Prospects of realizing two dimensional chiral p-wave fermionic superfluids

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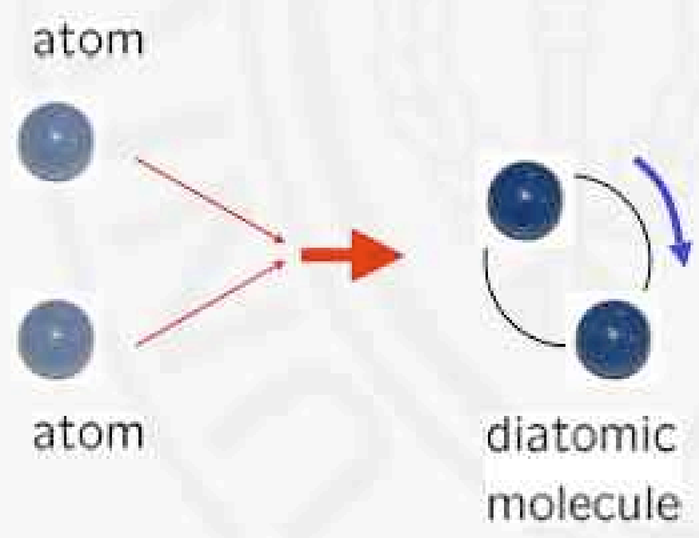
Paris, 2008

BCS-BEC, p-wave

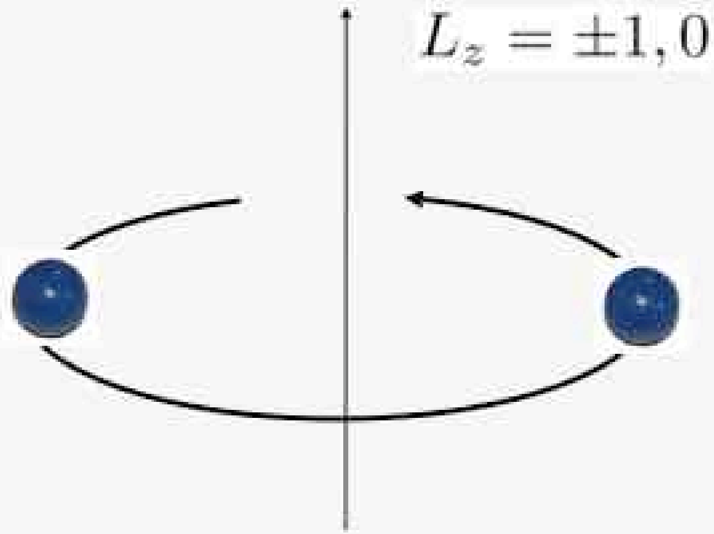
Fermions of one species (spinless) with attractive interactions

$$H = \sum_p \frac{p^2}{2m} a^\dagger_{\mathbf{p}} a_{\mathbf{p}} + \sum_{\mathbf{p},\alpha} \left(\epsilon_\alpha + \frac{p^2}{4m} \right) b^\dagger_{\alpha\mathbf{p}} b_{\alpha\mathbf{p}} + \sum_{\mathbf{p},\mathbf{q},\alpha} \frac{g}{\sqrt{V}} \left(b_{\alpha\mathbf{q}}\ p_\alpha\ a^\dagger_{\mathbf{p}+\frac{\mathbf{q}}{2}}\ a^\dagger_{-\mathbf{p}+\frac{\mathbf{q}}{2}} + h.c. \right).$$



New features of p-wave:

1. Phases



(condensed) molecule

Superfluids are Bose-condensed molecules (both in the BCS and in the BEC regime)

$L_z = \pm 1$ Chiral superfluid

$L_z = 0$ Polar superfluid

New features of p-wave:
2. BCS -> BEC is not a crossover

G. Volovik, early 90s: BCS -> BEC is a transition, not a crossover

$$E_p = \sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + 4g^2 \left|\sum_\alpha p_\alpha b_\alpha\right|^2}$$

