

## *Vague Entailment*

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**ABSTRACT:** On the dominant view of vagueness, if it is vague whether Harry is bald, then it is unsettled, not merely epistemically, but metaphysically, whether Harry is bald. This view entails the following proposition: that clear vagueness as to whether Harry is bald clearly does not entail that Harry is bald. I give an argument against the proposition, and thus against the dominant view.

On the dominant view of vagueness, if it is vague whether Harry is bald, then all the specific facts about the distribution of hair on Harry's head, together with all the facts about Harry's comparison class, together with all the facts about our community-wide use of the word 'bald', fail to settle whether Harry is bald. On the dominant view, if it is vague whether Harry is bald, then *nothing* settles whether Harry is bald—it is unsettled, not merely epistemically, but *metaphysically*, whether Harry is bald.<sup>1,2</sup> Call this view *vagueness-as-indeterminacy*. On vagueness-as-indeterminacy, if it is *clearly* vague whether Harry is bald, then it is *clearly* metaphysically unsettled whether Harry is bald. (Here and throughout my paper, I use 'clearly' in the following neutral manner, to signify the absence of vagueness: *a* is *clearly F* iff (i) *a* is *F* and (ii) it is not vague whether *a* is *F*.) Of course, if it is clearly metaphysically unsettled whether Harry is bald, then, clearly, nothing settles that Harry is bald. Thus, clear vagueness as to whether Harry is bald clearly does not itself settle that Harry is bald. Thus, clear vagueness as to whether Harry is bald clearly

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<sup>1</sup> Some proponents of vagueness-as-indeterminacy hold that all vagueness-related indeterminacy has its source in semantic indeterminacy. On this version of the view, the reason that it is metaphysically unsettled whether Harry is bald is that our linguistic practices fail, in the first place, to settle which question is the question of whether Harry is bald.

<sup>2</sup> Proponents of vagueness-as-indeterminacy include Halldén 1949, Mehlberg 1958, Körner 1960, Zadeh 1965, Cargile 1969, Przelecki 1969, Lewis 1970, Campbell 1974, Kamp 1975, Fine 1975, Burgess 1990, Horwich 1990, 2005, Tye 1994, McGee and McLaughlin 1995, Field 2000, 2003, 2004, Schiffer 2000, 2003, Dorr 2003. Eklund 2005 argues that vagueness entails, but is not a type of, indeterminacy. Smith 2005 defends a degree-theoretic account that appears to entail vagueness-as-indeterminacy (though Smith himself suggests that his theory is neutral between vagueness-as-ignorance and vagueness-as-indeterminacy).

does not *entail* that Harry is bald. So, the following initially appealing proposition is a consequence of vagueness-as-indeterminacy:

**No Entailment**      Clear vagueness as to whether Harry is bald clearly does not entail that Harry is bald.

My aim in this paper is to refute **No Entailment** by arguing for a rival proposition:

**Vague Entailment**      It is vague whether clear vagueness as to whether Harry is bald entails that Harry is bald.

Before I give my argument, I need to make three preliminary points.

The first concerns two ordinary notions that my argument will employ: the notion of *vagueness* and the notion of one thing's *entailing* another. You might be tempted to interpret these notions theoretically. You might for instance be tempted to interpret my use of 'vague' according to vagueness-as-indeterminacy, so that when I say that it is *vague* whether *p*, you interpret me as saying that it is *indeterminate* (for such and such reason) whether *p*. Likewise, you might interpret my use of 'entails' according to some philosophical theory. This would be a mistake. My argument employs the ordinary notions of *vagueness* and *entailment*. Speaking ordinarily, having exactly two hamsters on a leash entails having an even number of hamsters on a leash; the existence of water on Mars entails the existence of H<sub>2</sub>O on Mars; and getting hit by a moving car does not entail getting injured (though it does make it very likely). Because the ordinary notions of vagueness and entailment may differ from their various theoretical interpretations, it is important, when evaluating my argument, that you not interpret either of them theoretically.

