

6. MARKET POWER

We have studied competitive markets where there are a large number of firms and each firm takes market prices as given. When a market contain only a few relevant firms, firms may no longer be price takers and they can often affect market prices. These are situations where we say that firms have market power. How are prices (output) determined when firms have market power? How may economic efficiency be affected by the presence of market power? What does all of this mean for public policies? These are some of the economic questions that we are interested in.

6.1 Monopoly

We start from the most familiar case of market power, a firm which is the monopoly of a market. Let the market demand be $x(p)$, the monopolist's cost function be $c(q)$, then the monopolist's problem is

$$\max_p px(p) - c(x(p)).$$

Alternatively, let the inverse demand function be $p(x) = x^{-1}(p)$. We can write the monopolist's problem as

$$\max_{q \geq 0} \pi(q) = qp(q) - c(q).$$

This is the formulation that is usually used.

Assume: $p(\cdot)$ and $c(\cdot)$ are continuous, twice differentiable, and $p'(\cdot) \leq 0$ at all $q \geq 0$; $p(0) > c'(0)$; there exists a unique $q_o \in (0, \infty)$ such that $p(q_o) = c'(q_o)$ (Thus q_o is the competitive, or social optimal output level.).

The first-order condition is

$$p(q_m) + q_m p'(q_m) - c'(q_m) \leq 0, \text{ with equality if } q_m > 0.$$

From the assumption $p(0) > c'(0)$, $q_m > 0$. From $p(q_o) = c'(q_o)$ and $p'(\cdot) \leq 0$, we have

$$p(q) + qp'(q) - c'(q) \leq 0, \text{ for all } q \geq q_o.$$

Therefore we can restrict the monopolist's optimal choice of q to the compact set $[0, q_o]$, and, since $\pi(q)$ is continuous in q , there must exist some q_m that maximizes $\pi(q)$, and q_m satisfies

$$p(q_m) + q_m p'(q_m) = c'(q_m).$$

This is, of course, the familiar condition that marginal revenue equals marginal cost. (Notice that I have not imposed restrictions on $p(\cdot)$ and $c(\cdot)$ that would ensure $\pi(q)$ to be concave, although in applications we often do that.)

Now if $p'(\cdot) < 0$ for all $q \geq 0$, (the demand curve is downward-sloping, as we usually assume,) then

$$p(q_m) > c'(q_m),$$

which implies

$$q_m < q_o.$$

Thus the monopolist charges a price higher than the competitive price, and produces a level of output lower than the competitive one. This creates a welfare loss, called the deadweight loss, to the society, given by

$$\begin{aligned} & \int_0^{q_o} [(p(s) - c'(s))] ds - \int_0^{q_m} [(p(s) - c'(s))] ds \\ &= \int_{q_m}^{q_o} [(p(s) - c'(s))] ds. \end{aligned}$$

The results above about monopoly pricing is well known. One interesting area of research about monopoly in recent years, instead of studying optimal pricing by the monopolist, deals with optimal selling mechanism by a monopolist. Another area that has had significant advances in recent years is optimal regulation of monopolists.

6.2 Static Models of Oligopoly

A situation less familiar to us and also more difficult to analyze, compared to monopoly, is oligopoly. In an oligopoly market, there are only several firms there

exist strategic interactions between firms when they make decisions. Thus the proper tool of analysis is game theory. Unless otherwise indicated, we shall consider only pure strategies in our analysis.

When firms make only one-time simultaneous decisions, we have what are called static models of oligopoly. We start from one of the simplest oligopoly models:

The Bertrand model. Suppose that two firms, 1 and 2, produce a homogeneous product with constant marginal cost $c \geq 0$. Demand is given by $x(p)$, which is continuous and strictly decreasing in all p such that $x(p) > 0$, and there exists some $\bar{p} < \infty$ such that $x(\bar{p}) = 0$ for all $p \geq \bar{p}$. The two firms simultaneously name their prices, p_1 and p_2 , with the resulting sales for firm j given by

$$x_j(p_j, p_k) = \begin{cases} x(p_j) & \text{if } p_j < p_k \\ \frac{1}{2}x(p_j) & \text{if } p_j = p_k \\ 0 & \text{if } p_j > p_k \end{cases}$$

