Philosophy 5340 – Epistemology

Topic 4: Skepticism

Part 4: Can Skepticism Be Refuted?

1. Overview of a Possible Refutation of Skepticism

In my view, there is only one way of attempting to refute skepticism that is promising. The basic idea is to defend a theory of logical probability, and then to use that theory to compare the probability that the non-skeptical hypothesis in question has relative to the relevant evidence to the probability that the disjunction of the competing skeptical hypotheses has relative to that evidence.

1. Probability

1.1 Different Concepts of Probability

What is probability? There are different conceptions of probability, among which the following five are the more important.

Subjective Probability

The subjective probability of a proposition *p* for a person *S* is the degree to which *S* assents to *p*. One can think of it as being more or less defined by the choices that *S* would make over lotteries. Thus, suppose that if *S* is offered a free ticket in a lottery with 100 tickets, numbered from 1 to 100, *S* does not care which ticket he or she is given. Then *S* is assigning the same subjective probability to the following 100 propositions:

T_1 = Ticket 1 will win.

T_2 = Ticket 2 will win.

T_3 = Ticket 3 will win.

………………………..

T_{100} = Ticket 100 will win.

Now suppose that *S*, who has no tickets in the lottery, prefers getting a prize if any one of tickets 1 through 25 is the winning ticket to getting the same prize if proposition *p* is true. Then the subjective probability that *S* assigns to *p*'s being true is less than 25/100. If, on the other hand, *S* prefers getting a prize if proposition *p* is true to getting the same prize if one of tickets 1 through 24 is the winning ticket, then the subjective probability that *S* assigns to *p*'s being true is greater than 24/100. These two preferences together would then mean that the subjective probability that *S* assigns to proposition *p*'s being true is greater than 0.24 and less than 0.25.

The Relative Frequency Conception of Probability

A second concept of probability is that of relative frequencies. Suppose that a coin has been flipped 100 times, and has come down heads 47 times. Then the probability of the coin's coming down heads, according to the relative frequency conception of probability, is 0.47.

The Relative Frequency in the Limit Conception of Probability

The idea of defining a concept of probability in terms of relative frequencies has certain unappealing consequences. What is the probability that a certain coin will, if flipped, come down heads? If the coin is only going to be flipped once in its lifetime, the probability has to be either 1 or 0, on the relative frequency conception. If it is going to be flipped three times in its lifetime, the probability has to be either 1, or 2/3, or 1/3, or 0, on the relative frequency conception. It does not really seem that this idea of probability as relative frequency is capturing some property of the coin. As a result, some people who are attracted to the empirically-based nature of relative frequencies shift to a conception of probability that equates the probability that a coin will come down heads, not with the relative frequency, but, instead, with the limit of the relative frequency with which the coin would come down heads if it were flipped an infinite number of times. But then the question arises as to what the truthmaker is for the subjunctive conditional statement about what **would** happen if the coin **were** flipped an infinite number of times.

Propensities and Objective Probabilities

A very different conception of probabilities, but one that is also an empirically based one, is that of propensities. Consider, for example, the isotope of uranium that has an atomic weight of 238. Uranium-238 has a half-life of about 4.47 billion years, so that for any given atom of uranium, the probability, on the propensity interpretation of probability, that that atom will undergo radioactive decay in about 4.47 billion years is equal to 0.5

Some philosophers think that propensities are fundamental properties that are irreducible to anything else. For reasons that I shall not go into here, that conception seems problematic, and I think it is much more plausible to view propensities as properties that logical supervene on categorical properties plus probabilistic laws of nature.

Logical Probabilities

Logical probabilities, unlike subjective probabilities, have nothing to do with subjective states of a person, and, unlike relative frequencies and propensities, logical probabilities are not based upon contingent facts about the world. The logical probability associated with a proposition is a necessary property of the proposition.

Logical probability will be discussed in more detail in the next section.

Some Suggestions for Further Readings on Probability

If you are interested in reading a little more about different conceptions of probability, a very comprehensive survey article is

"Probability" by Max Black, in *The Encyclopedia of Philosophy*, edited by Paul Edwards (New York: Macmillan, 1967), Volume 6, pages 464-79.

For a much fuller discussion, one book that can be recommended as worthwhile is

Ian Hacking, *An Introduction to Probability and Inductive Logic* (Cambridge: Cambridge University Press, 2001).

Finally, for a very good discussion of some different conceptions of probability, along with a defense of the idea of logical probability, see

Rudolf Carnap, Chapters 1 and 2 of *The Logical Foundations of Probability*, Second edition (Chicago: The University of Chicago Press, 1962), pages 1-51.

1.2 Logical Probability

What is logical probability? The basic idea is that for any two propositions, p and q, there is some number k that presents the likelihood that q is true given evidence that consists only of proposition p. Let us use "$\Pr(q/p) = k$" to say that the logical probability of q given p is equal to k.

Logical probability, if it exists, is a relation simply between two propositions – akin to the relation of logical entailment.

Moreover, because it is a relation simply between two propositions, it is **necessary** relation.

In the case where p logically entails q, $\Pr(q/p) = 1$. In the case where p logically entails the negation of q, $\Pr(q/p) = 0$.

In all other cases, the value of k is equal to or greater than 0, and equal to or less than 1. (If k can take on infinitesimal values, then there is reason to require that $\Pr(q/p) = 1$ only if p entails q, and that $\Pr(q/p) = 0$ only if p entails the negation of q.)

If p is a logically necessary truth, then "$\Pr(q/p)$" represents, in effect, the *a priori* logical probability of q – the probability that q is true given no evidence at all. This can be written as "$\Pr(q)$".