BCS, $\mu>0$

Gapless excitations (E_p=0) at

$$\frac{p^2}{2m} = \mu, \ \mathbf{p} \perp \mathbf{b}$$

BEC, $\mu<0$

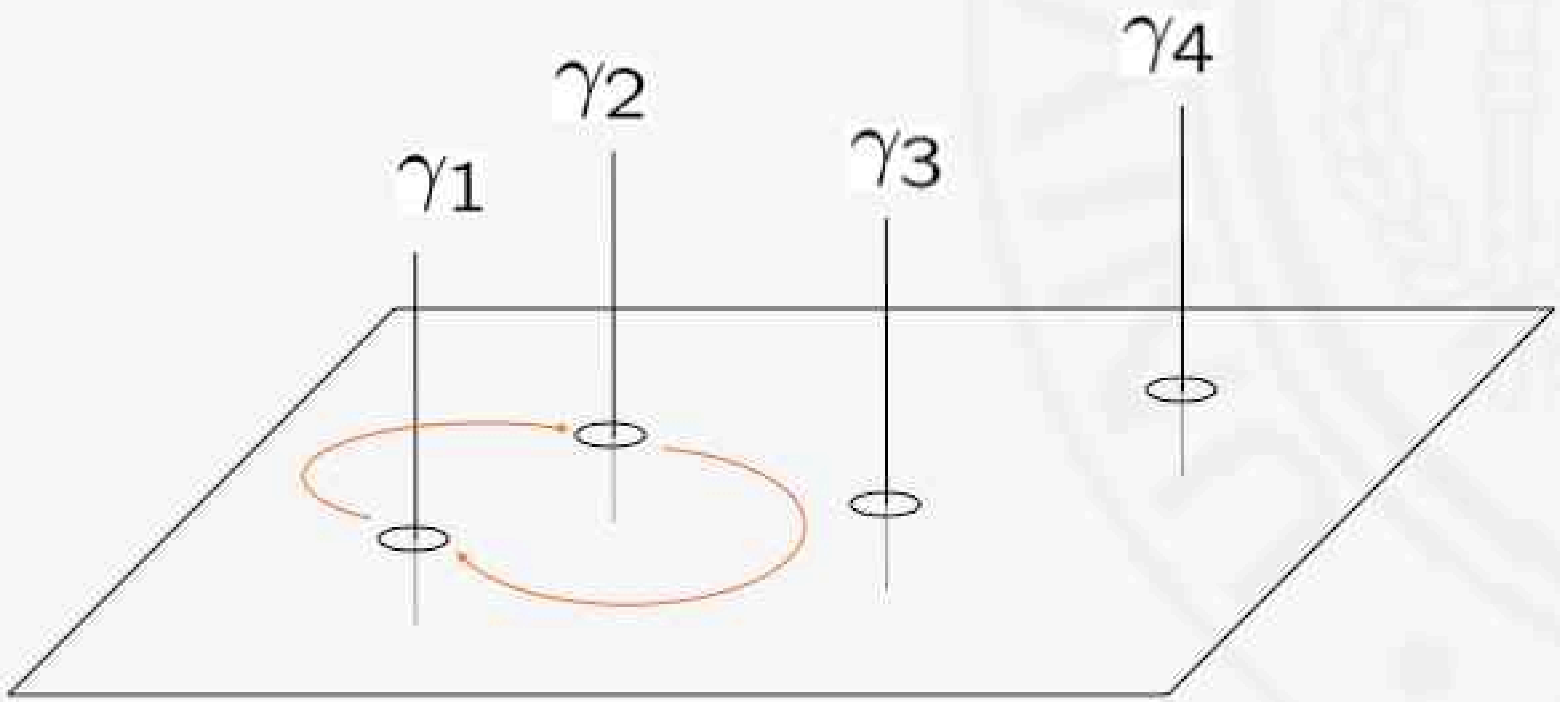
No gapless excitations

$$E_p > 0$$

New features of p-wave:

3. In 2D the BCS phase of the chiral superconductor is topological

N. Read and D. Green, 2000: in this phase, one can observe particles with non-Abelian statistics



These particles sit in the cores of vortices and are characterized by wavefunctions Ψ_α.

