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## INDETERMINACY AND INCOMPLETE DEFINITIONS\*

**I**ntuitively, a question is *indeterminate* just in case it is unsettled, not merely epistemically, but metaphysically. We ordinarily ascribe indeterminacy by saying that there is no fact of the matter. Some say, for instance, that there is no fact of the matter how many mountains exist. The topographical facts appear to settle that there are some mountains, but not how many.

While it can seem obvious that certain questions are indeterminate, it is remarkably easy, expanding on arguments by Paul Horwich and Timothy Williamson, to reduce an arbitrary ascription of indeterminacy to absurdity.<sup>1</sup> Suppose, for *reductio*, that it is indeterminate—that is, metaphysically unsettled—whether Harry is bald. Because it is metaphysically unsettled whether Harry is bald only if it is not metaphysically settled that Harry is bald and not metaphysically settled that Harry is not bald, it is not metaphysically settled that Harry is bald and not metaphysically settled that Harry is not bald. Because it is true that Harry is bald only if it is metaphysically settled that he is bald, and because it is true that Harry is not bald only if it is metaphysically settled that he is not bald, it is not true that Harry is bald and not true that Harry is not bald. Because Harry is bald only if it is true that he is bald, and because Harry is not bald only if it is true that he is not bald, Harry is not bald and Harry is not not bald. This is a contradiction. Hence, our supposition is false. Moreover, because the question of whether Harry is bald was chosen arbitrarily, the argument generalizes: indeterminacy is impossible.<sup>2</sup>

\*For helpful comments and discussion, I am indebted to David Christensen, Hartry Field, Kit Fine, Christian Lee, the editors of this JOURNAL and, in a special measure, to George Bealer and Adam Pautz.

<sup>1</sup>Horwich, *Truth* (Cambridge: Blackwell, 1990), p. 76; Williamson, *Vagueness* (New York: Routledge, 1994), pp. 187–95.

<sup>2</sup>My argument has a minor advantage over Horwich's and Williamson's. Horwich, *Truth*, p. 76, and Williamson, *Vagueness*, pp. 187–89, argue for bivalence. But, as Williamson

Rejecting this argument requires rejecting at least one of (a) – (c):

- (a) it is true that *S* only if it is metaphysically settled that *S*
- (b) *S* only if it is true that *S*
- (c) modus ponens, conditional contraposability, and reductio ad absurdum are valid rules of inference.

Suppose first that we reject (a). We would in particular need to deny that it is true that Harry is bald only if it is metaphysically settled that he is bald. But how could it possibly be true that Harry is bald if nothing in the fabric of reality *settles* that he is bald? If the more basic facts about Harry's hair situation, as well as those of his comparison class, together with all the facts about our use of the word 'bald', fail to settle that Harry is bald, then it is hardly plausible that it might be *true* that he is bald. Suppose, then, that we instead reject (b). We would in particular need to deny that Harry is bald only if it is true that he is bald. But how could Harry possibly be bald if it is not true that he is bald? What more could be required for it to be true that Harry is bald than for him to be bald? Suppose, then, that we instead reject (c). We would in particular need to deny, of at least one of three of the most basic forms of inference, that it is valid when reasoning about even the most mundane of topics. The costs of rejecting (c) hardly need emphasis.

We are faced with a dilemma: take the anti-indeterminacy argument at face value and look for a way to explain away our pro-indeterminacy intuitions, or take our intuitions at face value and give up on one or more of (a) – (c). While many are inclined toward the latter route, a significant few, including Williamson, go the former route.

Those of us who go the former route must explain away the pro-indeterminacy intuitions. The sources of these intuitions can be di-

acknowledges, pp. 194–95, the proponent of indeterminacy can accept bivalence by identifying a natural nonepistemic interpretation of 'determinately' according to which, possibly, some utterance is true or false but not determinately true or determinately false. Unable to find such an interpretation, Williamson concludes that indeterminacy is impossible. But a number of authors at least implicitly suggest such an interpretation: *x* is *determinately F* iff<sub>df</sub> it is metaphysically settled that *x* is *F*. (See James Cargile, "The Sorites Paradox," *British Journal for the Philosophy of Science*, xx (1969): 193–202; Richmond Campbell, "The Sorites Paradox," *Philosophical Studies*, xxvi (1974): 175–91; J.A. Burgess, "The Sorites Paradox and Higher-Order Vagueness," *Synthese*, lxxv (1990): 417–74; Horwich, *Truth*, pp. 88–90; Vann McGee and Brian McLaughlin, "Distinctions without a Difference," *Southern Journal of Philosophy*, xxxiii (1995, Supplement): 203–51; Harty Field, "Indeterminacy, Degree of Belief, and Excluded Middle," *Noûs*, xxiv (2000): 1–30.) This interpretation allows proponents of indeterminacy to escape Horwich's and Williamson's arguments. The argument I formulate in the text is designed to block this escape.

vided into two categories: vagueness and nonvagueness related. Williamson and others have tried to explain away the vagueness-related intuitions, in part by providing indeterminacy-free accounts of vagueness. On the epistemic account, for instance, vagueness is a type of ignorance.<sup>3</sup> On the contextualist account, vagueness is a type of shifting context.<sup>4</sup> And on the nonreductionist account, vagueness is *sui generis*.<sup>5</sup> In addition to providing a positive indeterminacy-free account of vagueness, proponents of this route need to locate the confusion that gives rise to pro-indeterminacy intuitions.<sup>6</sup> Suppose, then, that vagueness-related pro-indeterminacy intuitions can be explained away.

The next step toward justifying the anti-indeterminacy route is to explain away nonvagueness-related pro-indeterminacy intuitions. I aim in this paper to do this for intuitions related to explicitly incomplete definitions. Williamson attempts to explain away these intuitions, but my approach is different.<sup>7</sup>

First I state the intuitive case from incomplete definitions to indeterminacy, and I say why Williamson's approach is unsatisfactory (section I). Then I argue that incomplete definitions do not give rise to indeterminacy, and I provide my own explanation of our initial pro-indeterminacy intuitions. I do this first for singular terms (section II), then for predicates (section III). I apply our results to two cases: first I show how Robert Brandom and Hartry Field go wrong in drawing a pro-indeterminacy conclusion from the incomplete definition of a singular term;<sup>8</sup> then I show how Scott Soames goes wrong in drawing a pro-indeterminacy conclusion from the incomplete definition of a predicate (section IV).<sup>9</sup> Finally, I consider how my view bears on three issues: the apparent ubiquity of incomplete

<sup>3</sup> See, for example, Roy Sorenson, *Blindspots* (New York: Oxford, 1988), and *Vagueness and Contradiction* (New York: Oxford, 2001); and Williamson, *Vagueness*.

<sup>4</sup> See, for example, Diana Raffman, "Vagueness without Paradox," *Philosophical Review*, CIII (1994): 41–74; and Delia Graff, "Shifting Sands: An Interest-Relative Theory of Vagueness," *Philosophical Topics*, xxviii (2000): 45–81.

<sup>5</sup> See my "Is Vagueness *Sui Generis*?" *Australasian Journal of Philosophy* (forthcoming).

<sup>6</sup> For a sample of attempts, see Williamson, "Imagination, Stipulation, and Vagueness," in *Philosophical Issues, Volume 8: Truth*, E. Villanueva, ed. (Atascadero, CA: Ridgeview, 1997), pp. 215–28; Sorenson, *Vagueness and Contradiction*; and my "Is Vagueness *Sui Generis*?"

<sup>7</sup> Williamson, "Imagination, Stipulation, and Vagueness."