Given the *a priori* logical probability that a proposition is true – $\Pr(q)$ – the logical probability of one proposition relative to another can be defined via the standard definition of conditional probability, which is as follows:

$$\Pr(p/q) =_{\text{def}} \frac{\Pr(p \,\&\, q)}{\Pr(q)}, \text{ provided that } \Pr(q) \neq 0.$$

So if one can specify the *a priori* logical probabilities of every proposition – where this is the probability that a proposition is true given no evidence at all – then the logical probability of any proposition p relative to any proposition q whose *a priori* probability is not equal to 0 is automatically defined.

But how is the *a priori* logical probability that a proposition is true to be defined? This is a crucial, and very difficult question. One natural idea is that the *a priori* logical probability that p is true is equal to **the proportion of the totality of the logically (or analytically or metaphysically) possible worlds in which p is true**. But there are various difficulties that stand in the way of this answer. The most evident, perhaps, is that there are presumably an infinite number of possible worlds. So what sense can one make, for example, of the claim that p is true in, say, 75% of those worlds? Another, less familiar and less obvious problem is that Carnap set out an argument that appears to show that given this possible-worlds (or state-descriptions) conception of logical probability, it follows that one cannot learn from experience. So if, for example, one has drawn one thousand marbles from an urn, all of which were red, the probability that the one thousand and first marble drawn from the urn will be red will be precisely what it was before any marbles were drawn from the urn.

1.3 Some Axioms, Definitions, and Theorems of Logical Probability

Here are some axioms that logical probability must satisfy:

(1) If p and q are logically equivalent, then $\Pr(p) = \Pr(q)$.

(2) For any p, $0 \leq \Pr(p) \leq 1$.

(3) If p is necessarily true, then $\Pr(p) = 1$.

(4) If p and q are mutually exclusive (so that $p \Rightarrow$ not q and $q \Rightarrow$ not p), then

$\Pr(p \text{ or } q) = \Pr(p) + \Pr(q)$

Given axioms (1) and (4), one can then prove:

(5) Regardless of the relation between p and q,

$\Pr(p \text{ or } q) = \Pr(p) + \Pr(q) - \Pr(p \,\&\, q)$.

Next, conditional probability can be defined as follows:

(6) $\Pr(q/p) = \dfrac{\Pr(p \,\&\, q)}{\Pr(p)}$, provide $\Pr(p) \neq 0$.

Given (6), the following Multiplication Rule for logical probabilities then follows immediately:

(7) $\Pr(p \,\&\, q) = \Pr(q/p) \times \Pr(p)$, provided that $\Pr(p) \neq 0$.

Finally, the following Rule of Total Probabilities follows from axiom (1) together with the definition of conditional probabilities, via the Multiplication Rule:

(8) $\Pr(p) = \Pr(q/p) \times \Pr(p) + \Pr(q/\text{not } p) \times \Pr(\text{not } p)$, provided that $0 < \Pr(p) < 1$.

2. Skepticism concerning an External, Mind-Independent World

In the case of skepticism concerning the existence and nature of an external, mind-independent, spatial, physical world, a natural idea that many philosophers have had is that belief in the existence of such an external world can be justified by abduction (inference to the best explanation, hypothetico-deductive method, the method of hypothesis) – that is, by the method that is used to justify scientific theories. If one thinks of things in that way, there are two basic steps that must be carried out if skepticism is to be defeated:

Step 1: Justifying the Method

The first step involves showing that abduction (or inference to the best explanation, hypothetico-deductive method, etc.) is a legitimate form of non-deductive inference.

Step 2: Showing that that Method Justifies the Non-Skeptical Hypothesis

The second step involves showing that when that method is applied to the relevant evidence – which, in the present case, will be given by propositions about one's own present sensory experiences and memory-beliefs – the result is that a given non-skeptical hypothesis turns out to have a higher probability (or, as some would prefer to say, is a better explanation of the evidence) than any competing skeptical hypothesis (or, as I think is needed, than the **disjunction** of **all** relevant skeptical hypotheses).

2.1 Step 1: Justifying Abduction

Some philosophers hold, it seems, that the principle of abductive inference (or inference to the best explanation, hypothetico-deductive method, etc.) is a basic truth for which no justification can be offered. This view is rejected by Bas van Fraassen – and correctly in my opinion.

One way of thinking about this issue involves noticing that when philosophers talk about inference to the best explanation, the explanation in question is typically – though not always – a causal explanation. **Deductive** logical principles, however, are **content neutral**: they do not contain any descriptive terms. The question arises, then, of what account is to be given of the concept of explanation that is involved in inference to the best explanation. In particular, does it involve the idea of causation, or can it be cashed out in purely logical terms? No one, I think, has succeeded in doing the latter, and this suggests that the concept of causation is needed. But why should such a concept figure in an inductive principle? Shouldn't inductive principles, like deductive principles, be free of such descriptive terms?

My view is that abduction cannot be taken as a primitive form of non-deductive inference for which no proof can be offered. But what, then, could such a principle of induction be derived from? My suggestion is that it can be derived from a combination of two things:

(1) An analysis of causation – and, in particular, an analysis that connects up causation with logical probability;

(2) A theory of logical probability.

So far, however, no one has carried out such a derivation. So the task of justifying the inductive method that is needed to answer skepticism must be set aside as a task for the future.

2.2 Step 2: Showing that the Method Justifies the Non-Skeptical Hypothesis

If I'm right in thinking that the method of abduction needs to be justified, and that this is to be done by appealing, along with an analysis of causation, to a theory of logical probability, then what the method of abduction will generate, presumably, are the probabilities that different hypotheses have upon the relevant evidence. What needs to be shown, then, is that application of that method leads to the conclusion that the hypothesis that there is a mind-independent, spatial, physical world has an *a posteriori* probability relative to the propositions describing one's present experiential states and memory-beliefs that is greater than the *a posteriori* probability that any competing skeptical hypothesis has relative to that same body of evidence.

But that is not sufficient. The non-skeptical hypothesis must also be more likely to be true than to be false, and so the probability of the non-skeptical hypothesis must be greater than the probability of the disjunction of **all** of the competing skeptical hypotheses.