Exchanging two vortices leads to $\Psi_\alpha \rightarrow U_{\alpha\beta}\, \Psi_\beta$

A Kitaev, 1997:
One can use these particles to construct a decoherence-free quantum computer

Relationship to Quantum Hall Effect

Chiral 2D p-wave BCS ≡Pfaffian State of the QHE.
Indeed, compute the BCS wave function:

$$|\mathrm{BCS}\rangle = \prod_p \left(u_p + v_p a^\dagger_{-p} a^\dagger_p \right) |0\rangle \qquad \psi(r_1, r_2, \ldots) = \langle 0| \, a(r_1) a(r_2) \ldots |\mathrm{BCS}\rangle$$

$$\psi\left(r_1, r_2, \ldots\right) = \mathcal{A}\left[g(r_1 - r_2) g(r_3 - r_4) \ldots \right] \qquad g(r) = \int \frac{d^d p}{(2\pi)^d} \frac{v_p}{u_p} e^{ipr}$$

$$\mu > 0, \; 2D, \; p_x + ip_y \;\; \rightarrow \;\; g(r) \sim \frac{1}{z}$$

$$\psi(z_1, z_2, \ldots, z_N) = \mathcal{A}\left[\frac{1}{z_1 - z_2} \frac{1}{z_3 - z_4} \cdots \frac{1}{z_{N-1} - z_N} \right]$$

and recognize in it the Pfaffian (Moore-Read)
wave function in the quantum Hall effect
The non-Abelian statistics in the latter is well known.

N. Read and D. Green, PRB, 2000

C. Nayak,
F. Wilczek,
1994

p-wave superfluid prefers to be chiral

a) BCS superconductor wants to maximize its gap. Anderson and Morel (1961).

Non-chiral

$$E = \sqrt{\left(\frac{p^2}{2m} - \mu\right) + \Delta^2 p_x^2}$$

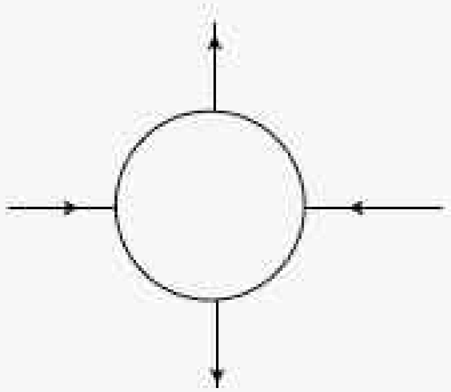
$p_y = \sqrt{2m\mu}$ gapless point

Chiral

$$E = \sqrt{\left(\frac{p^2}{2m} - \mu\right) + \Delta^2 p^2}$$

No gapless points

b) BEC molecules want to condense into $L_z = \pm 1$ state because they interact ferromagnetically. VG, A. Andreev, L. Radzihovsky, PRL (2005)



$$\left\langle b^\dagger_\alpha b^\dagger_\beta b_\gamma b_\delta \right\rangle_{\rm angles} \sim 2(b^\dagger_\alpha b_\alpha)^2 + b^\dagger_\alpha b^\dagger_\alpha b_\beta b_\beta$$

Ferromagnetic term

Conclusions so far

Phases of the p-wave superfluid

	Polar	Chiral
BCS		in 2D, non-Abelian
BEC		

- Atomic gases made of identical fermions with Feshbach resonances confined to 2D automatically form a chiral *p*-wave superconductor.
- To observe the topological phase, we will tune the chemical potential close to zero (to increase T_c), but above zero (to stay in the BCS phase).
- So why hasn't it been already done?

Experiments

PHYSICAL REVIEW A **70**, 030702(R) (2004)

P-wave Feshbach resonances of ultracold ^{6}Li

J. Zhang,[1,2] E. G. M. van Kempen,[3] T. Bourdel,[1] L. Khaykovich,[1,4] J. Cubizolles,[1] F. Chevy,[1] M. Teichmann,[1] L. Tarruell,[1]
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p-Wave Feshbach Molecules