The second preliminary point concerns a grammatical construction that my argument will employ: the indicative conditional, or 'if *p*, then *q*'. You might be tempted to interpret this construction as a material conditional—not-*p* or *q*. This would be a mistake. It would be a mistake, for instance, to think that confidence in *q* rationally requires confidence *that, if p, q*, as it would on the material conditional interpretation. My confidence that there are no unicorns does not rationally require confidence that, if

there are unicorns, there are no unicorns. Because I cannot in this limited space adequately argue against the material conditional theory of indicative conditionals, and because the only role that conditionals will play in my argument is to help express claims about necessary and sufficient conditions, readers may simply take it as stipulation that my use of ‘if  $p$ , then  $q$ ’ means ‘ $p$  is sufficient for  $q$ ’, and my use of ‘ $p$  iff  $q$ ’ means ‘ $q$  is necessary and sufficient for  $p$ ’. (To see that this stipulation is inconsistent with the material conditional interpretation, note that my confidence that there are no unicorns does not rationally require confidence that the existence of unicorns is sufficient for there to be no unicorns.)

The third preliminary point concerns a rule of inference that my argument will employ. I call the rule ‘VTC’, for *Vagueness Through (clearly true) Conditionals*:

- (1) Clearly, if  $p$ , then  $q$
- (2) It is vague whether  $p$

Therefore: (3) Either it is vague whether  $q$ , or it is clearly the case that  $q$ .

VTC is valid. For (1) tells us that  $p$  is clearly a sufficient condition for  $q$ , and (2) tells us that it is vague whether this condition obtains. If it is vague, of a condition that is clearly sufficient for  $q$ , whether that condition obtains, then it cannot *clearly* not be the case that  $q$ . It must either be vague whether  $q$  or clearly the case that  $q$ .

Here, then, is my argument for **Vague Entailment**. Throughout the argument, I will use the following abbreviations:  $B$  = that Harry is bald;  $V(B)$  = that it is vague whether Harry is bald;  $CV(B)$  = that it is clearly vague whether Harry is bald.

To help us to see what  $CV(B)$  entails, let us make two simplifying assumptions. First, let us assume that we have a specific context of use of ‘bald’ in mind, to avoid the possibility of shifting standards of use. Second, let us assume that whether one is bald depends only on the number of hairs on one’s head. So, to say that baldness is vague is just to say that it is vague what the maximum number of hairs for being bald is. Clearly, that number is greater than four and less than one trillion. But there is a range of numbers such that it is vague, of each number in that range, whether that number is the

maximum. Of course, due to higher-order vagueness, the identity of this range is itself vague. In other words, if we call the smallest number in this range ‘Low Vague’ and the largest number ‘High Vague’, then it is vague which number = Low Vague, and which number = High Vague. Of course, it is not *contingent* which number = Low Vague and which number = High Vague, for we have fixed on a specific context of use of ‘bald’. To have a number of hairs on one’s head that clearly falls between Low Vague and High Vague (inclusively) is to be such that it is clearly vague whether one is bald. Our question, then, is whether having such a number of hairs entails being bald. We know that it is vague, of each number between Low Vague and High Vague, whether that number is the maximum for being bald. So:

- (4) It is vague whether High Vague is the maximum.

Of course,

- (5) Clearly, if High Vague is the maximum, then having a number of hairs on one’s head that clearly falls between Low Vague and High Vague entails being bald.

By an application of VTC,

- (6) It is either vague whether having a number of hairs on one’s head that clearly falls between Low Vague and High Vague entails being bald, or it is clearly the case that having such a number of hairs entails being bald.