Nor does even that seem sufficient. For if the hypothesis that there is a mind-independent, spatial, physical world has an *a posteriori* probability relative to the propositions describing one's present experiential states and memory-beliefs that is only slightly greater than one half, one is still in an epistemologically unsatisfactory state – something that is especially clear when one notes that the likelihood that there are other minds is closely tied to the likelihood that there is a mind-independent, spatial, physical world. So it would seem that a satisfactory response to skepticism concerning the existence of a mind-independent, spatial, physical world requires that the *a posteriori* probability of there being such a world is quite close to one.

3. The Relation between *A Priori* Logical Probabilities and *A Posteriori* Logical Probabilities in this Case

It is often the case that the *a priori* probability of some proposition p is greater than the *a priori* probability of some other proposition q, but that things are switched when one considers the *a posteriori* probabilities relative to some evidence e: the *a posteriori* probability of p relative to e may be less than the *a posteriori* probability of q relative to e. But the situation is different when one considers a non-skeptical hypothesis and a corresponding skeptical hypothesis. There the order in the case of the *a posteriori* probabilities must be the same as in the case of the *a priori* probabilities.

3.1 The Argument

The relevant argument was set out earlier, in the second set of seminar notes on skepticism. It involves considering the relevant non-skeptical hypothesis, and a corresponding skeptical hypothesis. As before, let us take these to be

p = There is a certain sort of world of mind-independent objects.

m = Berkeley's view of the world is correct, but all of one's experiences and memory-beliefs are as they would be if proposition p were true.

Propositions p and m are to be understood in such a way that, first of all, they **are full descriptions** of the world as it would be under the relevant hypothesis, and, secondly, they are **experientially (and memory-belief) equivalent**, at least up until the point where one dies. If then we introduce the following proposition

e = The conjunction of all of the propositions about the sensory experiences and memory-beliefs that one has at the present time.

that proposition will be logically entailed by p and also by m, since those propositions are experientially equivalent, and they fully describe the world as it is under the respective hypotheses.

As in section 1 above, let us use the following abbreviations:

$\Pr(q)$ = the *a priori* logical probability that q is the case.

$\Pr(q/r)$ = the *a posteriori* logical probability that q is the case given that r is the case.

The argument in question is then as follows:

By the definition of conditional probability, given in section 1.3, one has the following four equations:

1. $\Pr(p/e) = \dfrac{\Pr(p \,\&\, e)}{\Pr(e)}$ and $\Pr(e/p) = \dfrac{\Pr(p \,\&\, e)}{\Pr(p)}$

2. $\Pr(m/e) = \dfrac{\Pr(m \,\&\, e)}{\Pr(e)}$ and $\Pr(e/m) = \dfrac{\Pr(m \,\&\, e)}{\Pr(m)}$

Multiplying through by the denominators of the fractions in these four equations – that is, by Pr(e), Pr(p), Pr(e), and Pr(m), respectively – then gives us:

3. $\Pr(p/e) \text{ x } \Pr(e) = \Pr(p \,\&\, e) = \Pr(e/p) \text{ x } \Pr(p)$

and

4. $\Pr(m/e) \text{ x } \Pr(e) = \Pr(m \,\&\, e) = \Pr(e/m) \text{ x } \Pr(m)$

But in view of the way that "e" is defined, one has the following two entailments:

5. $p \Rightarrow e$

6. $m \Rightarrow e$

These two entailments then entail, respectively, that

7. $\Pr(e/p) = 1$

and

8. $\Pr(e/m) = 1$

Substituting 5 and 6 into 3 and 4 then gives us, respectively:

9. $\Pr(p/e) \text{ x } \Pr(e) = \Pr(p)$

and

10. $\Pr(m/e) \text{ x } \Pr(e) = \Pr(m)$

Dividing equation 9 by equation 10 then yields

11. $\dfrac{\Pr(p/e)}{\Pr(m/e)} = \dfrac{\Pr(p)}{\Pr(m)}$

3.2 The Upshot

The conclusion of the above argument is that the ratio of the *a posteriori* logical probabilities is exactly equal to the ratio of the *a priori* logical probabilities. Among the important consequences of this are the following:

(1) It is logically impossible to establish that the *a posteriori* logical probability of the non-skeptical hypothesis is greater than the *a posteriori* logical probability of the skeptical hypothesis unless the *a priori* logical probability of the non-skeptical hypothesis is greater than the *a priori* logical probability of the skeptical hypothesis.

(2) It is logically impossible to establish that the *a posteriori* logical probability of the non-skeptical hypothesis is **much** greater than the *a posteriori* logical probability of the skeptical hypothesis unless the *a priori* logical probability of the non-skeptical hypothesis is **much** greater than the *a priori* logical probability of the skeptical hypothesis.

(3) On the positive side, if one can establish that the *a priori* logical probability of the non-skeptical hypothesis is (much) greater than the *a priori* logical probability of the skeptical hypothesis that then entails that the *a posteriori* logical probability of the non-skeptical hypothesis is (much) greater than the *a posteriori* logical probability of the skeptical hypothesis.

4. One Key Idea: The Probabilities of Laws of Nature Versus the Probabilities of Accidental Generalizations

4.1 The Probability that a Generalization Is True

But **how** can one establish that the *a priori* logical probability of the non-skeptical hypothesis is (much) greater than the *a priori* logical probability of any of the competing skeptical hypotheses? To begin to answer this question, we need to turn to the epistemology and metaphysics of laws of nature.

Suppose that four marbles are drawn from an urn, and that all four are a certain shade of red. Relative to that proposition, what is the probability that the fifth marble drawn from the urn will also be that same shade of red?

According to Rudolf Carnap's theory of logical probability, the answer depends upon the size of the family of mutually exclusive (and jointly exhaustive) color properties to which the property of being that shade of red belongs. Suppose, for simplicity, that there are only two color properties. Then the probability that the fifth marble drawn from the urn will be the same color as the first four = $\frac{4+1}{4+2} = \frac{5}{6}$.

