Physics 4430 and 5430 **John Price**

Acoustic Reflectometer Experiment

In this experiment you can explore some elementary and some not-so-elementary aspects of sound propagation in a cylindrical tube. The basic concepts include standing versus running waves, wave impedance, termination impedance, waves represented by complex amplitudes, and complex reflection coefficient. These wave concepts have applications to nearly all areas of science and technology, including electromagnetic waves and many problems in quantum mechanics.

The experiment is based on the AcousticVNA system, which is basically a loudspeaker connected to a tube with a set of microphones that measure the acoustic pressure along the tube. If the speaker is driven sinusoidally at a frequency below about 8 kHz, only one mode can propagate in the tube, so the sound field is completely described by the amplitude and phase of one right-going and one left-going plane wave traveling along the tube. The microphones, together with a data acquisition system and some MATLAB software, can measure the right- and left-going wave amplitudes and phases versus the excitation frequency. At the undriven end of the tube you can attach some object (a horn, an absorber, a resonant cavity, another transmission line…) and measure the reflection of sound off the object. If the object responds linearly, its behavior is completely described by the reflection coefficient, the (complex) ratio of the reflected to incident wave amplitudes. The software can make a plot of the reflection coefficient versus frequency, among other things. The hardware and software are documented in the AVNA User's Guide. A second document called Acoustic Waveguides explains the basic theory of sound propagation in tubes.

You will notice that this experiment runs under MATLAB, while most of the others use LabView and some use Mathematica for theory or analysis. It is valuable to get exposure to each of these systems. MATLAB combines fast and flexible linear algebra and signal processing with data acquisition, and it is widely used in industry. Python + SciPy is (roughly) an open-source substitute for MATLAB.

To begin, go through Chapters 1, 2 and 3 of the User's Guide. You will be able to skip most of 2.1 because the software is already installed. Some things will look slightly different than in the User's Guide because we have newer versions of Windows and MATLAB and also the hardware is a newer version than is shown in the photos. None of this should cause much difficulty. Be sure to linger a bit over the part of Section 2.2 that tells you how to get familiar with MATLAB, and set up your user folder as instructed so that your code and data won't interfere with other users. (We are using a 32-bit version of MATLAB. This was required in the past for compatibility with the pa_wavplay ASIO sound drivers, but now there is a version of pa_wavplay that is supposed to be 64-bit compatible. We have not tested it yet.)

Before doing the introductory experiments in Chapter 4 of the User's Guide, it is important to get familiar with some theory. Do the following exercises:

Exercise 1, Helmholtz equation:

Read Ch. 1 and Ch. 2 of Acoustic Waveguides. Eqn 2.6 is the wave equation for the acoustic pressure, and Eqn. 2.4 tells us how to get the acoustic velocity field once we know the acoustic pressure. Suppose that the acoustic pressure and velocity depend on time as $\exp(+i\omega t)$, as shown below Eqn. 2.12. Derive Eqn. 2.12, the Helmholtz equation. (Note that in quantum mechanics and some other areas we use $\exp(-i\omega t)$ time dependence. The plus sign is always used in electronics, so if we are going to speak about impedances and such things it is better to use plus.)

Exercise 2, complex waves:

a) Let $p = A\exp(-ikx)$, with *A* a complex number and $\exp(+i\omega t)$ time dependence assumed. (This means we multiply *p* by $\exp(+i\omega t)$ and take the real part to find the physical pressure.) Is this wave a running wave or a standing wave? Does it travel to the left or to the right?

b) Answer the same questions for $p = A\exp(+ikx)$.

c) Answer the same questions for $p = A(\exp(-ikx) + \exp(+ikx))$.

d) Suppose $A = R\exp(-i\theta)$ and $p = A\exp(+ikx)$. (R and θ are real.) Write the physical time-dependent pressure in terms of trig functions.

Exercise 3, characteristic impedance:

The waves in the lowest mode of an acoustic waveguide with uniform cross-section are just plane waves traveling along the axis of the waveguide. Suppose $p = A\exp(-ikx)$.

a) Use Eqn. 2.12 to find the velocity field $\vec{u} = u\hat{x}$. Notice that the pressure and the velocity are in phase in this case. This will not be true for more general waves.

b) In waveguides we generally use the volume velocity $U = uS$ as our variable rather than u (where S is the cross-section area of the guide). This is similar to using the current rather than the current density in an electrical circuit, and is more convenient because, according to the Kirchhoff law, it is conserved at a junction. For the case $p = A\exp(-ikx)$, find p/U and show that it is equal to $\rho v_s/S$. This quantity is called Z_0, the characteristic impedance of the waveguide. (If we were doing electronics instead of acoustics it would be the ratio of a voltage to a current.) The pressure to volume velocity ratio is independent of position and equal to the characteristic impedance *only* for this case, i.e. when there is only a right-going wave on the line. If there is only a left-going wave it has the opposite sign. In the general case, the ratio will be complex and will depend on position.

Exercise 4, reflection coefficient:

a) Look at Eqn. 3.11, the pressure for a general wave at frequency ω, a superposition of right-going and left-going parts. Show that the expression for the volume velocity in Eqn. 3.11 is correct.

b) Eqn. 3.12 is the definition of the reflection coefficient *S*. (We are using *S* also for the area of the waveguide. Sorry about that.) Note that the reflection coefficient depends on position! The position where it is evaluated is called the reference plane. Figure 3.5 shows an arbitrary impedance *Z* terminating the line at the position *l*. Derive Eqn. 3.13, the reflection coefficient with the reference plane at the position of the termination. This is the most important formula for reflectometry.

c) Suppose $Z = \infty$. This is called an open circuit in electronics because current cannot flow. In acoustics, it corresponds to a *closed* end, since flow is blocked and the volume velocity is zero. What is the reflection coefficient at the termination?

d) Suppose $Z = 0$. This corresponds to a short in electronics and an open pipe in acoustics. What is the reflection coefficient at the termination?

e) Derive Eqn. 3.14, which gives the reflection coefficient with the reference plane at any location on the line. Notice that the reflection coefficient goes around the complex plane in the clockwise direction as the reference plane moves away from the termination.

f) For what termination impedance *Z* is the reflection coefficient zero at the termination? In this case, what is the reflection coefficient at an arbitrary reference plane?

Now that you have learned about the theory of acoustic reflectometry, return to the User's Guide and do the four introductory experiments in Chapter 4. With perfect microphone calibration, perfect stability and an ideal closed end, you would get a reflection coefficient at the position of the closed end of exactly +1. Fig. 4.2 shows actual results that were obtained right after calibration with almost new microphone elements. You might find that your data is spread over a somewhat larger region around +1.

Talk to your instructor about what to do next. See the suggestions in Chapter 5 of the User's Guide. Note that the S-matrix (two-port) features mentioned in the User's Guide have not been implemented yet, so you should probably stick to reflection measurements.