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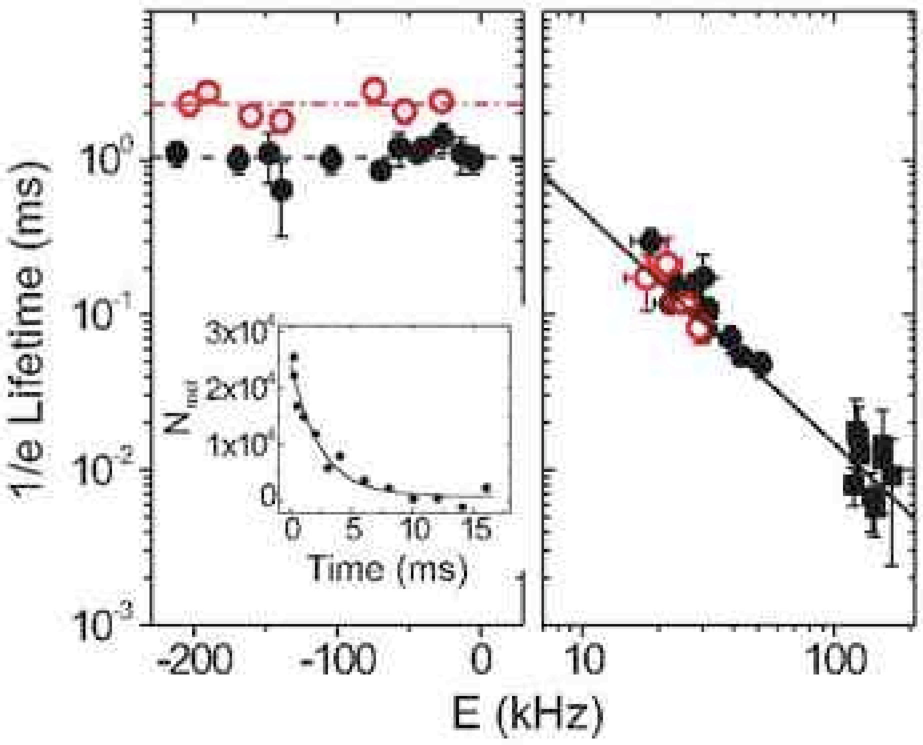
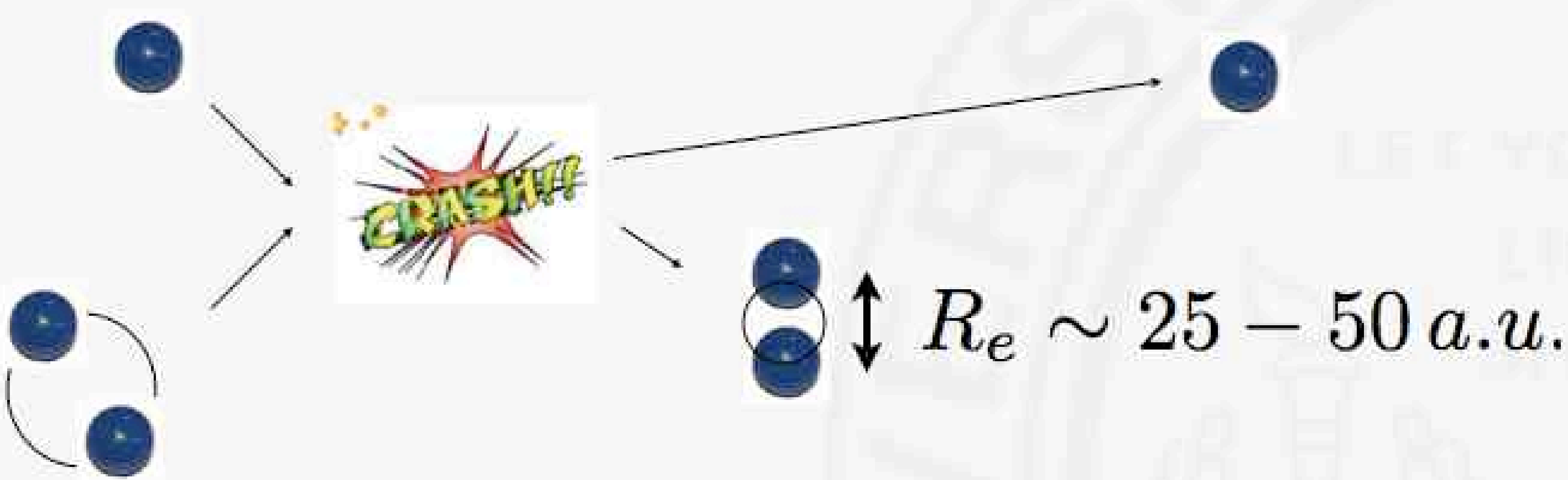
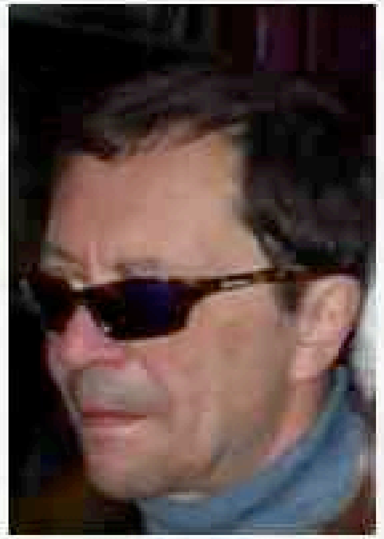


