FINE ON VAGUENESS

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1. EPISTEMICISM AND SUPERVALUATIONISM

Aside from its sheer intractability, the problem of vagueness has a historical dimension that lends it extra bite. The modern development of logic was primarily concerned to codify the canons of reasoning employed in mathematical proofs. A striking feature of the mathematical realm is that it's one which is *sharp:* there's no such thing as a number which is borderline odd, and the languages of pure mathematics don't contain vague expressions of other categories; for example, none of Hilbert's problems use 'many' in their formulation. So when we turn to the description of the empirical realm, we cannot avoid the question whether the apparent lack of sharpness requires that accommodations in logic be made.

Epistemicism is the view that *no* accommodations need to be made, since the empirical realm is no less sharp than the mathematical (Sorensen 1988, 2001; Williamson 1994). In particular, if *F* is a predicate whose application is typically persistent across small changes in some quantity but not persistent across some large ones, then if some such large change is decomposed into a series of small changes, there will be a particular small change which unseats the predicate. On this view, the term 'vague' simply marks the epistemic inaccessibility of *which* small change does the damage.

For example, take the concept of being well-paid$_C$ by one's employer, where C is a reference-class and we are concerned only with individuals belonging to it; C might be, say, the class of Full Professors in humanities departments in the current *US News and World Report* top fifty research universities in the USA. It is plausible that if any member of this group is well-paid$_C$, then any other member of the group who is paid less, but no more than $1.00 less, than the given member, is also well-paid$_C$. Suppose it is true that members of C are well-paid$_C$ if they are paid a nine-month salary of $150,000 or more, and not well-paid$_C$ if they are paid $75,000 or less. Then if $a_0,\ldots,a_{75000}$ are 75001 members of C such that a_0 is paid $150,000 and a_{i+1} is paid $1.00 less than a_i, we have our example of a predicate which can be unseated by a large change (drop) in magnitude of the relevant quantity (amount of salary). And since this change can be decomposed into a sequence of one-dollar drops in salary, one of these drops apparently marks the transition from being well-paid$_C$ to not being well-paid$_C$ (to being merely *adequately*-paid$_C$). For if none do, there would seem to be no transition, and so, contradicting our premise, a_{75000} (a_{75K}) having a salary of $75,000 makes a_{75K} well-paid$_C$. No doubt a_{75K} would beg to differ.

That some one-dollar drop marks the transition implies that there is a one-cent drop that marks the transition. And so we reach the conclusion that for all we know, a full professor in a humanities department in the current *USN&WR* top fifty national research universities is well-paid$_C$ iff he or she has a salary of at least $97,165.38, and that's just the way it is.[1] I respectfully suggest that it's because this is not credible that epistemicism has not caught on. The intuition that there are no abrupt transition-points is very powerful, and for good reason (Forbes 2011:93).

1. I have adjusted this number for inflation since I last used the example (Forbes 2011:92).

Theories of vagueness which allow for statements to have the status *neither true nor false* can claim to respect this intuition if they interpret it as opposing an abrupt transition from *true* to *false*. Fine's first major contribution to the theory of vagueness, the supervaluationist approach, is of this kind, and has appealed to many because in its most straightforward form it preserves classical logic, in the sense of having the same logically valid formulae over the standard logical constants (Fine 1975; Keefe 2000, Chs. 7–8).

To remind the reader of the details of a common version: sentences (of, let's say, a monadic first-order language $\mathscr{L}$) are evaluated with respect to a *specification space*, which is a set of points $\mathcal{T}$ partially ordered by $\leq$ with a root point t_0 ($\forall t \in \mathcal{T}$, $t_0 \leq t$). Each point in $\mathcal{T}$ is associated or identified with a partial assignment of truth-values to atomic sentences, subject to the constraint that if $t \leq t'$, then for any atomic sentence π, if π has a truth-value at t, it has the same truth-value at t'. When $t \leq t'$, t' is said to be a *sharpening* or *precisification* of t. t' is *complete* iff t' assigns a truth-value to every atomic sentence, and it's convenient to stipulate that in every space $\mathcal{T}$ there is a complete point t such that $t_0^{\mathcal{T}} \leq t$.

One would at this point expect a recursive definition of truth-at-t, one clause for each connective.[2] But the supervaluationist move is to reserve recursive calculation of truth-value for the complete points, where the semantics is classical, and stipulate that for any point t, t verifies ($\vDash$) σ iff for all complete t' such that $t \leq t'$, t' verifies σ according to classical semantics, *mutatis mutandis* for falsification ($\Dashv$). That is, we have the following clauses:

2. Fine illustrates one way of doing this with what he calls the 'bastard intuitionistic' definition, (1975:273).

(1) a. $\mathcal{T} \vDash \sigma$ iff $t_0^{\mathcal{T}} \vDash \sigma$.

b. $\mathcal{T} \Dashv \sigma$ iff $t_0^{\mathcal{T}} \Dashv \sigma$.

c. for any $t \in \mathcal{T}$, $t \vDash \sigma$ iff for all complete t' with $t \leq t'$, $t' \vDash \sigma$.

d. for any $t \in \mathcal{T}$, $t \Dashv \sigma$ iff $t' \Dashv \sigma$ iff for all complete t' with $t \leq t'$, $t' \Dashv \sigma$.

The clauses in (1) identify truth in a specification space with truth at all complete sharpenings of the root point of the space. The former notion can be labelled ‘truth’, the latter, ‘supertruth’. So supervaluationism identifies truth with supertruth. There is also a companion notion of semantic consequence: where $\Vdash$ means ‘entails’ and ‘$\mathcal{T} \vDash \Sigma$’ means that for every sentence σ in Σ, $\mathcal{T} \vDash \sigma$,

(2) $\Sigma \Vdash \sigma$ iff for any $\mathcal{T}$, if $\mathcal{T} \vDash \Sigma$ then $\mathcal{T} \vDash \sigma$.

According to (2), semantic entailment is preservation of supertruth.