<sup>8</sup> Brandom, "The Significance of Complex Numbers for Frege's Philosophy of Mathematics" (paper read at Proceedings of the Aristotelian Society, 1996); Field, "Indeterminacy, Degree of Belief, and Excluded Middle."

<sup>9</sup> Soames, "Presupposition," in F. Guenther and D. Gabbay, eds., *Handbook of Philosophical Logic*, Volume 4, *Topics in Philosophy of Language* (Boston: Reidel, 1989), pp. 553–616, and *Understanding Truth* (New York: Oxford, 1999).

definitions in natural language, the Problem of the Many, and the uniqueness problem for arithmetic (section v).

I. THE INTUITIVE CASE FROM INCOMPLETE DEFINITIONS  
TO INDETERMINACY

Let me introduce the expression ‘nice\*’ by uttering only two sentences:

- (1)  $n$  is nice\* if  $n > 15$   
 (2)  $n$  is not nice\* if  $n < 15$  (for natural numbers  $n$ ).<sup>10</sup>

Initially, it seems that 16 is nice\*, that 14 is not nice\*, and that it is indeterminate whether 15 is nice\*.

The intuition that some numbers, including 16, are nice\* motivates the standard view of incomplete definitions, on which predicates introduced by incomplete definitions typically succeed in expressing properties (by ‘properties’ I include relations). On this view, ‘is nice\*’ expresses a property, namely, that of being nice\*.

The question then arises: Which property is the property of being nice\*? Is it the property of being a natural number greater than 15 or being a natural number greater than 14? The intuition that it is indeterminate whether 15 is nice\* motivates the dominant answer to this question, namely, that there is no fact of the matter.<sup>11</sup>

Williamson defends a minority answer.<sup>12</sup> In line with the standard view that ‘is nice\*’ expresses a property, but in opposition to the pro-indeterminacy route, he suggests that being nice\* is the property of being a natural number greater than 15. He says, “...given the choice between regarding an atomic predicate as unspecific and so regarding its negation, we generally prefer the latter” (*ibid.*, p. 224). He then proposes the following principle:

SPEC: All other things equal, the application conditions of an atomic predicate are as specific as possible (*ibid.*, p. 224).

<sup>10</sup> I derive this example from an example in Kit Fine’s “Vagueness, Truth and Logic,” *Synthese*, xxx (1975): 265–300, p. 266.

<sup>11</sup> See, for example, Fine, “Vagueness, Truth and Logic”; Horwich, “The Nature of Vagueness,” *Philosophy and Phenomenological Research*, LVII (1997): 929–35; Soames, “Presupposition,” and *Understanding Truth*; Jamie Tappenden, “The Liar and Sorites Paradoxes: Toward a Unified Treatment,” this *JOURNAL*, XC, 11 (November 1993): 551–77.

For ease of exposition, I formulate the debate in terms of whether certain predicates express properties and, if so, whether there is indeterminacy as to the identities of these properties. The nominalistically inclined reader may reformulate the debate in ontologically more neutral terms, for example, as a debate over whether certain predicates have application conditions and, if so, whether incompleteness in these conditions leads to the possibility of indeterminacy in the application of the predicate.

<sup>12</sup> Williamson, “Imagination, Stipulation, and Vagueness.”

Williamson never says exactly what it takes for application conditions to be “as specific as possible.” He does however apply SPEC to a case similar to our own. From this application, I infer that, by an application of SPEC, ‘is nice\*’ should express the property of being a natural number greater than 15, which in some sense has a more specific range of application than the property of being a natural number greater than 14. According to Williamson, then, our initial pro-indeterminacy intuition arises out of our failure to recognize a key asymmetry between ‘nice\*’ and ‘not nice\*’, namely, SPEC.

For three reasons, I cannot accept Williamson’s proposal. First, the key notion of specificity has not been clearly defined. Second, SPEC strikes me as ad hoc. Third, SPEC has little or no impact on the force of my initial pro-indeterminacy intuition: even after considering SPEC, I still find it strongly intuitive that, if ‘is nice\*’ expresses a property, there is no fact of the matter which property that is.

I think the opponent of indeterminacy can do better. What is needed is a credible indeterminacy-free account of explicitly incomplete definitions that deflates the force of our initial pro-indeterminacy intuitions. In my view, such an account lies in the rejection of the standard view that predicates like ‘is nice\*’ express properties. In the next two sections I defend a positive indeterminacy-free rival to the standard view, and I show why our initial pro-indeterminacy intuitions either (i) do not commit us to genuine indeterminacy or (ii) are based on a simple confusion.

## II. INCOMPLETE DEFINITIONS OF SINGULAR TERMS

So far, we have considered an intuitive case for indeterminacy from incompletely defined predicates. Now I want to consider a similar case for indeterminacy from incompletely defined singular terms. I will show why it fails and explain away the intuition behind it. Then, in the next section, I will apply the same strategy to the predicate case.

Let me introduce the singular term ‘Bitz’ by uttering only one sentence:

- (3) Let Bitz be a resident of New York.

Now I ask you to consider whether Bitz is a resident of the U.S.A. and whether Bitz is female. Initially, it seems that Bitz *is* a resident of the U.S.A. and that there is no fact of the matter whether Bitz is female. This example illustrates the intuitive case for indeterminacy from incompletely defined singular terms.

In response to the case, let me begin by drawing your attention to the following uniqueness constraint: a singular term refers only if features of its use determine a unique referent for it (that is, only if

they determine, of some unique thing, that that thing is its referent.<sup>13</sup>) This constraint has the air of a truism. By definition a singular term purports to refer to a single thing: if it has a referent, it has a unique referent. And it is a platitude about meaning that words have their semantic features determined solely by features of their use (where use is construed broadly, to include both speaker intentions and relations to the environment). Hence, if a singular term refers, features of its use must determine a unique referent for it. (Do not confuse this constraint with the outright rejection of indeterminacy of reference; it does not exclude indeterminacy as to *which* object is so uniquely determined.) We have what appears to be a trivial constraint on reference for singular terms.<sup>14</sup>

Yet in certain contexts we are inclined to act as if there were no such constraint. For instance, even though it is blatantly obvious that the uniqueness constraint on reference for 'Bitz' is unmet, we are inclined to utter 'Bitz is a resident of the U.S.A.' and 'There is no fact of the matter whether Bitz is female'. What explains this inclination?

One hypothesis is that we have stumbled upon a counterexample to the constraint: although nothing in our use of 'Bitz' distinguishes among the millions of candidate referents—the residents of New

<sup>13</sup> Why is the parenthetical qualification needed? Because I understand the constraint to be incompatible with supervaluationism. Yet there may be a version of supervaluationism that is compatible with one understanding of the constraint. On this version, on each admissible precisification of our language, for every singular term *t*, the semantic value of 'determines that *t* refers to' is a relation that holds between our use of *t* and exactly one thing; accordingly, the sentence 'a singular term refers only if features of its use determine a unique referent for it' may be deemed *determinately true*. So, to capture my intent, it is not enough to say that a singular term refers only if features of its use determine a unique referent for it. To fully capture the constraint I have in mind, one must say that a singular term refers only if features of its use determine, of some unique thing, that *that* thing is its referent. On this understanding, the supervaluationist must reject the constraint, for it requires an *x* which, on every precisification, satisfies the open sentence 'Features of our use of "Bitz" determine that "Bitz" refers to *x*'; and there is no such *x*.

