ECON 7040
Spring '09
Assignment #1

Question.1

(*Recursive Competitive Equilibrium with Arrow Securities*)

Consider an economy which consists of two consumers indexed $i = 1, 2$. The endowments to consumers are given by $y_t^1 = s_t$ and $y_t^2 = 1$ where s_t is a random variable governed by a three-state Markov chain with values $s_t = s_1 = 0$, $s_t = s_2 = 1$ and $s_t = s_3 = 2$, and transition matrix

$$P = \begin{bmatrix} 0.4 & 0.4 & 0.2 \\ 0.3 & 0.4 & 0.3 \\ 0.5 & 0.4 & 0.1 \end{bmatrix}.$$

Also assume that household i orders consumption streams according to

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \ln[c_t^i(s^t)] \pi(s^t).$$

with $\beta = 0.95$.

a) Carefully define a recursive competitive equilibrium for this economy with one-period ahead Arrow securities. Define all of the objects of which such an equilibrium is composed.

b) Define the natural borrowing limits for both individuals if the initial state is given by $s_t = s_1 = 0$.

c) How many Arrow securities are traded in each period? Compute the prices of these securities for all states (when $s_t = 0$, 1 and 2).

d) Suppose that a new asset is introduced within this framework which promises to deliver one unit of consumption if $s_{t+1} = 0$, one unit of consumption if $s_{t+1} = 1$ and zero units of consumption if $s_{t+1} = 2$. Compute the equilibrium prices of this security when $s_t = 0$, $s_t = 1$ and $s_t = 2$.

Question.2

(*Habit Persistence*)

Consider the problem of choosing a consumption sequence c_t to maximize

$$\sum_{t=0}^{\infty} \beta^t (\ln c_t + \gamma \ln c_{t-1}), \quad 0 < \beta < 1, \quad \gamma > 0$$

subject to

$$\begin{aligned} c_t + k_{t+1} &\leq Ak_t^{\alpha}, \\ A &> 0 \\ 0 &< \alpha < 1 \\ k_0 &> 0, \quad \text{and } c_{-1} \text{ given.} \end{aligned}$$

Here the current utility function $\ln c_t + \gamma \ln c_{t-1}$ is designed to represent habit persistence in consumption.

a) Let $v(k_0, c_{-1})$ be the value of $\sum_{t=0}^{\infty} \beta^t (\ln c_t + \gamma \ln c_{t-1})$ for a consumer who begins at time 0 with capital stock k_0 and lagged consumption c_{-1} and behaves optimally. Formulate the dynamic programming problem.

b) Show that the solution of the Bellman's equation is of the form $v(k_t, c_{t-1}) = E + F \ln k_t + G \ln c_{t-1}$ and the optimal decision rule if of the form $\ln k_{t+1} = I + H \ln k_t$ where E, F, G, H and I are constants. Give explicit formulas for the constants E, F, G, H and I in terms of the parameters A, β, α and γ.

Question.3

(*Social Planner in a Discrete World*)

Consider the social planning problem in the one-sector stochastic growth model:

$$\begin{aligned} & \max E_s \sum_{t=s}^{\infty} \beta^{t-s} (-1/\theta) \exp\{-\theta c_t\} \\ s.t \qquad & c_t + k_{t+1} \leq f(k_t) \\ & k_{t+1}, c_t > 0 \end{aligned}$$

where $f(k_t) = a_t k_t^{\alpha}$. Suppose that the factor productivity parameter, a_t, follows a discrete Markov chain process with values $a_t = a_1 = 1.5$, $a_t = a_2 = 2.5$ and the transition matrix:

$$P = \begin{bmatrix} 0.35 & 0.65 \\ 0.55 & 0.45 \end{bmatrix}$$

Further assume that the choice set for saving decision is discrete. More specifically, $k_{t+1} \in \{0.1, 0.2.....1\}$, that is, k_{t+1} belongs to a set which consists of ten equidistant grid points on the interval $[0.1, 1]$. Suppose that $\beta = 0.96$, $\alpha = 0.45$ and $\theta = 0.75$.

a) Formulate the social planner's dynamic programming problem in the discrete state-space form.

b) Write a computer program to solve numerically for the optimal saving decisions when $a_t = 1.5$ and $a_t = 2.5$. In other words, find two (10×1) vectors which give the optimal capital accumulation decisions for all possible initial capital values when $a_t = 1.5$ and $a_t = 2.5$.