What if there are k color properties? Then, according to Carnap's theory of logical probability, the probability that the fifth marble drawn from the urn will be the same color as the first four = $\frac{4+1}{4+k} = \frac{5}{4+k}$. More generally, if n marbles have been drawn from an urn, and all of them have been a certain shade, the probability that the (n + 1)th marble drawn from the urn will be the same shade = $\frac{n+1}{n+k}$.

For simplicity, however, let us consider the case where the family of mutually incompatible properties has only two members. Then if four marbles have been drawn from the urn that are all the same color, the probability that the fifth will be the same color is, as noted above, $\frac{5}{6}$.

Next, let us consider what the probability is, if four marbles have been drawn from the urn, all of which are the same color, that both the fifth and the sixth marbles will be that color. The answer is that for that to be so, the fifth marble must match the color of the first four – the probability of which is $\frac{5}{6}$ – and then, if that happens, the color of the sixth marble must match the color of the first five. What is the probability of that? The answer is gotten by putting n = 5 and k = 2 in the general formula $\frac{n+1}{n+k}$, so that the probability is equal to $\frac{6}{7}$.

For both the fifth and the sixth marble to be the same color as the first four, two things have to happen, the first of which has a probability of $\frac{5}{6}$, and the second of which has a probability of $\frac{6}{7}$. To get the probability that both of these things will happen, one has to multiply the two probabilities. So if four marbles have been drawn from the urn, all of which are the same color, the probability that both the fifth and the sixth will be that color = $\frac{5}{6} \times \frac{6}{7}$.

In similar fashion, one can show that if four marbles have been drawn from the urn all of which are the same color, the probability that the fifth, the sixth, and the seventh marble will all be that color = $\frac{5}{6} \times \frac{6}{7} \times \frac{7}{8}$.

More generally, if four marbles have been drawn from the urn, all of which have the same color property, the probability that the next *m* marbles will be that color = $\frac{5}{6} \times \frac{6}{7} \times \frac{7}{8} \times \frac{8}{9} \times \frac{9}{10} \times \ldots \times \frac{4+m-1}{4+m} \times \frac{4+m}{4+m+1}$.

Looking at this formula, one can see that the denominator of any given fraction is equal to the numerator of the next fraction. This means that one can use cancellation to write the above product of fractions in a much simpler form. Thus we have that if four marbles have been drawn from the urn, all of which are the same color, the probability that the next *m* marbles will be that color = $\frac{5}{4+m+1} = \frac{5}{5+m}$.

Next, what is the situation as the number of marbles in the urn is larger and larger? In particular, what is the probability that **all** of the marbles in the urn are the same color as the first four? The answer is that since the probability = $\frac{5}{5+m}$, the probability will not only be smaller and smaller as *m* is larger and larger, but if the number of marbles in the urn is infinite, then, as *m* gets larger and larger ,the probability $\frac{5}{5+m}$ will approach 0 in the limit.

I have been focusing on the case where one is dealing with a family of properties that contains only two members. How would the argument be affected if there were more properties in the family?

If the family of properties contains three members – say, red, green, and blue – and four marbles have been drawn from the urn, all of which have been red, then the probability that the next marble will be red will be equal to $\frac{5}{7}$ rather than to $\frac{5}{6}$, and the probability that the next three marbles will all be red will be equal to $\frac{5}{7} \times \frac{6}{8} \times \frac{7}{9}$ rather than to $\frac{5}{6} \times \frac{6}{7} \times \frac{7}{8}$. What one can see is that cancellation still takes place, but now it occurs between the denominator of one fraction and the numerator, not of the next fraction, but of the one after that. What that means is that the probability that the next m marbles will all be red, rather than being equal to $\frac{5}{5+m}$ is equal to $\frac{5\times 6}{(4+m)(5+m)}$. But, once again, the limit of this fraction as m goes to infinity is still 0, so the argument is unaffected, and the same is true regardless of the number of properties in the relevant family of properties.

Conclusion

If there are an infinite number of marbles in the urn, and if four marbles have been drawn from the urn, all of which are the same color, then the probability that **all** of the marbles will be that color is **infinitesimally close to zero**.

Four marbles is, of course, a very small sample. Suppose, then that a trillion marbles have been drawn from the urn, all of which are the same color. Then, in the case where the family of properties contains only two members, the probability that the next m marbles are all that same color $= \frac{1{,}000{,}000{,}000{,}001}{1{,}000{,}000{,}000{,}001+m}$.

But if there are an infinite number of marbles in the urn, then the probability that **all** of them are that same color will still be infinitesimally close to 0, since $\frac{1{,}000{,}000{,}000{,}001}{1{,}000{,}000{,}000{,}001+m}$ approaches 0 as m tends to infinity.

4.2 The Problem of Justifying the Belief that a Law Obtains

This result looks disturbing. For consider a universe with non-probabilistic laws – for example, a Newtonian universe. Newton's Laws of Motion and the Newtonian Law of Gravitation will have an infinite number of instances if time never ends. They will also have an infinite number of instances if time ends, but the temporal series is dense – that is, there is a temporal instant between any two temporal instants. So how, in a Newtonian universe, could one ever be justified in believing that the Newtonian laws had no exceptions, and so were true?

If, as many philosophers following Hume maintain, laws of nature are merely certain **cosmic regularities**, then the answer is that, in view of the above result, whenever one was justified in believing that there would be an infinite number of instances falling under the cosmic generalization in question, one would **never** be justified in believing that the probability that the generalization in question was true was more than infinitesimally greater than zero, and so one would never be justified in believing that the exceptionless regularity in question obtained.