<sup>14</sup> Is the availability of the method of supervaluations evidence against this constraint? No. Of course, *if* the constraint is false, *then* the method of supervaluations might provide a satisfactory semantical account for singular terms (see Bas van Fraassen, "Singular Terms, Truth-Value Gaps, and Free Logic," this JOURNAL, LXIII, 17 (September 15, 1966): 481–95). But the mere existence of the method does not count against the constraint. To the contrary, the apparent triviality of the constraint counts against applying the method. To be sure, there is also initial reason to apply the method, namely, the presence of pro-indeterminacy intuitions. But it is the presence of these intuitions, and not the availability of the formal method, that must be addressed by the opponent of indeterminacy.

For a discussion of the uniqueness requirement on reference for theoretical terms, see David Lewis, "How to Define Theoretical Terms," this JOURNAL, LXVII, 13 (July 9, 1970): 427–46. In opposition to Carnap, Lewis contends that if a theory is either unrealized or multiply realized, its theoretical terms fail to refer.

York—one such candidate is nevertheless the referent of ‘Bitz’. It is of course indeterminate which such candidate is the referent, and this is why it seems indeterminate whether Bitz is female. Still, merely by uttering (3) I succeeded in securing a referent for ‘Bitz’. If correct, this is a remarkable result: not only can a singular term refer if features of its use fail to determine a *unique* referent, it can refer even if features of its use fail to narrow the candidate referents to fewer than 20 million!

Let me suggest a more plausible hypothesis. From the perspective of my audience, the most rational way to interpret my utterance of (3) is as an act of *supposition*, rather than as a genuine attempt at reference fixing. For if my utterance were a genuine attempt at reference fixing, I would be grossly incompetent: I would be under the illusion that I could secure a referent for a singular term simply by narrowing the candidate referents to 20 million. My audience might have reason to think that I am incompetent, but not *that* incompetent. A reasonable interpretation of my utterance of (3), then, is as an act of *supposing* that ‘Bitz’ refers to a resident of New York.

Given that my utterance was prefaced by, “Let me introduce a new term, ‘Bitz’,” it is clear that the purpose of my supposition is not to investigate the epistemic possibility that there is a unique person named ‘Bitz’ who resides in New York. Its purpose, rather, is to facilitate discussion of being a resident of New York. There is a well-established practice of investigating properties by reasoning *as if* we have concrete instances in mind. We prefer concrete instances because we are better at concrete reasoning; we avoid *actual* concrete instances because we do not want to risk tainting our investigation with extraneous detail. When a singular term is introduced with no attempt to narrow its candidate referents to one, it is reasonable in many contexts to interpret the introduction as an instance of this practice.

Here is a second illustration of the practice:

Maxwell: What do you know about the living conditions of children in Nigeria?

Emma: Let Frib be a five-year-old child in Nigeria. Can you guess what the chances are that Frib suffers from malnutrition?

Maxwell: I would say at least 30%.

Emma: Wrong! There is no chance at all. For there is no such person as Frib. Are you so naïve as to think that I can actually refer to a child in Nigeria just by saying ‘Let Frib be a five-year-old child in Nigeria’?

Maxwell: I was only cooperating. I did not take *you* to be so naïve as to think that you could refer to a child in Nigeria so easily; nor

did I take you to be out to deceive me. I could only figure that you wanted me to go along with you in *supposing* that 'Frib' referred to a five-year-old child in Nigeria. I thought that your aim was to teach me something about being a child in Nigeria by reasoning with me *as if* we had a concrete instance in mind. Clearly you anticipated that I would reason this way and thus act *as if* I thought there was such a person as Frib; otherwise you would not have set the trap in the first place. Of course, you would only have anticipated this had you believed that I was rational and cooperative. And so I am flattered.

For a third illustration of the practice, suppose that we are discussing mathematics and I say, "Let  $a$  be an odd number." It would seem misguided for you to interpret my utterance as a genuine attempt at reference fixing. It is not as if I am raising my arms to the heavens and asking the gods to make ' $a$ ' refer to some odd number.<sup>15</sup> Given that I make no attempt to narrow the candidate referents of ' $a$ ' to one, a rational interpretation of my act is as an act of *supposing* that ' $a$ ' refers to an odd number. The supposition is part of the preliminary groundwork for doing mathematics; its purpose is to set the background for reasoning about the property of being odd.

Two qualifications are in order. First, this proposal is not intended to apply to the mathematical practice of stipulating that a newly introduced function is *undefined* in certain cases. Following Frege, we can avoid postulating incompleteness here by taking *undefined* to be a further value (or by means of some other artificial assignment). Second, the proposal is not intended to apply in the context of formal logic. In this context, it might be suggested that constants are always treated as successfully referring singular terms. On this suggestion, an instance of ' $[\phi(a)]$ ' in an application of existential elimination qualifies, not as a supposition about a vacuous term, ' $a$ ', namely, that it refers to something that satisfies  $\phi(x)$ , but rather as a supposition about an actual referent of ' $a$ ', namely, that it satisfies  $\phi(x)$ . I doubt that this is what is going on informally when we say such things as "Let  $a$  be an odd number." It is not as if we have it in mind that  $a$  is actually some number, we know not which, and we are going to suppose, of this number, that it is odd. To emphasize the point, consider a proof that there is no largest prime which begins, "Let  $a$  be the largest prime number." Surely we do not think of ourselves as supposing, of some unknown number, that *it* is the largest prime.

<sup>15</sup> Fine, *Reasoning with Arbitrary Objects* (New York: Blackwell, 1985), would interpret my act as an attempt to secure an "arbitrary object" as the referent of ' $a$ '.

Returning to the ‘Bitz’ example, suppose that I were to make it clear that my act of uttering (3) was *not* an act of supposition: I did not mean to invite my audience to *suppose*, along with me, that ‘Bitz’ refers to a resident of New York. Rather, I meant to bring it about by my very act of uttering (3) that ‘Bitz’ *in fact* came to refer to some resident of New York. I suspect that the initial temptations to say that Bitz is a resident of the U.S.A. and that it is indeterminate whether Bitz is female would disappear. I suspect they would be replaced by the temptation to say that ‘Bitz’ fails to refer and that ‘Bitz is a resident of the U.S.A.’ and ‘Bitz is female’ are therefore either false or meaningless (or whatever is the right thing to say on the correct theory of vacuous names). It is too hard to believe that an utterance of (3) might by itself provide a referent for ‘Bitz’. One might as well hold that by uttering ‘Let “zibot” mean something’ an actual meaning might be secured for ‘zibot’; or that by uttering ‘Let “Ball #1” refer to one of the two balls in this urn’ an actual referent might be secured for ‘Ball #1’. Although it is easy to *suppose* that ‘Bitz’ refers to a resident of New York, that ‘zibot’ means something, or that ‘Ball #1’ refers to one of the two balls in the urn, surely some effort is required to *actually make these states of affairs obtain*.<sup>16</sup>

One might object that I have at most established that ‘Bitz’ fails to “refer determinately” and that although effort might be required to “secure determinately” a referent for a new term, little or no effort is required to secure a referent for a new term. On this picture, my utterance of ‘Let Bitz be a resident of New York’ sufficed to secure, but not to “secure determinately,” a referent for ‘Bitz’. This objection is misplaced. While a consequence of a certain view of indeterminacy is that indeterminacy as to the referent of ‘Bitz’ is consistent with ‘Bitz’ referring, just not determinately, my claim is that it is intuitively *not* indeterminate to whom ‘Bitz’ refers, as it is intuitive—in light of our considerations about supposition—that ‘Bitz’ fails to refer to anyone.