In application to natural language, the intended interpretation of expressions has to be taken into account, and this will rule out certain sharpenings as *inadmissible*. For example, if ‘a_{58000} is well-paid$_C$’ and ‘a_{57999} is well-paid$_C$’ are both undefined at the root point, we can’t allow a precisification which makes ‘a_{58000} is well-paid$_C$’ true and ‘a_{57999} is well-paid$_C$’ false, since a_{57999} makes \$1.00 more than a_{58000}, and so must be well-paid$_C$ if a less remunerated professor is. The official account of truth at the root point of $\mathcal{T}$ and truth with respect to $\mathcal{T}$ is therefore that it is truth at all the root point’s complete *admissible* sharpenings. Perhaps we can think of the assignment of truth-values to atomic sentences at a root point as reflecting a notion of correspondence truth: the undefined atomic sentences are those which neither correspond to the facts nor do not correspond to them. We then use the same notion of correspondence truth at the complete sharpenings, where it is recursively determined for all sentences, to extend the correspondence-based notion of truth for the root point to a more liberal one, that of corresponding to the facts, or being such as to correspond to them on every admissible complete sharpening of vague vocabulary.

One much-noted consequence of this account of truth (see, e.g., Fine 1975:284–5) is that disjunctions can be true at a root point t_0 without having a disjunct that is true at t_0, and similarly, an existential formula can be true at t_0 without having an instance that is true at t_0, even if there is a name in $\mathcal{L}$ for every object that exists. For example, perhaps some sharpenings of t_0 verify p and falsify q while the remainder do the opposite. Then $t_0 \vDash p \vee q$ since $p \vee q$ holds at all the complete sharpenings, yet $t_0 \nvDash p$ since p isn't true at all t_0's complete sharpenings, and $t_0 \nvDash q$ since q isn't true at all t_0's complete sharpenings. In the same way, an existential formula may be true at t_0 since at each complete sharpening there is a true instance of it, verifying the existential; but there may not be a particular instance which is true at every complete sharpening, so no instance need be true at t_0.[3]

This has consequences for the supervaluationist solution to the Sorites Paradox, say in the version in which there is some large number of conditional premises presented in an order reflecting the increasing or decreasing degree of possession of the relevant property. The conditionals all seem true, but by generalized transitivity they entail a false conditional (in our running example, that if a_0 is well-paid, so is a_{75K}), or, with the aid of a true minor premise ('a_0 is well-paid'), entail something false ('a_{75K} is well-paid') by repeated *modus ponens*. The formal logic is unobjectionable, so barring a fallacy of informal logic, it must be that some

3. For simplicity, I assume constant domains. If $\circ$ is a modal or epistemic operator, there can be a difference between '$\circ(\exists x)\mathrm{F}x$' and '$(\exists x)\circ\mathrm{F}x$'. For example, you may know that there are spies without there being someone you know to be a spy. 'Supertrue' and 'definitely' share this behavior, and so would 'true' if the identification of truth and supertruth were acceptable.

premise is not true. And this existential claim is indeed correct, since on each admissible complete sharpening of 'well-paid' there is a conditional premise that is false. But then supervaluationism licenses a stronger claim: that *some premise is false*. However, if we go through the premises one by one and ask 'Is this one false?' the answer will be 'No' every time, for there is no premise p_i such that p_i is false on every complete admissible sharpening. It is very difficult to maintain that *some* element of a domain has a property while rejecting each element in turn as a candidate for having the property (suppose there are three objects and I insist that at least one of them is red; but I don't accept that the first is red, don't accept that the second is red, and don't accept that the third is red, while I agree that that exhausts the inventory). This has no independent appeal, and some who embrace supervaluationism may be doing so rather loosely, keeping one eye open for a more attractive option.

2. LOGICO-GLOBALISM

Fine's new approach to vagueness (Fine 2015, 2016) is based on two principles, *logicalism* and *globalism*. Globalism is the view that a vague predicate such as 'well-paid' is not vague in application to a single case, but only in application to a range of cases. Fine gives the example of a garden path, whose evenness or otherwise is a property of the collection of stones. So we are to think of the indeterminacy in application of 'well-paid' as a collective property of some group to which it is applied, just as unevenness is a collective property of the stones. A single stone may be responsible for the unevenness (all the stones but it are on the same level) but the unevenness itself is still a basic feature of the collection and their arrangement in space.

Fine takes globalism to contrast with localism, of which his examples in (Fine 2015:2) are all views to the effect that a vague predicate is one for which there are actual or possible *borderline cases*. But I think analysis of 'borderline case' is likely to reintroduce globalism. It is plausible that typical vague predicates are mastered by reference to paradigm cases, and understood by competent subjects to be subject to the following vague semantic principles:

(3) a. The *Don't Stray* rule: don't stray too far from the paradigms in applying the predicate.

b. The *Don't Distinguish* rule: don't distinguish cases that are not significantly different in the relevant respect.[4]

In applying a vague predicate along a Sorites series, we hew to (3b) until pressure to observe (3a) overrides. Whether or not there is some 'incoherence' in this, it does suggest that the borderline cases are the ones where (3b) may be justifiably abandoned by a competent user under pressure from (3a). And because of the role of paradigms in the account, this analysis of 'borderline' is globalist rather than localist. So I do not think that simply because vague predicates are said to be those with borderline cases, we can conclude that we are dealing with a localist account.

What of logicalism? Here the idea is that the characteristic feature of a vague predicate is that of its logical behavior. This behavior is not be explained in terms of some underlying intrinsic feature of the predicate's semantics, but rather, its

4. This is not a trivial principle, correct simply because 'significant difference' is defined to mean 'any difference that justifies distinguishing cases'. What 'significant difference' amounts to will vary from case to case, but in the case of predicates applied on the basis of how things look, a significant difference is one that is observationally detectable in normal circumstances. In the case of salary, a significant difference is one that in not too many years accumulates to allow noticeably superior material quality of life, given similar purchasing patterns.

being vague consists just in the behavior in question. What, then, is the crucial behavior? To take a very simplified case, suppose we have three objects *a*, *b* and *c*, and a vague predicate *F* which is satisfied by *a* and not satisfied by *c*. *b* is intermediate between *a* and *c* in the respect relevant to determining wh¬ether or not it satisfies *F*. There are then eight possible *state-descriptions* (conjunctions of literals) for the domain $\{a,b,c\}$ and predicate *F*, namely (i) $Fa \wedge Fb \wedge Fc$, (ii) $\neg Fa \wedge \neg Fb \wedge \neg Fc$, (iii) $\neg Fa \wedge \neg Fb \wedge Fc$, (iv) $\neg Fa \wedge Fb \wedge Fc$, (v) $Fa \wedge \neg Fb \wedge Fc$, (vi) $\neg Fa \wedge Fb \wedge \neg Fc$, (vii) $Fa \wedge \neg Fb \wedge \neg Fc$, and (viii) $Fa \wedge Fb \wedge \neg Fc$. According to Fine, what makes it the case that *F* is vague in application to $\{a,b,c\}$ is that we reject *all* the state descriptions. We reject (i) through (vi), plainly enough, because *Fa* and $\neg Fc$ are both true. There is also another reason for rejecting (v) and (vi): supposing that *a*, *b* and *c* have decreasing amounts of the quantity that determines satisfaction or otherwise of *F* (e.g., salary, if *F* is 'well-paid'), (v) and (vi) are *a priori* impossible, since, for (v), *c*, who earns less than *b*, can't be well-paid for the same work if *b* isn't, and, for (vi), *b*, who earns less than *a*, can't be well-paid if *a* isn't. Finally, we reject (vii) and (viii), according to Fine, because they involve a switch in truth-value for atomic sentences, from $\top$ to $\bot$, that can't be justified in terms of change in the amount of the quantity to which *F*'s application is sensitive, such as salary.