Firm j 's profits are

$$\pi_j = (p_j - c)x_j(p_j, p_k).$$

Proposition 1 *There is a unique Nash equilibrium (p_1^*, p_2^*) in the Bertrand duopoly model. In this equilibrium, both firms set their prices equal to c .*

Proof. To begin, $(p_1^*, p_2^*) = (c, c)$ is a Nash equilibrium since no firm can benefit from deviating to a different price, given the price of the opponent. It remains to show that there is no other Nash equilibrium. First, notice that any price pair where one firm or both firm's prices are below c cannot be a Nash equilibrium. Next, suppose that the two prices named are such that $p_j > p_k = c$. Then by raising the price above c but still below p_j , firm k can increase its profits from 0 to a positive number. Therefore any pair of strategies where one firm's price is equal to c and another firm's price is higher than c cannot be a Nash equilibrium. Next, if the two prices named are such

that $p_j = p_k > c$. Then, by lowering its price by an infinitely small amount $\varepsilon > 0$, firm j would increase its profits from $\frac{1}{2}(p_j - c)x(p_j)$ to $(p_j - c - \varepsilon)x(p_j - \varepsilon)$. Also, if the two prices named are such that $p_j > p_k > c$. Then by lowering its price to some number that is smaller than p_k but bigger than c , firm j would be able to increase its profit from 0 to positive. Therefore any pair of prices that are both higher than c cannot be a Nash equilibrium. ■

The striking result that with only two firms the competitive price is obtained in the Bertrad model is troubling to observers of most markets, and it is often called the Bertrand Paradox.. There are several ways to resolve this paradox.. One is to assume that firms compete in quantities, which is the Cournot model. Another possibility is to assume that there are capacity constraints. Still another approach, the one I think is perhaps most useful, is to relax the extreme sensitivity of consumers regarding price differences between firms. One way to do this is to think firms actually produce differentiated products. Alternatively, beyond the static framework, we can consider dynamic models with repeated consumer purchases, and consumers may be less price sensitive then due to costs of switching suppliers or due to loyalties. This has been an active research area in recent years. Finally, repeated interactions between firms may also enable firms to collude on prices.

Within the static framework, the Cournot model generates equilibrium prices that are higher than marginal costs. Suppose that there are again two firms, 1 and 2, compete in quantities in a market with the inverse demand function being $p(\cdot)$. Assume $p(q)$ is continuous, differentiable, and $p'(q) < 0$ for all $q \geq 0$. The constant marginal cost of each firm is $c \geq 0$. The profits of firm j are

$$\pi_j = q_j p(q_j + q_k) - c q_j.$$

Each firm chooses its own output to maximize its own profit, taking the other firm's

output as given, which gives the following f.o.c.:

$$\frac{\partial \pi_1}{\partial q_1} = p(q_1 + q_2) + q_1 p'(q_1 + q_2) - c \leq 0, \text{ with equality if } q_1 > 0.$$

$$\frac{\partial \pi_2}{\partial q_2} = p(q_1 + q_2) + q_2 p'(q_1 + q_2) - c \leq 0, \text{ with equality if } q_2 > 0.$$

The condition $\frac{\partial \pi_j}{\partial q_j} \leq 0$ defines, implicitly, firm j 's best-response function $q_j = b_j(q_k)$.

A Nash equilibrium is any pair of (q_1^*, q_2^*) such that $q_1^* = b_1(q_2^*)$ and $q_2^* = b_2(q_1^*)$.

Suppose that $(q_1^*, q_2^*) \gg 0$ is a Nash equilibrium of the model, then

$$p(q_1^* + q_2^*) + q_1^* p'(q_1^* + q_2^*) - c = 0,$$

$$p(q_1^* + q_2^*) + q_2^* p'(q_1^* + q_2^*) - c = 0.$$

Therefore,

$$\frac{(q_1^* + q_2^*)}{2} p'(q_1^* + q_2^*) + p(q_1^* + q_2^*) = c.$$

Proposition 2 *In any Nash equilibrium of the Cournot Duopoly model with constant marginal cost c and with positive equilibrium outputs for both firms, the market price is greater than c and smaller than the monopoly price.*

