Global Sourcing under Uncertainty

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Abstract

I examine how firms' global sourcing strategies affect their responses to economic crises such as the 2008-2009 recession. I model firms' entry, exit and sourcing decisions (integrated production or outsourcing) under demand uncertainty. Uncertainty increases the option value of waiting, resulting in less integration as well as less entry and exit. Additionally, I show that trade decisions of integrated firms are less sensitive to uncertainty shocks. I test these predictions using detailed U.S. firm level exports during 2002-2011 and find that integration reduces the probability that a firm exits by as much as 8%, while moving from 25th percentile in the uncertainty distribution to the 75th increases this probability by 7%.

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1 Introduction

International trade takes place between firms that can either have an ownership relationship (i.e. related party trade) or not (i.e. arm's length trade). These alternative ways of trading imply differences in setup costs, in the degree of bargaining power between the trading parties and potentially in future profits, which in turn can affect how firms react to exogenous shocks. Despite the fact that trade between related parties represented 28.2% of U.S. exports and 50.2% of U.S. imports in 2012, most of the existing international trade models assume away these differences. Understanding both the determinants of global sourcing decisions (i.e. trade with a related party or at arm's length), and exploring whether different global sourcing schemes generate heterogeneous responses to shocks is particularly relevant in uncertain environments where shocks are frequent and/or large, such as the recent fall in international trade flows during the Great Recession (a phenomenon known as the Great Trade Collapse, GTC).

This paper makes the case that it is important to understand firms' sourcing decisions under uncertainty, an unexplored topic in the literature. First, uncertainty has played a central role in recent policy debates on the causes of the GTC: some argue that uncertainty is higher in recessions in general (cf. Bloom et al. (2012)), and that elevated uncertainty was one of the causes of the slow recovery during the recent Great Recession (cf. Stock and Watson (2012) and Baker et al. (2012)).[1] Second, uncertainty is more relevant if firms face significant sunk costs, as is the case for firms that engage in international trade in general (cf. Das, Roberts, and Tybout (2007)) and global sourcing strategies in particular (cf. Antràs and Helpman (2004)). Third, exporting firms are subject to additional sources of uncertainty, such as exchange rates and foreign market conditions. Fourth, the impact of uncertainty on sourcing decisions has yet not been explored in the literature. Fifth, heterogeneous responses to aggregate shocks across global sourcing strategies can potentially be important in terms of welfare and trade dynamics.

[1]For example, in words of Olivier Blanchard, chief economist at the IMF *"[Uncertainty] is largely behind the dramatic collapse in demand. [...] Given the uncertainty, why build a new plant, or introduce a new product now? Better to pause until the smoke clears."* Similarly, John C. Williams, president of Federal Reserve Bank of San Francisco, remarks that *"There is pretty strong evidence that the rise in uncertainty is a significant factor holding back the pace of recovery now."*

To address these issues, I first show that U.S. firms' adjustment to the GTC differs across sourcing schemes. Then, I develop a dynamic model with endogenous entry and exit combined with global sourcing decisions in which firms face demand uncertainty. Next, I construct a theory-based uncertainty measure and take the model predictions to U.S. firm level exports data for the period 2002-2011, with special focus on the exit decision. Finally, I use the estimated results to quantify the role of uncertainty and sourcing decisions. In the counterfactual analysis, I found that if all firms behave as related parties, the 2009 collapse of U.S. exports would have been 10% smaller. Also, the counterfactual analysis shows that reducing foreign demand uncertainty for all firms to the first tercile of the uncertainty would have reduced the 2009 collapse in U.S. exports by 8%.

The data required to examine the impact of uncertainty on global sourcing decisions and the impact of sourcing on responses to shocks are highly demanding. In addition to requiring detailed information on U.S. firms' export and import transactions at high frequency, such task requires information on the ownership relationships between U.S. trading firms and their foreign partners. This information is naturally scarce. However, U.S. firm-level international trade data is particularly well suited for the task under consideration, because it is one of the few datasets that records the relationship between trading firms for *every* transaction, and thus avoids the need to limit the analysis to a subsample of firms or to impose other restrictive assumptions.[2] Moreover, to the best of my knowledge, this paper is the first to exploit this data in order to analyze the heterogeneity in the impact of shocks across sourcing strategies.

Uncertainty rose sharply during the GTC. Bloom et al. (2012) show that both microeconomic uncertainty and macroeconomic uncertainty behave countercyclical for the period 1972-2010 in the U.S.. According to Bloom et al. (2012), microeconomic uncertainty increased by between 76% and 152% during the GTC, depending on the measure of uncertainty used, while macroeconomic uncertainty increased by 23%. These remarkable increases of uncertainty support the claim of policy makers that elevated uncertainty was a cause of the GTC. Given the significant role of uncertainty in policy debates and the evidence of its countercyclical behavior, it is striking that uncertainty has not been much explored as one

[2]For example, Corcos et al. (2009) used a survey of manufacturing firms that have foreign affiliates and trade more than 1 million euros in 1999.

of the potential factors behind the GTC in the academic trade literature. A notable exception is Carballo, Handley, and Limão (2013), who consider the impact of trade policy and economic uncertainty on the firm export decision. This current paper expands on their work by constructing a model where demand uncertainty plays a significant role. The interaction between uncertainty and global sourcing decisions is then examined. Finally, I quantify the impact of uncertainty and heterogeneity of responses by sourcing decision during the GTC.

Carballo, Handley, and Limão (2013) show that the extensive margin accounts for roughly one third of the collapse in U.S. exports during the GTC. Furthermore, the exit margin is the main force driving the adjustment along the extensive margin during this period. Most trade models are ill suited to understand the dynamics of episodes such as the GTC, since they are primarily focused on entry and disregard the exit decision. In order to overcome this limitation, I introduce an *endogenous* exit decision into a model where firms also make entry and sourcing decisions. Furthermore, I explore the impact of sourcing strategies on firms' exit decision and find that related party trade is more resilient to a large negative shock, such as the GTC.

As is standard in trade models, the model features heterogeneous firms that have to pay a sunk cost to start exporting (see Melitz (2003)). Following Antràs and Helpman (2004), I introduce incomplete contracts to model the sourcing decision and assume that integrating with a foreign firm requires to pay another sunk cost.[3] Firms also have to pay a fixed per period cost, which generates an endogenous exit decision. Additionally, I impose that the final good is consumed in the foreign destination.[4] Finally, I introduce uncertainty by assuming that firms do not know future foreign demand level. In this setting, I show that uncertainty leads firms to delay sunk investments, such as entry and integration, and makes them less responsive to demand level changes when compared to the deterministic framework. Firms internalize that demand level is going to change in the future, and thus, they do not fully respond to the current demand level. I further prove that sourcing strategies affect

[3]Transaction costs and incentive system are alternative approaches used to model the integration decision in the international trade literature. See Grossman and Helpman (2004) for an incentive system approach and McLaren (2000) and Grossman and Helpman (2002) for a transaction costs motive.

[4]According to Antràs and Yeaple (2013) a very small fraction of output is exported from foreign affiliates back to the headquarter country. Furthermore, Ramondo, Rappoport, and Ruhl (2011) show that most of foreign affiliates sell all their output in their host country.

the exit decision: related parties wait longer before leaving foreign markets. This impact arises due to the combination of uncertainty, additional sunk costs needed to export to a related party, and the higher profit flow associated with integration. Moreover, I prove that uncertainty generates heterogeneity on the impact of current demand changes across organizational forms and margins, therefore breaking the homogeneous impact of the deterministic framework. The reason behind this result is that in a deterministic framework, profits are log-separable in the current demand condition. However, this log-separability does not hold in a stochastic environment, wherein firms' response to changes in current demand level are affected by their sourcing strategy.

This work contributes to the literature of multinationals broadly (see Antràs and Yeaple (2013) for a recent survey) and more specifically to the topic of multinationals and option value. Early work by Rob and Vettas (2003) focus on the choice between FDI and exports when demand growth is uncertain. Fillat and Garetto (2010) analyze the relationship between stock market return and risk exposure of multinationals in a context of option value. More recently, Ramondo, Rappoport, and Ruhl (2013) consider the decision between FDI and exports in two period model with demand uncertainty. These papers focus on the choice between multinational production or exporting as substitutes. In contrast, my work allows for intra-firm exports and models firms' decisions between exporting through related party or arm's length. In this sense, this paper is closer to Irarrazabal, Moxnes, and Opromolla (2013) where they consider multinational production with intra-firm trade in a static context.[5] However, they focus on geography and multinational production and do not have information whether transactions take place through related parties or arm's length trade. Thus, this research adds by analyzing, for the first time, the role of demand uncertainty in global sourcing decisions using U.S. firm level export data for the period 2002-2011. The analysis is performed for the whole population of U.S. exporting firms, not just a sample, and without imposing any assumption on the organizational choice, since U.S. Census Bureau data provides the information about the type of relationship between trading firms.

This paper also contributes to the literature on the GTC in two ways.[6] First, this

[5]See also Keller and Yeaple (2013) that consider multinational production with tangible and intangible intra-firm trade.

[6]See Bems, Johnson, and Yi (2013) for a survey on the GTC literature on international trade.

paper highlights differences across organization forms in the dynamics of U.S. firm-level trade during the GTC and its recovery. Second, this research evaluates the contribution of uncertainty and its interaction with global sourcing decisions in explaining the dynamics of U.S. firms' exports during the GTC.

The remainder of the paper is organized as follows. Section 2 details the data used and provides descriptive evidence. Section 3 develops the theoretical model. Section 4 discusses the model's testable predictions and evaluates these predictions econometrically. Finally, section 5 concludes.

2 Data and Descriptive Evidence

In this section, I use data on firms' exports and characteristics to present descriptive evidence on the recent Great Trade Collapse (GTC). I focus on differences in dynamics across organizational forms during this period. First, I present aggregate evidence about related parties and arm's length trade. Second, I show evidence from several decomposition exercises that highlights differences in margins of adjustment across organizational forms when firms respond to shocks in foreign markets.

2.1 Data Description

The main data sources I use are the Longitudinal Foreign Trade Transactions Database (LFTTD) and the Longitudinal Business Database (LBD) for the period 2002-2011. Both databases are from the U.S. Census Bureau. The LFTTD provides detailed information on U.S. firms' export transactions with product and destination disaggregation. The LFTTD has a longitudinal identifiers variable that allows me to track firms over time. The LBD, meanwhile, is constructed based on administrative data and provides firm-level information over time such as employment, age and sector of activity.

The paper focuses on the sourcing decisions of U.S. exporters. Hence information about the ownership relationship between trading parties is key. Importantly, the LFTTD has a variable that allows one to identify whether the U.S. exporter and the foreign firm involved in a transaction are related parties. According to the Foreign Trade Regulations of the

Department of Commerce, a related party export transaction is "*a transaction involving trade between a U.S. principal party in interest and an ultimate consignee where either party owns directly or indirectly 10 percent or more of the other party.*" (see Foreign Trade Regulations, 2013). This is mandatory information that should be included in the *automatic electronic system* data filing for export transactions (see Ruhl (2013)).[7] Thus, the LFTTD provides a complete picture on the sourcing decisions of U.S. exporters at high frequency and over time, and avoids the problems that other studies in the literature have faced when considering the sourcing decision of firms. For example, Corcos et al. (2009) use a small sample of 4,305 French firms that respond to a survey in 1999 to analyze the determinants of sourcing decisions.

Other sources of information used in the paper include uncertainty measures introduced by Baker and Bloom [2013] and the International Financial Statistics from the International Monetary Fund for country characteristics. Additionally, I use the product concordances for the Harmonized Schedule developed by Pierce and Schott (2009) in order to avoid capturing spurious changes along the extensive margin due to schedule changes.[8]

2.2 Heterogeneous Responses during the GTC

The 2008-9 global recession and its associated trade contraction is known as the Great Trade Collapse in the literature. U.S. imports and exports begin falling in the third quarter of 2008. By the end of 2009, imports had fallen amounted to 22.7% and exports, 19.0%, generating a 20.1% decrease in U.S. international trade. This sudden trade collapse is remarkable since U.S. GDP only dropped 1.7% over the same period while world GDP contracted by 1.1%. Interestingly, the overall drop in total exports differed somewhat between related parties and arm's length trade present sat 16.92% and 18.74% respectively.[9] However, this difference

[7] This mandatory filing of the relationship between trading parties assures that the information covers virtually the whole universe of export transactions. The Census Bureau related parties data is very consistent when compared with the Bureau of Economic Analysis data on Multinational firms, which has been used more extensively in previous literature (see Ruhl (2013)).

[8] The U.S. harmonized product codes used to register imports and exports transactions are updated over time. Pierce and Schott (2009) developed an algorithm that matches revised codes to time-invariant identifiers that allow to follow products over time.

[9] According to BEA, data this difference between related parties exports and arm's length trade is significantly bigger. More specifically, exports to affiliated foreign firms contracted only by 5.32% while exports

across organizational forms is magnified when other dimensions of the collapse are considered, such as the number of firms and firm-varieties trading across organizational forms, or when the GTC collapse is decomposed into the extensive and intensive margins using U.S. firm level data distinguishing by organizational form.

Overall, the number of firms exporting fell by 12.16% during 2009. The number of firms trading to related parties fell by 8.51% in 2009, while for arm's length trade the collapse in the number of firms is significantly higher, at 12.52%. The fall in the number of firm-varieties traded by non-related parties during the GTC was almost double the fall for related parties. More specifically, at the peak of the collapse the number of related firm-varieties contracted by 5.73% while the contraction in the number of firm-varieties traded at arm's length reached the 11.38%. Moreover, the number of firm-varieties for related parties recovered more quickly after the 2008-2009 GTC. For example, by the end of 2009 the number of related firm-varieties was actually 0.64% higher than its level in 2008. In contrast, the number of firm-varieties for arm's length trade at the end of 2009 was 5.07% lower than its peak level in 2008, and did not recover its respective pre-crisis level until almost a year later. Naturally, this difference in the evolution of the number of varieties traded across organizational forms translates into differences in the exit rate of firm-varieties. The exit rate for related parties is roughly two-thirds of the exit rate for arm's length trade at quarterly frequency during the GTC, at 42.% and 62.2% respectively. Thus, although the overall drop in total exports during the GTC was similar between related parties and arm's length trade, the difference in the behavior of the number of varieties suggests that there is some heterogeneity in dynamics during the GTC across organizational form (See Table A.1in the appendix for more details.). To further explore this heterogeneity, I decompose firm level trade into its intensive and extensive margins.

Specifically, I perform the following decomposition exercise using highly disaggregated U.S. data. First, I compute the midpoint growth rate of exports at the firm-product-country level, in order to isolate the evolution of the extensive and intensive margins of exports and I do this separately for related parties and arm's length trade. I follow the literature on the GTC by working with data at quarterly frequency, motivated by the timing of the collapse, as annually frequency would mask many interesting dynamics(cf. Bricongne et al. (2012),

to unaffiliated foreign firms contracted by 22.10% over this period.

Behrens, Corcos, and Mion (2013) and Eaton et al. (2013)).

The midpoint decomposition breaks exports growth into its intensive and extensive margins at the firm-country-product level. I then decompose each of these terms into positive and negative margins. Finally, growth rates computed at the firm level are aggregated to compute the aggregate midpoint growth rate. The aggregate mid-point growth rate of exports is defined as follows:

$$G(q) = \frac{X(q) - X(q-4)}{\frac{1}{2}[X(q) + X(q-4)]}$$

$$G(q) = \sum_i \sum_c \sum_k \underbrace{\frac{[x_{ick}(q) + x_{ick}(q-4)]}{[X(q) + X(q-4)]}}_{s_{ick}(q)} \times \underbrace{\frac{[x_{ick}(q) - x_{ick}(q-4)]}{\frac{1}{2}[x_{ick}(q) + x_{ick}(q-4)]}}_{g_{ick}(q)}$$

$$G(q) = \sum_i \sum_c \sum_k s_{ick}(q) \times g_{ick}(q)$$

where x denotes exports and i, c, k, q index firm, country, product and quarter respectively; $g_{ick}(q)$ is the midpoint growth rate of firm i exports of product k to country c in quarter q; and $s_{ick}(q)$ is the weight corresponding to $g_{ick}(q)$ in total exports.

Changes in exports at the firm-product-country level can be classified into: (i) extensive positive ("Entry") where $x_{ick}(q) > 0$ and $x_{ick}(q-4) = 0$; (ii) extensive negative ("Exit") where $x_{ick}(q) = 0$ and $x_{ick}(q-4) > 0$; (iii) intensive positive ("Growers") where $x_{ick}(q) > x_{ick}(q-4) > 0$; and (iv) intensive negative ("Shrinkers") where $x_{ick}(q-4) > x_{ick}(q) > 0$. Thus, the aggregate midpoint growth rate can be expressed as

$$\begin{aligned} G(q) = & \sum_i^{NE_{ck}} \sum_c \sum_k [s_{ick}(q) \times g_{ick}(q)] + \sum_i^{NX_{ck}} \sum_c \sum_k [s_{ick}(q) \times g_{ick}(q)] \\ & + \sum_i^{CN1_{ck}} \sum_c \sum_k [s_{ick}(q) \times g_{ick}(q)] + \sum_i^{CN2_{ck}} \sum_c \sum_k [s_{ick}(q) \times g_{ick}(q)] \end{aligned}$$

where $NE_{ck}, NX_{ck}, CN1_{ck}$ and $CN2_{ck}$ denote respectively the sets of entering, exiting, growing and shrinking firms exporting product k to country c. This decomposition also allows me to compute the net extensive and net intensive margins, by adding up both positive and negative components of each margin. The following graph presents the evolution of the net extensive and net intensive margins for related parties and arm's length trade during the period surrounding the GTC:

Figure 1: Decomposition Related Party

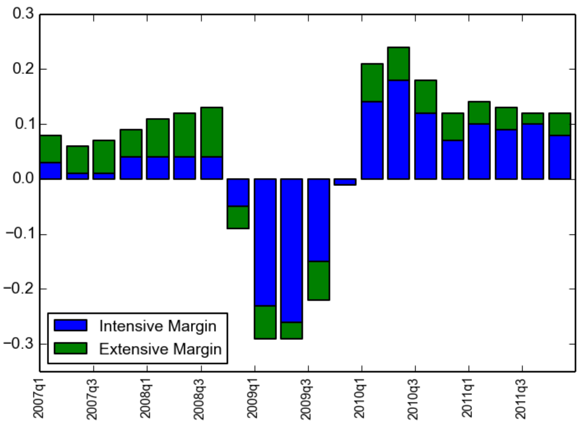
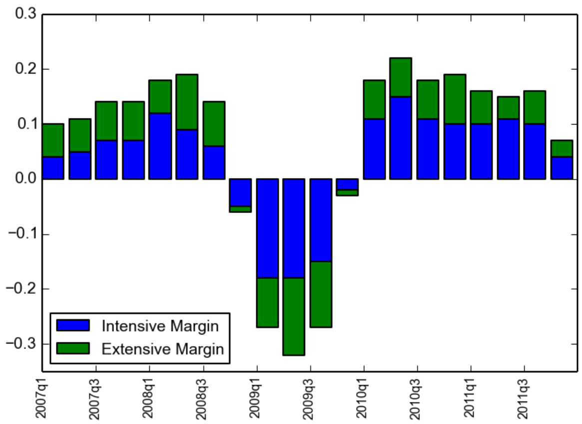


Figure 2: Decomposition Arm's Length



This decomposition exercise shows that the overall volume of related parties and arm's length trade followed a similar path during the GTC, but that their margin of adjustment differed substantially (see the appendix for detailed tables). In both cases, the intensive margin was the main margin of adjustment. However, the extensive margin took a more prominent role for arm's length trade than for related parties. More specifically, during 2009, the extensive margin for arm's length trade represented -9.90% points of growth on average, while for related parties the extensive margin contributed only -3.95% on average. Hence, the extensive margin contribution for arm's length trade was roughly two and a half times higher than the extensive margin contribution for related parties. The opposite holds true for the intensive margin, whose average contribution for related parties was 1.5 times the average intensive margin contribution for arm's length trade. Importantly, this pattern is robust to using lower frequency data and more aggregated firm-level data, in particular computing the decomposition using data at the half-yearly frequency and using firm-country exports aggregated over products. Furthermore, the differences between across related parties and arm's length trade are robust to controlling for firm size using employment level. When restricting the decomposition exercise to firms with more than 250 employees, I continue to find that arm's length trade adjusts significantly more through the extensive margin than related parties trade. Also, this result is robust to distinguishing between related-party and arm's length trade for both PTA partners and non-PTA partners. This robustness exercise is motivated by the fact that PTA agreements can protect firms from additional uncertainty as it is discussed in Carballo, Handley, and Limão (2013).