Of course, for a realistic example of a Sorites sequence we need many more than three objects. But the phenomenon illustrated in the previous paragraph is only more evident when the number of objects goes up to $\{a_1,\ldots,a_n\}$ for large *n*. It will still be the case that each state description σ is objectionable for at least one of the three reasons manifest in the three-object case: (A) σ is wrong about truth-value at the 'far ends', or (B) σ involves an application of *F* that conflicts with changes in the relevant underlying quantity, or (C) σ posits a switch from true to false at some point in the sequence of atomic sentences $Fa_1,\ldots,Fa_n$ which the underlying difference, say in salary, doesn't justify because it is too small.

In Fine's view we wish to assert the negation of each state-description. His view is logicalist, since he holds that the indeterminacy of F in application to $\{a_1,\ldots,a_n\}$ *consists in* the correctness of rejecting each state description. So in the three-object case, to spell it out, F is vague in application to $\{a,b,c\}$ precisely because

(4) $\neg((Fa \wedge Fb) \wedge Fc) \wedge \neg((Fa \wedge Fb) \wedge \neg Fc) \wedge \neg((Fa \wedge \neg Fb) \wedge Fc) \wedge \neg((Fa \wedge \neg Fb) \wedge \neg Fc) \wedge \neg((\neg Fa \wedge \neg Fb) \wedge Fc) \wedge \neg((\neg Fa \wedge Fb) \wedge \neg Fc) \wedge \neg((\neg Fa \wedge \neg Fb) \wedge Fc) \wedge \neg((\neg Fa \wedge \neg Fb) \wedge \neg Fc)$

is true. But (4) is logically false, classically and intuitionistically. So the phenomenon of vagueness, as manifested by the truth of sentences of the type of (4), forces a revision of logic. It is in this sense that vagueness is a logical phenomenon.

But before turning to Fine's proposed revision, it is worth pausing to ask how plausible it is that the vagueness of F over $\{a_1,\ldots,a_n\}$ consists in the truth of the analog of (4). For this account seems rather undiscriminating. Everyone agrees that state descriptions which are wrong about truth-values at the far ends should be rejected. And everyone agrees that state descriptions in conflict with the underlying quantity facts should be rejected. Disagreement with the epistemicist (and the supervaluationist) arises only over those state descriptions which violate *Don't Distinguish*, ones of the form

(5) $Fa_1,\ldots,Fa_k, \neg Fa_{k+1},\ldots,\neg Fa_n, 1 \lneqq k \lneqq n.$

I will refer to such descriptions as *cut-off* state descriptions. These are the state descriptions which, on the conventional view, are rejected *because* the predicate F is vague in application to $\{a_1,\ldots,a_n\}$. But Fine is saying that vagueness *consists in* the fact that we reject all such descriptions; that is, we take their negations to be true and the descriptions themselves to be false.

To hold instead that some cut-off descriptions are not false – though maybe not true either – is, according to Fine (2015:6), 'quite an extraordinary and counterintuitive position'. Since *Don't Distinguish* does all the work in producing this intuition, we should ask if recognizing that *Don't Stray* also governs our use of vague predicates changes anything. Classically, repeated use of *Don't Distinguish* leads eventually to gross violation of *Don't Stray*. So, if presented with $a_1,\ldots,a_n$, and asked to classify them in that order with respect to F, we will at some point be obliged by *Don't Stray* to stop classifying items as F.[5]

Does this mean that, *contra* Fine, some cut-off state description is indeed not false? Or is stopping properly classified as error because it creates a significant change in F-status not underpinned by a significant change in the relevant quantity (e.g., salary)? As before, we could think of stopping at some point as making an extension of correspondence truth for cases close to that point, an extension effected by a stipulation that generates truths which are not thought to be true by virtue of corresponding with the facts; and so we could even switch immediately to classifying the remaining cases as not-F. For example, the Administration of an impecunious university burdened with an athletics program it can neither afford nor discontinue may decree that no well-paid professor will be eligible for a salary increase next year, and will then stipulate a particular number, as low as can be chosen with a straight face, to fix the reach of 'well-paid'

5. There is a wealth of empirical data about actual speaker behavior in this context in (Raffman 2014:Ch.5).

(one hopes with a sense of 'artificiality and absurdity', Wright 1975). Ever-decreasing salaries are considered until *Don't Stray* forces the drawing of a line. That there is in fact such a line, and those who announce where they want it to be are simply guessing where it is, is incorrect. But *Don't Stray* justifies creative postulation. In this sense, drawing a line where there is no fact of the matter isn't making a mistake. Indeed, granted that correspondence truth is not at issue, the difference between cut-off classifications and classifications that leave one case in the middle undecided does not loom large, and rejecting the former while accepting the latter can seem arbitrary.

3. A NEW SEMANTICS OF VAGUENESS

If we are going to say that (4) and its lengthier cousins are legitimate assertions, we will have to find fault with the logic that shows they are inconsistent. The derivations of inconsistency are systematically related. A state-description is a conjunction of literals over a collection of simple states, and if there are n simple states there are 2^n such state descriptions. And for any n, rejecting all 2^n descriptions leads to inconsistency, as follows.

Suppose, for the base case, that there is only one simple state, so we have the one-conjunct 'conjunctions' of literals A and $\neg A$. The rejection of both produces $\neg A$ and $\neg\neg A$, which is evidently inconsistent.