Proof. That $p(q_1^* + q_2^*) > c$ follows from $q_1^* + q_2^* > 0$ and $p'(\cdot) < 0$. To show $p(q_1^* + q_2^*) < p_m$, we need to show that $q_1^* + q_2^* > q_m$. First, we argue that $q_1^* + q_2^* \geq q_m$. If not, then $q_1^* + q_2^* < q_m$. By increasing q_1 to $q_m - q_2^*$, the total profit of the two firms will be higher, while the market price will be lower. This means that firm 2's profit will be lower, while firm 1's profit must therefore be higher, contradicting the assumption that q_1^* is an equilibrium price. Next, if $q_1^* + q_2^* = q_m$, we would have

$$\frac{q_m}{2} p'(q_m) + p(q_m) = c,$$

contradicting the definition of q_m as the solution to

$$q_m p'(q_m) + p(q_m) = c.$$

Hence $q_1^* + q_2^* > q_m$. ■

You should also familiar yourself with the Cournot model with linear demand in the book (page 391-392).

Thus, if firms compete in quantities, two firms will not be enough to generate competitive prices. On the other hand, the joint profit in the Cournot duopoly is lower than the monopoly profit. This is because when a firm increases its output, it reduces the other firm's profit, which is not taken into account when a firm decides its own optimal output.

A nice feature of the Cournot model is that the equilibrium price decreases as the number of firms increases, with the competitive price as its limit. To see this, let the number of firms be J , and $Q = \sum_j q_j$. Then

$$\pi_j = q_j p(Q) - cq_j.$$

At a Nash equilibrium $(q_1^*, \dots, q_J^*) \gg 0$, we have

$$p(Q^*) + q_j^* p'(Q^*) = c, \quad j = 1, \dots, J.$$

Add these equations together, we obtain

$$\frac{Q^*}{J} p'(Q^*) + p(Q^*) = c.$$

Thus $p(Q^*) \rightarrow c$ as $J \rightarrow \infty$.

The problem with the Cournot model is that it is not very natural to think of firms competing in quantities. It is perhaps more natural to think of firms competing in prices. An important paper by Kreps and Scheinkman (*Rand*, 1983) suggests that the Cournot model should be viewed as a model of capacity competition. They show that if firms first compete in capacities and then compete in prices, the Cournot outcome emerges.

This is related to the idea that one way to resolve the Bertrand paradox is that

firms face capacity constraints in the short run. If the most a firm can produce is less than $x(c)$, then the market price will have to be higher than c .

Another way of resolving the Bertrand paradox is to relax the extreme sensitivity of consumers to a small price changes. Within the static framework, this is usually done by assuming that firms produce differentiated products. The best known model in this class is the Hotelling model:

Two firms, producing a homogeneous good, are located at the two ends of a line of unit length. Each firm has a constant marginal cost c . Call the firm on the left end 1 and the firm on right end 2. There is a continuum of consumers of measure M who are uniformly distributed on the line. The unit transportation cost of any consumer is t . Each consumer desires at most one unit of the good and values the good at v . Note that because the transportation cost and the locational differences, the two goods are actually differentiated viewed by consumers.

A consumer located at x is indifferent between purchasing from 1 and 2 if

$$v - p_1 - tx = v - p_2 - t(1 - x),$$

or

$$x = \frac{p_2 - p_1 + t}{2t}.$$

Assuming that $v > c + \frac{3}{2}t$, the demand for 1 and 2 are, respectively:

$$q_1(p_1, p_2) = \begin{cases} 0 & \text{if } p_1 > p_2 + t \\ \frac{p_2 - p_1 + t}{2t} \cdot M & \text{if } p_2 - t \leq p_1 \leq p_2 + t \\ M & \text{if } p_1 < p_2 - t. \end{cases}$$

$$q_2(p_1, p_2) = \begin{cases} 0 & \text{if } p_2 > p_1 + t \\ \frac{p_1 - p_2 + t}{2t} \cdot M & \text{if } p_1 - t \leq p_2 \leq p_1 + t \\ M & \text{if } p_2 < p_1 - t. \end{cases}$$