To establish an indeterminacy of reference one must perform two subtasks, which appear to be in tension. First, one must show that the term in question has properties sufficient to distinguish a *unique*

<sup>16</sup>To be sure, we can define a new relation, say, *reference\**, as that which obtains between a singular term and a thing iff<sub>df</sub> the thing is one of possibly many things described or demonstratively picked out as part of the introduction of the term. (Cf. Field’s notion of partial reference, “Quine and the Correspondence Theory,” *The Philosophical Review*, LXXXIII (1974): 200–28.) And we can say that a singular term *determinately-refers\** to a thing iff<sub>df</sub> it refers\* only to that thing. Note: *determinately-refers\** ≠ *determinately-refers\**. ‘Bitz’ then refers\* to every resident of New York and *determinately-refers\** to no one. Nothing, however, has been gained in terms of establishing an instance of indeterminacy, for no person is such that it is indeterminate whether ‘Bitz’ refers\* to her.

object as its referent. Second, one must show that the question of which object is so distinguished is *indeterminate*. In other words, one must establish that something is more qualified than all others to be the referent of the term *and* that there is no fact of the matter *which* thing is so qualified.<sup>17</sup> For singular terms like 'Bitz', 'Frib', and 'Ball #1', the first subtask cannot be completed.

How, then, should we treat our initial pro-indeterminacy intuition reports? This depends on whether the reports are made from within, or from outside, the scope of the supposition that 'Bitz' refers to a resident of New York. Reports made from within can be taken at face value. For they only commit us to genuine indeterminacy *on the supposition* that 'Bitz' refers to a resident of New York. And there is good reason to reject this supposition. Pro-indeterminacy reports made outside the scope of the supposition must be rephrased. Given that my audience and I have common knowledge that 'Bitz' is a new word that does not refer to anyone, we have common knowledge that the purpose of the supposition is not to explore the epistemic possibility that there is a person named 'Bitz' who resides in New York, but rather to investigate the property of being a resident of New York. Given this purpose, my asking a question that is obviously underdetermined by the supposition, for instance whether Bitz is female, is inappropriate. To draw attention to this fact, my audience might disengage from the supposition and misreport the sound intuition *that there is no suppositional fact of the matter* (that is, that the supposition underdetermines the matter) as the unsound intuition *that there is no fact at all of the matter*. Indeed, in light of the internal difficulties with the notion of indeterminacy, it is well advised that we disengage in this way rather than threaten the coherence of our discussion by remaining in the suppositional context and ascribing indeterminacy.

Let me summarize our results so far. The following constraint seems truistic: a singular term refers only if features of its use determine a unique referent for it. Yet, when a singular term is introduced with no effort to narrow its candidate referents to one, we are

<sup>17</sup> This task is pursued in a series of articles by Field. On the basis of difficulties resolving issues of reference prior to significant theoretical developments, Field argues that referential indeterminacy has plagued, and likely continues to plague, some of the most central terms of basic science and mathematics. See Field, "Theory Change and the Indeterminacy of Reference," this JOURNAL, LXX, 14 (August 16, 1973): 462–81, "Quine and the Correspondence Theory," pp. 200–28, "Are Our Logical and Mathematical Concepts Highly Indeterminate?" *Midwest Studies in Philosophy*, Volume 19 (Minneapolis: Minnesota UP, 1994), pp. 391–429, "Disquotational Truth and Factually Defective Discourse," *The Philosophical Review*, CIII (1994): 405–52, and "Indeterminacy, Degree of Belief, and Excluded Middle," *Noûs*, xxiv (2000): 1–30.

inclined to act as if this constraint does not exist: we act as if the newly introduced term successfully refers, but that there is indeterminacy as to the identity of its referent. This behavior can be reconciled with the truistic air of the constraint: in light of the fact that the introduction of the term explicitly lacks the degree of determinacy required of an act of reference fixing, we charitably interpret it as an act of supposition; then, in the spirit of cooperation, we suppose that the new term refers. Within the scope of this supposition, we are inclined to say things that would normally commit us to indeterminacy and to a counterexample to the uniqueness constraint; however, because these things are said within the scope of the supposition, and because the supposition is not true, no such commitments are made. Outside the scope of the supposition, we might misreport the sound intuition *that there is no suppositional fact of the matter* as the unsound intuition *that there is no fact at all of the matter*.

### III. INCOMPLETE DEFINITIONS OF PREDICATES

We began by considering the intuitive case for indeterminacy from incompletely defined predicates; I gave reasons to reject Williamson's treatment of this case. Next we considered the intuitive case for indeterminacy from incompletely defined singular terms. By focusing our attention on the uniqueness constraint for singular terms, and by explaining away the relevant pro-indeterminacy intuitions, I argued that this case fails.

Now I want to return to the case of predicates and argue, in similar fashion, that it fails. First I defend a uniqueness constraint on what it takes for a predicate to express a property. Then I acknowledge that, in the context of an explicitly incomplete definition, we are inclined to act as if we reject this constraint and accept indeterminacy as to which property is expressed by the new predicate. Using the same strategy as above, I argue that these dispositions need not commit us either to a violation of the uniqueness constraint or to an instance of indeterminacy.

Here is the constraint: a predicate expresses a property only if features of its use determine, of some unique property, that that property is expressed by the predicate.<sup>18</sup> To my mind, this constraint

<sup>18</sup>To stay neutral on whether there is a meaningful property/concept distinction, I formulate the constraint in terms of properties rather than concepts. One who believes in such a distinction may consider a stronger constraint: a predicate expresses a property only if features of its use determine, of some unique concept of some unique property, that that concept is expressed by the predicate. And one who doubts the existence of properties and/or concepts may reformulate the constraint in ontologically more neutral terms; see note 11.

has as much initial appeal as the corresponding constraint on reference for singular terms. After all, if a predicate '*F*' succeeds in expressing a property, there is a unique property such that '*F*' expresses it, namely, the property of *being F*. Because the expressing relation is just another meaning relation, and because words stand in meaning relations only by virtue of how they are used, a predicate expresses a property only if features of its use determine, of some unique property, that that property is expressed by the predicate. So it seems to me that the uniqueness constraint for predicates is on a par with that for singular terms.

Despite this reasoning, it might be suggested that the uniqueness constraint for predicates is less plausible than that for singular terms and should indeed be rejected. The suggestion might be defended as follows. Singular terms do not have genuine definitions (at most they have associated reference-fixing descriptions); predicates do. Some definitions are partial. Partial definitions give rise to incomplete application conditions. Because a partially defined predicate *has* application conditions, it expresses a property. Because its application conditions are *incomplete*, no property is such that the predicate uniquely expresses it. Hence, a predicate might express a property even though no property is such that our use of the predicate uniquely determines, of it, that it is expressed by the predicate. Because singular terms do not have genuine definitions, no corresponding argument impugns the uniqueness constraint for them; hence, we should reject the uniqueness constraint for predicates but not for singular terms.

This defense fails. For if there were such things as partially defined predicates as characterized above, then we could use them, not to *define* singular terms, but to *fix their reference*. And if we could do this, we could fix the reference of a singular term without our use determining, of any unique thing, that that thing is the referent of the term.

For illustration consider the predicate 'is nice\*'. Our use of 'nice\*' determines that, if 'is nice\*' expresses a property, it expresses either the property of being a natural number greater than 15 or the property of being a natural number greater than 14; but nothing in our use determines, of one property or the other, that 'is nice\*' expresses it. Now suppose that 'is nice\*' expresses a property, thus violating the uniqueness constraint for predicates. Then we can derive a violation of the uniqueness constraint for singular terms by introducing a new singular term, say, 'Dice\*', as follows:

(4) Let Dice\* be the least nice\* number.

'Dice\*' then refers either to 16 or to 15, yet no features of its use determine, of 16 or of 15, that 'Dice\*' refers to it. By supposing an

arbitrary counterinstance to the uniqueness constraint for predicates, we have derived a counterinstance to the uniqueness constraint for singular terms.

