ECON 7040
Spring '09
Assignment #2

Question.1

Each household in an economy maximizes a discounted sum of expected utilities which depend on consumption and leisure. Individual preferences are described by

$$E_0 \sum_{t=0}^{\infty} \beta^t \{\log c_t + \chi \log(1 - l_t)\}$$

with $\chi > 0$, where c_t and l_t respectively denote consumption and labor. The household constraints are given by

$$\begin{aligned} c_t + i_t + \phi(k_{t+1}, k_t) &\leq w_t l_t + r_t k_t \\ i_t &= k_{t+1} - (1 - \delta) k_t \end{aligned}$$

together with

$$k_{t+1},\ c_t \geq 0$$

where w and r respectively denote the real wage and capital rental rates. Households are subject to capital adjutment costs denoted by $\phi(k_{t+1}, k_t)$. Specifically,

$$\phi(k_{t+1}, k_t) = \frac{\phi}{2} \left(\frac{k_{t+1}}{k_t} - 1 \right)^2$$

where $\phi > 0$. Output is produced by identical firms that use the following production technology

$$y_t = a_t k_t^{\alpha} l_t^{1-\alpha}$$

where a_t is a stochastic productivity parameter which evolves according to the law of motion

$$\ln a_{t+1} = \rho \ln a_t + \varepsilon_{t+1}$$

with $|\rho| < 1$ and $\varepsilon_{t+1} \sim N(0, \sigma^2)$. Firms hire capital and labor services from perfectly competitive input markets.

a) Define a competitive equilibrium.

b) Solve the household and firm problems. Find the equations that characterize the competitive equilibrium.

c) Linearize the (non-linear) system of equations you found in part-b. Organize the linearized system of equations in the first-order difference form.

d) Suggest a method to solve the linear system of equations you found in part-c.

Question.2

Assume that the life-time utility of a typical agent is given by the function

$$\sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\phi}}{1+\phi} + \log\left(\frac{m_t}{p_t}\right) \right\}$$

in an economy with identical households. Households accumulate capital, k_t. The budget constraint faced by a typical household is given by

$$c_t + k_{t+1} + \frac{m_t}{p_t} = w_t l_t + r_t k_t + (1-\delta)k_t + \frac{m_{t-1}}{p_t}$$

where m_t denotes nominal balances and all other variables have their usual meanings. Output is produced using a standard Cobb-Douglas production technology ($y_t = k_t^{\alpha} l_t^{1-\alpha}$) and factor markets are perfectly competitive.

a) Formulate the household problem and find the money demand function.

b) Find the stationary (steady-state) equilibrium allocations.

c) Find the equilibrium inflation rate if the money supply grows at a constant rate μ, i.e. $m_t = (1+\mu)m_{t-1}$ where $\mu > 0$. Is money superneutral in this economy?