To look for a symmetric equilibrium, we need only consider each firm's profit function as

$$\pi_1 = (p_1 - c) \frac{p_2 - p_1 + t}{2t} \cdot M,$$

$$\pi_2 = (p_2 - c) \frac{p_1 - p_2 + t}{2t} \cdot M.$$

The first-order necessary conditions for a Nash equilibrium are:

$$\frac{p_2 - p_1 + t}{2t} \cdot M - \frac{p_1 - c}{2t} M = 0,$$

$$\frac{p_1 - p_2 + t}{2t} \cdot M - \frac{p_2 - c}{2t} M = 0.$$

These are also sufficient conditions since it is easy to check that $\frac{\partial^2 \pi_j}{\partial p_j^2} < 0$. Again, each of these two equations defines, implicitly, a best-response function. Solving the equilibrium prices from these two equations, (use the symmetry and let $p_1 = p_2$,) we obtain

$$p_j^* = c + t.$$

Note that under p_j^* all consumers indeed purchase as we have assumed, and therefore the prices do constitute a Nash equilibrium.

What happens if $v < c + \frac{3}{2}t$, or if $v < c + t$? Equilibria still exist but they are of different kinds. In most applications, v is assumed to be sufficiently large so that the equilibrium is the one we have characterized.

6.3 Repeated Interaction

An important element that is missing in a static model is considerations for repeated interactions. Firms often compete with each other more than one period. When a firm undercuts another firm in prices in one period, the other firm may respond by lowering prices in the next period. Oligopoly models of Multiperiod that incorporate such considerations are called dynamic models of oligopoly.

One of the simplest dynamic oligopoly models is a repeated play of a one-shot Bertrand duopoly. At each period, two duopolists, producing a homogeneous product with market demand $x(p)$ and having constant marginal cost c , compete in prices. Time starts at $t = 0$. The strategy of firm j at period t , $t \geq 1$, specifies a price p_{jt} as a function of all past price choices by the two firms, $H_{t-1} = \{p_{1\tau}, p_{2\tau}\}_{\tau=1}^{t-1}$. Suppose the total number of periods is T .

If T is a finite number, then the game is called a finitely repeated game. The finitely repeated Bertrand model has a unique *SPNE*: at every period, each firm charges a price equal to c . This result is obtained by backward induction. At the subgame of period T , the last period, the only *NE* is for both firms to charge c . Now consider the subgame starting at $T - 1$, the only *NE* in this subgame, given the *NE* payoffs at the subgame of the last period, is again for both firms to charge c . Continue this procedure, the only *SPNE* is thus for each firm to charge c at every period.

If T is infinite, the game is called an infinitely repeated game (or a super game). The result in the infinitely repeated Bertrand model is very different from its finitely repeated version. Suppose that the profit of firm j at period t is denoted as π_{jt} , each firm's discount factor is $0 < \delta < 1$, and the discounted sum of firm j 's profits is $\sum_{t=1}^{\infty} \delta^t \pi_{jt}$. Roughly speaking, when the discount factor is high enough, any price between c and the monopoly price charged by both firms at each period can be supported as a *SPNE*.

Proposition 3 *The strategies*

$$p_{jt}(H_{t-1}) = \begin{cases} p_m & \text{if all elements of } H_{t-1} \text{ equal } (p_m, p_m) \text{ or } t = 1 \\ c & \text{otherwise} \end{cases}$$

constitute a SPNE of the infinitely repeated Bertrand duopoly game if and only if $\delta \geq \frac{1}{2}$.

Proof. Since any subgame starting from any time t is itself an infinitely repeated

Bertrand game, we need to show that after any previous history of play, the strategies specified for the remainder of the game constitute a Nash equilibrium of an infinitely repeated Bertrand game.

Given the strategies, we need to be concerned with only two types of histories: those in which there has been a deviation (a price not equal to p_m) and those in which there has been no deviation.

First, if a subgame arises after there has been a deviation, then the strategies call for each player to charge c for each of all the future periods. This pair of strategies constitute a Nash equilibrium in a infinitely repeated Bertrand game since charging c at each period is optimal for a firm given that its opponent always charges c .