Obviously, it cannot clearly be the case that having such a number of hairs entails being bald, for that is incompatible with our supposition that it is vague, of each number in the given range, whether it is the maximum for being bald. So the first disjunct of (6) must be true: it is vague whether having such a number of hairs entails being bald. But this is just to say that it is vague whether clear vagueness as to whether one is bald entails being bald. Thus, it is vague whether  $CV(B)$  entails  $B$ .<sup>3</sup> Thus, **Vague**

### **Entailment.**

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<sup>3</sup> It is not surprising that it can be vague whether one thing entails another. Suppose that it is vague whether Harry is bald. Now consider the conjunction of all the specific facts about the distribution of hair on Harry’s head, together with any relevant facts about Harry’s comparison class, together with all the facts about our community-wide use of

*Objection.* On certain versions of vagueness-as-indeterminacy (typically those that reject the law of excluded middle), we cannot say that it is vague what the maximum number of hairs for being bald is. The reason we cannot say this is that our use of ‘the maximum number of hairs for being bald’ presupposes that there is a maximum number of hairs for being bald, and on these theories we are not justified in believing in such a thing. Hence, the preceding argument is ineffective, for it rests on the claim that it is vague what the maximum number of hairs for being bald is.

*Response.* The preceding argument *does* rest on the claim that it is vague what the maximum number of hairs for being bald is. And some theories of vagueness evidently cannot accommodate this claim. But surely the fact that these theories cannot accommodate the claim is evidence against the theories, and not evidence against the claim. For, at least prior to theorizing, it would seem to be a basic datum that it is vague what the maximum number of hairs for being bald is.

Nevertheless, for those readers who are not moved by this response, I now offer a completely different argument for **Vague Entailment**—one that does not rest on the claim that it is vague what the maximum number of hairs for being bald is. While I prefer the initial argument, I think that the following argument is also adequate.

This argument requires two further preliminary points.

The first concerns a rule of inference that my argument will employ. I call the rule ‘VTB’, for *Vagueness Through* (clearly true) *Biconditionals*:

(7) Clearly,  $p$  iff  $q$

(8) It is vague whether  $q$

Therefore: (9) It is vague whether  $p$ .

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the word ‘bald’. Whether Harry is bald depends on these more basic facts. And it is vague whether Harry is bald. So it must be vague whether these more basic facts *entail* that Harry is bald. To be sure, there may be theories of vagueness on which it can never be vague whether one thing entails another. But these theories founder on what evidently is a datum for a theory of vagueness: that it *can* be vague whether one thing entails another.

VTB is valid. For (7) tells us that  $q$  is clearly a necessary and sufficient condition for  $p$ . And if it is vague, of a condition that is clearly both necessary and sufficient for some proposition to hold, whether that condition obtains, then it must be vague whether the proposition holds.

The second preliminary point concerns a method for investigating whether a proposition,  $p$ , entails a proposition,  $q$ . The method is to (i) set aside our views on contingent matters, (ii) suppose  $p$ , and (iii) consider the question of whether  $q$ . Clearly, if the answer is  $q$ , then  $p$  entails  $q$ . Clearly, if it is not the case that the answer is  $q$ , then it is not the case that  $p$  entails  $q$ . Thus: clearly, the answer is  $q$  iff  $p$  entails  $q$ . By VTB, if it is vague whether the answer is  $q$ , then it is vague whether  $p$  entails  $q$ .

There is a complication with this method. At least typically (and perhaps without exception), when we suppose  $p$ , we *clearly* suppose  $p$ . For instance, when we suppose that Harry is bald, we *clearly* suppose that Harry is bald; it is not vague whether we are supposing that Harry is bald. Now, part of what it is to suppose  $p$  is to exclude the possibility that not- $p$ . So, clearly supposing  $p$  entails *clearly* excluding the possibility that not- $p$ . But clearly excluding the possibility that not- $p$  entails excluding the possibility that it is vague whether  $p$ . Thus, in effect, to clearly suppose  $p$  is to suppose clearly  $p$ . For illustration, suppose that Harry is bald. Now consider whether Harry is bald. Clearly, he is. How did we derive that something is clearly the case merely by supposing that it is the case? Answer: we *clearly* supposed that it is the case; in effect, we supposed that it is clearly the case. Because our acts of supposing  $p$  are typically acts of *clearly* supposing  $p$ , and because clearly supposing  $p$  is tantamount to supposing *clearly*  $p$ , typically when we take ourselves to be employing the preceding method for investigating whether a proposition,  $p$ , entails a proposition,  $q$ , what we are really doing is investigating whether *clearly*  $p$  entails  $q$ . For what we are really doing is employing the following method: (i) set aside our views on contingent matters, (ii) *clearly* suppose  $p$ , and (iii) consider the question of whether  $q$ . Clearly, the answer is  $q$  iff *clearly*  $p$  entails  $q$ . By VTB, if it is vague whether the answer is  $q$ , then it is vague whether *clearly*  $p$  entails  $q$ .