Now suppose there are two simple states, and thus four conjunctions of literals, whose rejections in truth-table order are (a) $\neg(A \wedge B)$, (b) $\neg(A \wedge \neg B)$, (c) $\neg(\neg A \wedge B)$, and (d) $\neg(\neg A \wedge \neg B)$. Restricting ourselves to intuitionistic rules in the language $\mathcal{L}^{\neg\wedge}$, specifically, the derived rules of *Reductio* ($\Gamma \vdash q, \Delta \vdash \neg q \Rightarrow (\Gamma, \Delta)\backslash p \vdash \neg p$) and *Conjunctive Syllogism* ($\Gamma \vdash \neg(p \wedge q), \Delta \vdash p \Rightarrow \Gamma, \Delta \vdash \neg q$), it's easy to see that from (a) and (b) we obtain $\neg A$ while from (c) and (d) we obtain $\neg\neg A$. These conclusion-formulae are precisely the rejections of the state-descriptions from the previous case, which we already know to be inconsistent.

The sequents $\neg(A \wedge B), \neg(A \wedge \neg B) \vdash \neg A$ and $\neg(\neg A \wedge B), \neg(\neg A \wedge \neg B) \vdash \neg\neg A$ are both proved in the logic of $\mathcal{L}^{\neg\wedge}$ by instances of the following proof schema:

(6)

$$\dfrac{\dfrac{\neg(p \wedge q) \qquad [p]}{\neg q}\,\text{CS} \qquad \dfrac{\neg(p \wedge \neg q) \qquad [p]}{\neg\neg q}\,\text{CS}}{\neg p}\,\text{R}$$

The same schema applies as we increase the number of simple states. For example, if there are three states, the denials of the state descriptions are the eight conjuncts of (4), replacing *Fa*, *Fb* and *Fc* with A, B and C respectively. The first two conjuncts entail $\neg(A \wedge B)$, the next two entail $\neg(A \wedge \neg B)$, the next two entail $\neg(\neg A \wedge B)$, and the last two entail $\neg(\neg A \wedge \neg B)$. Each entailment is proved by an instance of (6). The first two of these conclusions are the premises of the first sequent proved in the previous case (the one for two simple states), with conclusion $\neg A$, and the second two are the premises of the second sequent proved in the previous case, with conclusion $\neg\neg A$. This pattern repeats itself as n increases. Generally, $2^n/2$ uses of (6) allow us to infer the premises of the sequents from the previous case, sequents whose conclusions are the premises of the sequents in the case previous to that one, until we arrive back at the base case with its evident inconsistency. Thus Conjunctive Syllogism and *Reductio* are the only rules needed to establish that no matter the number of simple states, denying all state descriptions is inconsistent.

It is therefore of the first importance, if we wish to allow for denying all state descriptions, to find fault with the schema in (6): either at least one of the two rules is incorrect, or we are relying on some questionable structural feature of deductions. In fact, Fine rejects both derived rules. The primitive rules for $\mathscr{L}^{\neg\wedge}$ are $\wedge$E, $\wedge$I, $\neg$E, $\neg$I and EFQ.[6] With these rules we have the following canonical derivation of Conjunctive Syllogism, in which, to focus most easily on Fine's objection, we replace a formula at a tree-node with the sequent proved at that node:

(7)

$$\dfrac{\dfrac{\neg(A \wedge B) \vdash \neg(A \wedge B) \qquad \dfrac{A \vdash A \qquad B \vdash B}{A, B \vdash A \wedge B}\,{\scriptstyle \wedge\mathrm{I}}}{\neg(A \wedge B), A, B \vdash \text{⅄}}\,{\scriptstyle \neg\mathrm{E}}}{\neg(A \wedge B), A \vdash \neg B}\,{\scriptstyle \neg\mathrm{I}}$$

It is the final step that Fine rejects – indeed, it's the only real candidate for rejection, since it's hard to gainsay $\wedge$I, while $\neg$E is just bookkeeping.

What semantics will block (7) and at the same time apply in a convincing way to Sorites paradoxes? Fine adapts intuitionistic semantics to his purposes. Call a way of applying a vague predicate F to a range of appropriate objects a *use* of F. Uses may differ over how far along the range F is to be applied, and this gives rise to a reflexive symmetric relation of *compatibility* among uses. Fine defines 'u is compatible with v', which I will write '$u \| v$', as 'no sentence true under u is false under v'. So if I am willing to extend 'well-paid' down to those professors making at least \$120,000, but take no position on those making \$115,000, while you extend

6. $\neg$E is: $\Gamma \vdash p, \Delta \vdash \neg p \Rightarrow \Gamma, \Delta \vdash \text{⅄}$. $\neg$I is: $\Theta \vdash \text{⅄} \Rightarrow \Theta \backslash p \vdash \neg p$. EFQ is: $\Theta \vdash \text{⅄} \Rightarrow \Theta \vdash p$. ⅄ is a symbol for announcing an inconsistency, pronounced "Absurdity!". The difference between $\neg$I and *ex falso quodlibet* is that the former discharges an assumption. The p in EFQ stands for an arbitrary formula. Intuitionistic logic is minimal logic (the I and E rules) plus EFQ, while classical logic is intuitionistic logic with EFQ augmented by LEM or supplanted by DN ($\Theta \vdash \neg\neg p \Rightarrow \Theta \vdash p$).

'well-paid$_C$' down to \$110,000, your use of 'well-paid' is, so far, compatible with mine, since I am not saying that it's *false* that \$115,000 is enough to make you well-paid$_C$. But if some third person x extends 'well-paid' down to \$100,000, and I think \$100,000 is not enough for being well-paid, x's use of 'well-paid' is incompatible with mine, but perhaps not with yours, since you may take no position on \$100,000. This example indicates how transitivity of $\|$ may fail, since it can be filled in so that my use is compatible with yours, and yours with x's, while mine is incompatible with x's.

An interpretation $\mathcal{J} = (U, \|, T, u^*)$ is a set U of uses; a reflexive symmetric compatibility relation $\|$ on U; a function T such that for each $u \in U$, $T(u)$ is a set of sentence-letters, intuitively, those which are true under u (for every u, $\curlywedge \notin T(u)$); and a designated use u^*. Whether a sentence is true or not at a given u is determined by the following clauses, in which '$u \vDash$ _' can be read 'u verifies _' or '_ is true under u':

(8) a. $u \vDash \pi$ iff $\pi \in T(u)$, π atomic

b. $u \vDash p \wedge q$ ($p \vee q$) iff $u \vDash p$ and (or) $u \vDash q$

c. $u \vDash \neg p$ iff for all $v \in U$, if $u \| v$ then $v \nvDash p$

d. $u \vDash p \rightarrow q$ iff (i) $u \vDash p \wedge q$ or (ii) for every v such that $u \| v$, $v \vDash q$ if $v \vDash p$.[7]

We say $\mathcal{J} \vDash \sigma$ iff $u^* \vDash \sigma$, and that $\Sigma \Vdash \sigma$ iff for every $\mathcal{J}$, if $\mathcal{J}$ verifies every sentence in Σ, then $\mathcal{J}$ verifies σ. Fine calls this apparatus 'compatibilist semantics'.[8]

7. (ii) by itself guarantees $\neg A \Vdash A \rightarrow B$, but not $B \Vdash A \rightarrow B$: we may have $u^* \vDash B$, but $u^* \nvDash A \rightarrow B$ (in terms of (ii)) because for some compatible v, $v \vDash A$ and $v \nvDash B$. A weaker version of (i), which only requires $u \vDash q$, gets round this, but, as Fine pointed out to me (p.c.), has some unattractive consequences. For instance, we would then have $A \rightarrow B \nVdash A \rightarrow (A \wedge B)$.

