Abstract Algebra 1 (MATH 3140)

Worksheet 2: Groups, Subgroups, Cyclic Groups

1. Let $G = \langle a \rangle$ be a cyclic group of order 45.

(a) Draw the inclusion diagram for all subgroups of G.

(b) Which of the subgroups in the diagram is equal to

- $\langle a^7 \rangle$;
- $\langle a^{27} \rangle$?

2. Let G be an abelian group. Show that the set $H = \{g^2 : g \in G\}$ of the squares of all elements of G is a subgroup of G.

3. Let $k, n \in \mathbb{N}$, $k, n \geq 2$. Show that
(a) every 6-cycle in S_n has order 6 .
(b) every k-cycle in S_n has order k.

4. Find all permutations σ in S_5 such that $\sigma^2 = (1\ 2\ 3)$.