2.3 Uncertainty and Sourcing Decision

Demand uncertainty has not been considered in previous work as a determinant of the sourcing decision of whether to export to a related party or at arm's length. However, descriptive evidence suggests that there is a correlation between uncertainty and firms' decision regarding the organizational form of trade. Table 1 shows that U.S. firms export on average less via related parties to countries with high uncertainty, as measured by the volatility of the GDP following Bloom (2014). More specifically, I classified countries as low uncertainty if their standard deviation of GDP growth rate belongs to the first tercile; while I considered as countries with high uncertainty those whose standard deviation of GDP growth rate is in third tercile. Countries categories are time-invariant over the sample considered. The negative relationship holds both in terms of average total exports and the average number of products, as well as, more importantly, in terms of the average share of related parties exports in total exports and the average ratio of the number of related parties products to the total number of product exported to that country.[10] [11]

[10]Section 4.1 discuss a theory consistent measure of uncertainty. These patterns on the share of related party exports and uncertainty hold when I use this theory-consistent uncertainty measure.

[11]Figure A.1 shows that this correlation between uncertainty and the way U.S. firms choose to serve a foreign market holds for the distribution of related party share in exports.

Table 1: Related Parties and Uncertainty (2002-2011 Average)

	Uncertainty		Total
	Low	High	
Related Exports	8,770.00	1,000.00	4,510
	[1,780]	[2,030]	[10,130]
Share of Related Exports	0.2331	0.1294	0.193
	[0.1268]	[0.118]	[0.1331]
Number of Related Products Exported	264.79	140.28	195.09
	[120.93]	[118.49]	[126.33]
Share of Related Products Exported	0.6428	0.3877	0.5008
	[0.2385]	[0.2442]	[0.2551]
Observations	310	310	310

Means and standard deviations in brackets.
Low and High refer to the bottom and top tercile of the uncertainty measure.
Total includes the full sample at country level.

In this section, I present evidence that suggests that related parties and arm's length trade had different responses to the GTC in 2008/9. In particular, decomposition exercises show that arm's length exporters were more likely to exit foreign markets than related parties during the GTC. Thus, the exit decision is a key margin along which the organizational form affects the response of firms. This evidence on heterogeneous responses combined with the existing literature on uncertainty during the GTC and the descriptive evidence above on the relationship between uncertainty and the organizational form chosen by U.S. exporters are the key insights that motivate me to build a dynamic model with endogenous entry, exit and organizational choice when firms face demand uncertainty.

3 Theory

This section develops a dynamic model with incomplete contracts in which firms endogenously choose when to start and stop exporting and make global sourcing decisions under demand uncertainty. The novel features of the model are the stochastic demand process and its interaction with the exit and global sourcing decisions. In the first subsection, I derive the basic elements of the model for exporting firms under incomplete contracts: optimal demand, supply, pricing and profits. Then I consider firms' entry, exit and global sourcing decisions under demand uncertainty.

3.1 Incomplete Contracts

The incomplete contracts setting of the model follows the standard approach of Antràs and Helpman (2004), in which this incompleteness affects how the revenue of a trading relationship is distributed between the firms involved. Interestingly, this setup simplifies the analysis because the optimal quantity and price decisions are invariant across global sourcing decisions. There are two countries, N and S, where S is the foreign country. I assume that wages are higher in N, $w^N > w^S$.

Preferences in the foreign country are represented by a Cobb-Douglas utility function over a homogeneous good, denoted x_0, and a CES sub-utility index defined over differentiated goods X with constant expenditure share μ, where $0 < \mu < 1$. The homogeneous good is the numeraire of the model and is freely traded. Formally,

$$U = x_o^{1-\mu} X^{\mu} \tag{3.1}$$

$$X = \left[\int x(i)^{\alpha} di \right]^{1/\alpha} \tag{3.2}$$

where $0 < \alpha < 1$. Optimal demand for variety i when aggregate income is equal to Y is given by $x(i) = \mu Y \left[\frac{p(i)}{P^{\alpha}}\right]^{-\frac{1}{1-\alpha}}$ where P denotes the price index and $p(i)$ is the price of the variety.

There are two types of agents in the economy, entrepreneurs (H) and manufacturers (M). Entrepreneurs provide headquarter services and are located only in N, while manufacturers are located in S and provide assembly services. For simplicity, firms in the North can only outsource or integrate with firms in the South.[12] The final good production is represented by $x(i) = \theta \left[\frac{h(i)}{\eta}\right]^{\eta} \left[\frac{m(i)}{1-\eta}\right]^{1-\eta}$, where $0 < \eta < 1$. θ is the productivity parameter, while functions $h(i)$ and $s(i)$ denote headquarters and assembly services respectively. As in Antràs (2014), headquarter services is meant to include high-tech manufacturing or assembly, while assembly services also encompass distribution, packaging or marketing services. Following Ramondo and Rodríguez-Clare (2013) and Antràs (2003), I assume that there is trade in intermediates but no trade in final goods and the final good is consumed

[12] Fernandes and Tang (2012) use the same assumption in the context of a static model.

in the foreign destination. This assumption is motivated by the fact that most foreign affiliates do not export back to their headquarter country (see Antràs and Yeaple (2013)) and that most foreign affiliates do not export to third countries (see Ramondo, Rappoport, and Ruhl (2011)). The entrepreneur (i.e. the firm located in N) can choose either to integrate vertically with a firm located in S, or to outsource its demand for assembly services to a firm located in S. In this setup, an organizational form consists of an ownership structure $k \in \{Outsourcing\,(O), Vertical\,Integration\,(V)\}$.

As is standard in trade models, firms are required to pay a sunk cost to start exporting (cf. Melitz (2003)). More specifically, firms have to pay f_e in order to start exporting via outsourcing, which is the default option to start exporting. If firms want to integrate, they have to pay an additional sunk cost f_v as is the case in AH. All firms also must pay fixed per period cost f_p to operate in the foreign market. This fixed per period cost allows me to consider the optimal decision to stop exporting. Finally, to preserve the symmetry of the model, I assume that firms have to pay an sunk cost f_x to exit the foreign market.[13]

Following AH, I assume that parties cannot write enforceable contracts contingent on outcomes. Instead the entrepreneur and manufacturer bargain over surplus from the relationship. Ex-post bargaining is modeled as a generalized Nash bargaining game, in which the final-good producer obtains a fraction $\zeta \in (0, 1)$ of the ex-post gains from the relationship. Importantly, the ownership structure does not affect whether or not there is ex-post bargaining. More specifically, the space of contracts is independent of the ownership structure and the same is true for the ex-post bargaining process. In the incomplete contract setting, the outside options for the two parties determine the incentives that each party has ex-post. I assume that the outside option for the manufacturing firm is zero in all cases, while the outside option for the entrepreneur depends on the organizational form. In the case of outsourcing, the outside option is zero while under vertical integration, the entrepreneur H can seize a share of the final good δ, where $0 < \delta < 1$.

The mode of ownership is chosen at the beginning of the period by H to maximize its profits. The contract includes an up-front fee (positive or negative) that is paid by M in order to participate in the relationship. Under the assumption that the supply of M is infinitely elastic, in equilibrium M's profits from the relationship net of the participation

[13] Results do not depend on the sunk cost to exit, although this sunk cost simplifies the exposition.

fee should be equal to its outside option, zero. Under outsourcing, when parties reach an agreement ex-post, H gets $\zeta R(i)$ while M gets $(1-\zeta)R(i)$, where $R(i)$ denotes the potential revenue of the trade relationship.[14] On the other hand, if parties fail to reach an agreement, both parties get zero under outsourcing. In contrast, when parties fail to reach an agreement under vertical integration, H can sell an amount $\delta x(i)$ of output which yields revenue $\delta^\alpha R(i)$. Hence the ex-post gains from trade are $[1-\delta^\alpha]R(i)$. Accordingly in the bargaining, H receives its outside option plus its share of the ex-post gains, or $\delta^\alpha R(i) + \zeta[1-\delta^\alpha]R(i)$. This implies that M receives $(1-\zeta)[1-\delta^\alpha]R(i)$. Hence

$$\zeta_V = \delta^\alpha + \zeta[1-\delta^\alpha] \geq \zeta_o = \zeta$$

In other words, H is able to appropriate a higher fraction of revenue under integration than under outsourcing.

Given the nature of the contract, parties choose their quantities of inputs noncooperatively, since inputs are not contractible ex-ante. Thus, firms' problems conditional on organizational form k are

$$H: \max_{h(i)} \zeta_k R(i) - w^N h(i)$$
$$M: \max_{m(i)} (1-\zeta_k) R(i) - w^S m(i)$$

After solving these two problems, I obtain the following expression for the total current period profit:

$$\pi_k(\mathcal{A}, \eta, \theta) = \mathcal{A}\theta^{\frac{\alpha}{1-\alpha}} \psi_k(\eta)$$

The profit function is the product of a term capturing the demand level ($\mathcal{A} = (\mu Y) P^{\frac{\alpha}{1-\alpha}}$), the modified productivity of the firm ($\theta^{\frac{\alpha}{1-\alpha}}$), a term capturing the impact of the incomplete contracts mechanism ($\psi_k(\eta) = \frac{(1-\alpha[\zeta_k\eta+(1-\zeta_k)(1-\eta)])}{\left(\frac{1}{\alpha}\left[\frac{w^N}{\zeta_k}\right]^\eta \left[\frac{w^S}{(1-\zeta_k)}\right]^{1-\eta}\right)^{\frac{\alpha}{1-\alpha}}}$) and the fixed per period costs $w^N f_p$.

The setup implies that H chooses the organizational form that maximizes $\pi_k(\theta, A, \eta)$. Operating profits for the two firms are:

$$\pi_{Hk} = \zeta_k R(i) + t - w^N h(i)$$
$$\pi_{Mk} = (1-\zeta_k) R(i) - t - w^S m(i)$$

Given that the outside option for M is zero, then the fee t is set such that $\pi_{Mk} = 0$. Hence

[14] The potential revenue of the trade relationship is $R(i) = (\mu Y)^{1-\alpha} P^\alpha \theta^\alpha \left[\frac{h(i)}{\eta}\right]^{\alpha\eta} \left[\frac{m(i)}{1-\eta}\right]^{\alpha(1-\eta)}$.

$\pi_{Hk} = R(i) - w^N h(i) - w^S m(i)$ and in a subgame-perfect equilibrium $\pi_{hk} = \pi_k(\theta, \mathcal{A}, \eta)$. As in AH, there are no means to commit ex ante to a division rule of the surplus. The choice of ownership structure is the only instrument for affecting the division rule. The final good producer then can choose between $\zeta_k = \{\zeta_V, \zeta_0\}$ which determines whether the H receives $\pi_v(\theta, \mathcal{A}, \eta)$ or $\pi_o(\theta, \mathcal{A}, \eta)$. In general. $\pi_v(\theta, \mathcal{A}, \eta)$ and $\pi_o(\theta, \mathcal{A}, \eta)$ cannot be ranked without further assumptions on η (intensity of headquarters services).

Note that the division rule of the profits ζ_k affects the slope of the profits function with respect to the productivity parameter θ and the parameter capturing the demand level $\mathcal{A}$. Furthermore, note the fact that $\zeta_V > \zeta_0$ is not enough to unequivocally rank whether $\psi_v(\eta)$ is greater or lower than $\psi_o(\eta)$. The intensity of the headquarter services is key to determining which function $\psi_k(\eta)$ is larger. Intuitively, the explanation is that the incompleteness of contracts implies that neither party appropriates the full marginal return on its investments. Hence both H and M underinvest, although this underinvestment is ameliorated by the fraction of the surplus that they receive. Thus, ex ante efficiency requires that the higher the intensity of headquarter services (i.e. high η), the higher the fraction of the surplus that should be allocated to H. This relationship between the optimal ζ and η, combined with the assumption that ζ_k are fixed, implies that for η, sufficiently large higher values of ζ_k generate more profits. This, in turn, implies that for η sufficiently large, $\psi_v(\eta) > \psi_o(\eta)$ given that $\zeta_v > \zeta_o$. In contrast, for low enough η, $\zeta_o > \zeta_v$ implies that $\psi_v(\eta) < \psi_o(\eta)$. Note that given that vertical integration requires an additional sunk cost, $\psi_v(\eta) < \psi_o(\eta)$ is sufficient for outsourcing to be the optimal choice. In contrast, $\psi_v(\eta) > \psi_o(\eta)$ is not sufficient for vertical integration to be optimal given the additional sunk cost. In this case, firms with different θ will choose different organizational forms in equilibrium. Since this is the case I want to focus on, I will assume that η is sufficiently large enough so that $\psi_v(\eta) > \psi_o(\eta)$ whenever $\zeta_v > \zeta_o$. So far the setup of the model is standard and uncertainty has not played any role. This is due to the fact that uncertainty about current period state variables is resolved before firms take any decisions. In the next subsection, I discuss how uncertainty is incorporated into the setup.

3.2 Demand Uncertainty and Firms' Decisions

Firms face uncertainty when considering the decision to enter or to exit a market and when choosing their ownership structure. More specifically, firms have to deal with uncertainty about foreign demand. Firms do not know next period's value of $\mathcal{A}$, and today's demand level is only partially informative about the future values of $\mathcal{A}$. This uncertainty is captured by a stochastic foreign demand process. The foreign demand level is a random variable with CDF $G(\mathcal{A})$, with shocks to the path of foreign demand arriving with probability $\gamma > 0$. Furthermore, I assume that the CDF $G(\mathcal{A})$ is stable across time and that the arrival of shocks implies that a new demand level is drawn from this stable underlying distribution. I also assume that $G(\mathcal{A})$ has support $[\underline{\mathcal{A}}, \bar{\mathcal{A}}]$.

This uncertainty implies that firms solve an optimal stopping problem. In a deterministic framework, firms make decisions by comparing the profit flows of each status with the cost of changing status. However, in a stochastic framework, this approach, called the *naive* approach by Dixit and Pindyck (1994), ignores the possibility of waiting. More specifically, comparing the flow of profits across states leaves out the possibility that the optimal decision may be to switch status in some future period when the environment has different conditions.

Firms in the model endogenously decide when to start exporting and when to stop exporting and what ownership structure to employ. Thus, firms that currently only produce domestically must decide whether or not to start exporting, and under what ownership structure. Firms that currently export via outsourcing must decide whether to keep exporting, integrate or exit; and firms that currently export under integration must decide whether to continue or exit. In each transition from one status to another, firms compare the difference in value between each status with the cost of changing the status. For example, non-exporting firms at the margin of considering exporting via outsourcing will compare the fixed cost of entry to the difference between the expected value of being an exporter and the expected value of being a non-exporter. It is worth noting that the expected value of not exporting in the current period includes the possibility of becoming an exporter in the future, because non-exporters in the current period could begin exporting in some future period when conditions improve. In other words, the value of not exporting implicitly includes the value of waiting. Similarly, the value of exporting via outsourcing includes the possibility that the firm integrates in the future. Formally, non-exporting firms solve:

$$V = \max\{V_o - f_e, V_v - f_e - f_v, V_w\}$$

where V_k denotes the value function of each possible status (o, exporting via outsourcing; v, integrated exporter; and w non-exporter). Note that this problem can be decomposed into two simpler problems, given the assumption that the profit flow from outsourcing is lower than the profit flow from integration conditional on the demand level.[15] More specifically, a non-exporting firm will prefer exporting via outsourcing to being a non-exporter if $V_o - f_e > V_w$. Hence, equalizing the value of exporting via outsourcing minus the sunk cost to the value of waiting implicitly defines a demand level that makes a firm with given productivity θ indifferent between these two options:

$$V_w(\mathcal{A}_o^e) = V_o(\mathcal{A}_o^e) - f_e \tag{3.3}$$

where $\mathcal{A}_o^e$ is the *demand entry threshold with outsourcing*. This condition, when evaluated at $\mathcal{A}_o^e$ implies that the difference in current period profits (note that when the firm is not exporting, the current period profits *of exporting* are zero) plus the difference in expected future value should be equal to the fixed cost of entering. Similarly, non-exporting firms will prefer exporting with integration to being a non-exporter as long as $V_v - f_e - f_v > V_w$. However, note that the condition $V_v - f_e - f_v > V_w$ is not relevant to a firm's decision because for any demand level that fulfills this condition it is also true that $V_o - f_e > V_w$. Hence the optimal alternative to integration for a firm on the margin is to export via outsourcing than rather not exporting.

A firm exporting via outsourcing needs to consider whether to continue exporting via outsourcing, integrate or exit the market. Formally, firms exporting via outsourcing solve:

$$V = \max\{V_o, V_v - f_v, V_w - f_x\}$$

which can be separated into two decisions, whether or not to integrate and whether or not to exit. Each of these decisions determines a threshold demand level. A firm that currently exports via outsourcing will integrate if $V_v - f_v > V_o$. Hence the following equation determines the demand integration threshold $\mathcal{A}_v^e$:

$$V_o(\mathcal{A}_v^e) = V_v(\mathcal{A}_v^e) - f_v \tag{3.4}$$

[15]This is due to the assumption that the sector is intensive in headquarter services, which assures that $\pi_v > \pi_o$.

