Philosophy 5340 - Epistemology

Topic 5 – The Justification of Induction

The Proofs of the Anti-Reductionist Results

Result 1: State Descriptions and the Next Instance

According to the classical definition of probability, the logical probability of *p* given *e* is equal to the ratio of the number of cases – state descriptions, possible worlds – in which both *p* and *e* are true to the number of cases – state descriptions, possible worlds – in which *e* is true.

Consider then, for illustration, an urn that contains one million marbles, each of which is either red or green, so that there are two possibilities with regard to the color of each marble. There are then $2^{1,000,000}$ possibilities with regard to the colors of the one million marbles. Given no further information, what is the probability that some particular marble – say, the millionth marble drawn from the urn – is red? The answer is that if that marble is red, there are $2^{999.999}$ possibilities with regard to the colors of the other 999,999 marbles, so the probability that the millionth marble drawn is red is, by the classical definition of probability, equal to $\frac{2^{999,999}}{2^{1,000,000}}$, which is equal to $\frac{1}{2}$.

Suppose, now, that 999,999 marbles are drawn from the urn, and all of them are red. What is the probability that the millionth marble is red, given that information? Of the $2^{1,000,000}$ possibilities there originally were with regard to the colors of the one million marbles, the only ones that remain are the one where all one million marbles are red, and the one were the first 999,999 are red, and the millionth marble is green. So there are now two state descriptions that remain possible, in one of which the millionth marble is red, and in the other of which that marble is green. So by the classical definition of probability, the probability that the millionth marble is red, given that the first 999,999 are all red, is equal to $\frac{1}{2}$.

Conclusion: If strong laws of nature are logically impossible, and all state descriptions are equally probable, then one cannot learn from experience. So a very strong form of inductive skepticism is true, if strong laws of nature are logically impossible.

Result 2: State Descriptions and Cosmic Regularities

Consider, now, any generalization of the form $(x)(Fx \supset Gx)$, where the property Q that something must have if the predicate '*G*' is to be true of it is not a property that belongs to a family of positive properties: the only possibilities are either having property Q or not having property Q. Then precisely the same calculation applies as in the case of the marbles and the urn. Suppose now that, throughout history, the number of things that are *F* is equal to $(1,000,000 + n)$. If one has observed 1,000,000 things that are *F*, and all of them have been *G*, what is the probability that the other *n* things that are *F* are also *G*? The answer is that that probability is equal to $\frac{1}{2^n}$.

Accordingly, if there are an infinite number of things that are *F*, the probability that all of them are *G*, given only the information that one million things that are *F* have been observed to be G, will be equal to the limit of $\frac{1}{2^n}$ as n goes to infinity, and that limit is zero.

This is also the probability that $(x)(Fx \supset Gx)$ will be true, given the information (1) that there are m things that are F, all of which are G, (2) that there are an infinite number of things that are F, and (3) that there are no laws of nature other than ones that supervene upon the Humean base.

Suppose that, somewhat disheartened by those two theorems, a reductionist with regard to laws of nature follows Carnap's lead, and defines logical probability based on the proposition that it is **structure** descriptions, not state descriptions, which are equally likely. Then one has the following theorem:

Result 3: Structure Descriptions and the Next Instance

The idea that all state descriptions are equally likely is a very natural notion indeed, and as Rudolf Carnap points out in the second edition of his *Logical Foundations of Probability* (Chicago: University of Chicago Press, 1962), page 565, it is the approach that was adopted by some very well known thinkers, including Charles Sanders Peirce in his paper "A Theory of Probable Inference" (1883), John Maynard Keynes in his book *A Treatise of Probability* (1921), and Ludwig Wittgenstein in his *Tractatus Logico-Philosophicus* (1922). Carnap's view is that it generates, in view of the sort of argument that I just sketched, the result that one can never learn from experience, and so Carnap concludes, very plausibly, but, as we shall see, mistakenly, that an approach that treats all state descriptions as equally likely is an "entirely inadequate" approach to logical probability.

Carnap's approach, then, was to abandon the view that state descriptions are equally likely in favor of the view that structure descriptions are equally likely. Doing that enables one to follow Laplace in deriving the following rule for the probability that the next F is G, given that the first n things that are F are G, and given that G does not belong to a family of positive properties:

The probability that the $(n + 1)$th F is a G, given that the first n things that are F are G, $= \frac{n+1}{n+2}$.