Next, consider any subgame starting at time t , following a history that has had no deviation by either firm. We want to show that the specified strategies constitute a Nash equilibrium in this subgame. If firm j plays the proposed strategy, given the proposed strategy of the opponent, j 's sum of profits in this subgame, discounted to time t , is

$$\frac{1}{2}(p_m - c)x(p_m)\frac{1}{1 - \delta}.$$

Now consider a possible deviation by j . Given that any subgame starting from any period has the same structure, if j can benefit from a deviation, then it should make the deviation at the beginning of the subgame, t , which would maximize the benefit from the deviation discounted to t . Moreover, if j ever deviates (charging a price different from p_m), it will receive zero profits for any period after the deviation period. Therefore, if j deviates, the best j can do is to charge a price that is slightly lower than p_m at t , and charge c each period thereafter. The sum of j 's profits discounted to t with such an optimal deviation is

$$(p_m - c - \varepsilon)x(p_m - \varepsilon),$$

where $\varepsilon > 0$ can be arbitrarily small, and thus $(p_m - c - \varepsilon)x(p_m - \varepsilon)$ can be arbitrarily

close to

$$(p_m - c)x(p_m).$$

Therefore, firm j will prefer no deviation to deviation in any subgame starting from t if and only if

$$\frac{1}{2}(p_m - c)x(p_m)\frac{1}{1 - \delta} \geq (p_m - c)x(p_m),$$

or

$$\delta \geq \frac{1}{2}.$$

Thus the proposed strategies constitute an *SPNE* if and only if $\delta \geq \frac{1}{2}$. ■

The type of strategy in the proposition above is called a Nash reversion strategy: firms cooperate until someone deviates, and any deviation triggers a permanent retaliation in which both firms charge c at every period thereafter, the one-period Nash strategy. For this reason, the strategy is also called a trigger strategy.

Note that both firms always charging p_m is not the only outcome that can occur in a *SPNE* of the infinitely repeated Bertrand model. Both firms always charging c at every period is also a *SPNE*, since this pair of strategies constitute a *NE* in any subgame starting from any time t . In fact, we have:

Proposition 4 *In the infinitely repeated Bertrand duopoly model, when $\delta \geq \frac{1}{2}$, repeated choice of any price $p \in [c, p_m]$ can be supported as a *SPNE* outcome path using Nash reversion strategies. On the other hand, if $\delta < \frac{1}{2}$, any *SPNE* outcome path must have all sales occurring at a price equal to c in every period.*

Proof. For the first part of the result, the proof is the same as in the case of $p = p_m$, which has been shown earlier. We thus need only show that if $\delta < \frac{1}{2}$, any *SPNE* outcome path must have all sales occurring at a price equal to c in every period.

Let $v_{jt} = \sum_{\tau \geq t} \delta^{\tau-t} \pi_{j\tau}$ denote firm j 's profits in any subgame starting from period t and discounted to period t , when the equilibrium strategies are played from t onward. Define also $\pi_t = \pi_{1t} + \pi_{2t}$.

First, we must have $\pi_t \leq v_{jt}$ for $j = 1, 2$ and every t , since by slightly undercutting the lowest price in the market at period t , j can obtain a payoff that is arbitrarily close to π_t .

Suppose that there exists at least one period t in which $\pi_t > 0$. We will derive a contradiction. There are two cases to consider:

Case 1. Suppose that there is a period τ with $\pi_\tau > 0$ such that $\pi_\tau \geq \pi_t$ for all t . If so, then

$$2\pi_\tau \leq (v_{1\tau} + v_{2\tau}).$$

But

$$\begin{aligned} v_{1\tau} + v_{2\tau} &= \sum_{s \geq \tau} \delta^{s-\tau} (\pi_{1s} + \pi_{2s}) = \sum_{s \geq \tau} \delta^{s-\tau} \pi_s \\ &\leq \sum_{s \geq \tau} \delta^{s-\tau} \pi_\tau = \frac{1}{1-\delta} \pi_\tau \\ &< 2\pi_\tau, \text{ if } \delta < \frac{1}{2}, \text{ a contradiction.} \end{aligned}$$