Similarly, a firm that is currently exporting via outsourcing will stop exporting if $V_o > V_w - f_x$. The exit with outsourcing demand threshold $\mathcal{A}_o^x$ is determined by

$$V_o(\mathcal{A}_o^x) = V_w(\mathcal{A}_o^x) - f_x \tag{3.5}$$

Finally, a firm currently exporting under vertical integration will solve the following problem:

$$V = \max\{V_o - f_o, V_v, V_w - f_x\}$$

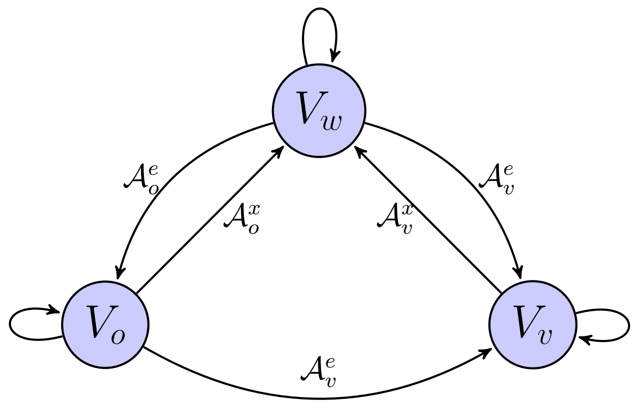
Note that since $V_v > V_o$ for all demand levels, it is never optimal for a currently integrated firm to switch to outsourcing. Thus, the relevant decision for an integrated firm is whether or not to continue exporting, which will be the case if $V_v > V_w - f_v$. Then the exit with integration threshold $\mathcal{A}_v^x$ is determined by

$$V_v(\mathcal{A}_v^x) = V_W(\mathcal{A}_v^x) - f_x \tag{3.6}$$

In summary, firms can be in one of three states in the model: non-exporter, exporting via outsourcing and exporting via integration. These three states imply four relevant margins which firms can be indifferent : (i) between being a non-exporter and exporting via outsourcing, (ii) between exporting via outsourcing and integration, (iii) between exporting under integration to being a non-exporter and (iv) from exporting via outsourcing to being a non-exporter.[16] These four conditions determine four demand thresholds $(\mathcal{A}_o^e, \mathcal{A}_v^e, \mathcal{A}_o^x, \mathcal{A}_v^x)$ for each firm that completely describe the firm's policy function. The next step is solve the value functions for each of the possible states. Figure 3 shows all the transitions across the states in the model with their respective demand threshold:

[16] The other two potential margins that are left out are (v) between exporting with integration and exporting via outsourcing and (vi) between being non-exporter and exporting with integration. The former is irrelevant because exporting with integration is always more profitable than exporting via outsourcing, hence no firm is going to optimally do this transition. The latter is irrelevant because the following holds. In order to integrate, the non-exporter firm needs that $V_v - f_e - f_v > V_w$ and $V_v - f_e - f_v > V_o - f_e$ but since for any demand level satisfying $V_v - f_e - f_v > V_w$ it is also true that $V_o - f_e > V_w$ so that the two initial conditions collapse to $V_v - f_v > V_o$ which is the same comparison for the transition (ii).

Figure 3: Transitions and Demand Thresholds



Starting with the value of exporting via outsourcing, note that the firm perceives a current profit from exporting equal to $\pi_o(\mathcal{A}_t) - f_p$ and a continuation value that depends on the optimal decision for the next period. Formally, the value function for a firm with productivity θ_i that exports with outsourcing is:

$$V_o(\mathcal{A}, \theta_i) = \pi_o(\mathcal{A}, \theta_i) - f_p + \beta \mathbb{E} \max\{V_v(\mathcal{A}', \theta_i) - f_v, V_o(\mathcal{A}', \theta_i), V_W(\mathcal{A}', \theta_i) - f_x\}$$

where V_k is the expected value with respect to the demand level conditional on $\mathcal{A}$ and β is the assumed discounted factor of the firm. From now on, I will drop the productivity level for simplicity. Exploiting the structure of the stochastic demand process and the threshold demand levels defined above, value function can be expressed as follows:

$$\begin{aligned} V_o(\mathcal{A}) = & \pi_o(\mathcal{A}) - f_p + \underbrace{\beta(1-\gamma)V_o(\mathcal{A})}_{no\, shock} + \underbrace{\gamma G(\mathcal{A}_o^x)\beta[\mathbb{E}V_W(\mathcal{A}' < \mathcal{A}_o^x) - f_x]}_{shock\, below\, exit} \\ & + \underbrace{\gamma[G(\mathcal{A}_v^e) - G(\mathcal{A}_o^x)]\beta\mathbb{E}V_o(\mathcal{A}_o^x < \mathcal{A}' < \mathcal{A}_v^e)}_{shock\, between\, o\, and\, v} \\ & + \underbrace{\gamma[1 - G(\mathcal{A}_v^e)]\beta[\mathbb{E}V_v(\mathcal{A}' > \mathcal{A}_v^e) - f_v]}_{shock\, above\, v} \end{aligned} \tag{3.7}$$

where the value of exporting via outsourcing is equal to current profits, plus the value of remaining in the same status if no shock arrives, which happens with probability $(1-\gamma)$, plus the value if a shock arrives. Note that the latter can be decomposed into three terms using the demand thresholds. With probability $\gamma G(\mathcal{A}_o^x)$, a shock arrives such that the new demand level is below the exit threshold, and the optimal decision is to pay the sunk cost, f_x, and exit. With probability $\gamma[1 - G(\mathcal{A}_v^e)]$, a shock arrives such that $\mathcal{A}'$ is greater than the integration threshold and the firm decides to integrate after paying the sunk cost f_v. Finally, with probability $\gamma[G(\mathcal{A}_v^e) - G(\mathcal{A}_o^x)]$ the shock is between the exit threshold and the integration threshold, and the optimal decision is to remain an exporter with outsourcing.

Similarly, the value of exporting with integration can be expressed as follows:

$$\begin{aligned} V_v(\mathcal{A}) =& \pi_v(\mathcal{A}) - f_p + \underbrace{\beta(1-\gamma)V_v(\mathcal{A})}_{no\,shock} + \underbrace{\beta\gamma G(\mathcal{A}_v^x)[\mathbb{E}V_W(\mathcal{A} < \mathcal{A}_v^x) - f_x]}_{shock\,below\,exit} \\ &+ \underbrace{\beta\gamma[1 - G(\mathcal{A}_v^x)]\mathbb{E}V_v(\mathcal{A} > \mathcal{A}_v^x)}_{shock\,above\,exit} \end{aligned} \tag{3.8}$$

Note that in this case, there are only two possible choices conditional on the arrival of a demand shock. As discussed above, this is because once a firm has paid the sunk cost to integrate, it is never optimal to go back to outsourcing.

Finally, I consider the value of a non-exporting firm. In this case, the firm does not earn profits in the current period from exporting, and the value of being in this status stem from the the possibility that the demand level changes in the future, so that the firm would find it profitable to start exporting. Formally, the value of waiting as a non exporter is:

$$\begin{aligned} V_w(\mathcal{A}_t) =& \underbrace{\beta(1-\gamma)V_w(\mathcal{A}_t)}_{no\,shock} + \underbrace{\beta\gamma G(\mathcal{A}_o^e)\mathbb{E}V_w(\mathcal{A}_t)}_{shock\,below\,entry} \\ &+ \underbrace{\beta\gamma[G(\mathcal{A}_v^e) - G(\mathcal{A}_o^e)][\mathbb{E}V_o(\mathcal{A}_o^e < \mathcal{A} < \mathcal{A}_v^e) - f_e]}_{shock\,between\,o\,and\,v} \\ &+ \underbrace{\beta\gamma[1 - G(\mathcal{A}_v^e)][\mathbb{E}V_v(\mathcal{A} > \mathcal{A}_v^e) - f_e - f_v]}_{shock\,above\,v} \end{aligned} \tag{3.9}$$

Note that this is a flexible formulation where firms are allowed to start exporting via outsourcing or to start exporting as an integrated firm. Thus, I am not imposing any assumption of sequential entry to export markets.

3.2.1 Entry and Organizational Choice

Equations (3.7), (3.8) and (3.9) are a linear system in the value functions that can be solved for each value function. After some manipulations, I obtain an implicit solution for the entry threshold for a firm exporting via outsourcing, $\mathcal{A}_o^e$:[17]

$$f_e = \frac{\pi_o(\mathcal{A}_o^e) - f_p}{1-\beta\tilde{\lambda}_o^{:x}} + \frac{\beta\gamma}{1-\beta\tilde{\lambda}_o^{:x}} \int_{\mathcal{A}_o^x}^{\mathcal{A}_o^e} \frac{[\pi_o(\mathcal{A}) - \pi_o(\mathcal{A}_o^e)]dG}{1-\beta+\beta\gamma} - \frac{\beta\gamma G(\mathcal{A}_o^x) f_x}{1-\beta\tilde{\lambda}_o^{:x}} \tag{3.10}$$

where $\tilde{\lambda}_o^{:x} \equiv [1-\gamma G(\mathcal{A}_o^x)]$ represents the probability that the firm remains an active exporter in the next period. Hence at $\mathcal{A}_o^e$, the entry sunk cost is equal to the discounted flow of current profits (as in the deterministic case) plus two additional terms.[18] The first additional term is the discounted difference in profits resulting from the arrival of a demand shock below the entry threshold, such that the firm continues as an exporter via outsourcing. The second term reflects the discounted cost of exit when the new shock is below the exit threshold.

Notice that the flow of profits from any future demand shock above the entry threshold does not show up in the condition. This is the *bad news principle* in action: gains from realizations above the entry threshold also accrue to the firm that waits to become an exporter. Also, note that since firms can opt to stop exporting in the case that a very bad realization arrives (i.e. a realization such that $\mathcal{A} < \mathcal{A}_o^x$), profits under these realizations are replaced by the sunk cost of exit.

Finally, comparing (3.10) with the entry condition in the deterministic framework, I show in the appendix (see A.4.4) that $\mathcal{A}_o^e > \mathcal{A}_o^{eD}$ because $\pi_o(\mathcal{A}_o^e) > \pi_o(\mathcal{A}_o^{eD})$. Thus, a firm requires a higher demand realization to be willing to pay the cost of exporting via outsourcing when there is demand uncertainty.

Considering the decision to integrate, after some algebra I obtain an implicit solution for the integration threshold for a firm currently exporting via outsourcing, $\mathcal{A}_v^e$:

$$\begin{aligned} f_v = &\frac{\Delta_{vo}\pi(\mathcal{A}_v^e)}{1-\beta\tilde{\lambda}_v^{:x}} + \frac{\beta\gamma}{1-\beta\tilde{\lambda}_v^{:x}} \int_{\mathcal{A}_o^x}^{\mathcal{A}_v^e} \frac{[\Delta_{vo}\pi(\mathcal{A}) - \Delta_{vo}\pi(\mathcal{A}_v^e)]}{1-\beta+\beta\gamma} dG \\ &+ \frac{\beta\gamma}{1-\beta\tilde{\lambda}_v^{:x}} \int_{\mathcal{A}_v^x}^{\mathcal{A}_o^x} \frac{[\pi_v(\mathcal{A}) - \pi_o(\mathcal{A}_o^x)] - \Delta_{vo}\pi(\mathcal{A}_v^e)}{1-\beta+\beta\gamma} dG \end{aligned} \tag{3.11}$$

where $\Delta_{vo}\pi(\mathcal{A}) \equiv \pi_v(\mathcal{A}) - \pi_o(\mathcal{A})$ is the difference in profits between integration and outsourcing for a given demand level $\mathcal{A}$ (See Appendix A.4.5 for derivation). This condition implies that the integration sunk cost must be equal to the discounted difference in the flow of current profits between the two organizational forms, plus two additional terms that

[17]For a detailed derivation, see Appendix A.4, in general, and A.4.4, in particular.

[18]Note, however, that in the deterministic framework, the discount factor is $(1-\beta)$ while in this stochastic framework, the discount factor is $(1-\beta\tilde{\lambda}_o^{:x})$. It is simple to show that $\tilde{\lambda}_o^{:x} < 1$.

capture differences in the impact of future shocks below the integration threshold. The first additional term captures the discounted value of the difference between organizational forms from changes in profits due to the arrival of shocks in the inaction band $[\mathcal{A}_o^x, \mathcal{A}_v^e]$. The second term captures the discounted value of losses under integration due to the arrival of shocks that trigger exit under outsourcing but not integration. Note that the integration condition is similar to the entry condition (3.10) with the key difference that in (3.11), firms earn profits in both states , which explains why the differences and double differences show up in the condition. Since I allow firms to exit directly from integration, severe negative shocks (below the threshold for exit for integration) do not show up in the integration condition, since such shocks would trigger exit under both integration and outsourcing, and, i accordance with the bad news principle, realizations above the integration threshold are irrelevant for the decision to integrate.

Following the same strategy as before, I show in the appendix (see A.4.5) that $\mathcal{A}_v^e > \mathcal{A}_v^{eD}$ for all firms (or more precisely, for all productivity levels). Under uncertainty, therefore, firms delay the decision to integrate because of the possibility that the demand level will change in the future.

3.2.2 Exit from Foreign Destinations

Up to now I have considered the decision to start exporting and the choice of organizational form. The next step is to examine the exit decision for both integrated exporters and firms exporting via outsourcing. Starting with the latter, note that the exit threshold for firms exporting via outsourcing depends on the difference between the value of exporting and the value of being a non-exporter, similar to the entry threshold. In this case, the expression that explicitly defines the exit threshold for firms exporting via outsourcing is similar to (3.10).[19] In particular:

$$f_x = -\frac{\pi_o(\mathcal{A}_o^x) - f_p}{1 - \beta\tilde{\lambda}_o^{\bar{\mathcal{A}}e}} - \frac{\beta\gamma}{1 - \beta\tilde{\lambda}_o^{\bar{\mathcal{A}}e}} \int_{A_o^x}^{\mathcal{A}_o^e} \frac{[\pi_o(\mathcal{A}) - \pi_o(\mathcal{A}_o^x)]dG}{1 - \beta + \beta\gamma} - \frac{\beta\gamma[1 - G(\mathcal{A}_o^e)]f_e}{1 - \beta\tilde{\lambda}_o^{\bar{\mathcal{A}}e}} \tag{3.12}$$

This equation shows that at the exit threshold demand level, the sunk cost of exiting should be equal to the present discounted value of current flow losses (where losses are the per period fixed cost minus the flow variable profits), minus the potential profits that the

[19] (See Appendix A.4.6 for derivation)

firm gives up in the case that a shock between the entry and exit triggers arrives, minus the cost of reentering the export market in the case that a shock above the entry trigger arrives in some future period. When there is no uncertainty, last two terms disappear and the discount factor becomes $(1-\beta)$.

Equation (3.12) has no counterpart in Handley and Limão (2012), since their model focuses only on the entry side of the extensive margin. However, this exit condition is qualitatively similar to the one obtained for the entry decision. The first two terms in (3.12) are identical in form to their counterparts in (3.10), while the third term takes into account the sunk cost of entering instead of the corresponding cost of exit. In addition, the discount factor is different in the exit decision, since it takes into account the probability that a shock arrives above the entry threshold. Note also that a *good news principle* applies to the exit decision. Bad shocks with respect to the exit threshold, i.e. shocks below $\mathcal{A}_o^x$, are not included in the expected losses, since firms that do not exit today retain the option of exiting in the future.

Note that since $f_x \geq 0$ and the second and third terms are negative, it has to be the case that $\pi_o(\mathcal{A}_o^x) - f_p < 0$.[20] Thus, firms earn negative profits at the exit threshold and will earn negative flow profits throughout the entire demand interval $[\underline{\mathcal{A}}, \mathcal{A}_o^x]$. Hence the exit option allows firms to discard a part of the demand support, where flow profits are negative.

Next, I consider the exit decision for an integrated firm. Note that in this case, firms compare the difference between the value of exporting with integration and the value of being a non-exporter. After finding an expression for this difference (see Appendix A.4.3), I plug it into (3.6) and get the following implicit solution for the demand exit threshold for an integrated exporter, $\mathcal{A}_v^x$:

$$f_x = -\frac{\pi_v(\mathcal{A}_v^x) - f_p}{1-\beta\tilde{\lambda}_v^{\bar{\mathcal{A}}e}} - \frac{\beta\gamma}{1-\beta\tilde{\lambda}_v^{\bar{\mathcal{A}}e}}\left[\int_{\mathcal{A}_v^x}^{\mathcal{A}_v^e} \frac{\pi_v(\mathcal{A}) - \pi_v(\mathcal{A}_v^x)}{1-\beta+\beta\gamma} dG - \int_{\mathcal{A}_o^e}^{\mathcal{A}_v^e} \frac{\pi_o(\mathcal{A}) - \pi_o(\mathcal{A}_o^e)}{1-\beta+\beta\gamma} dG\right] - \frac{\beta\gamma[1-G(\mathcal{A}_v^e)][f_v + f_e]}{1-\beta\tilde{\lambda}_v^{\bar{\mathcal{A}}e}} \tag{3.13}$$

[20]Notice that this result does not stem from the sunk exit costs. Since, even in the absence of sunk cost to exit, current profits at the exit threshold have to be negative in order to compensate for the expected cost of reentering. However, the sunk exit cost does create more incentives for the firm to sustain negative profits before making the decision to exit.

Note that this condition implies that firms sustain negative profits before deciding to exit, since the last two terms are negative, and this implies that $\pi_v(\mathcal{A}_v^x) - f_p < 0$. Then, the exit decision allows firms to avoid states where profits are negative. Note also that expected profits from exporting via outsourcing appear in this condition. This is because the firm is comparing the future profits of remaining as an integrated exporter with the future profits that accrue in the case that a demand shock above the entry threshold for outsourcing ($\mathcal{A}_o^e$) arrives after the firm has exited, where the optimal decision in that case would be to pay the sunk cost to start exporting again.

A visual inspection of the exit conditions for each organizational structure shows clearly that the organizational form impacts the exit decisions. Furthermore, I prove in the appendix (see A.4.8)

$$\pi_o(\mathcal{A}_o^x) - \pi_v(\mathcal{A}_v^x) > 0$$

this implies that $\mathcal{A}_o^x > \mathcal{A}_v^x$ since $\pi_v > \pi_o$. Thus, integrated firms wait longer to exit than firms that outsource; this is true even conditional on the productivity level.

Summing up, firms optimally choose when to start exporting, how to export (integrated or outsourcing) and when to stop exporting. All of these decisions are completely described by the four demand thresholds $\mathcal{A}_v^e$, $\mathcal{A}_v^x$, $\mathcal{A}_o^e$, $\mathcal{A}_o^x$ and the current status of the firm.