There is a systematic method for such derivations. It employs a connection between predicates and certain of their nominalizations: if a predicate ' $F$ ' expresses a property, it expresses the property of being  $F$ . In other words, 'being  $F$ ' nominalizes the predicate ' $F$ ': it is a singular term that purports to refer to whatever is expressed by ' $F$ '. Suppose, then, that ' $F$ ' expresses a property even though its use does not determine, of any unique property, that that property is expressed by it. Then there is such a property as being  $F$ . Hence 'being  $F$ ' refers to a property, yet nothing in its use determines, of a unique property, that that property is its referent. We can systematically derive counterinstances to the uniqueness constraint for singular terms given counterinstances to the uniqueness constraint for predicates.

The rejection of the uniqueness constraint for predicates leads, then, to the rejection of the uniqueness constraint for singular terms. Unless we are willing to admit that a singular term can refer without features of its use narrowing its candidate referents to one, we should not admit that a predicate can express a property without features of its use narrowing the candidate properties to one. Our earlier defense of the uniqueness constraint for singular terms thus stands as a defense of the uniqueness constraint for predicates.

To be sure, one might draw the conclusion that something must be wrong with our earlier defense of the uniqueness constraint for singular terms. But it is incumbent on the proponent of this conclusion to say what exactly is wrong. After all, our defense consists of more than just the intuitive appeal of the uniqueness constraint; it includes an explanation of apparent violations of the constraint. And there is available a similar explanation of apparent violations of the uniqueness constraint on predicates. Why, in the context of an explicitly incomplete definition, are we inclined to act as if we reject the uniqueness constraint for predicates and accept indeterminacy as to which property is expressed by the new predicate? The answer is the same as for singular terms: we are conditioned by mathematics and other domains of abstract discourse to treat those acts of word introduction that explicitly lack the degree of determinacy required of successful acts of meaning fixing as acts of supposition.

Applied to the original 'nice\*' example, we are inclined to interpret (1) and (2) as suppositions. In the spirit of cooperation, we *suppose* (1) that  $n$  is nice\* if  $n > 15$  and (2) that  $n$  is not nice\* if  $n < 15$ . We are then disposed to accept that 16 is nice\*, that 14 is not nice\*, and that it is indeterminate whether 15 is nice\*. Outside the scope

of any supposition, this disposition would commit us to a violation of the uniqueness constraint for predicates and to an instance of indeterminacy. However, because it obtains within the scope of a supposition, and because the supposition is not true, no such commitment is made. The supposition is not true because there is no such property as being nice\*: the uniqueness constraint for 'is nice\*' is not satisfied; hence 'is nice\*' fails to express a property.

As in the case of singular terms, rather than ascribing indeterminacy from within the scope of the supposition, we might disengage from the suppositional context and say that there is no fact of the matter whether 15 is nice\*, in which case we would be misreporting the sound intuition that there is no *suppositional* fact of the matter—that is, that the supposition fails to settle the matter.

Suppose, then, that I were to make it clear that my utterance of (1) and (2) was *not* an act of supposition, but a genuine attempt at securing a property as the meaning of 'is nice\*' and thus as the referent of 'being nice\*'. Merely by uttering (1) and (2), I intended to make them true by bringing it about that 'is nice\*' *in fact* came to express either the property of being a natural number greater than 15 or the property of being a natural number greater than 14. I suspect that the initial temptations to say that 16 is nice\* and that it is indeterminate whether 15 is nice\* would disappear. I suspect they would be replaced by the temptation to say that 'is nice' fails to express a property and that '16 is nice\*' and '15 is nice\*' are therefore either false or meaningless (or whatever is the right thing to say on the correct theory of predicates that fail to express properties).

This concludes my treatment of incomplete definitions.

#### IV. BRANDOM, FIELD, AND SOAMES ON INDETERMINACY AND INCOMPLETE DEFINITIONS

I have defended what I shall call *the strict view* of incomplete definitions, on which incompletely defined singular terms fail to refer, and incompletely defined predicates fail to express properties. Now I want to apply the strict view to two examples from the literature.

The first concerns singular terms. Robert Brandom and Hartry Field ask us to imagine that a population of speakers separates from ours and that our respective mathematicians independently develop complex-number theory.<sup>19</sup> The only difference is that our mathematicians choose '*i*' and '*-i*', whereas theirs choose '*\*' and '*/*', to signify

<sup>19</sup> Brandom, "The Significance of Complex Numbers for Frege's Philosophy of Mathematics"; Field, "Indeterminacy, Degree of Belief, and Excluded Middle."

the two square roots of  $-1$ . Because the two roots are structurally identical, it is alleged by Brandom and Field to be indeterminate how ' $i$ ' and ' $-i$ ' translate ' $\backslash$ ' and ' $/$ '. Field says, "It isn't that there is a subtle fact as to 'the correct translation' that we can never know, it is that there is simply no determinate fact of the matter: the whole idea of a unique 'correct translation' is misconceived" (*ibid.*, p. 3). This claim, Field notes, goes hand in hand with the claim that it is indeterminate, of each of the four symbols, to which square root of  $-1$  it refers.

To me it seems doubtful that ' $i$ ' and ' $-i$ ' even purport to refer in the first place (though whether they do is clearly an empirical matter). What a term purports to do depends on the intentions of those who introduce it and, if changes in meaning and reference are possible, of those experts who subsequently use it. Given that the experts who use ' $i$ ' recognize the impossibility of distinguishing the two square roots of  $-1$ , it seems to me unlikely that they intend to use ' $i$ ' so as to *refer* to one of the two roots. After all, if the experts recognize *their* inability to distinguish between two objects, how could they reasonably expect *their terms* to do it for them? More likely, I think, their use of ' $i$ ' involves some sort of linguistic supposition. Richard Feynman for instance makes explicit his intention to use ' $i$ ' under the supposition that it refers to a specific square root of  $-1$ . He says:

Let us suppose that a specific solution of  $x^2 = -1$  is called something, we shall call it  $i$ ;  $i$  has the property, by definition, that its square is  $-1$ .... Someone could write  $i$ , but another could say, 'No, I prefer  $-i$ . My  $i$  is minus your  $i$ .' It *is* just as good a solution, and because the only definition that  $i$  has is that  $i^2 = -1$ , it must be true that any equation we can write is equally true if the sign of  $i$  is changed everywhere.<sup>20</sup>

Upon considering Brandom's hypothetical scenario, Feynman might either (i) correctly report *from within the scope of his supposition* that there is no fact of the matter whether ' $i$ ' and ' $\backslash$ ' corefer; in which case he would commit to the proposition that the supposition underdetermines whether ' $i$ ' and ' $\backslash$ ' corefer, but not to an instance of indeterminacy; or (ii) misreport *from outside the scope of his supposition* the sound intuition *that there is no suppositional fact of the matter whether ' $i$ ' and ' $\backslash$ ' corefer* as the unsound intuition *that there is no fact of the matter whether ' $i$ ' and ' $\backslash$ ' corefer*. On the strict view, speaking outside the

<sup>20</sup> Feynman, *The Feynman Lectures on Physics*, Volume 1 (New York: Addison-Wesley, 1963), p. 188.

supposition, the fact of the matter is that ‘*i*’ and ‘\’ do not refer and therefore do not corefer. This view is consistent with the fact that practicing mathematicians would never, and should never, deny that  $i^2 = -1$ ; for because the supposition is part of the preliminary groundwork for doing mathematics, practicing mathematicians are always practicing within its scope.

We turn to an example concerning predicates. Soames discusses an explicitly incomplete definition similar to that of ‘nice\*’.<sup>21</sup> He says:

Imagine the predicate ‘smidget’ being introduced into a language by the following semantic stipulation...