8. Fine does without designated uses. I have included them with a view to the discussion of intended models to follow.

It's important to note that although uses classify some sentences as true, some as false, and some as neither, compatibilist semantics isn't a three-valued logic in the usual sense. Uses do not explicitly assign 'undefined' and 'false' as the semantic values of atomic sentences: only 'true' is assigned explicitly, while 'false' and 'undefined' are applied following the clauses in (8), and so depend on what other uses there are and how compatibility is stipulated for them. Consequently, uses should not be *identified* with the assignments of $\top$ they stipulate, for two uses may agree on which atomic sentences are true, but disagree on where the line is to be drawn between the undefined and the false. For example, let $U = \{u^*, u, v\}$, and take $\|$ as the least reflexive symmetric relation on U including $u^* \| u$ and $u^* \| v$. u^*, u and v all assign $\top$ to A, while u also assigns $\top$ to B, or in the terms of the definition of interpretation, $T(u) = \{A, B\}$, $T(u^*) = T(v) = \{A\}$. So u^* and v agree on which atomic sentences are true. But B remains undefined at u^* because $u^* \| u$ and $u \vDash B$, so $\neg B$ isn't true at u^*, i.e., B isn't false there. However, B *is* false ($\neg B$ is true) at v, since B isn't true at either u^* or v. Hence u^* and v disagree on where the line is to be drawn between the undefined and the false.

To see how revisionist the resulting logic is for principles relevant to vagueness, we can easily check that LEM, DN and the de Morgan law $\Gamma \vdash \neg(A \wedge B) \Rightarrow \Gamma \vdash \neg A \vee \neg B$ are semantically incorrect for $\mathcal{L}^{\neg\wedge}_{AB}$, so unsound as rules of proof (as in intuitionistic logic). For LEM and DN, we simply need two uses u^* and u with $u^* \| u$, A untrue under u^* ($T(u^*) = \varnothing$), and A assigned $\top$ under u ($T(u) = \{A\}$). Then by stipulation u^* does not verify A, and by (8c) u^* does not verify $\neg A$, since u^* is compatible with a use verifying A, namely u. So $u^* \nvDash A$ and $u^* \nvDash \neg A$; hence by (8b), $u^* \nvDash A \vee \neg A$. Since we also have $u \nvDash \neg A$, we have $u^* \vDash \neg\neg A$, so $\neg\neg A \nVdash A$.

For the de Morgan principle in $\mathcal{L}^{\neg\wedge}_{AB}$, we let $U = \{u^*, u, v\}$, and take $\|$ as the least reflexive symmetric relation on U including $u^* \| u$ and $u^* \| v$. No sentence-letter is true under u^*, while u verifies A but not B and v verifies B but not A ($T(u^*) = \varnothing$, $T(u) = \{A\}$, $T(v) = \{B\}$). So by (8b), $u \nvDash A \wedge B$ since $u \nvDash B$, and $v \nvDash A \wedge B$ since $v \nvDash A$. Consequently, using (8c), $u^* \vDash \neg(A \wedge B)$. However, $u^* \nvDash \neg A$, because $u^* \| u$ and $u \vDash A$, and $u^* \nvDash \neg B$, because $u^* \| v$ and $v \vDash B$. So $u^* \nvDash \neg A \vee \neg B$, therefore $\neg(A \wedge B) \nVdash \neg A \vee \neg B$.

What of ¬I, which is sound for intuitionistic semantics? (7)'s last step is from $\neg(A \wedge B), A, B \vdash \curlywedge$ to $\neg(A \wedge B), A \vdash \neg B$. Obviously, $\neg(A \wedge B), A, B \Vdash \curlywedge$ (no use verifies all three premises), so to defeat (7) we have to show $\neg(A \wedge B), A \nVdash \neg B$. Let $U = \{u^*, v\}$, and for $\|$ take the least reflexive symmetric relation on U including $u^* \| v$. $T(u^*) = \{A\}$, $T(v) = \{B\}$. Then $u^* \nvDash A \wedge B$ and $v \nvDash A \wedge B$, so $u^* \vDash \neg(A \wedge B)$. Also, $u^* \vDash A$. But $u^* \nvDash \neg B$, because $u^* \| v$ and $v \vDash B$. So $\neg(A \wedge B), A \nVdash \neg B$.[9] Therefore compatibilist semantics, if coherent, establishes the consistency of denying all state-descriptions, given that every proof of its inconsistency using only primitive-rules uses ¬I.[10]

At this point, I wish to raise a question about the compatibilist clause for negation. It's a frequent objection to the intuitionistic treatment of negation, according to which $\neg p$ is warranted in the current state of information iff no development of that state warrants p, that it unacceptably epistemicizes the

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9. The interpretation defined here wouldn't be legitimate intuitionistically, even though $\|$ is transitive in it, since a sentence-letter that is warranted in a state of information i in an intuitionistic model remains warranted in all states of information extending i.