3.3 Impact of Uncertainty at Industry Level

The previous section derives implicit expressions for the demand thresholds that describe the policy function at the firm level. The next step is to use these conditions to describe behavior at the industry level. My approach is to start from a realization of industry demand and then determine the productivity level of the marginal firm for each decision. This parametrization will allow me to perform key comparative statics exercises and will also allow me to compare industry behavior in the stochastic framework with the deterministic framework.[21]

In the stochastic framework, after some manipulations (see A.5.1 for the detail derivation), I get the following expression for the productivity cutoff for entry with outsourcing:

$$\theta_o^e = \Psi_o^e \theta_o^{eD} > \theta_o^{eD} \tag{3.14}$$

[21] A.4.8 solves the corresponding deterministic framework to the model.

where

$$\Psi_o^e = \left[1 + \frac{\beta\gamma G(\mathcal{A}_t\xi_o^x)[f_x + f_e]}{(1-\beta)f_e + f_p}\right]^\rho \Big/ \left[1 + \frac{\beta\gamma\Delta_{\mathcal{A}}(\mathcal{A}_t, \mathcal{A}_t\xi_o^x)/\mathcal{A}_t}{1-\beta+\beta\gamma}\right]^\rho \tag{3.15}$$

and ξ_k^m denotes the parameters relating $\mathcal{A}_t$ with the $m \in \{e, x\}$ from $k \in \{O, V\}$ threshold and $\Delta_{\mathcal{A}}(\mathcal{A}_i, \mathcal{A}_j) = \int_{\mathcal{A}_j}^{\mathcal{A}_i}(\mathcal{A} - \mathcal{A}_i)dG$ is a function capturing the expected difference in profits between the specific demand realization $\mathcal{A}_i$ and potential new realizations over the interval $(\mathcal{A}_i, \mathcal{A}_j)$. In this case $\Delta_{\mathcal{A}}(\mathcal{A}_t, \mathcal{A}_t\xi_o^x) < 0$, and this term reflects the loss if the new demand level is below the entry threshold but high enough not to force the firm to exit.

Ψ_o^e captures the ratio between the sunk costs of entry and exit when a bad shock arrives, on the one hand, and the profits lost relative to profits at the entry threshold in case that a shock in the inaction band arrives. Intuitively, Ψ_o^e compares the cost of becoming an exporter and exiting in the future, i.e. the sunk cost that the firm has to pay in the case a new shock arrives forcing the firm to exit, with the *relative* cost of exporting when a shock arrives in the inaction band, i.e. the profit loss relative to profits at the entry threshold. Firms with productivity $\theta_i > \theta_o^e$ will find it profitable to pay the sunk costs to start exporting. $\Psi_o^e > 1$ since the numerator is greater than 1, while the denominator is less than 1 because the second term in the denominator is negative but less than 1 in absolute value. Uncertainty, therefore, delays firms' decision to start exporting via outsourcing. Note that in the absence of demand uncertainty, Ψ_o^e collapses to 1 and $\theta_o^e = \theta_o^{eD}$.

Similarly, an expression for the productivity cutoff for exit from outsourcing can be derived in the stochastic framework in terms of the respective deterministic productivity cutoff (see A.5.2):

$$\theta_o^x = \Psi_o^x \theta_o^{xD} < \theta_o^{xD} \tag{3.16}$$

where $\Psi_o^x = [1 - \frac{\beta\gamma[1-G(\mathcal{A}_t\xi_o^e)][f_e+f_x]}{f_p-[1-\beta]f_x}]^\rho/[1 + \frac{\beta\gamma\Delta_{\mathcal{A}}(\mathcal{A}_t\xi_o^e,\mathcal{A}_t)/\mathcal{A}_t}{1-\beta+\beta\gamma}]^\rho$. Outsourcing firms with $\theta_i \leq \theta_o^x$ will exit and firms with $\theta_i > \theta_o^x$ will keep exporting. Ψ_o^x captures the ratio between the sunk cost of entry, on the one hand, and exit in the case that a shock arrives above the entry threshold, and the profits gained if a shock arrives in the inaction band, where $\Delta_{\mathcal{A}}(\mathcal{A}_t\xi_o^e, \mathcal{A}_t) > 0$. I show that $\Psi_o^x < 1$ in the appendix. Thus, compared to the deterministic framework, firms wait longer before exiting the foreign market in the stochastic framework.

Next, I derive the following expression for the productivity cutoff for entry with integration under the stochastic framework (see A.5.3):

$$\theta_v^e = \Psi_v^e \theta_v^{eD} > \theta_v^{eD} \tag{3.17}$$

where $\Psi_v^e = \left[1 + \frac{\beta\gamma G(\mathcal{A}_t \xi_v^x)}{1-\beta}\right]^\rho / \left[1 + \frac{\beta\gamma}{1-\beta+\beta\gamma}\frac{\Delta_{\mathcal{A}}(\mathcal{A}_t, \mathcal{A}_t\xi_o^x)}{\mathcal{A}_t} + \frac{\psi_o}{\psi_v - \psi_o}\frac{\beta\gamma}{1-\beta+\beta\gamma}\frac{\Delta_{\mathcal{A}}(\mathcal{A}_t\xi_o^x, \mathcal{A}_t\xi_v^x)}{\mathcal{A}_t}\right]^\rho$. Ψ_v^e captures the ratio between the cost of exiting in the case a bad shock arrives, and the profit loss in the case that a shock arrives in the inaction band for integrated firms. Note that the profit loss takes into account that the alternative optimal decision in the inaction band may either be integration or outsourcing, depending on the realization of the demand level. In the appendix, I show that $\Psi_v^e > 1$. Hence demand uncertainty delays the decision to integrate. I also show in the appendix that $\theta_o^e < \theta_v^e$, since I am focusing on sectors with high headquarters intensity where $\pi_v(\mathcal{A}) > \pi_o(\mathcal{A})$ for all $\mathcal{A}$. Thus, only relatively productive firms, i.e. firms with $\theta_i > \theta_o^e$ will export, and only the most productive of these will integrate, the ones with $\theta_i > \theta_v^e$.

The respective expression for the exit productivity cutoff for integrated exporters is as follows (see A.5.4):

$$\theta_v^x = \Psi_v^x \theta_v^{xD} < \theta_v^{xD}$$

where $\Psi_v^x = \left[1 - \frac{\beta\gamma[1-G(\mathcal{A}_t\xi_v^e)][f_v+f_e+f_x]}{f_p-[1-\beta]f_x}\right]^\rho / \left[1 + \frac{\beta\gamma}{1-\beta+\beta\gamma}\frac{\Delta_{\mathcal{A}}(\mathcal{A}_t\xi_v^e, \mathcal{A}_t)}{\mathcal{A}_t} - \frac{\psi_o}{\psi_v}\frac{\beta\gamma}{1-\beta+\beta\gamma}\frac{\Delta_{\mathcal{A}}(\mathcal{A}_t\xi_v^e, \mathcal{A}_t\xi_o^e)}{\mathcal{A}_t}\right]^\rho$ and $\Psi_v^x < 1$ (see Appendix). Thus, integrated exporters with $\theta_i < \theta_x^v$ will exit. Since $\theta_v^x < \theta_v^{xD}$, firms wait longer to exit foreign markets- this difference is due to the fact that firms internalize, that with some probability, things will improve in the future. Furthermore, I show in the appendix that $\theta_v^x < \theta_o^x$ (see A.5.5.1). Hence the productivity level needed to keep exporting with integration is lower than the productivity required to keep exporting via outsourcing. Thus, firms characterized by $\theta_i \in [\theta_v^x, \theta_o^x]$ will keep exporting if they are already exporting with integration but will stop exporting if they are exporting via outsourcing. The following propositions summarize the results for the productivity cutoffs:

Proposition 1. *(Exit and Organizational Choice under Uncertainty) Under foreign demand uncertainty $\mathcal{A}$, the productivity exit cutoffs are (i) proportional to the deterministic cutoff by an uncertainty factor, $\theta_k^x = \Psi_k^x \theta_k^{xD}$ where $k \in \{V, O\}$ and (ii) lower than its deterministic counterpart, $\theta_k^x < \theta_k^{xD}$; (iii) cutoffs are specific to organizational form, where vertical integration cutoff is lower than the outsourcing cutoff, $\theta_v^x < \theta_o^x$, and (iv) differences across organizational choice are higher than in the deterministic setting,* $(\theta_o^x - \theta_v^x) > (\theta_o^{xD} - \theta_v^{xD})$

Proof. See appendix. □

Proposition 2. *(Entry and Integration under Uncertainty) Under foreign demand uncertainty, the entry and integration productivity cutoffs are (i) proportional to the deterministic cutoffs by an uncertainty factor, $\theta_k^e = \Psi_k^e \theta_k^{eD}$ where $k \in \{V, O\}$ and (ii) higher than their deterministic counterparts, $\theta_k^e > \theta_k^{eD}$, (iii) the vertical integration cutoff is higher than the outsourcing cutoff, $\theta_v^e > \theta_o^e$.*

Proof. See appendix. □

The results derived so far consider the impact of demand uncertainty on productivity cutoffs compared to the deterministic framework. Additionally, I am interested in uncovering how the introduction of uncertainty modifies the responses of these productivity cutoffs to changes in the key parameters of the model. The following propositions discuss the effect of uncertainty on the productivity cutoffs.

Proposition 3. *(Delay) A higher arrival rate of demand shocks increases the productivity cutoff for entry ($\frac{\partial \ln \theta_o^e}{\partial \gamma} > 0$) and decreases the productivity cutoff for exit from outsourcing ($\frac{\partial \ln \theta_o^x}{\partial \gamma} < 0$). An increase in the arrival rate of demand shocks increases the entry productivity cutoff ($\frac{\partial \ln \theta_v^e}{\partial \gamma}|_{\gamma=0} > 0$) and decreases the exit productivity cutoff ($\frac{\partial \ln \theta_v^x}{\partial \gamma}|_{\gamma=0} < 0$) for exporting with integration when evaluated around the deterministic case, i.e. $\gamma = 0$. Moving away from $\gamma = 0$, a compensating factor, the impact of uncertainty on the productivity cutoff for outsourcing, kicks in and ameliorates the effect on integration productivity cutoffs.*

Proof. See appendix. □

This proposition implies that firms facing more uncertainty are more likely to delay their entry and exit decisions. Note that this result refers to one of the elements used to model uncertainty in this framework, namely the demand shock arrival rate, holding fixed the other component of the demand stochastic process, namely the cumulative distribution function. In the case of the integration productivity cutoffs , the first order effect of higher γ is to delay entry and exit.

Proposition 4. *(Heterogeneity in the responses by organizational form) The entry and exit cutoffs for exporting via outsourcing are more elastic to demand changes than the respective cutoffs for exporting with integration, i.e. ($|\frac{\partial \ln \theta_o^e}{\partial \ln \mathcal{A}_t}| - |\frac{\partial \ln \theta_v^e}{\partial \ln \mathcal{A}_t}| > 0$) and ($|\frac{\partial \ln \theta_o^x}{\partial \ln \mathcal{A}_t}| - |\frac{\partial \ln \theta_v^x}{\partial \ln \mathcal{A}_t}| > 0$).*

Proof. See appendix. □

The intuition for these results is that sunk costs dampen the response of the productivity cutoffs to shocks. Since integration requires higher sunk costs compared to outsourcing, it follows that demand elasticities for outsourcing cutoffs are higher (in absolute value). This higher elasticity of productivity cutoffs for outsourcing than integration sharply contrasts the deterministic framework. In the deterministic framework, elasticities are similar both within organizational form and across organizational form. The reason is that all productivity cutoffs in the deterministic framework are log-separable in the demand realization. This log-separability does not hold in the stochastic framework since the current demand realization affects the expected gains and losses of potential future changes in demand conditions. These differences by organizational form in the elasticity to demand level are potentially interesting, because productivity is the only motive for differences in the behavior between integrated and non-integrated firms in the deterministic framework. In the stochastic framework, however, demand uncertainty and partially irreversible costs create another channel to explain the differences in the margin of adjustment between integrated and non-integrated firms.

Recall that demand uncertainty is modeled as a two component stochastic processes: the demand shock arrival rate and the underlying demand distribution. Thus, comparative statics in terms of the arrival rate do not capture all of the possible effects of uncertainty in the model. Another approach to analyze the effect of uncertainty in the model is to consider changes in the distribution function $G(\mathcal{A})$. For example, a perceived worsening of demand conditions can be parametrized as a shift in mass towards the left tail of the distribution $G(\mathcal{A})$. The following propositions consider different scenarios in which I am able to identify the direction of the effect of changes in $G(\mathcal{A})$ on firms' decisions.

Proposition 5. *(Bad News I) Suppose that the distribution of demand $G(\mathcal{A})$ changes such that the new distribution $G'(\mathcal{A})$ is first order stochastic dominated by the initial distribution $G(\mathcal{A})$. Then productivity cutoffs for entry with outsourcing, integration, exit from outsourcing and exit from integration are higher under $G'(\cdot)$ That is, $\theta_k^m(G') > \theta_k^m(G)$ where $k \in \{V, O\}$and $m \in \{e, x\}$.*

Proof. See appendix. □

Proposition 6. *(Bad News II) Suppose that the distribution of demand $G(\mathcal{A})$ changes such that $G'(\mathcal{A})$ is a mean-preserving spread of $G(\mathcal{A})$ such that $G(\mathcal{A})$ and $G'(\mathcal{A})$ cross only once at $\tilde{\mathcal{A}}$. Then the exit cutoff increases, $\theta_k^x < \theta_k^{x\prime}$, for current realizations below $\tilde{\mathcal{A}}$ and declines, $\theta_k^x > \theta_k^{x\prime}$, above this threshold. Also, the entry cutoff increases, $\theta_k^e > \theta_k^{e\prime}$, for current realizations below $\tilde{\mathcal{A}}$ and decreases , $\theta_k^e > \theta_k^{e\prime}$, above this threshold.*

Proof. See appendix. □

These propositions imply that even when the current demand conditions remain constant, a change in the underlying demand distribution can lead firms to either enter, exit or change their organizational form. The is because an improvement in the demand distribution implies an increase of the expected gains of exporting. Therefore some firms -that had been waiting for good news before beginning to export - decide to stop waiting and start exporting. Similarly, some firms that were considering integration will decide to stop waiting, because their expected profits from a new shock are higher than before. This result, interpreted from the perspective of episodes such as the Great Trade Collapse, implies that shocks that change not only the current demand condition but also the underlying distribution in the same direction would have stronger impacts than shocks to the current realization only. This proposition can also provide a potential explanation for recoveries after a negative shock through an improvement of the underlying demand distribution even if current conditions are unchanged.

To summarize, this section shows theoretically that uncertainty affects both entry, exit and organizational form decisions at both the threshold levels and the marginal response of these thresholds to shocks. This suggests that uncertainty can play a significant role in episodes characterized by high and changing uncertainty such as the GTC. The next section presents the steps I follow to test the model empirically and to quantify the role of related parties and uncertainty in the GTC.

4 Testable Predictions and Empirical Approach

The model developed in the previous sections has several predictions. In this section, I focus on key predictions to test the model and to provide insights on heterogeneous responses of firms across organizational choice. I focus on the decision to exit from foreign markets for several reasons. First, the exit decision has not been explored in detail by the trade literature. Second, the exit decision is a key margin that highlights the differences between related parties and arm's length trade, which played a key role during the GTC. Third, the exit decision provides a better setting than entry to identify the impact of uncertainty, as no assumption is needed to know the level of uncertainty to which the firm is exposed. More specifically, since it is not known a priori which foreign destination a firm would enter, it is not straightforward to measure uncertainty in the case of entry.

Based on the theoretical results and the propositions in the previous section, I expect the following: (i) related parties should survive longer following negative aggregate shocks, (ii) an increase in foreign demand uncertainty, as measured by the share of GDP that would be lost if a severe shock arrives, should induce firms to exit, and (iii) the difference in exit threshold between related parties and arm's length trade should be bigger during times of higher uncertainty. The first testable prediction is based on proposition 1 and the result that even conditional on firm productivity the demand threshold to exit from vertical integration is lower than the exit threshold for outsourcing, $\mathcal{A}_v^x < \mathcal{A}_o^x$. The second prediction is based on proposition 5 with the definition of uncertainty that I construct. The third testable prediction comes from the fact that introducing uncertainty into the model expands the differences in the exit cutoff between vertical integration and outsourcing.

A straightforward approach to testing these predictions is to build a duration model where the probability that the firm stops exporting in the next period is modeled using a hazard function that depends on independent variables such as measures of uncertainty and organizational form, including firm and destination characteristics in order to control for potential differences across firms.

4.1 Uncertainty Measure

In order to test the model, the first step is to compute a measure of uncertainty. In the model, firms are uncertain about the future value of $\mathcal{A}$ where $\mathcal{A} = \mu \times Y(t+1) \times P(t+1)^{\frac{\alpha}{1-\alpha}}$ and $Y(t+1)$ is the income of the foreign country. Therefore, I compute uncertainty by modeling the stochastic process for countries' Gross Domestic Product. More specifically, I assume that $\ln gdp_c(t)$ for country c follows an AR(1) process in differences with a Gaussian distributed error term:

$$\Delta \ln gdp_c(t+1) = a_c + \rho_c \Delta \ln gdp_c(t) + \epsilon_c(t+1)$$

After estimating this AR(1) process for all countries with at least 20 annual observations in the 1988-2011 period, I then compute the uncertainty measure as the share of GDP that a country will lose in the next period if a bad shock arrives.

$$unc_c(t) = 1 - \frac{\exp(\ln gdp_c(t) + \hat{\rho}_c \Delta \ln gdp_c(t) + \hat{\epsilon}_{c,0.05})}{gdp_c(t)}$$

This uncertainty measure assumes that firms base their decision to exit by forming an expectation of how much profit would be lost if a severe shock arrives. In the theoretical model, changes in foreign GDP are the only factor affecting $\mathcal{A}$. In reality, there are many other sources of uncertainty specific to destinations besides GDP; however, destination GDP is surely one important factor about which exporting firms are uncertain. Implicitly, the measure is approximating the expected profit loss using a two state process, involving GDP today and a bad shock at the 0.05 percentile of the distribution. This approach simplifies the construction of the measure and highlights the role of severe shocks, such as the GTC, in firms' decisions. Note that the country-specific uncertainty measure is varying over time. However, most of variation comes from the country dimension compared to the time dimension. More specifically, country is the source of around 90% of the variation while time accounts for the remaining variation.

As an alternative, I follow the literature on uncertainty at the macro level and use stock market volatility in the foreign market as my measure of uncertainty (cf. Bloom et al. (2012)). In particular, I use yearly stock market volatility in the destination market, as constructed by Baker and Bloom (2013).