- (i) Any adult human being under three feet in height is a smidget.
- (ii) Any adult human being over four feet in height is not a smidget (or is such that it is not the case that he/she is a smidget).

... An assertive utterance of ‘Jack is a smidget’ will convey to one’s hearers the information that Jack is an adult under three feet tall and an assertive utterance of ‘Jack is not a smidget’ will convey the information that Jack is an adult over four feet tall. In short, ‘smidget’ will enter the language as a useful and meaningful predicate.

The interesting thing about the predicate is, of course, that the defining conditions for something to be a smidget, and for something to fail to be a smidget, are not jointly exhaustive. Adults between three and four feet tall cannot be correctly characterized either as being smidgets or as not being smidgets.<sup>22</sup>

There is a temptation to go along with Soames in saying that there is no fact of the matter whether adults between three and four feet tall are smidgets. But we need to ask whether this temptation arises in the context of a supposition, and if so whether that supposition is true.

Are we merely supposing (i) and (ii), or are we genuinely convinced of their truth? Before accepting (i) and (ii), do we seriously consider whether Soames has the wherewithal to *stipulate* them—to make them true merely by uttering them? Or do we simply go along with him in the spirit of cooperation? Soames prefaces (i) and (ii) by saying, “Imagine the predicate ‘smidget’ being introduced into a language by the following semantic stipulation.” On a reasonable interpretation, he is asking us to *suppose* that ‘smidget’ is introduced by (i) and (ii) *and* that this act of introduction qualifies as a stipulation. It would therefore be reasonable to react to the example by supposing, rather than evaluating, (i) and (ii).

<sup>21</sup> Soames, “Presupposition,” and *Understanding Truth*.

<sup>22</sup> Soames, “Presupposition,” pp. 584–85.

It is true that, within the scope of this supposition, an assertive utterance of ‘Jack is a smidget’ will convey to one’s hearers the information that Jack is an adult under three feet tall, and an assertive utterance of ‘Jack is not a smidget’ will convey the information that Jack is an adult over four feet tall. It is also true that, within the scope of this supposition, an assertive utterance of ‘It is indeterminate whether Jack is a smidget’ will convey the information that Jack is an adult between three and four feet tall. And so even if our entire language community were to acknowledge that ‘smidget’ is introduced by supposition, rather than by stipulation, Soames would be correct in saying that “‘smidget’ will enter the language as a useful and meaningful predicate.” It would enter the language as a *meaningful* predicate, not in the sense that it would actually express a property, but in that acts of predicating it would—in the context of the community-wide supposition—be *meaningful*: they would convey information. All of this is consistent with the impossibility of indeterminacy: if the supposition is false, then ascriptions of indeterminacy within its scope do not carry genuine commitments to indeterminacy.

The question remains: Is the supposition false, or does Soames have the wherewithal to stipulate (i) and (ii)? Soames defends versions of (i) and (ii): “Surely there is no a priori reason why the advantages of introducing a predicate by stipulations of the sort just illustrated must always be outweighed by the potential disadvantages.”<sup>23</sup>

If by ‘advantages’ Soames has in mind *practical* advantages, and if by ‘stipulations’ he simply has in mind the *acts* of uttering (i) and (ii) regardless of whether they really qualify as stipulations, then we may agree. For we need not deny the potential practical value of introducing a predicate by supposing that it expresses a property without supposing of any particular property that the predicate expresses it.

But if by ‘advantages’ Soames has in mind features conducive to the success of an act of meaning fixing, and if by ‘stipulations’ he has in mind acts that rightly qualify as stipulations, then we should disagree. For there *is* an a priori reason why the advantages must always be outweighed by the disadvantages: (i) and (ii) fail to meet the uniqueness constraint on predicates. Contrary to what Soames appears to believe, there are substantive a priori constraints on stipulations involving new words. Speakers cannot just stipulate anything they want about the meanings of new words. I for instance cannot stipulate that ‘Bitz’ refers to a resident of New York simply

<sup>23</sup> Soames, *Understanding Truth*, p. 164.

by uttering ‘Let Bitz be a resident of New York’. I can *suppose* that ‘Bitz’ refers to a resident of New York, but some effort is required to actually make it true. Applying our earlier results to Soames’s example, we know that if he can stipulate (i) and (ii), then he can also stipulate of a certain singular term that it refers simply by narrowing its candidate referents to a certain uncountable infinity; the relevant term is ‘being a smidget’; the candidate referents are all properties of the form *being an adult human under  $k$  feet in height*—where  $k$  ranges over the reals in the closed interval  $[3,4]$ . But if he could do this, then the uniqueness constraints on both predicates and singular terms could be grossly violated. And we have not only an a priori argument in defense of these constraints but an explanation as to why, in contexts just like the one Soames creates, we are initially inclined to behave as if we reject the constraints. I conclude that the best treatment of Soames’s example does not require positing indeterminacy.

#### V. IMPORT FOR A THEORY OF NATURAL AND ARITHMETICAL LANGUAGE

The strict view of incomplete definitions provides for a plausible indeterminacy-free treatment of a certain family of cases, including the ‘nice\*’, ‘Bitz’, ‘*i*’, and ‘smidget’ cases. Still, one might worry that the view has unacceptable consequences for a theory of natural and arithmetical language. In particular, one might worry that it precludes a credible treatment of three issues: the apparent ubiquity of incomplete definitions in natural language; the Problem of the Many; and the uniqueness problem for arithmetic.

The first worry is that, because natural language is replete with implicit incompleteness of the sort explicit in our ‘nice\*’ example, the strict view leads to a widespread error theory: If the correct thing to say about ‘is nice\*’ is that it fails to express a property, then the correct thing to say about most predicates of our language is that they too fail to express properties. And if the correct thing to say about ‘Bitz’ is that it fails to refer, then the correct thing to say about most singular terms of our language is that they too fail to refer. But then much of what we say is either false or meaningless; and this is unacceptable.

I suspect that this worry depends on the assumption that vagueness is a type of incompleteness in the implicit definitions of our words. Vagueness is pervasive in natural language. But remember that I began the paper by saying that my goal was to show that incomplete definitions present no obstacle to the anti-indeterminacy route *on the assumption that vagueness presents no obstacle*. Moreover, positive incompleteness-free accounts of vagueness are available: the indeterminacy-free accounts of vagueness mentioned at the start of

our discussion all avoid postulating incompleteness as the source of vagueness. Finally, just to whet the reader's appetite, I want to give a phenomenological consideration for thinking that vagueness never in the first place presents as a type of incompleteness.

An appearance of incompleteness manifests itself in the form of missing intuitions: in certain cases, the word neither seems to apply nor seems not to apply; initially, the number 15 *clearly does not seem nice\** and *clearly does not seem not nice\**. By contrast, an appearance of vagueness manifests itself in the form of vagueness in intuition: for certain cases, the word *sort of* seems to apply and *sort of* seems not to apply; a person who seems borderline bald *sort of* seems bald and *sort of* seems not bald. For emphasis, consider a borderline case of 'seems bald'. Suppose that it *sort of* seems to you that Harry is bald. Now I ask: Does it seem to you that it seems to you that Harry is bald? Your candidate answers are as follows:

- (i) clearly so
- (ii) clearly not
- (iii) sort of.

(iii) is the obvious answer. Unlike incompleteness, vagueness presents as vagueness in intuition. Indeed, on the not implausible supposition that meanings of predicates like 'bald' depend on community-wide dispositions to have intuitions about candidate application-conditions, if it is vague whether such a condition suffices for the application of the predicate, it is reasonable to expect vagueness as to whether, upon consideration, the condition would *seem* to us to suffice for the application of the predicate. The fact that vagueness presents as vagueness in, rather than as an absence of, intuition is an initial reason to doubt that vagueness is a type of incompleteness. I should emphasize, however, that while I am persuaded by this consideration, its success is not required for the thesis of this paper.