(This is Laplace's famous Rule of Succession.)

Now this is a cheering result. The more Fs one observes, all of which have property G, the more likely it is that the next F has property G. So one can learn from experience.

But one also has the following, rather less appealing result.

Result 4: Structure Descriptions and Cosmic Regularities

Suppose now that there are m things that are F. What is the probability that all of them are G? By Laplace's rule of succession, the probability that the first F is a G is equal to $\frac{0+1}{0+2}$, or $\frac{1}{2}$. Similarly, the probability that the second F is a G, given that the first F is a G, is equal to $\frac{1+1}{1+2}$, or $\frac{2}{3}$, while the probability that the third F is a G, given that the first two things that are F are G is equal to $\frac{2+1}{2+2}$, or $\frac{3}{4}$. To arrive at the probability that the first three things that are F are all G, one has to multiply those three probabilities, so that probability is equal to $\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)$. Finally, the probability that all m things that are F are G is equal to $\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right) \ldots \left(\frac{m-1+1}{m-1+2}\right)$, which is equal to $\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right) \ldots \left(\frac{m}{m+1}\right)$. Then, since the denominator of each fraction is equal to the numerator of the next fraction, we can cancel all the way along, which gives us that the probability that all m things that are F are G is equal to $\frac{1}{m+1}$.

Finally, if there are an infinite number of things that are F, the probability that all of them are G is equal to the limit of $\frac{1}{m+1}$ as m goes to infinity, which is zero.

Conclusion: The use of Laplace's Rule of Succession enables one to avoid inductive skepticism, but it still entails the result that the probability that $(x)(Fx \supset Gx)$ will be true, given the information (1) that there are m things that are F, all of which are G, (2) that there are an infinite number of things that are F, and (3) that there are no laws of nature other than ones that supervene upon the Humean base, is infinitesimally close to zero.

Results 2 and 4 look depressing if one holds that strong laws of nature are logically impossible. But Hans Reichenbach offered an interesting argument for the following proposition:

> **If probabilistic laws of nature are logically possible, then no evidence can ever make it likely that a *non-probabilistic* law obtains.**

Result 5: Hans Reichenbach's Challenge: Epistemology and Non-Probabilistic Laws

Hans Reichenbach argued that in considering the question of whether one can confirm the existence of a non-probabilistic law, such as the law that $(x)(Fx \supset Gx)$, one needs to consider the possibility of **probabilistic laws**. For, in the first place, there does not seem to be any obvious incoherence in the view that it could be a law of nature that the probability that an F is a G is equal to some number k between zero and one. In the second place, quantum mechanics, as standardly formulated, involves probabilistic laws.

The problem now is that once the possibility of probabilistic laws is admitted, one is admitting a large number of possibilities, since the value of the number k in a law to the effect that the probability that an F is a G is equal to some number k can take on absolutely any of the non-denumerably infinite values between zero and one. But it is not just between zero and one that there are an infinite number of possibilities for the value of k. Choose any number as close to the number one as you want – say, the number 0.99999 . . . for a sequence of a billion 9's. That number is very close to one, but there are still an uncountably infinite number of probabilistic laws expressed by statements of the form "The probability that an *F* is a *G* is equal to *k*" where *k* lies between that number and one.

The problem now is this. Suppose one has observed 1,000 *F*s, and all of them were *G*s. This is the result that one would expect not only if it was a law that all *F*s are *G*s, but also if it was a law that the probability that an *F* is a *G* is equal to the number 0.99999 . . . for a sequence of a billion 9's. So how can one be justified even in believing that it is more likely that it is a law that all *F*s are *G*s than that **that particular law obtains**, let along in believing that it is more likely that it is a law that all *F*s are *G*s than that **some one or other** of the intervening, non-denumerable infinity of possible probabilistic laws obtains?

The upshot is that it ***seems*** that one has to abandon the idea that one can confirm the existence of a non-probabilistic law, and adopt instead the idea that what one should be attempting to do is to confirm the existence of a law that is either non-probabilistic, or that is probabilistic, and that lies in a specified interval, (*a*, 1), where the number *a* can be as close to one as one likes.

I think that Reichenbach's argument is unsound. But this is a complicated matter, so let us assume, for the sake of argument, that Reichenbach is right. If so, one can never confirm any non-probabilistic law, and Results 2 and 4 need not trouble the reductionist with regard to laws.