4.2 Survival Approach

The standard approach in duration analysis when working with annual data is to use a discrete time model.[22] A discrete time model has the advantage that unobserved factors

[22] In the duration literature it also common to use the Cox proportional hazard estimator for duration model. The Cox proportional has the disadvantage to assume that time is continuous and that failures can occur at any point. This is clearly not the case when using annual frequency data.

which may affect the estimation can be control for relatively easily. These factors, such as firm productivity and other unobserved firms characteristics, are controlled by introducing a "frailty" term that varies at the firm level. Formally, the hazard function is:

$$h_{ipc}(t) = Prob[exit \in [t-1, t] | survive > t-1]$$

where $h_{ipc}(t)$ is the probability that firm i exporting product p to country c stops exporting in period t. Under the proportional hazard assumption, the hazard function becomes:

$$h_{ipc}(t) = 1 - \exp\left(-\exp\left(X_{ipc}(t)\beta + j_t + \nu_{ipc}\right)\right)$$

where $X_{ipc,t}$ is a set of covariates that includes: (i) a dummy variable for related parties, (ii) an uncertainty measure, (iii) the interaction of uncertainty and the related parties dummy, and (iv) additional controls. j_t is the non-parametric baseline hazard and ν_{ipc} is assumed to follow a normal distribution, which implies that the individual effect is distributed log-normal. The assumption of log-normality is particularly appealing for modeling the productivity of the firm, since there is evidence that at least the right tail of firm productivity follows a log-normal distribution (see Head, Mayer, and Thoenig (2014)).

As described in section 2, I use the LFTTD and LBD database from the U.S. Census Bureau as the main source of information. The LFTTD provides detailed information on all U.S. export transaction at firm level disaggregating by product, destination and ownership relationship between the trading firms. The LBD collects firm characteristics that I used as controls in the regression analysis. The unit of analysis is firm-destination-product-year and the period is 2002-2011.[23]

According to the model predictions discussed above, a dummy variable capturing whether firms engage in related party trade should have a negative impact on the hazard function, while uncertainty should have a positive impact. Additionally, uncertainty and GDP in the destination country should have a heterogeneous impact across organizational form. More specifically, related party trade should be less affected by uncertainty and the same level of GDP should have a stronger impact on the reduction of the probability of exiting. These heterogeneous effects imply that interactions between the related party indicator and the respective covariate should be negative for both cases.

The set of control variables includes firm and market characteristics. Firm age, size and

[23]The period of analysis is determined by the data availability since 2011 is that last year of export transaction available at the Census Bureau.

previous export levels are introduced to control for firm productivity and export history, since previous studies show that more productive and more experienced firms survive longer (see e.g. Dunne, Roberts, and Samuelson (1988), Mata and Portugal (1994), Pérez, Llopis, and Llopis (2004) and Volpe Martincus and Carballo (2009)). Also, control variables are motivated by the results on determinants of intra-firm imports for a sample of French firms by Corcos et al. (2009). They show that importing from a related-party firm is more probable for more productive firms, firms more capital and skill intensive. Also, they show that French firms are more likely to import through related-party from countries with better judicial systems. First, the individual effects captures firms' constant characteristics and in doing so control for the constant component of skill intensity, capital intensity and firm productivity. Second, employment size controls at least partially for time-varying productivity. Also, in the robustness section I perform several exercises that further control for these determinants.[24] Destination country GDP is also included to control for current demand conditions.

Table 2

Depvar: Exit	**(1)**	**(2)**	**(3)**
Related Party	0.791***	0.807***	0.946***
	(0.001)	(0.001)	(0.001)
GDP (log)		0.957***	0.963***
		(0.001)	(0.001)
Employees (log)			0.955***
			(0.001)
Age (log)			1.006***
			(0.001)
Exports (log)			0.747***
			(0.001)
Observations (rounded)	17,600,000	17,600,000	17,600,000

Clustered standard errors at firm-destination-product in parenthesis
Hazard function is non-parametric.
Coefficient are presented in their exponential form.
$***p < 0.01$, $**\ p < 0.05$

[24]First, I introduce firm TFP for a sample of manufacturing firms. Second, I introduce industry-specific baseline hazard that control for capital intensity and skill intensive differences across industries. Third, I add measures for country rule of law to control for the quality of judicial system. Results are robust to introduce all these controls.

Results are presented in Table 2 in the exponential form as is standard in duration model. These coefficients in exponential form should be interpreted using 1 as the reference. Coefficients lower than one reduce the probability of exiting while coefficients above one increase it. The results confirm the prior predictions. The indicator for related party trade is negative and significant in all specifications. This implies that related parties firms have a lower probability of exiting compared to arm's length trade. In the specification where all controls are included, the presence of related parties implies a reduction of 5.6% of the baseline exit hazard. Note that the controls have the expected effects; bigger firms in terms either of exports or employment are less likely to stop exporting and a higher GDP in the foreign destination also reduces this probability.

Table 3

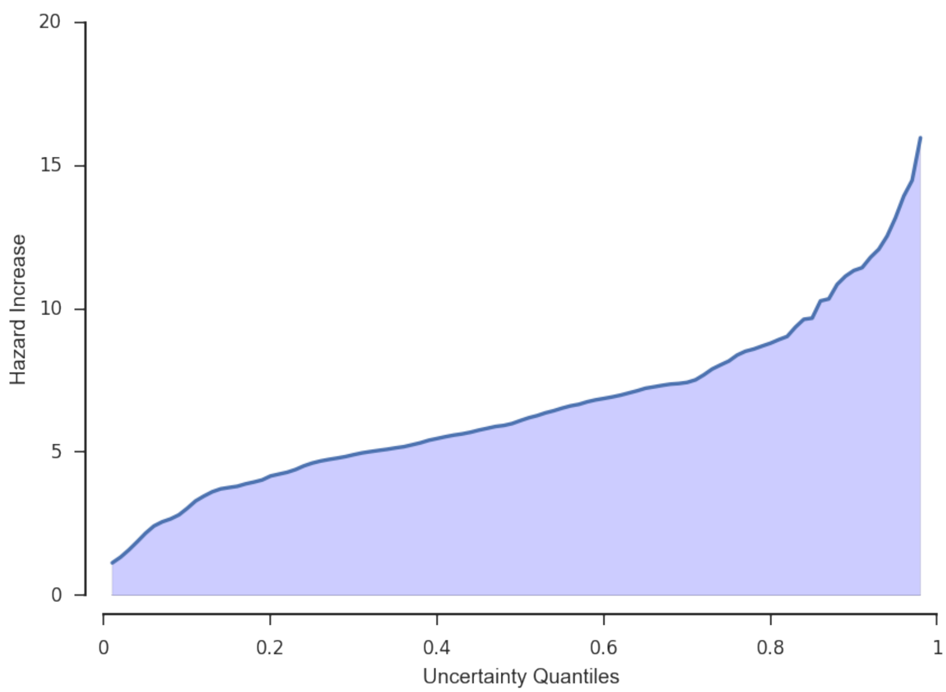
Dep var: Exit	**(1)**	**(2)**	**(3)**
Related Party	0.804***	0.889***	0.917***
	(0.001)	(0.001)	(0.001)
Uncertainty AR(1)	1.046***	1.080***	1.232***
	(0.004)	(0.005)	(0.004)
Crisis (2009)	1.082***	1.084***	1.099***
	(0.001)	(0.001)	(0.001)
GDP (log)		0.957***	0.968***
		(0.001)	(0.001)
Employees (log)			0.965***
			(0.001)
Age (log)			1.009***
			(0.001)
Exports (log)			0.777***
			(0.001)
Observations (rounded)	14,300,000	14,300,000	12,700,000

Clustered standard errors at firm-destination-product in parenthesis
Hazard function is non-parametric.
Coefficient are presented in their exponential form.
$***p < 0.01$, $**\ p < 0.05$

The results from estimating the impact of uncertainty on the firm decision to exit are presented in Table 3. The measure of uncertainty used is the expected GDP loss if a bad shock arrives, as we explain above. The impact of uncertainty is significant, as it increase the probability of exiting in the next period by 20% in the preferred specification in which all covariates and controls are included. Note that the impact of trading to related parties remains negative and significant when we control for uncertainty and also a dummy capturing whether the year corresponds to the GTC.

Results presented in the previous tables show that uncertainty has a negative impact on firms' survival. However they do not provide with a measure of how much uncertainty increases the probability of exiting. Hence, I estimate the marginal impact of uncertainty for the full sample of firms. Figure 4 plots the marginal effect for the full distribution of firms exporting in 2005. I find that for at least 50% of firm-destination-product flows, uncertainty increases the probability of exiting by at least 5 percentage points and that for the upper tail of the distribution the increases is higher than 10 percentage points.

Figure 4: Uncertainty Impact



The third testable prediction is the heterogeneity in the impact of uncertainty across organizational form. More specifically, uncertainty has a smaller impact on the probability of exiting for related party exports according to the model. In order to test this, I reestimate the model allowing for an interaction between uncertainty and related parties. The specification in the case of allowing for heterogeneity in the impact of uncertainty is as follows:

$$h_{ipc}(t) = 1 - \exp(-\exp(\beta_{unc} unc_c(t) + \beta^R_{unc} unc_c(t) \times R + \beta_R R + X_{ipc}(t)\beta + j_t + \nu_{ipc}))$$

and results are presented in the following table

Table 4

Depvar: Exit	**(1)**	**(2)**
Related Party (β_R)	0.972***	0.976***
	(0.00876)	(0.0053)
Uncertainty AR(1) (β_{unc})	1.307***	1.152***
	(0.0305)	(0.0102)
RP x Uncertainty AR(1) (β^R_{unc})	0.910***	
	(0.0113)	
GDP (log) (β_{gdp})	0.956***	0.957***
	(0.007)	(0.007)
RP x GDP (log) (β^R_{gdp})		0.988***
		(0.00225)
Firms Characteristics	Yes	Yes
Individual Effect	Yes	Yes
Observations (rounded)	14,300,000	14,300,000

Clustered standard errors at firm-destination-product in parenthesis
Hazard function is non-parametric.
$***p < 0.01$, $**\ p < 0.05$

Results show that trading to related parties rather than arm's length reduces the impact of uncertainty on the probability from a 30.7% increase to a 18.9% increase. This represent a reduction of the effect of almost 40%. To further explore the heterogeneity in the impact of uncertainty, I compute the marginal effect for the full sample under analysis and plot the impact for percentiles of the uncertainty distribution. Figure 5 shows that the difference in the increase of the hazard between related parties and arm's length trade increases as uncertainty increases. For example, uncertainty increases the probability of exiting by 5 percentage points or more for more than 80% of firms trading at arm's length but only 50% of firms trading with related parties.

Figure 5: Heterogenous Uncertainty Impact

Note that the identified effects correspond to the average industries. However, industries are heterogeneous in their production process. From the perspective of the model, the η parameter is the one capturing this heterogeneity. According to it, the impact of trading to related parties should be stronger the higher the η parameter is, i.e. the more relevant headquarters inputs are in the production process. In the robustness section, I introduce industry-specific hazard rates that control for these potential differences in the production process as long as they are constant over time, as the theory section assumes.

A key assumption of the model is that the final good is consumed in the foreign destination. However, firms may have a multi-stage production process involving several countries. In particular, U.S. firms may be more likely to have a multi-stage production involving countries such as Canada and Mexico, due to NAFTA integration, or China, where the wage differential is higher. Unfortunately, it is not possible to track multi-country production process given the information available. However, I control for this potential issue by restricting the sample to countries less likely to be part of multi-stage production process (see results in the robustness section).

4.2.1 Quantification

To demonstrate the economic significance of the mechanisms highlighted by the model, I quantify the impact of trading to related parties and the role of uncertainty in the exit decision. The first counterfactual analysis considers the role of trading with a related parties. More specifically, I compute how many additional exports would result if all firms had traded to related parties in 2009. Under this scenario, all firms originally trading to arm's length would have a lower probability of exiting in 2009, which in turns generates additional exports. Then, I assume that these additional surviving firms have exports level similar to average firms exporting to non-related firms in 2009. As an alternative scenario, I assume that these firms experience the average contraction in exports experienced by firms exporting to related parties. Results from performing these counterfactual analysis show that the 2009 contraction in U.S. exports would have been between 10% and 12% smaller under this scenario.

The second counterfactual analysis assumes that uncertainty in 2009 drops to its first tercile for all firms with uncertainty above this third tercile. In this scenario, all firms facing uncertainty above the third tercile have a higher probability of surviving, and this, in turns, generates additional exports. Assuming that additional firms surviving in this scenario have average exports level in their respective country in 2009, I compute how would have been exports in 2009. Results indicate that 2009 would have been reduced by an 8%.

4.3 Robustness Exercises

There are a number of potential concerns about these results. The main concern is whether these results are specific to the uncertainty measure used. In order to test this, I use the annual average stock market volatility in the foreign destination over the period as an alternative measure of uncertainty. The results confirm that uncertainty increases the exit probability. An increase of one standard deviation of stock market volatility increase the probability of exiting by 2.5% while moving from the 5th percentile to the 75th increase this probability by 4.4%.

Table 5

Dep var: Exit	**(1)**	**(2)**	**(3)**	**(4)**
Related Party	0.804***	0.917***	0.799***	0.914***
	(0.001)	(0.001)	(0.001)	(0.001)
Crisis (2009)	1.082***	1.099***	1.079***	1.05***
	(0.001)	(0.001)	(0.002)	(0.002)
GDP (log)	0.959***	0.968***	0.974***	0.974***
	(0.001)	(0.001)	(0.001)	(0.001)
Uncertainty AR(1)	1.046***	1.232***		
	(0.004)	(0.004)		
Stock Market Volatility			1.027***	1.041***
			(0.003)	(0.001)
Stock Market Return			0.9539**	0.856***
			(0.021)	(0.009)
Observations (rounded)	15,300,000	12,700,000	12,700,000	11,900,000
Firms characteristics	No	Yes	No	Yes

Clustered standard errors at firm-destination-product in parenthesis
Hazard function is non-parametric.
Coefficient are presented in their exponential form.
$***p < 0.01$, $**\ p < 0.05$

Second, a key identification assumption is that the final good is consumed in the foreign country, such that uncertainty over the foreign country demand affects the firm's decision to export through related parties or at arm's length. If instead U.S. firms export via related parties to a foreign country with the intention to ship back the good, then foreign demand conditions in general, and uncertainty in particular, are irrelevant to the organizational choice. Note than if reshipment is more likely for related parties than arms-length trade, the impact of uncertainty would be higher than the one estimated above. Such a multi-stage production process involving U.S. exports is more probable for exports within NAFTA or China. Hence, I reestimate the model taking out exports to NAFTA and China from the sample. Results are reported in Table A.4. These regressions confirm all previous results; moreover, the impact of uncertainty is stronger when I eliminate all export flows to Canada, Mexico and China.

Third, a potential concern is whether the estimated impact of uncertainty is driven solely by the 2008-9 recession (GTC), with no impact of uncertainty before 2008. However, reestimating the model for the 2002-2007 period, before the GTC started, confirms the baseline results and shows that uncertainty plays a significant role in firms' exit decisions even in periods of relative stability, see Table A.5 for detailed results.

Fourth, results may depend on unobserved factors, such as productivity and other firm's characteristics. Controlling for a timer-varying firm-level TFP measure, as computed by the Census Bureau, does not affect previous findings.[25] This result is reported in the first column of Table A.6; the other columns include other firm characteristics such as whether the firm is importing and total domestic sales. All results and conclusions remain the same after introducing these controls. Similarly, results may be affected by unobserved characteristics at the industry or destination level. In order to control for these unobserved characteristics, I reestimate the model using industry and country-specific baseline hazards, where industries are defined using the 2-digits harmonized classification. These industry-specific and country-specific baseline hazards incorporate permanent characteristics of industries and destinations. Results are reported in A.7 and are robust to incorporating these additional controls. Note that country-specific hazard rates allow to control for the impact of PTA agreements on the probability of exiting. Results are robust to incorporating these additional controls.

Fifth, results may be affected by firms reentering in the future. Empirically, this does not seem to be the case, as most firms exit permanently. Moreover, adding a dummy for previous spells to control for this reentry behavior does not change results. Additionally, results could depend on the frequency of the data or the estimator used. However, results are robust to using data at the semi-annual frequency and to using the Cox proportional hazard model, instead of the discrete approach followed in the main specification; results are reported in Table A.8.

Sixth, the empirical analysis uses firm-destination-product as it level of analysis. However, firms do not take decision independently across destinations and markets. In order to control for this, I run several robustness exercises: first, I included a dummy to check

[25] The TFP index measure is constructed using the following formula:

$$\ln TFP_e(t) = \ln Q_e(t) - \alpha_K \ln K_e(t) - \alpha_L \ln L_e(t) - \alpha_M \ln M_e(t)$$

where Q is real output, K is real capital, L is labor input, M is real materials, α denotes factor elasticities, the subscript e denotes individual establishments and t denotes time. Factor elasticities are industry-level cost shares for each input. See more details in Foster, Grim, and Haltiwanger (2014).

whether the firm is exiting in another foreign market; second, I allowed standard errors to be clustered at firm-level to account for common shocks at firm level; third, I included domestic sales as a control in the regression to include information about the domestic market. In all cases, the results remain the same qualitatively.

5 Conclusions

I examine how firms' global sourcing strategies affect their responses to economic crises such as the 2008-2009 recession. I model firms' entry, exit and sourcing decisions (integrated production or outsourcing) under demand uncertainty. Uncertainty increases the option value of waiting, resulting in less integration as well as less entry and exit. Additionally, I show that trade decisions of integrated firms are less sensitive to demand and uncertainty shocks. This heterogeneous responses to shocks highlights the role of sourcing strategies in the way firms adjust and contrasts the homogeneous responses that the deterministic model shows.

I develop a theory consistent measure of foreign demand uncertainty following closely the model. Then, I combine this dynamic model with U.S. firm level export data for the 2002-2011 period to test the predictions of the model for the exit decision. In doing so, I exploit the fact that U.S. customs data is one of the few databases that records the ownership relation between trading parties for every transaction. I find that integration reduces the probability that a firm exits by as much as 8%, while uncertainty increases this probability by 22%. Quantifying the impact of related parties, I find that if all firms traded to related parties, the 2009 collapse would have been reduced between 10% and 12%. Also, if uncertainty was reduced to the first tercile for all firms, the reduction on exports in 2009 would have been 8% smaller.