10. In rejecting ¬I, Fine is not being idiosyncratic. ¬I is also invalid in three-valued logics of vagueness such as the one developed in Parsons 2000 (see p.25). The general point is that in Parsons' three-valued system, we cannot conclude $\Theta \backslash p \vdash \neg p$ from $\Theta \vdash \curlywedge$ because p, hence $\neg p$, may be undefined, so ¬I produces sequents which are not truth-preserving. However, one special feature of Fine's semantics is that ¬I is reliable when $\Theta \backslash p$ is empty. In both systems, $A \wedge \neg A \vdash \curlywedge$, but in Parsons', we cannot conclude $\vdash \neg(A \wedge \neg A)$, since if A is undefined so is $\neg(A \wedge \neg A)$, while in Fine's, the theorem is correct. In both systems, $\curlywedge$ is never true, but in Fine's only, this implies $\neg\curlywedge$ is always true.

meaning of 'not'. For it seems we have little difficulty understanding how there may be no development of our current state of information which warrants p, yet p is true. Is there an analogous problem with (8c), that it unacceptably pragmatizes the meaning of 'not'? The thought is that whether or not an object x is a borderline case of an F or instead an outright satisfier of not-F doesn't depend on what other uses of F there are or could be, but rather depends on the meaning of F and the properties of x (salary, height, age, etc.), just like any sharp predicate ('earns at least $100,000'). This is not the perspective of compatibilist semantics. For example, if there are only three uses, pairwise compatible, they cannot all leave A undefined, but that they should be able to seems quite coherent. In the same vein, if U contains just one use u^*, then it will be classical, since for any sentence letter π, either π is true under u^* or u^* is not compatible with any use under which π is true, so $\neg\pi$ is true under u^*. But shouldn't an isolated use be able to leave some atomic propositions undecided?

We might reply that behavior in artificially restricted interpretations doesn't matter: what matters is behavior in *intended* interpretations. In an intended interpretation for 'well-paid' in application to our chosen group of professors $a_0,\ldots,a_{75K}$, U is the set of *all* admissible uses of 'well-paid', all the ones in which it is applied to those professors in accordance with its actual meaning, and compatibility is defined so as to realize all ways of classifying professors as well-paid, not well-paid, or neither, that are in accordance with the actual meaning of 'well-paid'.[11]

This solves the problems above, which arose from leaving admissible uses out.

11. Ignoring vagueness, intended interpretations for a fixed language-fragment and range of objects differ only over which $u \in U$ is designated. But taking account of vagueness, there will be a range of interpretations with equal title to the claim *intended*, as I have characterized it, and for some interpretations there will be no fact of the matter whether they merit the title.

But there may be a similar problem even if they are all included. For some professor, say a_{i+1}, is the best-paid professor whom *no* use in U puts in the extension of 'well-paid' (no use assigns $\top$ to 'a_{i+1} is well-paid'). But then by (8c), each u in U verifies 'a_{i+1} is not well-paid'. It follows that no u in U assigns $\top$ to 'a_i is well-paid', since no admissible u verifies a cut-off state-description of the form of (5). Consequently, a_i, who earns \$1.00 more than a_{i+1}, is omitted from the extension of 'well-paid' under each u in U. However, this contradicts the choice of a_{i+1} as the best-paid of the universally omitted professors.

Fine considers a related argument (2015:10) and concludes that we have to admit uses which are compatible with admissible uses, but which are not themselves admissible because they verify cut-off state-descriptions. The above argument is therefore blocked, because an inadmissible u may assign $\top$ to 'a_i is well-paid' and be compatible with the designated use u^*. So in intended interpretations of a fragment of English, the domain U will include not just the admissible uses of the vague predicates of the fragment, but also such inadmissible uses as are needed to undercut the inconsistency argument of the previous paragraph.

This means, I think, that no natural notion of intended interpretation is available in compatibilist semantics. One way in which this comes out is in the consequences of including inadmissible uses for certain sentences one might have expected to be logical truths in the semantics. For example, instances of (#), 'not(a_i is well-paid and a_{i+1} is not well-paid)', reflect Fine's insistence that cut-off state-descriptions are false. If we make instances of (#) the consequents of conditionals whose antecedents state that a_{i+1} is paid only \$1.00 less than a_i, the results are not merely true, as Fine says they are (Fine 2015:15–16): they are *a priori*, even analytic. But they will fail under admissible uses in intended interpretations if the latter include inadmissible uses: we only need an admissible u to be compatible with an inadmissible u' verifying 'a_i is well-paid and a_{i+1} is not well-paid'.

The way things are and the way the language is understood should constitute a special case of the definition of model on a semantics if that semantics is to have any authority in settling logical consequence in the language. The need to include inadmissibe uses, in my opinion, justifies scepticism that (8c) properly captures the meaning of negation in languages with vague predicates.

4. 'THE' SORITES PARADOX

Fine distinguishes three formulations Sorites paradoxes might be given, which differ over whether the major premises are disjunctions, conditionals, or negated conjunctions. I shall rehearse his analysis of each formulation, and compare it with the account afforded by fuzzy logic.[12] It's indisputably a very great strength of compatibilist semantics that it deals decisively with each way of formulating a Sorites argument, but I think fuzzy logic shares that strength and so is a genuine rival to compatibilist semantics.[13]

The Sorites formulation with least force is the disjunctive one, where the transitions have the form

(9) a_i is well-paid; either a_i is not well-paid or a_{i+1} is well-paid; so a_{i+1} is well-paid,

that is, from A and $\neg A \vee B$ to B, a version of Disjunctive Syllogism. The problem is

12. See (Forbes 1983) for the use of fuzzy logic to solve a special kind of Sorites paradox in modal logic, and (Forbes 2011) for a logical construction of fuzzy logic's 'degrees of truth'.

13. Wright has for some time insisted that any apparatus for resolution of conditional formulations is inadequate if not applicable to negated-conjunction formulations (Wright 1987 *passim*).

not that DS is invalid in compatibilist semantics: it's valid. Rather, the problem, Fine says, is that in a framework without LEM and DN, there is no reason to accept the disjunction $\neg A \vee B$. Suppose we assume (†) $u^* \vDash Wa_i$ or $u^* \vDash \neg Wa_i$. If $u^* \vDash Wa_i$, then if we also assume (‡) $u^* \vDash Wa_{i+1}$ or $u^* \vDash \neg Wa_{i+1}$, it must be that $u^* \vDash Wa_{i+1}$, on pain of a cut-off, so the disjunction in (9) is true. If instead $u^* \vDash \neg Wa_i$, then the disjunction in (9) is true anyway. But of course, we may *not* assume (†) or (‡). For it may be that neither 'a_i is well-paid' nor 'a_i isn't well-paid' is true under u^*, *mutatis mutandis* for a_{i+1}.