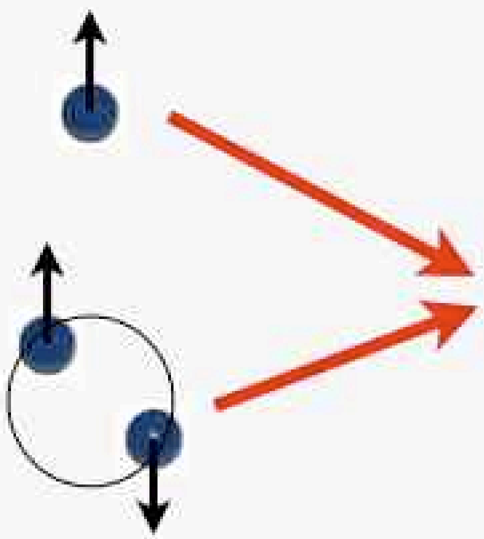
Bottom line:
the molecules are unstable,
with $\tau \sim 2ms$

Origins of instability close to resonance: 3-body recombination



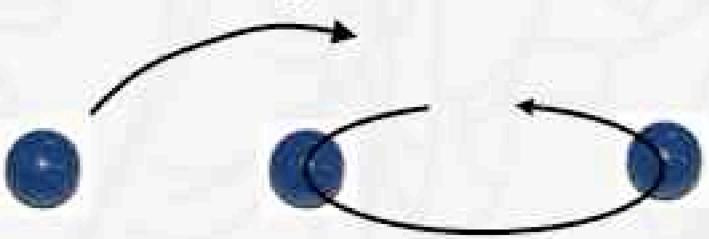


s-wave



Protected by the Pauli principle

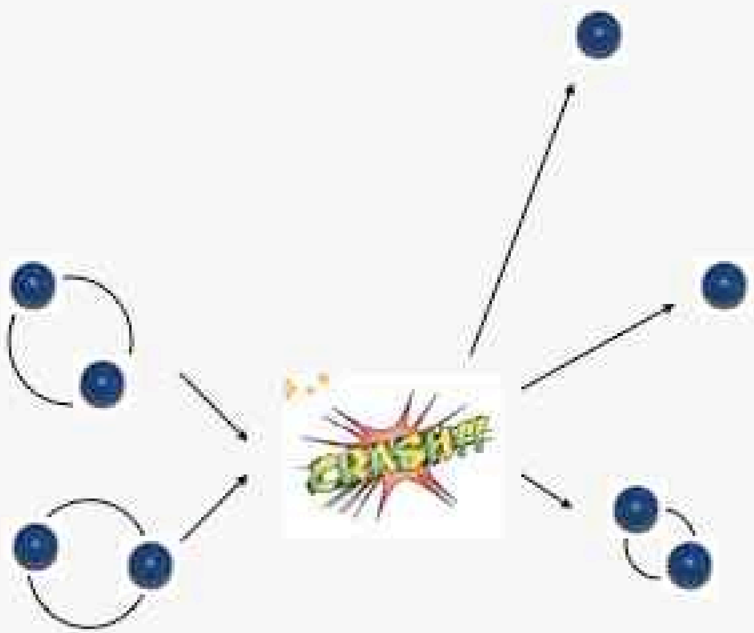
p-wave



By having opposite angular momenta, *p*-wave fermions beat the Pauli principle

Molecule-molecule relaxation, 3D s-wave

Take molecules of size ℓ at an average distance ℓ from each other



$$\Gamma \sim \frac{\hbar}{m\ell^2}\left(\frac{R_e}{\ell}\right)^{2d+2\gamma-2}$$

$$\frac{\ell}{R_e} \sim 200 - 1000$$

$$\Psi_{3\ \mathrm{body}} \sim r^{\gamma}$$

3D s-wave fermions

$$\gamma \approx -0.22$$

$$\Gamma \sim \frac{\hbar}{ma^2}\left(\frac{R_e}{a}\right)^{3.55}$$

Petrov, Salomon, Shlyapnikov (2005)