Case 2. Suppose, instead, that for any period t , there is a period $\tau > t$ such that $\pi_\tau > \pi_t$. Define $\tau(t)$ recursively as follows: $\tau(1) = 1$; and for $t \geq 2$, let $\tau(t) = \min\{\tau > \tau(t-1) : \pi_\tau > \pi_{\tau(t-1)}\}$. Since for any t , π_t is bounded above by $\pi_m = (p_m - c)x(p_m)$, and the sequence $\{\pi_{\tau(t)}\}_{t=1}^\infty$ is monotonically increasing. Hence when $t \rightarrow \infty$, $\pi_{\tau(t)}$ must converge to some $\bar{\pi} \in (0, \pi_m]$ such that $\pi_t < \bar{\pi}$ for all t . Therefore, we have

$$v_{1\tau(t)} + v_{2\tau(t)} \leq \frac{1}{1-\delta} \bar{\pi} \text{ for all } t.$$

On the other hand, since

$$2\pi_{\tau(t)} \leq v_{1\tau(t)} + v_{2\tau(t)},$$

we have

$$2\pi_{\tau(t)} \leq \frac{1}{1-\delta} \bar{\pi},$$

or,

$$\pi_{\tau(t)} \leq \frac{2}{1-\delta} \bar{\pi}, \text{ for all } t,$$

which, for sufficiently large t , cannot hold if $\delta < \frac{1}{2}$. ■

The result that there are infinitely many equilibria is bothersome, and it is a problem with infinitely repeated game in general. In fact, the Folk Theorem says that any outcome in a infinitely repeated game that gives a player a discounted payoff of more than the lowest payoff he can guarantee himself in a one period game can be supported by a *SPNE*. This makes it difficult to make predictions in an infinitely repeated game.

One possible treatment is to look at the equilibrium that obtains the collusive outcome in the game, using a focal point argument. Within this framework, much of the research has been concerned with the issue of when firms are more likely to sustain a collusive outcome: how easy a defection may be detected; at what stages of a business cycle; in a expanding market or a declining market, etc.

Another approach is to restrict to Markov strategies, strategies that can depend on history only through some state variables in the most recent period. The sub-game perfect equilibrium is then called a Markov perfect equilibrium. This can often significantly narrow down the set of equilibrium strategies.

6.4 Entry

In our analysis of monopoly and oligopoly so far, we have assumed that the number of firms is given. We now relax this assumption and consider two questions: how is the equilibrium number of firms determined in these markets, and how does this number compare to the social optimal one?

Suppose that there are infinitely many identical potential firms that can enter a market. We model the entry and competition as a two stage game. In the first stage, each firm simultaneously decides “In” or “Out”. In the second stage, the firms who are in the market play some oligopoly game.

Assume that an entrant incurs a sunk cost K , and at the second stage oligopoly

game there is a unique Nash equilibrium in which each firm's equilibrium profit, excluding K , is π_J , where J is the number of firms in the market. Assume that π_J decreases in J , and $\pi_J \rightarrow 0$ as $J \rightarrow \infty$. The subgame perfect Nash equilibrium number of J , J^* , is determined by

$$\pi_{J^*} \geq K$$

and

$$\pi_{J^*+1} < K.$$

Under our assumption, J^* exists uniquely. (Notice, though, that the identity of the J^* firms is not unique- there are many equilibria.)

We now consider equilibrium entry with Cournot competition. Suppose that at the post-entry game each firm's cost is $c(q) = cq$, market demand is $p(q) = a - bq$, where $a > c \geq 0$, and $b > 0$. The stage 2 output per firm, q_J , and the profit per firm, π_J , are

$$q_J = \left(\frac{a-c}{b} \right) \left(\frac{1}{J+1} \right),$$

$$\pi_J = \left(\frac{a-c}{J+1} \right)^2 \left(\frac{1}{b} \right).$$

Notice that π_J decreases in J and approaches zero as J approaches ∞ . Now let \hat{J} solves

$$\left(\frac{a-c}{\hat{J}+1} \right)^2 \left(\frac{1}{b} \right) = K,$$

or

$$\hat{J} = \frac{a-c}{\sqrt{bK}} - 1.$$

Then

$$J^* = \max\{N : N \leq \hat{J} \text{ and } N \text{ is an integer}\}.$$

Next, consider equilibrium entry with Bertrand competition. Again assume that at the post-entry game each firm's cost is $c(q) = cq$, market demand is $p(q) = a - bq$, where $a > c \geq 0$, and $b > 0$. Now $\pi_1 = \pi_m$, but $\pi_J = 0$ for $J \geq 2$ (so this is not the

case of π_J we typically assume of). If $\pi_m > K$, we must have $J^* = 1$. Thus, when the number of firms is endogenous, the perspective of more intense competition can actually result in lower competition.