In fuzzy logic, sentence-letters are assigned degrees of truth, which are standardly represented by real numbers in [0,1], with $0 = \bot$ and $1 = \top$. The degree of truth $d[\varphi]$ of a complex formula φ is calculated by:

(10) a. $d[\neg p] = 1 - d[p]$;

b. $d[p \wedge q]$ ($d[p \vee q]$) $= \min\{d[p], d[q]\}$ ($= \max\{d[p], d[q]\}$);

c. $d[p \to q] = 1 - (d[p] \dot{-} d[q])$.[14]

(10c)'s rationale is that in the cases of interest, where $d[p] > d[q]$, the degree of truth of the conditional should fall as the gap between antecedent and consequent rises, until at the limit, where $d[p] = 1$ and $d[q] = 0$, we have $d[p \to q] = 0$.

There are two notions of semantic consequence in fuzzy logic, $\Vdash_\top$ and $\Vdash_d$. $p_1,\ldots,p_n \Vdash_\top q$ means that if $p_1,\ldots,p_n$ all have degree of truth 1, so does q, and $p_1,\ldots,p_n \Vdash_d q$ means that $d[q] \geq \min\{d[p_1],\ldots d[p_n]\}$. DS is $\top$-valid: if $d[A] = 1$, then $d[\neg A \vee B] = 1$ only if $d[B] = 1$. By the lights of $\top$-validity, the disjunctive Sorites is simply unsound, for if $\top \gneqq d[A] \gneqq \bot$, $d[\neg A \vee B] \neq \top$. But DS is not d-valid: if $d[A] = 0.5$, $d[B]$

14. $\dot{-}$ is cut-off subtraction, whose result is 0 when $d[p] < d[q]$.

$= 0.4$, then $\min\{d[A], d[\neg A \vee B]\} = 0.5$. So by the lights of d-validity, the argument relies on incorrect logic. The critique of the disjunctive Sorites in terms of $\top$-validity is like Fine's.

A difference emerges over the standard conditional formulation. *Modus ponens* (MP) is valid in Fine's system, so he has to reject some of the conditional premises. The premises are supported by the *Don't Distinguish* principle,[15] at least so long as not distinguishing cases is understood as requiring that of any two sufficiently similar objects a and b, we either have $Fa \wedge Fb$ or we have $\neg Fa \wedge \neg Fb$, for F the vague predicate in question. Since this would be based on LEM and deletion of inadmissible uses, Fine can say that some conditional premises are not true, and that failure to notice this is just a matter of being hoodwinked by *Don't Distinguish*.

In fuzzy logic, we again find that the status of Sorites reasoning varies with choice of consequence-concept: MP is $\top$-valid but not d-valid. Using $\top$-validity, the problem is as Fine says: some conditional premises are not $\top$. The reason, in terms of degrees of truth now, is that given drops of \$1.00 in salary, some of the conditionals will have a degree of truth very slightly lower than $\top$, since their consequents will be very slightly less true than their antecedents. But using d-validity, MP is to be rejected (apply (10c) to $d[A] = 0.9$, $d[B] = 0.8$). Either way, the Sorites reasoning incorrectly detaches consequents that steadily fall in degree of truth to 0.

However, I think fuzzy logic has an advantage over compatibilist semantics in dealing with the conditional Sorites. For *Don't Distinguish* can have a stronger reading than Fine in effect gives it: it can be taken to exclude the transition from

15. Fine calls his version of this principle for conditional Sorites 'Tolerance': 'if two cases are sufficiently alike and the first is [F] then the second is also [F]'.

asserting 'a_i is well-paid' to asserting neither 'a_{i+1} is well-paid' nor 'a_{i+1} is not well-paid'. Fine's view of this transition is that it marks 'a distinction in our response to the cases; it does not…commit one to a distinction in the cases themselves' (Fine 2015:15). Presumably the same has to be said about the transition from asserting neither 'a_{i+1} is well-paid' nor 'a_{i+1} is not well-paid' to asserting 'a_{i+2} is not well-paid'. Yet Fine also wants to hold that the transition from 'a_i is well-paid' to 'a_{i+2} is not well-paid' *does* mark 'a distinction in the cases themselves'. How is this possible, when this last transition is the combination of two transitions neither of which marks a distinction in the cases themselves? This seems different from summing two insignificant changes of the same kind to get a significant change *again of that kind*. In terms of our running example, it instead seems that all three transitions mark distinctions in the cases themselves, since all three involve changes in use of 'well-paid' answering to changes in salary. It's just that the change is bigger comparing a_i and a_{i+2}. If correct, this would mean that *Don't Distinguish* has a true reading that generates the conditional Sorites: if *a* is well-paid and *b* earns only $1.00 less, you cannot politely decline to say that *b* is well-paid too.[16]

On the other hand, applied to fuzzy logic, the strong reading of *Don't Distinguish* means that we should not assign different degrees of truth when salary differences are insignificant. It's unclear why we should accept this prescription. A correct

16. This seems to me to be even clearer for predicates applied on the basis of how things look. If two people side by side cannot be told apart in terms of height without very sensitive measuring apparatus, what would we make of someone who, just on the basis of looking, assents to 'the one on the left is tall' but refuses to assent to 'the one on the right is tall' though he also accepts 'I cannot see the slightest difference between them'? To make this intelligible we'd have to suppose that such an observer equivocates on 'tall', or has been persuaded by epistemicist literature and is taking a guess, or thinks some kind of trick is being played on him, or simply anticipates a Sorites paradox coming down the tracks. (With an isolated pair, *Don't Stray* will not play a role. See Raffman 2014:173–5 for an argument to the effect that focus on isolated pairs explains the force of Sorites reasoning.)

principle is that we should not assign *significantly* different degrees of truth unless there is a significant salary difference, which is compatible with assigning very slightly different degrees to mark insignificant salary differences. This means that assigning very slightly different degrees of truth is consistent with not distinguishing, so perhaps we should reword *Don't Distinguish* to "don't *significantly* distinguish cases that are not significantly different in the relevant respect". The claim is then that assigning very slightly different degrees of truth does not signal a significant difference in cases, while going from asserting that a_i is well-paid to *declining* to assert that a_{i+1} is well-paid *does* signal a significant difference.[17]

However, when we turn to negative-conjunction ($\neg\wedge$) formulations of Sorites reasoning, compatibilist semantics diagnoses the problem more straightforwardly than fuzzy logic. Fine takes premises of the form $\neg(A \wedge \neg B)$ to be true, since they reflect the version of *Don't Distinguish* that he endorses, the one which rules out transitions from 'a_i is well-paid' to 'a_{i+1} is not well-paid'. However, each step in a $\neg\wedge$-formulation carries us from A and $\neg(A \wedge \neg B)$ to B, and this use of CS is invalid.[18] So $\neg\wedge$-formulations of the Sorites rest on faulty logic, by contrast with the two previous formulations, which had untrue premises on Fine's account.

