiting *a priori* probabilities has proven extremely difficult, so much so that one might be forgiven for doubting that such principles exist. However, if we accede to this doubt, we must also surrender to skepticism: if there are no *a priori* epistemic probabilities, then there are no epistemic probabilities whatsoever. One therefore could not say—no matter what evidence one had—that a scientific theory, or any other inductive conclusion, was likely to be true.

I have offered a first step towards a solution of this problem—a step, that is, towards articulating the principles governing *a priori* objective probabilities. In brief, in the absence of evidence favoring one alternative over another, one should assign equal epistemic probabilities to the explanatorily basic alternatives. A great deal of additional work needs to be done, both in attempting to apply this principle to problem cases, and in seeking additional principles governing *a priori* probabilities. Nevertheless, the approach can be shown in simple examples to yield inductivist probability distributions and to rule out inductive skepticism. This suggests that a hybrid Explanationist-Bayesian treatment along these general lines may offer the most promising avenue of attack on the problem of induction.[33]

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