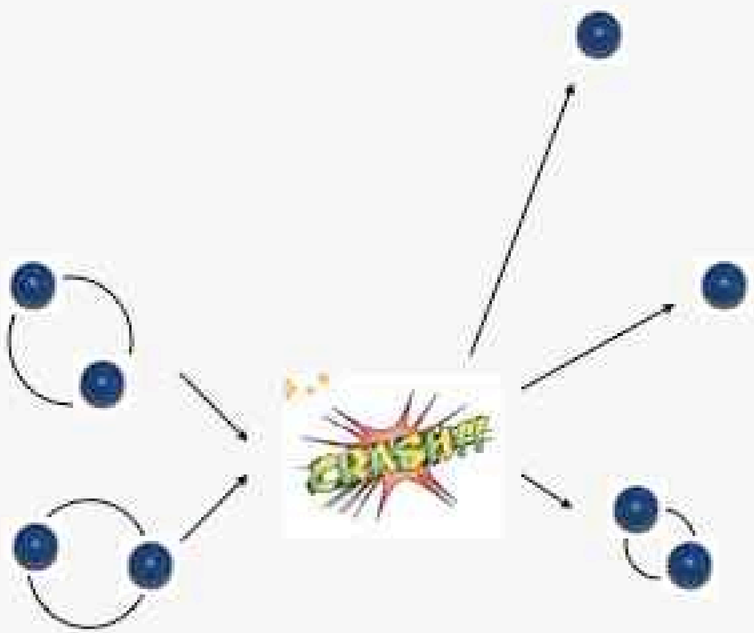
3D s-wave bosons

$$\gamma = -2 \quad \Gamma \sim \frac{\hbar}{m\ell^2}$$

Shlyapnikov (96); Greene (99); Braaten (01)

Molecule-molecule relaxation, 3D p-wave

Take *p*-wave molecules at an average distance ℓ from each other



$$\Gamma = nv\sigma \sim \frac{1}{\ell^3} v R_e^2 \frac{R_e}{v} \frac{\hbar}{m R_e^2}$$

$$\Gamma = \frac{\hbar}{m\ell^2} \frac{R_e}{\ell}$$

$$\frac{\ell}{R_e} \sim 200 - 1000$$

J. Levinsen, N. Cooper, V. Gurarie, PRL (2007)

Also: M. Jona-Lasinio, L Pricoupenko, Y. Castin, PRA (2008)

Summary of the decay rates so far

	3D s-wave bosons	3D p-wave fermions	3D s-wave fermions
Theory: decay rate	$\Gamma \sim \frac{\hbar}{ma^2}$	$\Gamma \sim \frac{\hbar}{ma^2}\frac{R_e}{a}$	$\Gamma \sim \frac{\hbar}{ma^2}\left(\frac{R_e}{a}\right)^{3.55}$
Theory +experiment: lifetime	$\frac{1}{\Gamma} = \tau \sim 0.1ms$	$\frac{1}{\Gamma} = \tau \sim 40ms$ $\tau_{\text{exp}} \sim 2ms$	$\frac{1}{\Gamma} = \tau \sim 10^5 s$

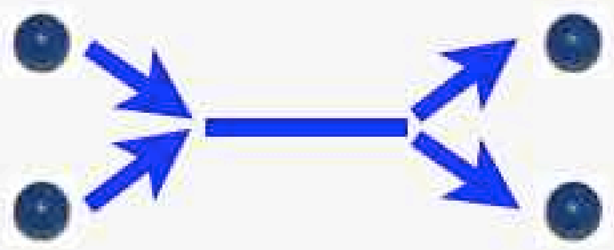
$$\frac{\hbar}{ma^2} \sim 0.1ms$$

"Fermi energy"

$$a \sim 10^4 au \qquad R_e \sim 25au$$

Which lifetime is good enough?

One guesses, long enough for the superfluid to form and to reach the thermodynamic equilibrium. That is, at least, long enough for two atoms to interact...