There are, however, other views of laws. According to one, which I favor, laws, rather than being regularities, are second order relations between the relevant properties. Suppose, for example, that it is a law that all *F*s and *G*s. Then according to the view in question, what makes this a law is that the property of *F*-ness and the property of *G*-ness stand in a certain relation – call it the relation of nomic necessitation. If this relation obtains, then it entails that all *F*s are *G*s, so that the cosmic regularity obtains. But the cosmic regularity is **not itself** the law.

How does this help? The answer is that the state of affairs that consists of the property of *F*-ness standing in the relation of nomic necessitation to the property of

G-ness is an **atomic** state of affairs – in sharp contrast to the regularity, which consists of an infinite number of states of affairs of the form '*a* has the property of *F*-ness and also the property of *G*-ness'. It is then possible to argue that the atomic state of affairs that consists of the property of *F*-ness standing in the relation of nomic necessitation to the property of *G*-ness has an *a priori* logical probability that is greater than zero, and that is not an infinitesimal. This in turn enables one to show that as the number of *Fs* that have been observed to be *G*s increases, with no counterexamples appearing, the *a posteriori* logical probability that it is a law that all *F*s are *G*s not only increases, but increases quite rapidly and, moreover, is soon very close to 1.

4.3 Conclusions

If this is right, we can draw the following important conclusions:

(1) If laws are simply cosmic regularities, then the *a posteriori* logical probability that a non-probabilistic law with an infinite number of instances will obtain is infinitesimally close to zero.

(2) By contrast, if laws are second-order relations between universals, then the observation of instances falling under the law, with no counterexamples, will result in a rapid increase in the *a posteriori* logical probability that the law obtains, and, moreover, that *a posteriori* logical probability will also rapidly approach the value of 1.

5. The Relevance of this to the Problem of Skepticism

How is this relevant to our problem? Though I am not confident that all the details are in place, the basic ideas are as follows.

5.1 The Earlier Skeptical Argument

Recall the earlier skeptical argument that I set out. The key idea was that since there is a very extensive mapping, for example, from propositions concerning how things would be in a certain mind-independent physical world at a given time to an exactly corresponding representation in the mind of a Berkeleian deity, there are **extremely extensive similarities between the two theories**, and this, together with the fact that **they differ very little in complexity**, is grounds for holding that their *a priori* logical probabilities should not be very different. That conclusion, together with the results of the argument set out in section 3 above, then entails that the *a posteriori* logical probability of the hypothesis that there is a mind-independent, physical world should, at best, be **only slightly greater** than the *a posteriori* logical probability of the hypothesis that one lives in a Berkeleian world.

The discussion in section 4 enables one to see why that argument is problematic. For compare the following two hypotheses:

Hypothesis 1	**Hypothesis 2**
There are an infinite number of Fs.	There are an infinite number of Fs.
All Fs are Gs.	All Fs are Gs.
Object 1 is both F and G.	Object 1 is both F and G.
Object 2 is both F and G.	Object 2 is both F and G

…………………………..	…………………………..
Object 1 trillion is both F and G.	Object 1 trillion is both F and G.
It is a 'metaphysically robust' law that all Fs are Gs.	There are no 'metaphysically robust' laws that entail that all Fs are Gs.

(The laws involved here need to be construed as 'metaphysically robust' if one is to show that Hypothesis 1 has a much higher *a priori* logical probability than Hypothesis 2. In addition, if reference to 'metaphysically robust' laws were, replaced by, reference to mere cosmic regularities, then Hypothesis 2 would make no sense, since on Hypothesis 2 it is surely a cosmic regularity that all Fs are Gs, given that there are an infinite number of Fs, all of which are Gs.)

Hypotheses 1 and 2 agree with regard to 1,000,000,000,002 statements, and disagree only with regard to one claim. So one might very well think that hypotheses 1 and 2 differ only by the slightest amount with regard to simplicity. Nevertheless, the probability of hypothesis 1 upon the one trillion statements concerning objects 1 through 1 trillion is extremely high, whereas the probability of hypothesis 2 upon that same set of one trillion statements concerning objects 1 through 1 trillion is infinitesimally close to zero. Two theories that differ only in that one postulates a law where the other postulates an accidental regularity can therefore differ enormously in probability.

5.2 The Brain-in-a-Vat Skeptical Hypothesis

1. Consider, for simplicity, a deterministic world – such as a Newtonian world. In such a world, suppose that the state of the universe at some time t is I, while the state of the universe at some later time u is J. Then the existence of state I at time t together with the laws of that deterministic universe **logically entail** that the universe at time u must be in state J. States of the universe at different times are, then, **nomologically linked**, and the state of the universe at any earlier time **nomologically (or causally) <u>necessitates</u>** that the universe will be in the relevant state at any later time that one chooses.

2. Consider, now, the skeptical hypothesis that one is a brain in a vat. If that hypothesis is to be **experientially equivalent** to the hypothesis that there is a real physical world located in space and time, then it would seem that one of the following things much be the case.

(1) One possibility is that, first of all, the computer that is controlling the experiences that one is having as a brain in a vat contains some sort of map that represents how things would be at a given time in a physical world of the sort that the person associated with the brain in the vat is apparently experiencing. So at time t, the computer contains some representation of state I – call it $R(I)$. Secondly, the computer then constantly applies rules to update that representation – **rules that parallel the laws** that would exist if the world were as it seems to be to the person associated with the brain in the vat – so that, at time u, the computer contains instead a representation of state J – call it $R(J)$. Then, thirdly, the computer calculates the type of experience – call it $E(I)$ – that the person associated with the brain in the vat would be having at time t if he were located in a physical world in the way that it seems to him he is, and the computer then stimulates the person's brain to produce

an experience of type $E(I)$. Similarly, at time u, the computer produces an experience of type $E(J)$.