We now turn to the question of how the equilibrium number of firms compares to the one that would maximize social welfare, taking as given the form of competition in the post-entry game.

Suppose that firms compete in a homogeneous-good market. Let q_J be the symmetric equilibrium output per firm when there are J firms in the market. The inverse demand function is $p(\cdot)$. Each firm's equilibrium profit is $\pi_J = p(Jq_J)q_J - c(q_J)$. Assume $c(0) = 0$.

Social welfare is

$$W(J) = \int_0^{Jq_J} p(s)ds - Jc(q_J) - JK.$$

Let the J that maximizes $W(J)$ be J^o .

Suppose that the post-entry game is Cournot competition. Treat, for the moment, J as a continuous variable.

$$\begin{aligned} W'(J) &= p(Jq_J)[q_J + J\frac{dq_J}{dJ}] - [c(q_J) + Jc'(q_J)\frac{dq_J}{dJ}] - K \\ &= \frac{a+cJ}{J+1} \left[\left(\frac{a-c}{b}\right)\left(\frac{1}{J+1}\right) - J\left(\frac{a-c}{b}\right) \cdot \frac{1}{(J+1)^2} \right] - c\left(\frac{a-c}{b}\right)\left(\frac{1}{J+1}\right) \\ &\quad + Jc\left(\frac{a-c}{b}\right)\frac{1}{(J+1)^2} - K \\ &= \frac{a+cJ}{J+1}\left(\frac{a-c}{b}\right)\left(\frac{1}{J+1}\right)^2 - c\left(\frac{a-c}{b}\right)\left(\frac{1}{J+1}\right)^2 - K \\ &= \left(\frac{a-c}{b}\right)\left(\frac{1}{J+1}\right)^2 \frac{a+cJ-cJ-c}{J+1} - K \\ &= \frac{1}{b} \cdot \frac{(a-c)^2}{(J+1)^3} - K = 0. \end{aligned}$$

That is,

$$(\bar{J} + 1)^3 = \frac{(a-c)^2}{bK}.$$

Or

$$(\hat{J} + 1) = (\bar{J} + 1) \cdot \sqrt{(\bar{J} + 1)}$$

Therefore, for $\bar{J} \geq 2$, $\hat{J} \geq (\bar{J} + 1) \cdot \sqrt{3} - 1 > \bar{J} + 1$, which implies that $J^* > J^o$.

This tells us that with imperfect competition in the production stage, there can also be distortions in entry. Government interventions that reduce entry could be welfare improving as a second best solution. In general, though, depending on industry conditions, both too much or too little entry can occur in oligopoly markets.

The two-stage entry model we have developed so far has uniqueness in equilibrium number of firms, but the equilibria we have characterized say very little about which firms are “In” and which are “Out”. That is, the equilibria are not symmetric. An alternative approach is to look at a symmetric equilibrium, in which each firm randomizes between “In” and “Out”.

Another way of modeling entry is to think that firms make entry and price (or output) decisions at the same time. That is, there is no sunk cost. The fixed cost is incurred only if a firm produces. This collapses the two-stage model into one stage. In this case, the cost function of a firm is

$$C(q) = \begin{cases} K + c(q) & \text{if } q > 0 \\ 0 & \text{if } q = 0 \end{cases}$$

Now if firms compete in Bertrand fashion, and assume that

$$p > [K + cx(p)]/x(p)$$

for some p , the only Nash equilibrium will have one firm setting price equal to

$$p^* = \min\{p : p \geq [K + cx(p)]/x(p)\}$$

and supplying the entire market, with zero profit. (This is because if price is above p^* , some firm can benefit by setting a price slightly lower and take the entire market;

if price is lower than p^* , some firm must earn negative profit; and if two or more firms sell a positive amount at p^* , they will all earn negative profit.) This result is in contrast to the one obtained in the two-stage entry model with Bertrand competition. The crucial assumption here is that firms need not incur sunk cost with entry, this encourages entry and thus competition. This one-stage entry model with price competition provides a formalization of what is called a contestable market.