The second worry is that the strict view precludes a credible solution to Peter Unger's "Problem of the Many."<sup>24</sup> Imagine that from the ground there appears to be a single cloud in the sky. I point toward the sky and say, "Let us call that cloud 'McCloud'." On closer examination, what we find is a relatively dense swarm of water droplets. The swarm has no sharp boundaries; it gradually fades away as the density of its water droplets decreases. The question arises: Which collection of water droplets constitutes McCloud? One is inclined to say that there are many equally good candidate collections. But if the

<sup>24</sup> Unger, "The Problem of the Many," *Midwest Studies in Philosophy*, v (1980): 411–67.

candidates are equally good candidates for constituting McCloud, then they are equally good candidates for constituting *a* cloud: if one of them constitutes a cloud, so do the others. Thus, where there initially appeared to be a single cloud, there are either no clouds or many. The problem, moreover, has nothing in particular to do with clouds; where there initially appears to be a single table, ship, cat, human, or just about any other ordinary kind of physical object, there are either no such objects or many. This is the Problem of the Many.

There is a spectrum of positions on the problem, all but two of which are consistent with the strict view of incomplete definitions. There is the *nihilistic* position, defended by Unger himself: there are no clouds because the concept of a cloud is incoherent (*ibid.*). Clearly this position is compatible with the strict view. Then there are three positions that appear to require some theory of vagueness, but none in particular. First, there is the *restricted composition* position, defended by Ned Markosian: one and only one of the many sets of water droplets composes anything at all, and that thing is a cloud.<sup>25</sup> Second, there is the *relative identity* position, defended by P.T. Geach: the many sets of water droplets are distinct collections of water droplets but one and the same cloud.<sup>26</sup> Third, there is the *constitution is not identity* position, defended by E.J. Lowe and Mark Johnston: the many sets of water droplets compose many distinct collections of water droplets, one and only one of which constitutes a cloud.<sup>27</sup> None of these three positions qualifies as a complete solution, for each leaves open a crucial question. The restricted-composition position leaves open *which* set of water droplets composes a cloud; the relative-identity position leaves open *which* collections of water droplets are one and the same cloud; and the constitution-is-not-identity position leaves open *which* collection of water droplets constitutes a cloud. To each of the three questions, a plausible answer is *that it is vague*. Insofar as incompleteness-free analyses of vagueness are available, these three positions are compatible with the strict view.

Two positions are incompatible with the strict view. The first invokes a particular theory of vagueness. By it:

- (5) Vagueness is a type of incompleteness in the implicit definitions of words.
- (6) The proper semantics of incompletely defined words is super-valuational.

<sup>25</sup> Markosian, "Brutal Composition," *Philosophical Studies*, xcii (1998): 211–49.

<sup>26</sup> Geach, *Reference and Generality* (Ithaca: Cornell, 1980).

<sup>27</sup> Lowe, "The Paradox of the 1,001 Cats," *Analysis*, xlii (1982): 27–30; Johnston, "Constitution Is Not Identity," *Mind*, ci (1992): 89–105.

By the method of supervaluations:

- (7) Clearly, there is one and only one cloud.
- (8) Clearly, McCloud is this cloud.
- (9) It is vague which collection of droplets constitutes McCloud.<sup>28</sup>

Evaluated simply as a proposal for solving the Problem of the Many, and not as a theory of vagueness, this proposal is appealing because it allows us to maintain (7) – (9). But so do all of the aforementioned indeterminacy-free alternatives to treating vagueness as a type of incompleteness. And so, debates over the nature of vagueness aside, it is not a problem for the strict view that it precludes this particular proposal for solving the Problem of the Many.

David Lewis proposes a second solution that is incompatible with the strict view.<sup>29</sup> He maintains (10) – (13):

- (10) Each of the many relevant collections of water droplets constitutes a cloud.

Thus, strictly speaking, where there initially appeared to be a single cloud, there are actually many. However, because the collections of droplets largely overlap one another, they are “almost identical,” and for the purposes of ordinary conversation, this is sufficient for us to count them as one cloud:

- (11) There is one and only one cloud.

So far, the proposal is compatible with the strict view. The question arises, however, whether ‘McCloud’ refers to a cloud, and if so, which one. The proposal continues:

- (12) ‘McCloud’ refers to a cloud.
- (13) It is indeterminate to which cloud ‘McCloud’ refers.<sup>30</sup>

Now the proposal is incompatible with the strict view (insofar as it requires a violation of the uniqueness constraint on reference for singular terms). But I do not see this as a problem for the strict view. I pointed at what appeared to be a single cloud and said, “Let us call

<sup>28</sup> A supervaluationist solution is defended by Brian Weatherson, “The Problem of the Many,” *The Stanford Encyclopedia of Philosophy*, Edward Zalta, ed. (Winter 2005 Edition, <http://plato.stanford.edu/archives/win2005/entries/problem-of-many/>) and, to some extent, by Lewis, “Many, but Almost One,” in John Bacon, Keith Campbell, and Lloyd Reinhardt, eds., *Ontology, Causality and Mind: Essays in Honour of D M Armstrong* (New York: Cambridge, 1993), pp. 164–82; and by McGee and McLaughlin, “The Lessons of the Many,” *Philosophical Topics*, xxviii (2000): 129–51.

<sup>29</sup> Lewis, “Many, but Almost One.”

<sup>30</sup> Lewis, “Many, but Almost One,” p. 36, suggests a supervaluational treatment of denoting expressions such as ‘McCloud’.

that cloud ‘McCloud’.” Supposing now that what appeared to be a single cloud was, strictly speaking, a massive group of partly overlapping clouds, it seems to my mind absurd to think that I successfully referred to one of those clouds. Surely the initial appeal of (12) is contingent on the appearance of a *unique* cloud in the sky. Once we accept (10) and thus accept that, strictly speaking, there was no unique cloud at which I pointed, the initial appeal of (12) disappears. So, rather than presenting a problem for the strict view of incomplete definitions, Lewis’s proposal faces the problem of conflicting with an intuitive core principle of the strict view, namely, the uniqueness constraint on reference for singular terms.

I doubt, then, that the Problem of the Many poses a serious difficulty for the strict view.

The third worry is that the strict view precludes a credible treatment of the uniqueness problem for arithmetic. The problem is that our arithmetical practice appears to underdetermine the identities of the natural numbers. Because the Peano axioms are satisfied by every countable progression of elements of any kind, they alone do not appear rich enough to determine the identities of the numbers, and it is not obvious what other features of our practice might help. The problem is a special underdetermination problem for two reasons: first, standard causal accounts of reference plausibly do not apply to the case of numbers; second, it is hard to see what epistemic access we could have to numbers. There are various versions of the problem. Paul Benacerraf puts it in terms of questions of cross-theory identification;<sup>31</sup> for instance, is 1 identical to  $\{\{0\}\}$  or to  $\{0, \{0\}\}$ ? Harold Hodes puts it in terms of the question of what, if anything, in our use of arithmetical terms could distinguish unique referents for them.<sup>32</sup>

I will not attempt an exhaustive survey of positions on this problem. For present purposes, it will suffice to consider a few broad categories and show that a number of credible options are compatible with the strict view.<sup>33</sup>

First consider positions on which the numerals do not refer. On this view, speaking strictly, literally, and outside the scope of all suppositions, there are no numbers. Proponents of this view may inter-

<sup>31</sup> Benacerraf, “What Numbers Could Not Be,” *Philosophical Review*, LXXIV (1965): 47–73.