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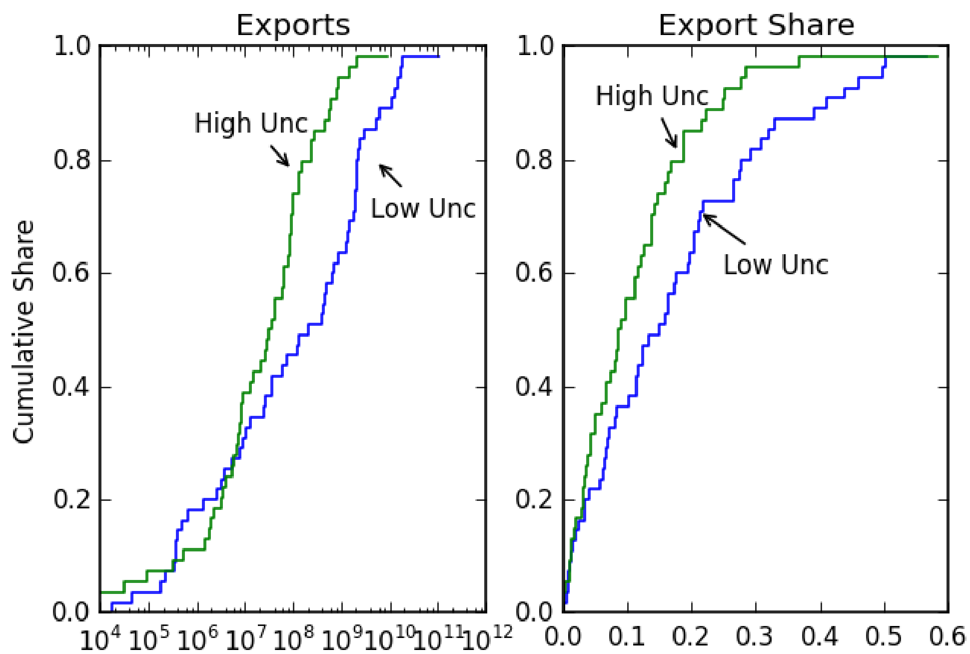
Appendix

Figure A.1: Related Party Exports and Uncertainty



A.1 GTC and Organizational Forms

A.1.1 Net Extensive Margin during the GTC

Table A.1: Net Extensive Margin: Related-Party vs Arm's Length Trade

	Firm		
	Related-Party	Arm's Length Trade	Mid-point Difference
Fall at Through	-11.11%	-15.47%	32.81%
Fall at Q4:2009	-6.86%	-8.92%	26.11%
Recovery Peak	0.01%	-0.53%	207.69%
Quarters until Recovery	13	>14	-7.41%
	Firm-Country-Product		
	Related-Party	Arm's Length Trade	Mid-point Difference
Fall at Through	-7.2%	-12.64%	54.84%
Fall at Q4:2009	0.64%	-5.2%	256.14%
Recovery Peak	15.94%	7.6%	70.86%
Quarters until Recovery	4	8	-66.67%

Mid-point difference computes the mid-point difference rate between related-party and arm's length trade. More specifically, the formula is $(x^{RP} - x^{AL})/(0.5 * (x^{RP} + x^{AL}))$

A.1.2 Midpoint Decompositions during the GTC

Table A.2: Midpoint Decomposition - 2006-2011 - Related Party Trade

Year	Quarter	Intensive Margin			Extensive Margin		
		Growers	Shrinkers	Net	Entry	Exit	Net
2007	1	0.238	-0.194	0.044	0.280	-0.241	0.039
2007	2	0.231	-0.200	0.031	0.277	-0.262	0.015
2007	3	0.236	-0.206	0.030	0.293	-0.278	0.015
2007	4	0.248	-0.205	0.043	0.317	-0.273	0.044
2008	1	0.216	-0.202	0.014	0.318	-0.263	0.055
2008	2	0.234	-0.232	0.002	0.338	-0.241	0.097
2008	3	0.232	-0.229	0.003	0.352	-0.238	0.114
2008	4	0.209	-0.278	-0.069	0.300	-0.284	0.016
2009	1	0.153	-0.368	-0.215	0.302	-0.335	-0.033
2009	2	0.154	-0.353	-0.199	0.284	-0.358	-0.074
2009	3	0.158	-0.290	-0.132	0.299	-0.378	-0.079
2009	4	0.203	-0.234	-0.031	0.350	-0.322	0.028
2010	1	0.285	-0.182	0.103	0.353	-0.271	0.082
2010	2	0.314	-0.170	0.144	0.344	-0.262	0.082
2010	3	0.272	-0.155	0.117	0.335	-0.260	0.075
2010	4	0.252	-0.173	0.079	0.316	-0.257	0.059
2011	1	0.246	-0.181	0.065	0.300	-0.230	0.070
2011	2	0.243	-0.197	0.046	0.304	-0.212	0.092
2011	3	0.237	-0.189	0.048	0.291	-0.218	0.073
2011	4	0.233	-0.208	0.025	0.302	-0.238	0.064

Midpoint decomposition of the quarterly log growth rate for U.S. firms exporting to related parties. See Section 2.2 for detailed formulas. Growers denotes the positive intensive margin and shrinkers denotes the negative intensive margin.

Table A.3: Midpoint Decomposition - 2006-2011 - Arm's Length Trade

Year	Quarter	Intensive Margin			Extensive Margin		
		Growers	Shrinkers	Net	Entry	Exit	Net
2007	1	0.228	-0.197	0.031	0.439	-0.363	0.076
2007	2	0.235	-0.187	0.048	0.446	-0.370	0.076
2007	3	0.237	-0.183	0.054	0.453	-0.361	0.092
2007	4	0.257	-0.186	0.071	0.444	-0.362	0.082
2008	1	0.258	-0.168	0.090	0.458	-0.359	0.099
2008	2	0.252	-0.173	0.079	0.459	-0.347	0.112
2008	3	0.239	-0.182	0.057	0.473	-0.369	0.104
2008	4	0.185	-0.243	-0.058	0.436	-0.417	0.019
2009	1	0.142	-0.297	-0.155	0.396	-0.487	-0.091
2009	2	0.137	-0.292	-0.155	0.375	-0.528	-0.153
2009	3	0.162	-0.278	-0.116	0.373	-0.513	-0.140
2009	4	0.211	-0.212	-0.001	0.418	-0.433	-0.015
2010	1	0.260	-0.170	0.090	0.453	-0.369	0.084
2010	2	0.275	-0.157	0.118	0.457	-0.362	0.095
2010	3	0.257	-0.173	0.084	0.444	-0.353	0.091
2010	4	0.264	-0.171	0.093	0.427	-0.340	0.087
2011	1	0.254	-0.181	0.073	0.435	-0.334	0.101
2011	2	0.256	-0.175	0.081	0.435	-0.343	0.092
2011	3	0.260	-0.172	0.088	0.432	-0.342	0.090
2011	4	0.233	-0.200	0.033	0.410	-0.349	0.061

Midpoint decomposition of the quarterly log growth rate for U.S. firms exporting to related parties. See Section 2.2 for detailed formulas. Growers denotes the positive intensive margin and shrinkers denotes the negative intensive margin.

A.2 Robustness Checks

Table A.4: Estimation without China and NAFTA

Dep var: Exit	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Related Party	0.845***	0.913***	0.934***	0.841***	0.908***	0.932***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
GDP (log)	0.969***	0.966***	0.98***	0.98***	0.974***	0.98***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Uncertainty AR(1)	1.046***	1.076***	1.252***			
	(0.003)	(0.004)	(0.004)			
Crisis (2009)	1.06***	1.062***	1.081***	1.055***	1.052***	1.032***
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)
Employees (log)		0.963***	0.973***		0.964***	0.975***
		(0.001)	(0.001)		(0.001)	(0.001)
Age (log)		1.016***	1.004***			1.009***
		(0.001)	(0.001)			(0.001)
Exports (log)			0.803***			0.803***
			(0.001)			(0.001)
Stock Market Volatitly				1.016***	1.02***	1.025***
				(0.001)	(0.001)	(0.001)
Stock Market Return				0.89***	0.896***	0.811***
				(0.009)	(0.009)	(0.009)
Observations (rounded)	8,130,000	8,130,000	8,130,000	7,030,000	7,030,000	7,030,000

Clustered standard errors at firm-destination-product in parenthesis
Hazard function is non-parametric.
Coefficient are presented in their exponential form.
$***p < 0.01$, $**\ p < 0.05$

Table A.5: Estimation without GTC

Dep var: Exit	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Related Party	0.824***	0.904***	0.932***	0.814***	0.896***	0.924***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
GDP (log)	0.963***	0.961***	0.97***	0.979***	0.974***	0.99***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Uncertainty AR(1)	1.05***	1.046***	1.183***			
	(0.006)	(0.006)	(0.006)			
Employees (log)		0.956***	0.969***		0.956***	0.97***
		(0.001)	(0.001)		(0.001)	(0.001)
Age (log)		1.018***	0.994***		1.018***	0.993***
		(0.001)	(0.001)		(0.001)	(0.001)
Exports (log)			0.79***			0.787***
			(0.001)			(0.001)
Stock Market Volatility				1.012***	1.033***	1.065***
				(0.00179)	(0.00184)	(0.00179)
Stock Market Return				0.884***	0.848***	0.887***
				(0.0119)	(0.0123)	(0.0121)
Observations (rounded)	7,822,000	7,822,000	7,822,000	6,568,000	6,568,000	6,568,000

Clustered standard errors at firm-destination-product in parenthesis
Hazard function is non-parametric.
Coefficient are presented in their exponential form.
$***p < 0.01$, $**\ p < 0.05$

Table A.6: Additional Firms' Characteristics

Dep var: Exit	**(1)**	**(2)**	**(3)**
Related Party	0.920***	0.920***	0.919***
	(0.003)	(0.003)	(0.003)
GDP (log)	0.973***	0.973***	0.98***
	(0.001)	(0.001)	(0.001)
Uncertainty AR(1)	1.155***	1.156***	1.156***
	(0.010)	(0.010)	(0.010)
Crisis	1.08***	1.08***	1.079***
	(0.003)	(0.003)	(0.003)
Employees (log)	0.974***	0.974***	0.971***
	(0.001)	(0.001)	(0.001)
Age (log)	0.993***	0.993***	0.992***
	(0.002)	(0.002)	(0.002)
Exports (log)	0.776***	0.776***	0.776***
	(0.001)	(0.001)	(0.001)
TFP (log)	0.982***	0.982***	0.982***
	(0.002)	(0.002)	(0.002)
Importer		0.9933**	0.985***
		(0.003)	(0.003)
Domestic Sales (log)			0.988***
			(0.001)
Observations (rounded)	7,822,000	7,822,000	7,822,000

Clustered standard errors at firm-destination-product in parenthesis
Hazard function is non-parametric.
Coefficient are presented in their exponential form.
$***p < 0.01$, $**\ p < 0.05$

Table A.7: Estimation using Industry-specific Baseline Hazard

Dep var: Exit	**(1)**	**(2)**	**(3)**
Related Party	0.885***	0.926***	0.936***
	(0.003)	(0.003)	(0.003)
GDP (log)	0.965***	0.961***	0.98***
	(0.001)	(0.001)	(0.001)
Uncertainty AR(1)	1.033***	1.055***	1.138***
	(0.009)	(0.009)	(0.009)
Crisis	1.031***	1.042***	1.061***
	(0.002)	(0.002)	(0.002)
Employees (log)		0.965***	0.977***
		(0.001)	(0.001)
Age (log)		1.013***	0.988***
		(0.002)	(0.002)
Exports (log)			0.792***
			(0.001)
Observations (rounded)	14,300,000	14,300,000	12,700,000

Clustered standard errors at firm-destination-product in parenthesis
Hazard function is non-parametric.
Coefficient are presented in their exponential form.
$***p < 0.01$, $**\ p < 0.05$

Table A.8: Estimation using Cox Proportional Hazard

Dep var: Exit	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Related Party	0.919***	0.953***	0.964***	0.916***	0.951***	0.963***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
GDP (log)	0.986***	0.985***	0.99***	0.99***	0.988***	0.99***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Uncertainty AR(1)	1.025***	1.036***	1.097***			
	(0.002)	(0.002)	(0.002)			
Crisis (2009)	1.025***	1.026***	1.033***	1.03***	1.027***	1.015***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Employees (log)		0.982***	0.987***		0.981***	0.988***
		(0.001)	(0.001)		(0.001)	(0.001)
Age (log)		1.006***	0.997***			0.995***
		(0.001)	(0.001)			(0.001)
Exports (log)			0.89***			0.887***
			(0.001)			(0.001)
Stock Market Volatility				1.01***	1.072***	1.016***
				(0.001)	(0.001)	(0.001)
Stock Market Return				0.967***	0.971***	0.942***
				(0.004)	(0.004)	(0.004)
Observations (rounded)	14,300,000	14,300,000	12,700,000	12,700,000	12,700,000	11,900,000

Clustered standard errors at firm-destination-product in parenthesis
Hazard function is non-parametric.
Coefficient are presented in their exponential form.
$***p < 0.01$, $**\ p < 0.05$

A.3 Incomplete Contracts

Taking the initial preferences and the setup (see section 3), I obtain that the quantity demanded of the differentiated product is:

$$x(i) = \mu Y \frac{p(i)^{-\frac{1}{1-\alpha}}}{P^{-\frac{\alpha}{1-\alpha}}} = \mu Y \left[\frac{p(i)}{P^{\alpha}}\right]^{-\frac{1}{1-\alpha}} \tag{A.3.1}$$

$$p(i) = P^{\alpha}\left[\frac{\mu Y}{x(i)}\right]^{1-\alpha} \tag{A.3.2}$$

Hence the potential revenue is

$$R(i) = P^{\alpha}(\mu Y)^{1-\alpha}\theta^{\alpha}\left[\frac{h(i)}{\eta}\right]^{\alpha\eta}\left[\frac{m(i)}{1-\eta}\right]^{\alpha(1-\eta)} \tag{A.3.3}$$

Since output if not contractable, then parties (entrepreneur and manufacturer) choose noncooperatively their factor demands:

$$H: \max_{h(i)} \zeta_k R(i) - w^N h(i)$$
$$M: \max_{m(i)} (1-\zeta_k) R(i) - w^S m(i)$$

Hence the optimal demands are

$$w^N = \zeta_k \frac{\partial R(i)}{\partial h(i)} = \frac{\zeta_k \alpha \eta R(i)}{h(i)}$$
$$w^S = (1-\zeta_k)\frac{\partial R(i)}{\partial h(i)} = \frac{(1-\zeta_k)\alpha(1-\eta)R(i)}{m(i)}$$

Then total profits are

$$R(i) = (\mu Y)\left(P\theta\alpha\left[\frac{\zeta_k}{w^N}\right]^{\eta}\left[\frac{(1-\zeta_k)}{w^S}\right]^{1-\eta}\right)^{\frac{\alpha}{1-\alpha}} \tag{A.3.4}$$

$$\pi_k(\mathcal{A},\eta,\theta) = \mathcal{A}\theta^{\frac{\alpha}{1-\alpha}}\psi_k(\eta) \tag{A.3.5}$$

where

$$\mathcal{A} =(\mu Y)P^{\frac{\alpha}{1-\alpha}} \tag{A.3.6}$$

$$\psi_k(\eta) =\frac{(1-\alpha\left[\zeta_k\eta+(1-\zeta_k)(1-\eta)\right])}{\left(\frac{1}{\alpha}\left[\frac{w^N}{\zeta_k}\right]^{\eta}\left[\frac{w^S}{(1-\zeta_k)}\right]^{1-\eta}\right)^{\frac{\alpha}{1-\alpha}}} \tag{A.3.7}$$

A.4 Demand Threshold

In order to solve for the demand thresholds that describe the decision rule of a firm, I need- expressions for either each of the value functions V_w, V_o, V_v or alternative find expressions for the difference of the value functions. I follow the second approach and construct the differences between the relevant pair of value functions for each decision. The three relevant difference between value function are: $(V_o - V_w)$, $(V_v - V_w)$ and $(V_v - V_o)$.

A.4.1 Value of Outsourcing - Value of waiting

Combining the expressions for $V_o(\mathcal{A})$ and $V_w(\mathcal{A})$, (3.7) and (3.9) respectively,

$$\begin{aligned}V_o(\mathcal{A})-V_w(\mathcal{A}) =&\frac{\pi_o(\mathcal{A})-f_p}{1-\beta+\beta\gamma}+\frac{\beta\gamma[G(\mathcal{A}_v^e)-G(\mathcal{A}_o^x)]}{1-\beta+\beta\gamma}\mathbb{E}V_o(\mathcal{A}_o^x<\mathcal{A}<\mathcal{A}_v^e)\\&-\frac{\beta\gamma[G(\mathcal{A}_v^e)-G(\mathcal{A}_o^e)]}{1-\beta+\beta\gamma}[\mathbb{E}V_o(\mathcal{A}_o^e<\mathcal{A}<\mathcal{A}_v^e)-f_e]\\&+\frac{\beta\gamma G(\mathcal{A}_o^x)}{1-\beta+\beta\gamma}[\mathbb{E}V_w(\mathcal{A}<\mathcal{A}_o^x)-f_x]\\&-\frac{\beta\gamma G(\mathcal{A}_o^e)}{1-\beta+\beta\gamma}\mathbb{E}V_w(\mathcal{A}<\mathcal{A}_o^e)+\frac{\beta\gamma[1-G(\mathcal{A}_v^e)]}{1-\beta+\beta\gamma}f_e\end{aligned}$$

Note that $\mathbb{E}V_w(\mathcal{A}<\mathcal{A}_o^x)=\mathbb{E}V_w(\mathcal{A}<\mathcal{A}_o^e)=V_w(\mathcal{A})$ since the value of waiting for non-exporter is independent of the current realization. Focusing on the value of exporting via outsourcing, the next step is to express $\mathbb{E}V_o(\mathcal{A}_o^x<\mathcal{A}<\mathcal{A}_v^e)$ and $\mathbb{E}V_o(\mathcal{A}_o^e<\mathcal{A}<\mathcal{A}_v^e)$ as a function of $V_o(\mathcal{A})$ using (3.7). After some manipulations:

$$V_o(\mathcal{A}) - V_w(\mathcal{A}) = \frac{\pi_o(\mathcal{A}) - f_p}{1-\beta+\beta\gamma} + \frac{\beta\gamma}{1-\beta+\beta\gamma}\frac{\int_{\mathcal{A}_o^x}^{\mathcal{A}_o^e}[\pi_o(\mathcal{A}) - f_p]dG}{1-\beta+\beta\gamma\lambda_o^{ex}} + \frac{\beta\gamma[1-G(\mathcal{A}_o^e)]f_e - \beta\gamma G(\mathcal{A}_o^x)f_x}{1-\beta+\beta\gamma\lambda_o^{ex}} \tag{A.4.1}$$

where $\lambda_k^{rs} \equiv [1 - G(\mathcal{A}_k^r) + G(\mathcal{A}_k^s)]$, $k \in \{o, v\}$ and $r, s \in \{e, x\}$. λ_o^{ex} captures the probability that a shock outside of the inaction band for an exporter with outsourcing arrives.

This expression A.4.1 is intuitive. The difference between the value of exporting via outsourcing and the value of waiting is equal to discounted current profits, plus the discounted profits from the inaction band plus the difference between the sunk costs to start and stop exporting adjusted by their probability.

A.4.2 Value of Integration - Value of Outsourcing

Combining the expressions for $V_v(\mathcal{A})$ and $V_o(\mathcal{A})$, (3.7) and (3.8) respectively and after some algebra,

$$V_v(\mathcal{A}) - V_o(\mathcal{A}) = \frac{\pi_v(\mathcal{A}) - \pi_o(\mathcal{A})}{1-\beta+\beta\gamma} + \frac{\beta\gamma[1-G(\mathcal{A}_v^e)]f_v}{1-\beta+\beta\gamma\lambda_v^{ex}} + \frac{\beta\gamma}{1-\beta+\beta\gamma}\left[\frac{\int_{\mathcal{A}_o^x}^{\mathcal{A}_v^e}[\pi_v(\mathcal{A}) - \pi_o(\mathcal{A})]dG}{1-\beta+\beta\gamma\lambda_v^{ex}} + \frac{\int_{\mathcal{A}_v^x}^{\mathcal{A}_o^x}[\pi_v(\mathcal{A}) - \pi_o(\mathcal{A}_o^x)]dG}{1-\beta+\beta\gamma\lambda_v^{ex}}\right] \tag{A.4.2}$$

The intuition is similar to A.4.1. The difference between the organizational forms to export is equal to the discounted difference of today profits plus the discounted difference in profits across the inaction band of both organizational forms and the discounted difference in the sunk costs.