(2) A second possibility is that the computer that is controlling the experiences that one is having as a brain in a vat, rather than containing some sort of map that represents how things would be **at one given time** in a physical world of the sort that the person associated with the brain in the vat is apparently experiencing – a map which the computer then constantly updates – contains instead **a total map** of **what the relevant world would be like at absolutely every time**. The computer, in that case, does not have to update its map at any time. But it has, instead, to access different parts of that total map, at different times, in order to work out how to stimulate the brain at those different times to produce the appropriate experiences.

(3) A third possibility is this. The computer is somehow running a program that causes it to be in a state at time t – call it $S(t)$ – that is not a representation of state I, but that nevertheless causes the computer to cause the person who is associated with the brain in the vat to have an experience of type $E(I)$. Similarly, the computer winds up in state $S(u)$ at time u, a state that is not a representation of state J, but that nevertheless causes the computer to cause the person who is associated with the brain in the vat to have an experience of type $E(J)$ at the relevant time.

3. The crucial point is now this. First, in the case where the computer is in different representational states at different times, the computer's being in representational state $R(I)$ at time t does **not nomologically (or causally) necessitate** that the computer will be in state $R(J)$ at the later time u. That is to say, the computer's being in state $R(I)$ at time t, together with the laws of nature, does **not logically entail** that the computer is in state $R(J)$ at time u.

Secondly, and similarly, in the case where the computer has a complete map of how the apparent physical world would be at absolutely every time, the part of that total map that represents how the apparent world would be at time t does **not nomologically (or causally) necessitate** the part of that total map that represents how the apparent world would be at the later time u. That is to say, the computer's having a total map that contains state $R(I)$,together with the laws of nature, does **not logically entail** that the computer's total map also contains state $R(J)$.

Nor does the computer's accessing one part of the total map – namely, state $R(I)$ – at time t, together with the laws of nature, **logically entail** that the computer will access the $R(J)$ part of the total map at time u. So the computer's accessing one part of the total map at time t **nomologically (or causally) necessitate** its being the case that the computer will access the right part of the total map at the later time u.

Finally, and again similarly, in the case where the computer does not have states that represent how the apparent physical world is at any time, the non-representational state $S(t)$ of the computer at time t, together with the laws of nature, does **not logically entail** its being the case that the computer will be in state $S(u)$ at the later time u. So the non-representational state $S(t)$ of the computer at time t does **not nomologically (or causally) necessitate** its being the case that the computer will be in state $S(u)$ at the later time u.

One reason these things are not the case is that it is nomologically possible that there are things outside of the computer that can interact with the computer to shut it down, or to modify its program, or to damage it, etc. Another reason is that there may be a failure of the computer's memory at any point, or a failure of its processing system.

4. In contrast to the case of the two hypotheses discussed in section 5.1, the difference between the non-skeptical, mind-independent, spatial world hypothesis and the skeptical brain-in-a-vat hypothesis is not that where the former postulates laws of nature connecting states, the latter postulates **simply a cosmic coincidence**. The brain-in-a-vat hypothesis is different, and the reason is that laws of nature **do** play **a role** in linking together a computer's states at different times, even though earlier states do **not**, together with laws of nature, **logically necessitate** later states, and so earlier states do not **nomologically (or causally) necessitate** later states. For earlier states of the computer do causally give rise to the later states. But they do so **only as long as the correct background conditions continue to be satisfied**. So later states of the computer are a function of earlier states of the computer plus laws of nature **<u>together with</u> the obtaining of a <u>non-nomological</u> generalization concerning the continued existence of the relevant background conditions –** such as no failure of memory, or of processing, etc.

One has, in short, a generalization that is not nomically true, that is not entailed by laws of nature, but that, at the same time is **not a purely accidental** generalization. It holds partly as a matter of accident, and partly as a matter of laws of nature.

The upshot is that there is therefore **some non-zero probability** that that generalization will fail at any point of time. The problem is to assess the likelihood of that.

5. How likely is such a failure? That depends on a number of factors. One factor that appears to be relevant is the nature of time. If time is infinitely divisible, then between any two moments of time there will be an infinite number of instants, and so one might think that the probability that there will be no failure of the background conditions, in any period of time, no matter how short, will be infinitesimally close to zero. That would seem, however, not to be right – at least given only what we have considered so far – since it seems clear that the probability of something going wrong with any given computer depends upon a number of factors – such as how well it is protected against outside interference, whether it involves backup systems, etc. Computers break down, but they can also go for long periods of time without that happening, and with sophisticated backup systems, it might seem that one could make the chance of failure very small.

6. But there is a second factor that is surely relevant, and that is **the complexity of the world** that the computer is simulating. The larger that world is, the more complicated the computer's states have to be, regardless of whether the states are representational or non-representational. The intuition is then that as those states become larger and larger, the greater the likelihood of a breakdown is – a breakdown that will have the result that the computer fails to bring about the type of experience in the person associated with the brain in the vat that the person would have if he or she were located in the a physical world of the relevant type. Some such divergences may, of course, be small and not noticeable. But others, surely, will be noticeable.

7. The idea, in short, is that if one is a brain in a vat, apparently experiencing a very complex physical world – such as the world that you and I think we are experiencing – one should expect that there will be failures in the computer at some points that will have the result that one's experiences will not be what they should be: there will be, for example, discontinuities in successive visual experiences, or there will be gaps in one's visual field, or one's visual field will freeze at some point. There will, in short, be computer glitches.

8. There can, of course, be disturbances and discontinuities in one's visual experiences. Migraines can affect one's visual experiences, and strokes may cause double vision. But in such cases there is an explanation within the physical world that appears to exist for those changes in one's visual experiences. In addition, the types of disturbances are both limited in number, and much less dramatic than could occur if a computer were causing one's experiences.

9. This much, then, seems to me plausible: the probability that the brain-in-vat hypothesis is true is **less than** the probability of the non-skeptical, mind-independent, spatial world hypothesis, given that the relations of nomological (or causal) necessitation between state *I* and state *J* that exist if the non-skeptical hypothesis is true get replaced by relations between representational states $R(I)$ and $R(J)$, or between non-representational states $S(t)$ and $S(u)$ that are **not relations of nomological (or causal) necessitation**, since they depend upon the obtaining of a background generalization that does not express a law.