<sup>32</sup> Hodes, “Logicism and the Ontological Commitments of Arithmetic,” this JOURNAL, LXXXI, 3 (March 1984): 123–49.

<sup>33</sup> For ease of presentation, I will blur certain distinctions among views in a given category.

pret the Peano axioms as suppositions, or as stipulations in a story of fiction, as Field and Steven Wagner do.<sup>34</sup> Speaking outside the supposition/fiction, there are no such things as numbers. Speaking within the scope of the supposition/fiction, there are numbers, but their identities are indeterminate. Because this ascription of indeterminacy is made within the scope of the relevant supposition/fiction, it does not carry a commitment to genuine indeterminacy, but only to the underdetermination of the question by the supposition/fiction. Clearly this view is compatible with the strict view.

Next consider positions on which the numerals do refer. These positions divide over whether our use of the numerals determines a unique set of referents for them.

There are at least four sorts of views on which our use succeeds in determining a unique set of referents. First, there are the nonreductionist Platonist views—held perhaps by Kurt Gödel<sup>35</sup> and perhaps even by certain formalists, such as David Hilbert<sup>36</sup>—which appeal to a special mode of access to a unique set of primitive entities (namely, the natural numbers) that serve as the referents of the numerals.<sup>37</sup> Second, there is the nonreductionist view of Kit Fine, which appeals to the method of postulation to secure a unique set of primitive entities as the referents of the numerals.<sup>38</sup> Third, there are the reductionist views of Gottlob Frege, Bertrand Russell, Russell and Alfred Whitehead, Mark Steiner, George Bealer, and others, which appeal to features of use beyond the Peano axioms—for instance, ordinary-language idioms of cardinality and perhaps other categorial considerations—to provide evidence for the reduction of the refer-

<sup>34</sup> Field, *Science without Numbers* (New York: Blackwell, 1980); Wagner, “Arithmetical Fiction,” *Pacific Philosophical Quarterly*, LXIII (1982): 255–69.

<sup>35</sup> Gödel, “Russell’s Mathematical Logic,” in Paul A. Schilpp, ed., *The Philosophy of Bertrand Russell* (Evanston, IL: Northwestern, 1944), pp. 125–53.

<sup>36</sup> Hilbert, “Über das Unendliche,” English translation in J. van Heijenoort, ed., *From Frege to Gödel: A Source Book in Mathematical Logic, 1897–1931* (Cambridge: Harvard, 1967), pp. 367–92, originally published in 1926.

<sup>37</sup> By ‘Platonist view’ I mean to pick out a position readily attributed to Plato by those who are willing to set aside certain scholarly issues. Whether Plato actually held that numbers were primitive entities is unclear; for discussion, see Anders Wedberg, *Plato’s Philosophy of Mathematics* (Stockholm, Sweden: Almqvist and Wiksell, 1955); Julia Annas, *Aristotle’s Metaphysics Books M and N* (New York: Oxford, 1976), Introduction; C.D.C. Reeve, *Philosopher-Kings: The Argument of Plato’s Republic* (Princeton: University Press, 1988).

<sup>38</sup> Fine, “The Limits of Abstraction,” in M. Schirn, ed., *The Philosophy of Mathematics Today: Proceedings of an International Congress in Munich* (New York: Oxford, 1988), pp. 503–629, *The Limits of Abstraction* (New York: Oxford, 2002), and “Our Knowledge of Mathematical Objects,” in John Hawthorne and Tamar Gendler, eds., *Oxford Studies in Epistemology: Volume 1* (New York: Oxford, 2006), pp. 89–110.

ents of the numerals to unique reducing entities (for example, sets, properties).<sup>39</sup> Fourth, there are the structuralist views of Benacerraf, Michael Resnik, Stewart Shapiro, and others, which appeal simply to Peano's axioms to identify a unique structure whose "places" or "positions" serve as the referents of the numerals.<sup>40</sup> All four of these views are compatible with the strict view, for none posits a relevant incompleteness in the definitions of the numerals.

Next consider views on which the numerals successfully refer, but their use fails to determine a unique set of referents for them. Here one might hold, for instance, that it is indeterminate which of the rival set-theoretic reductions contemplated by Benacerraf is correct.<sup>41</sup> Or one might endorse a more extreme indeterminacy, as Field suggests, on which it is indeterminate, not only which sets numbers are, should they be sets, but what general sorts of things numbers are in the first place: primitive entities, concepts, properties, sets, or positions in  $\omega$ -sequences.<sup>42</sup> Both positions are incompatible with the strict view, for both entail that the numerals successfully refer even though nothing in our use of them determines a unique set of referents for them. Again, I do not see this as a problem for the strict view. To the contrary, it appears to be a problem for the two positions. If the choice among the candidate sets of referents is indeed arbitrary, then it seems wrong to conclude that one of the sets is in fact the correct candidate, regardless of whether one is then forced to add the further claim that it is indeterminate which such set is the correct candidate. On the supposition that our use underdetermines the referents of numerals, the sensible thing to say is that the numerals fail to refer. To be sure, they might still bear reference-like relations to the candidate referents.<sup>43</sup> But it seems wrong to maintain that they genuinely refer. Field himself seems to acknowledge as much in his move away from his original suggestion (*ibid.*), and toward fictionalism.<sup>44</sup> And let

<sup>39</sup> Frege, *Die Grundlagen der Arithmetik: eine logisch-mathematische Untersuchung über den Begriff der Zahl* (Breslau: W. Koenner, 1884), translated as *The Foundations of Arithmetic: A Logico-Mathematical Enquiry into the Concept of Number* by J.L. Austin (Oxford, UK: Blackwell, 1974, second revised edition); Russell, *The Principles of Mathematics* (New York: Cambridge, 1903); Russell and Whitehead, *Principia Mathematica*, 3 volumes (New York: Cambridge, 1910, 1912, 1913); Steiner, *Mathematical Knowledge* (Ithaca: Cornell, 1975); Bealer, *Quality and Concept* (New York: Oxford, 1982).

<sup>40</sup> Benacerraf, "What Numbers Could Not Be"; Resnik, *Mathematics as a Science of Patterns* (New York: Oxford, 1997); Shapiro, *Philosophy of Mathematics: Structure and Ontology* (New York: Oxford, 1997).

<sup>41</sup> Benacerraf, "What Numbers Could Not Be."

<sup>42</sup> Field, "Quine and the Correspondence Theory."

<sup>43</sup> Cf. Field's notion of partial reference in "Quine and the Correspondence Theory."

<sup>44</sup> Field, *Science without Numbers*, and *Realism, Mathematics, and Modality* (New York: Blackwell, 1989), Introduction.

us not forget that each of the four aforementioned indeterminacy-free alternatives is still very much alive as far as anyone has shown.

I conclude that the uniqueness problem for arithmetic poses no serious difficulty for the strict view of incomplete definitions.

#### VI. CONCLUSION

I have argued that the standard view of incomplete definitions is incorrect: singular terms introduced by incomplete reference-fixing acts do not refer, and predicates introduced by incomplete definitions do not express properties. In place of the standard view, I have developed an indeterminacy-free rival: the strict view of incomplete definitions. My defense of the strict view includes a strategy for explaining away our initial pro-indeterminacy intuitions. If my account is sound, incomplete definitions present no obstacle to taking our opening anti-indeterminacy argument at face value.

One could not at this stage show decisively that indeterminacy can be avoided universally. But if the hardest challenges can be credibly met, we are forced to take a serious look at the view that indeterminacy is impossible.

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