A.4.3 Value of Integration - Value of Waiting

Combining the expressions for $V_v(\mathcal{A})$ and $V_w(\mathcal{A})$, (3.9) and (3.8) respectively, and following the same approach as the previous expressions for the differences between value functions, I obtain

$$V_v(\mathcal{A}) - V_w(\mathcal{A}) = \frac{\pi_v(\mathcal{A}) - f_p}{1-\beta+\beta\gamma} + \frac{\beta\gamma[1-G(\mathcal{A}_v^e)][f_v+f_e] - \beta\gamma G(\mathcal{A}_v^x) f_x}{1-\beta+\beta\gamma\lambda_v^{ex}} + \frac{\beta\gamma}{1-\beta+\beta\gamma}\left[\frac{\int_{\mathcal{A}_v^x}^{\mathcal{A}_v^e}[\pi_v(\mathcal{A}) - f_p]dG}{1-\beta+\beta\gamma\lambda_v^{ex}} - \frac{\int_{\mathcal{A}_o^e}^{\mathcal{A}_v^e}[\pi_o(\mathcal{A}) - \pi_o(A_e^o)]dG}{1-\beta+\beta\gamma\lambda_v^{ex}}\right] \tag{A.4.3}$$

Notice that this difference correspond to the cases where $\mathcal{A} < \mathcal{A}_o^e$ which is the case for $\mathcal{A}_v^x$.

A.4.4 Entry with Outsourcing

After computing the difference between the value functions $V_o(\mathcal{A})$ and $V_w(\mathcal{A})$, the expression for entry cutoff is quite direct since $f_e = V_o(\mathcal{A}_o^e) - V_w(\mathcal{A}_o^e)$. Formally:

$$f_e = \frac{\pi_o(\mathcal{A}_o^e) - f_p}{1-\beta\tilde{\lambda}_o^x} + \frac{\beta\gamma}{1-\beta+\beta\gamma}\frac{\int_{\mathcal{A}_o^x}^{\mathcal{A}_o^e}[\pi_o(\mathcal{A}) - \pi_o(\mathcal{A}_o^e)]dG}{1-\beta\tilde{\lambda}_o^x} - \frac{\beta\gamma G(\mathcal{A}_o^x) f_x}{1-\beta\tilde{\lambda}_o^x} \tag{A.4.4}$$

where $\tilde{\lambda}_o^x = 1 - \gamma G(\mathcal{A}_o^x)$. Note that $\gamma \in (0,1)$ and $G(\mathcal{A}_o^x) < 1$ then $\tilde{\lambda}_o^x < 1$.

From the expression above is easy to show that $\mathcal{A}_o^e > \mathcal{A}_o^{eD}$. Noting that $f_e = \frac{\pi_o(\mathcal{A}_o^{eD})}{1-\beta}$ (see A.4.8 for the derivation of the expression on the deterministic framework) and rearranging the entry condition, I obtain:

$$\pi_o(\mathcal{A}_o^e) - \pi_o(\mathcal{A}_o^{eD}) = \beta\gamma G(\mathcal{A}_o^x)[f_e + f_x] + \frac{\beta\gamma}{1-\beta+\beta\gamma}\int_{\mathcal{A}_o^x}^{\mathcal{A}_o^e}\left[\pi_o(\mathcal{A}_o^e) - \pi(\mathcal{A})\right] dG(\mathcal{A})$$

Then $\pi_o(\mathcal{A}_o^e) - \pi_o(\mathcal{A}_o^{eD}) > 0 \Rightarrow \mathcal{A}_o^e > \mathcal{A}_o^{eD}$.

A.4.5 Integration Decision

Combining (3.4) and (A.4.2) and after some algebra:

$$
\begin{aligned}
f_v =& \frac{\pi_v(\mathcal{A}_v^e) - \pi_o(\mathcal{A}_v^e)}{1-\beta\tilde{\lambda}_v^x} - \frac{\beta\gamma[G(\mathcal{A}_v^e) - G(\mathcal{A}_x^v)]}{1-\beta+\beta\gamma}\left[\frac{\pi_v(\mathcal{A}_v^e) - \pi_o(\mathcal{A}_v^e)}{1-\beta\tilde{\lambda}_v^x}\right] \\
&+ \frac{\beta\gamma[G(\mathcal{A}_v^e) - G(\mathcal{A}_x^v)]}{1-\beta+\beta\gamma}\left[\frac{\mathbb{E}\pi_v(\mathcal{A}_x^v < \mathcal{A} < \mathcal{A}_v^e) - f_p}{1-\beta\tilde{\lambda}_v^x}\right] \\
&- \frac{\beta\gamma[G(\mathcal{A}_v^e) - G(\mathcal{A}_o^x)]}{1-\beta+\beta\gamma}\left[\frac{\mathbb{E}\pi_o(\mathcal{A}_o^x < \mathcal{A} < \mathcal{A}_v^e) - f_p}{1-\beta\tilde{\lambda}_v^x}\right] \\
&+ \frac{\beta\gamma[G(\mathcal{A}_o^x) - G(\mathcal{A}_x^v)][f_e + f_x]}{1-\beta\tilde{\lambda}_v^x} - \frac{\beta\gamma[G(\mathcal{A}_x^v) - G(\mathcal{A}_o^x)]}{1-\beta+\beta\gamma}\left[\frac{\pi_o(\mathcal{A}_e^o) - f_p}{1-\beta\tilde{\lambda}_v^x}\right]
\end{aligned} \tag{A.4.5}
$$

where $\tilde{\lambda}_v^x = 1 - \gamma G(\mathcal{A}_v^x)$.

This condition allows U.S. to show that $\mathcal{A}_v^e > \mathcal{A}_v^{eD}$ as follows. Reexpressing the integration condition, recalling that $f_v = \frac{\pi_v(\mathcal{A}_v^{eD}) - \pi_o(\mathcal{A}_v^{eD})}{1-\beta}$ (see A.4.8 for the derivation of this expression) and using the functional form of the profit function

$$
\begin{aligned}
\mathcal{A}_v^e - \mathcal{A}_v^{eD} \propto & \frac{\beta\gamma[G(\mathcal{A}_v^e) - G(\mathcal{A}_x^v)]}{1-\beta+\beta\gamma}\left[\frac{\pi_v(\mathcal{A}_v^e) - \mathbb{E}\pi_v(\mathcal{A}_x^v < \mathcal{A} < \mathcal{A}_v^e)}{1-\beta\tilde{\lambda}_v^x}\right] \\
&- \frac{\beta\gamma[G(\mathcal{A}_v^e) - G(\mathcal{A}_o^x)]}{1-\beta+\beta\gamma}\left[\frac{\pi_o(\mathcal{A}_v^e) - \mathbb{E}\pi_o(\mathcal{A}_o^x < \mathcal{A} < \mathcal{A}_v^e)}{1-\beta\tilde{\lambda}_v^x}\right] \\
&+ \frac{\beta\gamma[G(\mathcal{A}_o^x) - G(\mathcal{A}_x^v)]}{1-\beta+\beta\gamma}\left[\frac{\pi_o(\mathcal{A}_o^x) - f_p}{1-\beta\tilde{\lambda}_v^x}\right] + \beta\gamma G(A_x^v) f_v
\end{aligned}
$$

Hence $\mathcal{A}_v^e > \mathcal{A}_v^{eD}$ since the right hand side of the equation is positive. Note that showing that the first term is higher that the second term is sufficient to prove that $RHS > 0$.

$$
\begin{aligned}
\Gamma =& \beta\gamma[G(\mathcal{A}_v^e) - G(\mathcal{A}_x^v)]\left[\pi_v(\mathcal{A}_v^e) - \mathbb{E}\pi_v(\mathcal{A}_x^v < \mathcal{A} < \mathcal{A}_v^e)\right] \\
&- \beta\gamma[G(\mathcal{A}_v^e) - G(\mathcal{A}_o^x)]\left[\pi_o(\mathcal{A}_v^e) - \mathbb{E}\pi_o(\mathcal{A}_o^x < \mathcal{A} < \mathcal{A}_v^e)\right] \\
=& \beta\gamma[G(\mathcal{A}_v^e) - G(\mathcal{A}_o^x)]\left[\pi_v(\mathcal{A}_v^e - \bar{\mathcal{A}}) - \pi_o(\mathcal{A}_v^e - \bar{\mathcal{A}})\right] \\
&+ \beta\gamma[G(\mathcal{A}_o^x) - G(\mathcal{A}_x^v)]\left[\pi_v(\mathcal{A}_v^e) - \mathbb{E}\pi_v(\mathcal{A}_x^v < \mathcal{A} < \mathcal{A}_o^x)\right]
\end{aligned}
$$

where I apply the second mean theorem for integration in the last step. By construction, $\mathcal{A}_v^e \geq \bar{\mathcal{A}}$ since $\bar{\mathcal{A}} \in [\mathcal{A}_o^x, \mathcal{A}_v^e]$. Then $\Gamma > 0$ and $\mathcal{A}_v^e > \mathcal{A}_v^{eD}$.

A.4.6 Exit from Outsourcing

Since the exit condition for an outsourcing exporter is $f_x = V_w(\mathcal{A}_o^x) - V_o(\mathcal{A}_o^x)$ and replacing the RHS using A.4.1, I obtain

$$
\begin{aligned}
f_x = & -\frac{\pi_o(\mathcal{A}_o^x) - f_p}{1 - \beta\tilde{\lambda}_o^x + \beta\gamma[1 - G(\mathcal{A}_o^e)]} - \frac{\beta\gamma[G(\mathcal{A}_o^e) - G(\mathcal{A}_o^x)]\left[\frac{\mathbb{E}\pi_o(\mathcal{A}_o^x < \mathcal{A} < \mathcal{A}_o^e) - \pi_o(\mathcal{A}_o^x)}{1-\beta+\beta\gamma}\right]}{1 - \beta + \beta\gamma[1 - G(\mathcal{A}_o^e)]} \\
& - \frac{\beta\gamma[G(\mathcal{A}_v^e) - G(\mathcal{A}_o^e)]f_e}{1 - \beta + \beta\gamma[1 - G(\mathcal{A}_o^e)]}
\end{aligned}
\tag{A.4.6}
$$

where $\tilde{\lambda}_o^x = 1 - \gamma(1 - G(\mathcal{A}_o^e))$. Reorganizing this conditionand replacing the exit condition in the deterministic framework (see A.4.8 for the expression), I get

$$
\begin{aligned}
\pi_o(\mathcal{A}_o^x) - \pi_o(\mathcal{A}_o^{xD}) = & -\frac{\beta\gamma[G(\mathcal{A}_e^o) - G(\mathcal{A}_o^x)]}{1 - \beta + \beta\gamma}\left[\mathbb{E}\pi_o(\mathcal{A}_o^x < \mathcal{A} < \mathcal{A}_e^o) - \pi_o(\mathcal{A}_o^x)\right] \\
& - \beta\gamma[G(\mathcal{A}_v^e) - G(\mathcal{A}_e^o)]f_e - \beta\gamma[1 - G(\mathcal{A}_e^o)]f_x \\
\pi_o(\mathcal{A}_o^x) - \pi_o(\mathcal{A}_o^{xD}) < & 0
\end{aligned}
$$

This is the case since $\mathcal{A}_o^e > \mathcal{A}_o^x$ and $\mathcal{A}_v^e > \mathcal{A}_o^e$. Hence $\frac{\partial\pi}{\partial\mathcal{A}} > 0$ implies $\mathcal{A}_o^x < \mathcal{A}_o^{xD}$.

A.4.7 Exit from Vertical Integration

A vertical integrated exporter make the decision to exit if $f_x = V_w(\mathcal{A}_v^x) - V_v(\mathcal{A}_v^x)$, applying (A.4.3) and doing some algebra

$$
\begin{aligned}
f_x = & -\frac{\pi_v(\mathcal{A}_v^x) - f_p}{1 - \beta\tilde{\lambda}_v^x} - \frac{\beta\gamma}{1 - \beta + \beta\gamma}\frac{\int_{\mathcal{A}_v^x}^{\mathcal{A}_v^e}[\pi_v(\mathcal{A}) - \pi_v(\mathcal{A}_v^x)]\,dG}{1 - \beta\tilde{\lambda}_v^x} \\
& + \frac{\beta\gamma}{1 - \beta + \beta\gamma}\frac{\int_{\mathcal{A}_o^e}^{\mathcal{A}_v^e}[\pi_o(\mathcal{A}) - \pi_o(\mathcal{A}_o^e)]\,dG}{1 - \beta\tilde{\lambda}_v^x} - \frac{\beta\gamma[1 - G(\mathcal{A}_v^e)][f_v + f_e]}{1 - \beta\tilde{\lambda}_v^x}
\end{aligned}
\tag{A.4.7}
$$

where $\tilde{\lambda}_v^x = 1 - \gamma(1 - G(\mathcal{A}_v^e))$.

Reorganizing this condition, I obtain

$$\begin{aligned}
\pi_v(\mathcal{A}_v^x) - \pi_v(\mathcal{A}_v^{xD}) =& -\frac{\beta\gamma}{1-\beta+\beta\gamma}\int_{\mathcal{A}_o^e}^{\mathcal{A}_v^e}[\pi_v(\mathcal{A}) - \pi_o(\mathcal{A})]\,dG + \frac{\beta\gamma}{1-\beta+\beta\gamma}\int_{\mathcal{A}_o^e}^{\mathcal{A}_v^e}[\pi_v(\mathcal{A}_v^x) - \pi_o(\mathcal{A}_o^e)]\,dG \\
& -\frac{\beta\gamma}{1-\beta+\beta\gamma}\int_{\mathcal{A}_v^x}^{\mathcal{A}_o^e}[\pi_v(\mathcal{A}) - \pi_v(\mathcal{A}_v^x)]\,dG \\
& -\beta\gamma[1-G(\mathcal{A}_v^e)][f_v + f_e + f_x] \\
\pi_v(\mathcal{A}_v^x) - \pi_v(\mathcal{A}_v^{xD}) <& 0
\end{aligned}$$

since first term is negative because $\pi_v > \pi_o$ and the first term is higher, in absolute value, than the second term, and all remaining terms are negative. Since $\frac{\partial\pi}{\partial\mathcal{A}} > 0$ then $\mathcal{A}_v^x < \mathcal{A}_v^{xD}$.

A.4.8 Exit across Sourcing Decision

In order to compare the exit threshold across sourcing decisions, combining (A.4.6) and (A.4.7) to write an expression for $\pi_o(\mathcal{A}_x^o) - \pi_v(\mathcal{A}_v^x)$:

$$\begin{aligned}
\pi_o(\mathcal{A}_o^x) - \pi_v(\mathcal{A}_v^x) =& \frac{\beta\gamma\int_{\mathcal{A}_v^x}^{\mathcal{A}_v^e}[\pi_v(\mathcal{A}) - \pi_v(\mathcal{A}_v^x)]dG}{1-\beta+\beta\gamma} - \frac{\beta\gamma\int_{\mathcal{A}_e^o}^{\mathcal{A}_v^e}[\pi_o(\mathcal{A}) - \pi_o(\mathcal{A}_o^e)]dG}{1-\beta+\beta\gamma} \\
& + \beta\gamma[1-G(\mathcal{A}_e^v)][f_v + f_e + f_x] \\
& - \frac{\beta\gamma\int_{\mathcal{A}_o^x}^{\mathcal{A}_o^e}[\pi_o(\mathcal{A}) - \pi_o(\mathcal{A}_o^x)]dG}{1-\beta+\beta\gamma} - \beta\gamma[1-G(\mathcal{A}_e^o)][f_e + f_x] \\
\pi_o(\mathcal{A}_o^x) - \pi_v(\mathcal{A}_v^x) =& \frac{\beta\gamma\int_{\mathcal{A}_o^x}^{\mathcal{A}_v^e}[\Delta_{vo}\pi(\mathcal{A}) - \Delta_{vo}\pi(\mathcal{A}_o^x)]dG}{1-\beta+\beta\gamma} \\
& + \frac{\beta\gamma\int_{\mathcal{A}_o^x}^{\mathcal{A}_v^e}[\pi_v(\mathcal{A}_o^x) - \pi_v(\mathcal{A}_v^x)]dG}{1-\beta+\beta\gamma} + \frac{\beta\gamma\int_{\mathcal{A}_v^x}^{\mathcal{A}_o^x}[\pi_v(\mathcal{A}) - \pi_v(\mathcal{A}_v^x)]dG}{1-\beta+\beta\gamma} \\
& + \beta\gamma[1-G(\mathcal{A}_e^v)]f_v \\
\pi_o(\mathcal{A}_o^x) - \pi_v(\mathcal{A}_v^x) >& 0
\end{aligned}$$

Since $\pi_v > \pi_o$ for a given $\mathcal{A}$ and $\frac{\partial\pi}{\partial\mathcal{A}} > 0$ then $\mathcal{A}_o^x > \mathcal{A}_v^x$

Note that this result also implies that introducing foreign demand uncertainty expands the difference between the exit threshold across. This is the case since $\pi_v(\mathcal{A}_v^{xD}) = \pi_o(\mathcal{A}_o^{xD})$ and this implies that $[\pi_o(\mathcal{A}_x^o) - \pi_v(\mathcal{A}_v^x)] > [\pi_o(\mathcal{A}_x^{oD}) - \pi_v(\mathcal{A}_v^{xD})]$. Note that since uncertainty reduce both threshold then it has to be the case that the reduction on integration exit threshold is bigger than the reduction on the exit from outsourcing threshold.Deterministic

Framework

A.4.8.1 The Deterministic Framework as Benchmark

In the deterministic framework firms compare the discounted value of the profits of their current state to the sunk cost and resulting profits from switching states. This implies that for a given demand level $\mathcal{A}_t$ the productivity cutoff for entry with outsourcing θ_o^{eD} is defined as

$$\frac{\pi_o(\mathcal{A}_t, \theta_o^{eD}) - f_p}{1-\beta} = f_e \Leftrightarrow \theta_o^{eD} = \left[\frac{[1-\beta]f_e + f_p}{\psi_o \mathcal{A}_t}\right]^{\rho} \tag{A.4.8}$$

where $\rho = (1-\alpha)/\alpha$. Hence firms with productivity θ_i just above θ_o^{eD} will pay the sunk cost and start exporting via outsourcing. However, since firms can integrate and get a higher flow of profits by paying an additional sunk cost, firms with high enough productivity will integrate rather than outsource. More specifically, firms with $\theta_i > \theta_v^{eD} > \theta_o^{eD}$ will start exporting with integration. Formally, θ_v^{eD} is defined as follows for a given demand level of $\mathcal{A}_t$:

$$\frac{\pi_v(\mathcal{A}_t, \theta_v^{eD}) - \pi_o(\mathcal{A}_t, \theta_v^{eD})}{1-\beta} = f_v \Leftrightarrow \theta_v^{eD} = \left[\frac{[1-\beta]f_v}{(\psi_v - \psi_o)\mathcal{A}_t}\right]^{\rho} \tag{A.4.9}$$

In the case of the exit decision, for a given demand level $\mathcal{A}_t$, firms that are currently exporting via outsourcing will exit if their productivity level is below the exit productivity cutoff, i.e. $\theta_i < \theta_o^{xD}$. The exit productivity cutoff satisfies the following expression:

$$-\frac{\pi_o(\mathcal{A}_t, \theta_o^{xD}) - f_p}{1-\beta} = f_x \Leftrightarrow \theta_o^{xD} = \left[-\frac{[1-\beta]f_x - f_p}{\psi_o \mathcal{A}_t}\right]^{1/\rho} \tag{A.4.10}$$

Note that since $\theta > 0$, it will be optimal for some firms to exit if and only if $(1-\beta)f_x - f_p < 0$.[26] Similarly, the exit productivity cutoff for integrated firms θ_v^{xD} is as follows:

$$-\frac{\pi_v(\mathcal{A}_t, \theta_v^{xD}) - f_p}{1-\beta} = f_x \Leftrightarrow \theta_v^{xD} = \left[-\frac{[1-\beta]f_x - f_p}{\psi_v \mathcal{A}_t}\right]^{\rho} \tag{A.4.11}$$

It is easy to prove that the rankings of the productivity cutoffs satisfy $\theta_v^{xD} < \theta_o^{xD} < \theta_o^{eD} < \theta_v^{eD}$.