10. There is another issue that is very relevant, and that needs to be considered. To this point, I have simply supposed that the computer that is controlling the experiences that one is having as a brain in a vat somehow is in states – either representational states $R(I)$, or non-representational states $S(t)$ – that it makes use of to determine what experiences to cause at any given time. But let us now ask what the computer must be like to have such states. Suppose, in particular, that the way it operates is by being in representational states. If the brain-in-a-vat hypothesis is to be experientially equivalent to the non-skeptical, real-physical-world hypothesis, then it would seem that state $R(I)$ must contain **complete information** concerning what properties would be present in the spatiotemporal universe at the time in question if there really were a physical world of the sort in question.

This is an enormous amount of information. Indeed, given that every property, every relation, and every spatial location would need to be represented, I think it could be argued that the space required for storing that information could not be less than the space that would be occupied by the physical world in question if the latter were real. How large is that? In an article in a recent issue of *Scientific American* (November, 2007, page 55) the diameter of the observable universe was given as 10^{26} meters.

Moreover, if space is infinitely divisible, there will be a non-denumerable infinity of locations, and in each case the computer would have to contain information about what particles, or fields, etc., were or were not present at each of those locations. It may be, however, that space is not infinitely divisible, since the article just mentioned also states that there is a "minimum meaningful length in nature". But that length is very short – namely 10^{-18} meters. (An atomic nucleus has a diameter of the order of 10^{-15} meters, and the smallest distance probed by particle accelerators is meters.)

The upshot is that the amount of information that the computer would need to store would seem to be extraordinary, and it would seem that the size of the computer, at the very least, would have to be as great as the physical universe that exists if the non-skeptical hypothesis is true.

11. I have considered only the possibility of representational states. I suspect, however, that one can argue that there is no way that one can use non-representational states – $S(t)$ – to encode information that will enable the computer to calculate what experiences to produce, and that will require less space.

12. It is not enough, however, for the computer to store information about how the physical universe, if it existed as it seems to the person associated with the brain in the vat, is at any given time. The computer has, under **the first** of the three scenarios mentioned above, **to update** that information with the passage of time, and the information covers how things are at every apparent spatial location. In updating that information, then, the computer must be accessing information that, if the earlier conclusion concerning the size of the regions where that information is stored is correct, is distributed throughout a space that is at least as large as our physical universe is, (or would be, if the latter were real). How could a computer perform calculations involving such widely scattered information at any given time?

The answer is that the processor in the computer would have to be causally connected with all of the locations where the information was stored. All of those causal connections would depend, however, upon pathways that could break down, either due to interference from without, or by internal deterioration. So again we have places where things can fail, and these places, moreover, and as we have just seen, would seem to constitute a region at least as large as our universe, assuming it exists.

13. Moreover, consider **how quickly** the computer must work. If time is infinitely divisible, the computer cannot take any finite length of time to update how things are at any given spatial location: it must work infinitely fast. But even if time is not infinitely divisible, even if there is a **smallest** temporal interval – related, perhaps, to the Planck constant – the speed of the computer would have to be enormous in the extreme.

14. On the second scenario mentioned above, the computer has a total map representing how the apparent world would be at every moment, if it were real, so no updating is needed. But this saving is bought at the cost of requiring sufficient space to store information about the complete state of the universe at every time. The computer would thus need space whose size is comparable to that of our apparent **spatiotemporal** world. Moreover, this increase in the amount of information stored requires a comparable increase in the **accessing networks**, which in turn translates into many more possibilities for breakdowns. Indeed, if time is infinitely divisible, the possibilities for breakdowns will be infinitely greater.

15. These processes either of accessing information, or of performing calculations to update it, or both, are not ones in which successive states stand in relations of causal necessitation. Consequently, there must be some non-zero probability that there will

be a failure, either with regard to the original creation and storing of the information, or with regard to the calculations by which the information is updated, and then, in either case, with regard to the retrieving of information. Then, given the overwhelming amount of information involved, and the lengthy causal processes needed for its retrieval – and also, in the first case, the astounding nature of the calculations involved – it would seem **extremely likely** that something would go wrong, even in a brief stretch of time.

Conclusion

Here, in contrast with the comparison of laws of nature with regularities that involve cosmic accidents, discussed in section 4, I am unable to calculate a probability. Nevertheless, it seems to me that there are excellent grounds for concluding that the brain-in-a-vat hypothesis has a much lower probability than the hypothesis that there is a mind-independent, physical world governed by natural laws.

5.3 A Second Skeptical Hypothesis: Berkeley's Immaterial World

1. But mightn't one be able to avoid the arguments that I have offered for thinking that the brain-in-a-vat hypothesis is very improbable by shifting to a different skeptical hypothesis? In particular, what if one switched from the brain-in-a-vat hypothesis to Berkeley's hypothesis? Can it be argued that God, as an immaterial being, will have no problem storing all of the information, accessing it, and then updating it infinitely quickly?

2. If, as I am inclined to think, the idea of an omnipotent being is logically coherent, then Berkeley's hypothesis does seem more difficult to criticize. But if that's right, and if, as some writers, such as Richard Swinburne, maintain, the idea of God is quite a simple idea, may not one be able to argue that that *a priori* logical probability of the existence of God is not that low?

3. Reflections on what is involved in the storing, accessing, and updating of information in the computer case, however, makes me wonder whether the *a priori* logical probability of there being an immaterial thing that could do these things isn't in fact extremely low, contrary to what some philosophers have claimed, and contrary to what one might initially think oneself. For mustn't any immaterial being that stores information, that accesses that information, and that updates it, involve causal processes? Mustn't there, then, be causal networks, and won't those causal networks be equally complex? Won't any calculations involve just as many steps?