Calculate the atom-atom scattering amplitude
VG, L. Radzihovsky, Ann. Phys. (2007)

$$f(k) = \frac{k^2}{\frac{m}{R_e}\left(\omega_0 - \frac{k^2}{m}\right) - ik^3}$$

The pole of the scattering amplitude:

$$\frac{k^2}{m} = \omega_0 - i\sqrt{m}R_e\omega_0^{\frac{3}{2}}$$

Typical detuning

$$\omega_0 \sim \frac{\hbar^2}{m\ell^2}$$

Typical interaction rate

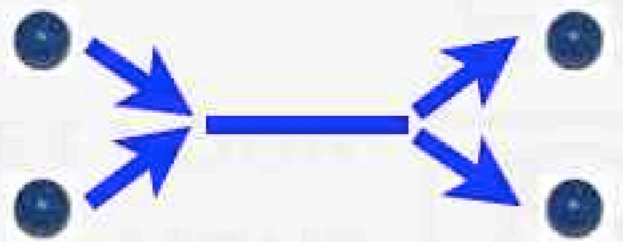
$$\Gamma_{\rm int} = \mathrm{Im}\,\frac{k^2}{m} = \frac{\hbar^2}{m\ell^2}\frac{R_e}{\ell}$$

Disaster: typical interaction rate equals typical decay rate

$$\Gamma_{\rm int} \sim \Gamma$$

What happens for the 2D p-wave

In 2D the p-wave molecules are large



$$\Gamma \sim \frac{\hbar}{m\ell^2}\left(\frac{R_e}{\ell}\right)^{2d+2\gamma-2}$$

Calculate the atom-atom scattering amplitude for 2D p-wave

$$f = \frac{1}{-\sqrt{\frac{k}{2\pi}}\left(\log\left[\frac{1}{R_e^2 k^2}\right] - \frac{m\omega_0}{k^2}\log\left[\frac{1}{R_e^2 m\omega_0}\right]\right) - i\sqrt{\frac{\pi k}{2}}}$$

In 2D γ=-1

$$\frac{k^2}{m} \approx \omega_0 - i\frac{\pi\omega_0}{\log\left[\frac{1}{R_e^2 m\omega_0}\right]}$$

$$\Gamma \sim \frac{\hbar}{m\ell^2}$$

$$\Gamma_{\rm int} = \frac{\hbar}{m\ell^2}\frac{\pi}{\log\left[\frac{\ell^2}{R_e^2}\right]}$$

$$\Gamma \gg \Gamma_{\rm int}$$

The situation is even worse than 3D: the decay rate is faster than the interaction rate.

J. Levinsen, N. Cooper, VG, submitted to PRA

Quasi-2D geometry

In a quasi-2D geometry (with the confinement "width" *d*), the interaction rate is the same as in 2D, while the decay rate is the same as in 3D.

$$\Gamma = \frac{\hbar}{ma^2}\frac{R_e}{a} = \frac{\hbar n_{3D} R_e}{m} = \frac{\hbar n_{2D} R_e}{md} = \frac{\hbar}{ma_{2D}^2}\frac{R_e}{d}$$

Transverse confinement width

$$\Gamma_{\text{int}} = \frac{\hbar}{ma_{2D}^2}\frac{\pi}{\log\left[\frac{a_{2D}^2}{R_e^2}\right]}$$

$$\frac{R_e}{d} \ll \frac{\pi}{\log\left[\frac{a_{2D}^2}{R_e^2}\right]}$$

$$\Gamma \ll \Gamma_{\text{int}}$$

In this geometry, the decay rate is much smaller than the interaction energy

J. Levinsen, N. Cooper, VG (2008), submitted to PRA

Summary: from 3D to quasi-2D down to 2D

	3D	Quasi-2D	2D
Decay rate	$\Gamma \sim \frac{\hbar}{ma^2}\frac{R_e}{a}$	$\Gamma \sim \frac{\hbar}{ma^2}\frac{R_e}{d}$	$\Gamma \sim \frac{\hbar}{ma^2}$
Interaction rate	$\Gamma_{\text{int}} \sim \frac{\hbar}{ma^2}\frac{R_e}{a}$	$\Gamma_{\text{int}} = \frac{\hbar}{ma^2}\frac{\pi}{\log\left[\frac{a^2}{R_e^2}\right]}$	$\Gamma_{\text{int}} = \frac{\hbar}{ma^2}\frac{\pi}{\log\left[\frac{a^2}{R_e^2}\right]}$
Conclusion	$\Gamma_{\text{int}} \sim \Gamma$ Likely unstable	$\Gamma_{\text{int}} \gg \Gamma$ Could be stable	$\Gamma_{\text{int}} \ll \Gamma$ Likely unstable