Happiness for the reductionist, however, is short-lived. For, first of all, if one returns to the idea of defining logical probabilities based on the proposition that all **state** descriptions are equally likely, one can then prove the following theorem:

Result 6: State Descriptions and Regularities Falling within a Range

Let us shift, then, from considering the probability that all *F*s are *G*s, given that *m* *F*s have been examined, and all of them have turned out to be *G*s, to considering the probability that it is a law that the probability that an *F* is a *G* is equal to *k*, where *k* lies in a specified interval, (*a*, 1), given that *n* *F*s have been examined, and that all of them turned out to be *G*s.

Suppose, for concreteness, that one has examined 1000 things that are *F*s, and all 1000 have turned out to be *G*s. Rather than asking, as I did earlier, what the probability is that all *F* are *G*s, let us ask instead what the probability is that the proportion of *F*s that are *G*s lies in the range from 90% to 100% – the idea being that if that is the case, then, other things being equal, it will be reasonable to believe that there is some number *k* in the range from 0.9 to 1 such that it is a reductionist law of nature that the probability (or chance) that an *F* is a *G* is equal to *k*.

If 1000 *F*s have all turned out to be *G*s, what is the logical probability, using the method of state descriptions, that in the next 400 *F*s, enough will be *G*s that the proportion of *F*s in the combined set of 1400 *F*s that are *G*s is equal to or greater than 90%? For that to be the case, at least 1260 of those 1400 *F*s will need to be *G*s, and for that to be the case, at least 260 of those next 400 *F*s must be *G*s. So we need to ask what the probability of that is.

The probability that out of *n* *F*s, *m* or more will be *G*s, given that *G* belongs to a two-member family of properties, is, if one accepts the method of state descriptions, given by the following formula:

Probability that at least *m* out of *n* *F*s will be *G*s =

$$\sum_{k=m+1}^{n} {}_kC_n \left(\frac{1}{2}\right)^n = \sum_{k=m+1}^{n} \frac{n!}{k!(n-k)!}\left(\frac{1}{2}\right)^n$$

What is the value of this when, as in the present case, $m = 260$ and $n = 400$? The answer, as calculated by *Mathematica*, is 5.4232×10^{-10}. This is a rather small number.

Suppose, once again, that 1000 *F*s have all turned out to be *G*s. What is the logical probability, using the method of state descriptions, that in the next 1000 *F*s, enough will be *G*s that the proportion of *F*s that are *G*s in the combined set of 2000 *F*s is equal to or greater than 90%? For that to be the case, at least 1800 of the total set of 2000 *F*s need to be *G*s, and for that to be the case, at least 800 of those next 1000 *F*s must be *G*s. When the probability of that is calculated using *Mathematica*, the result is 2.04944×10^{-86}.

The picture, I hope, is clear, even from just these two examples. If one adopts a reductionist approach to laws of nature, the probability that there is some number *k* in the range from 0.9 to 1 such that it is a reductionist law of nature that the probability (or chance) that an *F* is a *G* is equal to *k* is extraordinarily low, even given the evidence that a large number of *F*s have been examined, and all of them have turned out to be *G*s.

Suppose, finally, that one shifts, once again, from a formulation of logical probability that treats state descriptions as equally likely to a formulation that treats **structure** descriptions as equally likely. Does that save the reductionist? The answer is that it does not, since one can prove the following theorems:

Result 7: Structure Descriptions and Regularities Falling within a Range

Suppose that, like Carnap, one shifts from a formulation of logical probability that treats state descriptions as equally likely to a formulation that treats structure descriptions as equally likely. I have contended that defining probability in terms of state descriptions is the more natural of those two choices, and that defining probability in terms of structure descriptions can only be supported by the observation that it allows one to avoid inductive skepticism – an epistemological strategy that I think one should regard as having the usual advantages of theft over honest toil. But let us set aside that objection, and see how a reductionist approach to laws of nature fares when one does that.

Suppose, then, that one has examined 1000 things that are *F*s, and all 1000 have turned out to be *G*s. Once again, as we did earlier in the case of a state description approach, rather than asking what the probability is that all *F*s are *G*s, let us ask instead what the probability is that the proportion of *F*s that are *G*s lies in the range from 90% to 100% – the idea being, once again, that if that is the case, then, other things being equal, it will be reasonable to believe that there is some number *k* in the range from 0.9 to 1 such that it is a reductionist law of nature that the probability (or chance) that an *F* is a *G* is equal to *k*.