4. Consequently, I'm inclined to think that the situation is not really different when one shifts from the brain-in-a-vat skeptical hypothesis to the Berkeleian skeptical hypothesis. For imagine describing the storing of information, the accessing of it, the updating of it, and the performing of relevant calculations, all in a way that does not mention **what sort of entity** is doing all of that. In all of this, earlier states do not causally or nomically necessitate later states, and, because of this, there are many – and perhaps literally countless – opportunities for failures and breakdowns to occur. If someone then tells you that, by the way, an immaterial being is doing all of this, are you then inclined to think that all of those problems then vanish?

5.4 Can Berkeley's Immaterial World Hypothesis Be Refuted in Some Other Way?

1. According to Berkeley's worldview, the only basic entities that exist are God and finite immaterial minds, so that the skeptical possibility that one is considering is that one is an immaterial mind being appropriately stimulated by God. So why not argue that Berkeley's hypothesis is unlikely to be true either because it is unlikely that God exists, or unlikely that immaterial minds exist, or unlikely that one is oneself an immaterial mind?

2. I hold that there is a version of the argument from evil that shows that it is unlikely that there is an omnipotent, omniscient, and morally perfect person. So why not appeal to that to refute Berkeley? The answer is that there are closely related skeptical hypotheses that would be untouched by such a response. In particular, a skeptic can suggest that the only things that exist are finite immaterial minds plus an omnipotent and omniscient being who may not even be good, let alone perfectly good.

3. What about arguing that there are good reasons for thinking either that there are no finite immaterial minds, or, at least, that one is not oneself an immaterial mind? Here, too, I think that there are good reasons for holding that other human persons, if there are such – on Berkeley's view it is not clear that one has good reasons for thinking that there are – do not have immaterial minds, and that one does not oneself have an immaterial mind. Relevant here, for example, is ordinary information about what can happen when the brain is damaged, along with more refined scientific information on localization of psychological capacities and mental states.

4. But I don't want to adopt that approach, for two reasons. First, such a refutation of skepticism of a Berkeleyan sort seems intellectually unsatisfying, because of its limited scope. For one can, I think, easily imagine a slightly different world in which neither the brain not any other part of one's body, seemed to have anything to do with psychological capacities or mental states. If one were in such a world, one could not refute Berkeley by arguing that there were good reasons for thinking that one was not oneself an immaterial mind. But wouldn't one still be inclined to think that there was a mind-independent, physical world, and thus that skepticism concerning the existence of such a world was mistaken?

5.5 A Third Skeptical Hypothesis: God Plus a Material World

There is, also, however, a second reason for not attempting to answer Berkeley by arguing that it is very unlikely that one is an immaterial mind, and a reason that is very strong indeed. The reason is that there are not just the following two types of skeptical hypotheses:

(1) I am, or am associated with, something like a brain in a vat, and there is some physical device that is capable of storing, retrieving, and manipulating information, and that is creating in me the illusion that I live in a mind-independent, physical, spatial world.

(2) I am an immaterial mind, and there is some very powerful immaterial mind that is capable of storing, retrieving, and manipulating information, and that is creating in me the illusion that I live in a mind-independent, physical, spatial world.

For there is also the following, mixed hypothesis:

(3) I am, or am associated with, something like a brain in a vat, and there is some very powerful immaterial mind that is capable of storing, retrieving, and manipulating information, and that is creating in me the illusion that I live in a mind-independent, physical, spatial world.

Arguments supporting the view that I am not an immaterial mind may refute the second hypothesis, but they leave the third hypothesis completely untouched.

5.6 A Final Skeptical Possibility: A Skeptical Hypothesis that Involves Nomological/Causal Necessitation

Central to my line of argument has been the idea that the mapping that takes one from the non-skeptical hypothesis that there is a mind-independent, physical, spatial world into skeptical alternatives is a mapping that takes one from states of affairs that stand in relations of nomological (or causal) necessitation to states of affairs that are not thus related. Is it possible for the skeptic to avoid this by introducing a skeptical hypothesis where the connections are ones involving nomological (or causal) necessitation. Couldn't there be, for example, an immaterial mind whose later states were nomologically (or causally) necessitated by its earlier states?

6. Summing Up

The result is that while the matter is far from clear cut, I am inclined to think that, given further development, the above lines of thought will provide a refutation of skepticism concerning the existence and nature of an external, mind-independent, physical world. There is, as we have just seen, a serious question of whether the considerations that I appealed to earlier, in the case of the brain-in-a-vat skeptical hypothesis, in order to argue that that hypothesis has a very low probability can be applied to Berkeley's hypothesis that we live in an immaterial world. But if it can be successfully argued that that can be done, then I think that we will have a refutation of skepticism concerning the existence of an external, mind-independent physical world of the sort that most people believe exists. The reason is, first, that while variations are possible, it does seem that the only skeptical alternatives are an illusion maintained by a physical thing with extraordinary capacities for storing, retrieving, and manipulating information, and an illusion enjoyed by a person who is either an immaterial mind, or associated with a brain in a vat, and that is created and maintained by an immaterial being with extraordinary capacities for storing, retrieving, and manipulating information. So if one can show that the brain-in-a-vat scenario is unlikely, and that Berkeley's immaterial-world hypothesis is unlikely **on similar grounds**, I think that one can show that **any** given skeptical hypothesis is unlikely. Secondly, I think that one has good reasons for thinking that the brain-in-a-vat hypothesis is not only unlikely, but **extremely unlikely**. If the same is true of Berkeley's hypothesis, and of the mixed hypothesis, then I think one can conclude that the non-skeptical, mind-independent, physical world hypothesis is not only much more probable than any **given** skeptical hypothesis, but also much more probable than the disjunction of all possible skeptical hypothesis.

Finally, all of this assumes, as I noted earlier, that the problem of induction can be solved, and that, in particular, one can justify abduction.