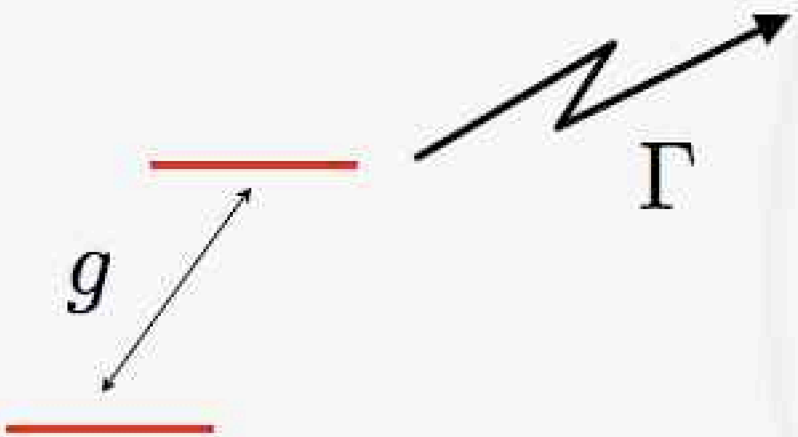
a interparticle separation

$R_e \sim 25\, a.u.$ interaction range

d confinement width

Novel idea: the use of the "quantum Zeno effect"

Consider a two-level system, one of whose levels is strongly unstable



$$H = \begin{pmatrix} 0 & g \\ g & -i\Gamma/2 \end{pmatrix} \qquad \Gamma \gg g$$

$$E_1 \approx -i\frac{2g^2}{\Gamma} \qquad E_2 \approx -i\Gamma/2$$

The two-level system decays at the rate $E_1 \ll g$

H. Bethe (1933); W. Lamb and R. Retherford (1950).

Proposed to use this to enhance stability, G. Rempe et al, Science (2008)

Optical lattices stabilize the superfluid via the "quantum Zeno effect"

Suppose the p-wave molecules move on the lattice. As soon as two bosons occupy one site, they decay at the rate Γ.

$$H_0 = -t \sum_{i,\mu} \left[b_i^\dagger b_{i+\mu} + b_{i+\mu}^\dagger b_i \right]$$

$$H = H_0 - i\frac{\Gamma}{2}P$$

P is the projection operator on states with at least doubly occupied lattices sites

Strategy: eliminate doubly occupied sites within the Brillouin-Wigner perturbation theory

$$H_{\mathrm{BW}} = (1-P)H_0(1-P) - \frac{2i}{\Gamma}(1-P)H_0 P H_0(1-P)$$

Expect the decay to be weak if $\Gamma \gg t$

P. Zoller (2007)

Estimates of the lifetime in 2D

We evaluate the average of the decay term over the ground state

$$\Gamma_{\text{eff}} \sim \frac{t^2}{\Gamma}\frac{N_p}{N}$$

N_p ← Number of particles

N ← Number of lattice sites

and compare it to the Fermi energy of the underlying fermions

$$E_F \sim t\frac{N_p}{N}$$

$$\frac{\Gamma_{\text{eff}}}{E_F} \sim \frac{t}{\Gamma}$$

Thus the decay is slow as long as $t \ll \Gamma$

VG, (2007)

Some realistic numbers (courtesy of E. Bloch)

Lattice site size as low as $l \sim 50nm$

Tunneling as low as $t \sim 50Hz$

Interaction range at least $R_e \sim 1nm$

The decay rates of two bosonic molecules Li_2 sharing a lattice site $\Gamma \sim \frac{\hbar}{ml^2}\frac{R_e}{l} \sim 50KHz$

$$\frac{t}{\Gamma} \sim \frac{1}{1000}$$

Under the conditions of current experiments where E_F=10 KHz, we expect the condensate lifetimes of at least .1s

Conclusions

- p-wave fermionic condensates are (arguably) much more interesting than their s-wave counterparts

- 2D chiral p-wave fermionic condensate is topological, has particles with non-Abelian statistics. Potential impact of observing these particles is hard to overestimate

- Currently p-wave molecules are unstable; confining them to 2D and (especially) putting them on an optical lattice should dramatically increase stability

The end