If 1000 *F*s, have all turned out to be *G*s, what is the logical probability, using the method of structure descriptions, that in the next collection of 1000 *F*s, enough will be *G*s that the proportion of *F*s that are *G*s in the combined set of 2000 *F*s is equal to or greater than 90%? For that to be the case, 1800 of those 2000 *F*s will need to be *G*s. So we need to ask what the probability of that is.

What structure descriptions are possible here, and in how many of those will it be the case that 1800 of the 2000 *F*s will be *G*s? The answer is that, given that at least 1000 of the 2000 *F*s are by hypothesis *G*s, the possibilities with respect to the number of the 2000 *F*s that are *G*s range from 1000 up to 2000, so that there are 1001 possible relevant structure descriptions. For 90% or more of the *F*s to be *G*s, the number of *F*s that are *G*s must fall in the range from 1800 up to 2000, and so there 201 relevant structure descriptions where this is the case. Hence, according to the method of structure descriptions, the probability that the proportion of *F*s that are *G*s in the sample of 2000 *F*s is equal to or greater than 90%, given that, of those 2000 *F*s, 1000 are known to be *G*s, is equal to $\frac{201}{1001}$, or just over 20%.

Next, suppose, once again, that 1000 *F*s have been examined, all of which have turned out to be *G*s. What is the logical probability, using the method of structure descriptions, that in the next collection of 10,000 *F*s, enough will be *G*s that the proportion of *F*s that are *G*s in the combined set of 11,000 *F*s is equal to or greater than 90%? For that to be the case, 9900 of the 11,000 *F*s will need to be *G*s. So we need to ask what the probability of that is.

What structure descriptions are possible here, and in how many of those will it be the case that 9900 of the 11,000 *F*s will be *G*s? The answer is that, given that 1000 of the 11,000 *F*s are by hypothesis *G*s, the possibilities with respect to the number of the 11,000 *F*s that are *G*s range from 1000 up to 11,000, so that there are 10,001 possible relevant structure descriptions. For 90% or more of the *F*s to be *G*s, the number of *F*s that are *G*s must fall in the range from 9900 up to 11,000, and so there 1101 relevant structure descriptions where this is the case. Hence, according to the method of structure descriptions, the probability that the proportion of *F*s that are *G*s in the combined set of 10,000 *F*s is equal to or greater than 90%, given that, of those 10,000 *F*s, 1000 are known to be *G*s, is equal to $\frac{1101}{10001}$, or just over 11%.

What is the general pattern here? Suppose that *m F*s have been examined, and all of them have turned out to be *G*s. What is the probability that, if *n* more *F*s are examined, *x*% of the total of (*m* + *n*) *F*s will be *G*s? The general formula for that probability, for which a derivation is given in an appendix, is $\frac{\left(1-\frac{x}{100}\right)(m+n)+100}{n+1}$.

So if one adopts a reductionist view of laws of nature, and has examined a billion *F*s, and found all of them to be *G*s, and has reason to believe that there are at least a billion more *F*s in the world, the probability that there is a law of nature to the effect that the probability that an *F* is a *G* is equal to *k*, where *k* lies in the range from

0.9 to 1, is approximately 20%, whereas if *k* is instead in the range from 0.99 to 1, the probability is approximately 2%.

Given the general formula, it is easy to show that as *n* increases without limit, this tends to the limit of $\left(1-\frac{x}{100}\right)$ as *n* increases without limit. The means that, on a reductionist approach to laws of nature, if the universe contains an **infinite** number of *F*s, *m* of which are known to be *G*s, the probability that *x*% of all of the *F*s are *G*s is equal to $\left(1-\frac{x}{100}\right)$, and this is so regardless of the value of *m*.

Suppose, for illustration, that one billion *F*s have been examined, and all of them have turned out to be *G*s. Suppose, further, that in the total history of the universe, there are an **infinite** number of *F*s. What is the probability that 90% of all the *F*s will be *G*s? The answer is that the probability is equal to $\left(1-\frac{90}{100}\right)$, or 10%.

Summing Up

The moral, I suggest, seems clear: if one embraces a reductionist approach to laws of nature, then regardless of whether one adopts a state description approach to inductive logic or a structure description approach, one will not be able to avoid the following conclusion: **No interesting scientific hypothesis concerning laws of nature can be confirmed**.