The problem for fuzzy logic's treatment of $\neg\wedge$-formulations is not that the premises are true and CS valid in both senses. For although CS is $\top$-valid (if $d[A] = 1$, $d[\neg(A \wedge \neg B)] = 1$ only if $d[B] = 1$), the premises $\neg(A \wedge \neg B)$ are not in general true

17. This is not the same as objecting that the switch postulates a mysterious cut-off, i.e., a precise fact about how low 'well-paid' goes. That kind of objection can also be made to fuzzy logic (see, for example, Sainsbury 1991), since in any interpretation there is always a lowest salary earning which makes it wholly true that you are well-paid. In (Forbes 2011) I respond to this problem (p.106) by appeal to Kaplan's distinction between representational and artefactual features of models (Kaplan 1975:722). If this distinction is helpful in the current context, Fine could also appeal to it.

18. That there is an embedded use of DN in this version of CS ('classical' CS) is not fundamental to the problem with Sorites reasoning – see Fine on 'running the reasoning backwards' (2015:12–14).

($\top$). The problem is rather that their degrees of truth fluctuate in an unintuitive way. For example, when $d[A] = 0.9$ and $d[B] = 0.8$, $d[\neg(A \wedge \neg B)] = 0.8$, which is an acceptable result though not as plausible as $(A \rightarrow B)$'s degree of truth, 0.9. If $d[\neg(A \wedge \neg B)]$ were stable at 0.8 whenever adjacent members of a Sorites sequence are under consideration, that would be good enough to allay Wright's concerns (see note 13). But if $d[A] = 0.6$ and $d[B] = 0.5$, $d[\neg(A \wedge \neg B)] = 0.5$, which is unjustifiable: given that the degrees of truth in the two pairs are equally close, how can we have 0.8 in the one case and 0.5 in the other? This happens because by contrast with the clause for the conditional, the closeness of the two members of the sequence plays no role in fixing $d[\neg(A \wedge \neg B)]$. If $d[B] \geq 0.5$, then $d[\neg B]$ fixes $d[A \wedge \neg B]$; otherwise $d[A]$ fixes $d[A \wedge \neg B]$; so $d[A] - d[B]$ is irrelevant. But ideally, we would like all the relevant instances of $\neg(A \wedge \neg B)$ to have the same, high, degree of truth, because $d[A] - d[B]$ is so small.[19]

To solve this problem, we could try redefining conjunction to make the magnitude of $d[A] - d[B]$ decisive. One idea is that conjunction, now written '&', should be defined in terms of $\rightarrow$ in the usual way:

(11) $p \,\&\, q \stackrel{\text{def}}{=} \neg(p \rightarrow \neg q)$.

$\neg(A \,\&\, \neg B)$ is d-equivalent to $A \rightarrow B$, and a $\neg$&-formulation of Sorites reasoning may be faulted in the same way as the conditional formulation. However, though (11) does produce a commutative associative connective agreeing with classical conjunction for inputs restricted to $\top$ and $\bot$, the switch from $\wedge$ to & seems suspiciously *ad hoc*. Worse, there are respects in which & is 'un-conjunction-like'. For example, $A \nVdash_d A \,\&\, A$, for whenever $d[A] \leq 0.5$, $d[A \,\&\, A] = 0$, and though, for

19. This problem is ameliorated, but not ultimately dissolved, by the treatment of conjunction in (Edgington 1996).

inputs strictly between 0.5 and 1, $d[A \& A]$ approaches 1 as $d[A]$ does, the former lags the latter, since $d[A \& A] = d[A] - d[\neg A]$, which is always lower than $d[A]$. Therefore, we can assign *any* degree of truth strictly between 0 and 1 to A to show $A \nVdash_d A \& A$.

A better approach to negated conjunction formulations of Sorites is to observe that although there is no trace of ambiguity in 'and' according to whether or not it occurs in a Sorites context, there is a potential ambiguity in negation-*phrases* suggested by the wording of such a premise as

(12) It is not the case that (a_i is well-paid and a_{i+1} is not well-paid).

For here the first occurrence of 'not' is embedded in the context 'it is…the case' while the second is an adjective modifier. While the second 'not' is adequately captured by ¬, 'it is the case' appears to be a synonym of 'it is true'. So inserting 'not' will produce an operator that semantically is sensitive only to whether or not the value of its complement (the proposition expressed by 'that a_i is well-paid and a_{i+1} is not well-paid') is $\top$. That is to say, the prefix 'it is not the case' introduces a negation-operator ~, 'semantic negation', that is different from ¬ and is governed by the following rule:

(13) $d[{\sim}p] = 1$ if $d[p] \neq 1$; $d[{\sim}p] = 0$ if $d[p] = 1$.

A statement like (12), therefore, will be true if taken to have the form ${\sim}(A \wedge \neg B)$. For either 'a_i is well-paid' has a high degree of truth, whence 'a_{i+1} is not well-paid' will have a low degree of truth, or 'a_i is well-paid' has a middling to low degree of truth; so the conjunction in (12) is never 1, that is, never wholly true. This accounts for our intuition that all statements like (12) are true, as they are on compatibilist semantics. On other hand, the pseudo-CS sequent A, $\sim(A \wedge \neg B) \vdash B$ is not even $\top$-valid, for if A is $\top$ and B is slightly less than $\top$, both premises are $\top$ and the conclusion is slightly less than $\top$.[20] The distinction we are appealing to, between semantic negation and what is sometimes called 'fixed-point' negation (because in three-valued logic it maps the intermediate value to itself) is of course a standard one in many-valued theories of vagueness.

My conclusion about Fine's discussion of the three formulations of Sorites reasoning, then, is that it is a very great strength of compatibilist semantics that it diagnoses a problem in each of the three, using the same formal apparatus, and other approaches which cannot do something like this are at an immediate disadvantage. However, I also think that the fuzzy logic analysis of the three formulations is overall equally as effective, in particular because it is independently plausible that embedding fixed-point negation in 'it is the case' produces an operator expressing semantic negation. So the seductiveness of apparent $\neg\wedge$-formulations is explained by revealing them to be $\sim\wedge$-formulations.[21]

20. The $\top$-invalidity of pseudo-CS is obvious if $d[A]$ is $\top$ and $d[B]$ is low but not $\bot$. The explanation of the force of Sorites reasoning is that we only consider cases where $d[B]$ is *slightly* less than $d[A]$.

21. I thank Kit Fine for helpful discussion while I was writing this paper.

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