Let us then employ this method to investigate whether  $CV(B)$  entails  $B$ . First, we set aside our views on contingent matters and clearly suppose  $V(B)$ . In effect, we thereby suppose  $CV(B)$ . Next, we

consider the question of whether  $B$ . Clearly, the answer is  $B$  iff  $CV(B)$  entails  $B$ . By our supposition, it is vague whether the answer is  $B$ . By an application of VTB, it is vague whether  $CV(B)$  entails  $B$ . Thus, **Vague Entailment**.

This concludes my alternative argument for **Vague Entailment**. Unlike my original argument, this argument does not rest on the claim that it is vague what the maximum number of hairs for being bald is.

I have given two arguments for **Vague Entailment**. If we accept that either is sound, we must reject **No Entailment** and those theories of vagueness that recommend it, including vagueness-as-indeterminacy.

The question remains why **No Entailment** is initially appealing. Perhaps its initial appeal lies in our failure to distinguish the idea of one thing's *entailing* another from the idea of one thing's *clearly entailing* another. This failure might lead us to misreport the sound intuition (10) as the unsound intuition (11):

- (10) that vagueness as to whether Harry is bald clearly does not *clearly* entail that Harry is bald
- (11) that vagueness as to whether Harry is bald clearly does not *entail* that Harry is bald.

To my mind, the initial appeal of (11) disappears upon recognizing that we can accept (10), which is consistent with the preceding arguments for **Vague Entailment**, instead of (11), which is not.

If we reject vagueness-as-indeterminacy because we accept **Vague Entailment**, does that mean that we are forced to adopt an epistemic view of vagueness, as Sorenson (1988, 2001) and Williamson (1994) do? No. In my view, **Vague Entailment** fits best with a theory that I develop and defend elsewhere (forthcoming), on which vagueness is neither a species of indeterminacy nor an epistemic phenomenon, but is rather *sui generis*. On this theory, there is a close-knit family of concepts, none of which admits of analysis in terms of concepts outside the family, and at least some of which are essential to an adequate treatment of vagueness-related phenomena. One might hold a similar non-reductionism

about intentional, normative, or modal phenomena. For instance, one might hold that the notions of physical necessity, metaphysical necessity, and causation, together with that of a law of nature, admit of analysis only in terms of one another, if at all. To get a feel for the family of vagueness-related notions, suppose again that it is vague whether Harry is bald. Depending on the source of this vagueness, we might characterize Harry as a *borderline case* of baldness. And we might say that Harry is neither *clearly* bald nor *clearly* not bald. We might say that he is *sort of* bald and *sort of* not bald; and that he both *roughly* qualifies and *roughly* fails to qualify as bald. This, we might say, is due in part to the fact that baldness is a *rough* concept; it bears only a *rough* relation to more basic concepts; and so there is *vagueness* as to what its basic application-conditions are. According to *vagueness-as-sui-generis*, it is impossible to break out of this family of concepts by way of reduction. This view is consistent with the preceding arguments for **Vague Entailment**, for nothing in the idea that vagueness is *sui generis* recommends **No Entailment** over its rival, **Vague Entailment**.<sup>4</sup>

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