[26] Note that in the case of $f_x = 0$, the exit cutoff will be $\theta_o^{xD} = [f_p/\psi_o \mathcal{A}_t]^{1/\rho}$, and firms will exit as soon as the profit flow cannot cover the fixed per period costs.

A.5 Parametrizing Firms' Decisions

A.5.1 Productivity Cutoff Entry with Outsourcing

The entry condition is

$$[1-\beta]f_e = [\pi_o(\mathcal{A}_o^e) - f_p] + \frac{\beta\gamma \int_{A_x^o}^{A_e^o} [\pi_o(A) - \pi_o(A_e^o)]dG}{[1-\beta(1-\gamma)]} - \beta\gamma G(\mathcal{A}_o^x)[f_x + f_e]$$

For the marginal firm that $\mathcal{A}_o^e = \mathcal{A}_t$,

$$[1-\beta]f_e = \psi_o[\theta_e^o]^{\frac{\alpha}{1-\alpha}}\mathcal{A}_t - f_p + \frac{\beta\gamma\psi_o[\theta_e^o]^{\frac{\alpha}{1-\alpha}} \int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t} [\mathcal{A} - \mathcal{A}_t]dG}{[1-\beta(1-\gamma)]} - \beta\gamma G(\mathcal{A}_o^x)[f_x + f_e]$$

$$[\theta_e^o]^{\frac{\alpha}{1-\alpha}} = [\theta_e^{oD}]^{\frac{\alpha}{1-\alpha}} \times \frac{1 + \frac{\beta\gamma G(\mathcal{A}_t\xi_o^e)[f_x+f_e]}{(1-\beta)f_e+f_p}}{1 + \frac{\beta\gamma \int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t} [\mathcal{A}-\mathcal{A}_t]/\mathcal{A}_t dG}{1-\beta(1-\gamma)}}$$

Taking logs

$$\ln[\theta_e^o] = \ln[\theta_e^{oD}] + \rho \ln\left[1 + \frac{\beta\gamma G(\mathcal{A}_t\xi_o^e)[f_x + f_e]}{(1-\beta)f_e + f_p}\right] - \rho \ln\left[1 + \frac{\beta\gamma \int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t} \frac{[\mathcal{A}-\mathcal{A}_t]}{\mathcal{A}_t} dG}{1-\beta(1-\gamma)}\right] \quad \text{(A.5.1)}$$

where $\rho = (1-\alpha)/\alpha$.

A.5.2 Productivity Cutoff Exit from Outsourcing

For the marginal firm that $\mathcal{A}_o^x = \mathcal{A}_t$ it is the case that

$$\begin{aligned}(1-\beta)f_x = & -[\psi_o\mathcal{A}_t[\theta_o^x]^{1/\rho} - f_p] - \beta\gamma[1 - G(\mathcal{A}_t\xi_o^x)][f_e + f_x] \\ & - \frac{\beta\gamma[G(\mathcal{A}_t\xi_o^x) - G(\mathcal{A}_t)]}{1-\beta+\beta\gamma}[\mathbb{E}\pi_o(\mathcal{A}_t < \mathcal{A} < \mathcal{A}_t\xi_o^x) - \pi_o(\mathcal{A}_t)] \\ [\theta_o^x]^{1/\rho} = & [\theta_o^{xD}]^{1/\rho} \times \frac{1 - \frac{\beta\gamma[1-G(\mathcal{A}_t\xi_o^x)][f_e+f_x]}{f_p-[1-\beta]f_x}}{1 + \frac{\beta\gamma \int_{\mathcal{A}_t}^{\mathcal{A}_t\xi_o^x} [\mathcal{A}-\mathcal{A}_t]/\mathcal{A}_t dG}{1-\beta(1-\gamma)}}\end{aligned}$$

Taking logs

$$\ln\theta_o^x = \ln\theta_o^{xD} + \rho\ln\left[1-\kappa_1^o(x)\right] - \rho\ln\left[1+\kappa_2^o(x)\right] \tag{A.5.2}$$

where

$$\kappa_1^o(x) = \frac{\beta\gamma[1-G(\mathcal{A}_t\xi_o^x)][f_e+f_x]}{f_p-[1-\beta]f_x}$$
$$\kappa_2^o(x) = \frac{\beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi_o^x}[\mathcal{A}-\mathcal{A}_t]/\mathcal{A}_t dG}{[1-\beta(1-\gamma)]}$$

Note that $\kappa_1^o(x) > 0$ since $f_p - [1-\beta]f_x > 0$ and $G(\mathcal{A}_t\xi_o^x) \leq 1$; and $\kappa_2^o(x) > 0$ since $\mathcal{A} \geq \mathcal{A}_t$ for the integration interval.

A.5.3 Productivity Cutoff Integration

For the case that the firm $\mathcal{A}_v^e = \mathcal{A}_t$

$$\begin{aligned}
[1-\beta]f_v =& (\psi_v-\psi_o)\theta^{\frac{\alpha}{1-\alpha}}\mathcal{A}_v^e + \beta\gamma\frac{(\psi_v-\psi_o)[\theta_e^v]^{\frac{\alpha}{1-\alpha}}\int_{\mathcal{A}_o^x}^{\mathcal{A}_v^e}[\mathcal{A}-\mathcal{A}_v^e]dG}{[1-\beta+\beta\gamma]}\\
&+\beta\gamma\frac{\int_{\mathcal{A}_v^x}^{\mathcal{A}_o^x}[\pi_v(\mathcal{A})-\pi_o(\mathcal{A}_o^x)-(\psi_v-\psi_o)\theta^{\frac{\alpha}{1-\alpha}}\mathcal{A}_v^e]dG}{[1-\beta+\beta\gamma]}\\
&-\beta\gamma G(\mathcal{A}_v^x)f_v
\end{aligned}$$

Applying the inaction bands expressions and after some manipulations,

$$[\theta_e^v]^{1/\rho} = [\theta_e^{vD}]^{1/\rho} \times \frac{\left[1+\frac{\beta\gamma G(\mathcal{A}_t\xi_o^x)}{[1-\beta]}\right]}{\left[1+\frac{\beta\gamma\int_{\mathcal{A}_t\xi_o^x}^{\mathcal{A}_t}\frac{[\mathcal{A}-\mathcal{A}_t]}{\mathcal{A}_t}dG}{[1-\beta+\beta\gamma]}+\varphi\frac{\beta\gamma\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}\frac{[\mathcal{A}-\mathcal{A}_t\xi_v^x]}{\mathcal{A}_t}dG}{[1-\beta+\beta\gamma]}\right]}$$

where $\varphi = \frac{\psi_v}{\psi_v-\psi_o}$. Taking logs

$$\ln\theta_e^v = \ln\theta_e^{vD} + \rho\ln\left[1+\kappa_1^v\right] - \rho\ln\left[1+\kappa_2^v+\varphi\kappa_3^v(e)\right] \tag{A.5.3}$$

where

$$\kappa_1^v = \frac{\beta\gamma G(\mathcal{A}_t \xi_v^x)}{1-\beta}$$

$$\kappa_2^v = \frac{\beta\gamma \int_{\mathcal{A}_t \xi_v^x}^{\mathcal{A}_t} \frac{[\mathcal{A}-\mathcal{A}_t]}{\mathcal{A}_t} dG}{1-\beta+\beta\gamma}$$

$$\kappa_3^v(e) = \frac{\beta\gamma \int_{\mathcal{A}_t \xi_v^x}^{\mathcal{A}_t \xi_o^x} \frac{[\mathcal{A}-\mathcal{A}_t \xi_o^x]}{\mathcal{A}_t} dG}{1-\beta+\beta\gamma}$$

A.5.4 Productivity Cutoff Exit from Integration

In the case of the firm such that $\mathcal{A}_t = \mathcal{A}_v^x$, recalling that $[\theta_x^{vD}]^{\frac{\alpha}{1-\alpha}} = \frac{f_p - [1-\beta] f_x}{\psi_v \mathcal{A}_t}$ and exploiting the inaction band expression, the exit condition for an integrated firm is

$$\begin{aligned}
[1-\beta] f_x = & -[\psi_v [\theta_x^v]^{\frac{1}{\rho}} \mathcal{A}_t - f_p] - \frac{\beta\gamma\psi_v [\theta_x^v]^{\frac{1}{\rho}} \mathcal{A}_t \int_{\mathcal{A}_t}^{\mathcal{A}_t \xi_v^e} [\mathcal{A}-\mathcal{A}_t]/\mathcal{A}_t dG}{1-\beta+\beta\gamma} \\
& + \frac{\beta\gamma\psi_o [\theta_x^v]^{\frac{1}{\rho}} \mathcal{A}_t \int_{\mathcal{A}_o^e}^{\mathcal{A}_t \xi_v^e} [\mathcal{A}-\mathcal{A}_t \xi_o^e]/\mathcal{A}_t dG}{1-\beta+\beta\gamma} - \beta\gamma[1-G(\mathcal{A}_t \xi_v^e)][f_v + f_e + f_x] \\
[\theta_x^v]^{\frac{1}{\rho}} = & [\theta_x^{vD}]^{\frac{1}{\rho}} \times \frac{1-\kappa_1^v(x)}{1+\kappa_2^v(x) - \frac{\psi_o}{\psi_v}\kappa_3^v(x)}
\end{aligned}$$

where

$$\kappa_1^v(x) = \frac{\beta\gamma[1-G(\mathcal{A}_t \xi_v^e)][f_v + f_e + f_x]}{f_p - [1-\beta] f_x}$$

$$\kappa_2^v(x) = \frac{\beta\gamma \int_{\mathcal{A}_t}^{\mathcal{A}_t \xi_v^e} \frac{[\mathcal{A}-\mathcal{A}_t]}{\mathcal{A}_t} dG}{1-\beta+\beta\gamma}$$

$$\kappa_3^v(x) = \frac{\beta\gamma \int_{\mathcal{A}_t \xi_o^e}^{\mathcal{A}_t \xi_v^e} \frac{[\mathcal{A}-\mathcal{A}_t \xi_o^e]}{\mathcal{A}_t} dG}{1-\beta+\beta\gamma}$$

Then taking logs

$$\ln \theta_x^v = \ln \theta_x^{vD} + \rho \ln [1-\kappa_1^v(x)] - \rho \ln \left[1 + \kappa_2^v(x) - \frac{\psi_o}{\psi_v}\kappa_3^v(x)\right] \tag{A.5.4}$$

A.5.5 Productivity Cutoffs Ranking

A.5.5.1 Exit across Sourcing Decision

Proving that $\theta_o^x > \theta_v^x$ is trivially after showing that $\mathcal{A}_v^x < \mathcal{A}_o^x$ for any productivity level. For the marginal integrated that is going to exit such that $\mathcal{A}_v^x = \mathcal{A}_t$ with productivity θ_v^x, then it has to be the case that $\mathcal{A}_o^x > \mathcal{A}_t$. Similarly for the marginal firm exporting via outsourcing that is going to exit such that $\mathcal{A}_o^x = \mathcal{A}_t$ with productivity θ_o^x is true that $\mathcal{A}_v^x < \mathcal{A}_t$. Since $\frac{\partial \theta}{\partial \mathcal{A}} < 0$ then it is the case that $\theta_v^x < \theta_o^x$.

A.6 Comparative Statics

A.6.1 Entry - Outsourcing

A.6.1.1 Arrival Rate

$$\begin{aligned}
\frac{\partial \ln \theta_e^o}{\partial \gamma} =& \frac{\rho\beta G(\mathcal{A}_t\xi_o^e)[f_x + f_e]}{(1-\beta)f_e + f_p + \beta\gamma G(\mathcal{A}_t\xi_o^e)[f_x + f_e]} \\
&+ \frac{\rho}{1-\beta+\beta\gamma} \frac{\beta(1-\beta)\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A}_t - \mathcal{A}]/\mathcal{A}_t dG}{[1-\beta(1-\gamma)] + \beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A} - \mathcal{A}_t]/\mathcal{A}_t dG} \\
\frac{\partial \ln \theta_e^o}{\partial \gamma} =& \frac{\mathcal{A}_t - \mathcal{A}_t\xi_o^e}{\mathcal{A}_t} \frac{\rho\beta G(\mathcal{A}_t\xi_o^e)}{(1-\beta+\beta\gamma) + \beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A} - \mathcal{A}_t]/\mathcal{A}_t dG} \\
&+ \frac{\rho}{1-\beta+\beta\gamma} \frac{\beta(1-\beta)\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A}_t - \mathcal{A}]/\mathcal{A}_t dG}{[1-\beta(1-\gamma)] + \beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A} - \mathcal{A}_t]/\mathcal{A}_t dG}
\end{aligned}$$

Hence $\frac{\partial \ln \theta_e^o}{\partial \gamma} > 0$, when uncertainty increases the productivity required to start exporting is higher.

A.6.1.2 Current Realization

$$
\begin{aligned}
\frac{\partial \ln \theta_e^o}{\partial \mathcal{A}_t} =& \frac{\partial \ln \theta_e^{oD}}{\partial \mathcal{A}_t} + \frac{\rho\beta\gamma g(\mathcal{A}_t\xi_o^e)\left\{\xi_o^e + \mathcal{A}_t\frac{\partial \xi_o^e}{\partial \mathcal{A}_t}\right\}[f_x + f_e]}{(1-\beta)f_e + f_p + \beta\gamma G(\mathcal{A}_t\xi_o^e)[f_x+f_e]} \\
&+ \frac{\rho\beta\gamma g(\mathcal{A}_t\xi_o^e)\left\{\xi_o^e + \mathcal{A}_t\frac{\partial \xi_o^e}{\partial \mathcal{A}_t}\right\}[\mathcal{A}_t\xi_o^e - \mathcal{A}_t]/\mathcal{A}_t}{[1-\beta+\beta\gamma] + \beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A}-\mathcal{A}_t]/\mathcal{A}_t dG} \\
&+ \frac{\rho\beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A}/\mathcal{A}_t^2]dG}{[1-\beta+\beta\gamma] + \beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A}-\mathcal{A}_t]/\mathcal{A}_t dG} \\
\frac{\partial \ln \theta_e^o}{\partial \mathcal{A}_t} - \frac{\partial \ln \theta_e^{oD}}{\partial \mathcal{A}_t} =& \frac{\rho\beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A}/\mathcal{A}_t^2]dG}{[1-\beta+\beta\gamma] + \beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A}-\mathcal{A}_t]/\mathcal{A}_t dG} \\
=& \frac{\rho\beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A}/\mathcal{A}_t^2]dG}{[(1-\beta)f_e + f_p + \beta\gamma G(\mathcal{A}_t\xi_o^e)[f_x+f_e]]}\frac{[f_e+f_x]}{(1-\xi_o^e)}
\end{aligned}
$$

Hence $\frac{\partial \ln \theta_e^o}{\partial \mathcal{A}_t} - \frac{\partial \ln \theta_e^{oD}}{\partial \mathcal{A}_t} > 0$ and uncertainty reduces the response to changes in current realization. From here it easy to spot that the cross-partial between uncertainty and current realization is not null:

$$
\begin{aligned}
\frac{\partial^2 \ln \theta_e^o}{\partial \mathcal{A}_t \partial \gamma} =& \frac{[1-\beta]\left[\rho\beta\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A}/\mathcal{A}_t^2]dG\right]}{\left[1-\beta+\beta\gamma+\beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A}-\mathcal{A}_t]/\mathcal{A}_t dG\right]^2} \\
&- \frac{\left[\rho\beta\gamma\xi_o^e g(\mathcal{A}_t\xi_o^e)\frac{\partial \xi_o^e}{\partial \gamma}\right]\left[1-\beta+\beta\gamma-\beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A}_t^2 - \mathcal{A}(\mathcal{A}_t-1)]/\mathcal{A}_t^2 dG\right]}{\left[1-\beta+\beta\gamma+\beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A}-\mathcal{A}_t]/\mathcal{A}_t dG\right]^2}
\end{aligned}
$$

Note that this expression can be signed if it is evaluated around $\gamma = 0$,

$$
\frac{\partial^2 \ln \theta_e^o}{\partial \mathcal{A}_t \partial \gamma}|_{\gamma=0} = \frac{\beta\rho}{1-\beta}\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}\frac{\mathcal{A}_t}{\mathcal{A}_t^2}dG > 0
$$

This result is consistent with $\frac{\partial \ln \theta_e^o}{\partial \mathcal{A}_t} - \frac{\partial \ln \theta_e^{oD}}{\partial \mathcal{A}_t} > 0$ noting that $\frac{\partial \ln \theta_e^{oD}}{\partial \mathcal{A}_t} < 0$

A.6.2 Entry - Integration

A.6.2.1 Arrival rate

Totally differentiating with respect to γ

$$\begin{aligned}
d\ln\theta_e^v =& \frac{\rho}{1+\kappa_1^v}\left[\frac{\partial\kappa_1}{\partial\gamma}+\frac{\partial\kappa_1}{\partial\xi_v^x}\frac{\partial\xi_v^x}{\partial\gamma}\right]d\gamma \\
&-\frac{\rho}{1+\kappa_2^v+\varphi\kappa_3^v}\left[\frac{\partial\kappa_2}{\partial\gamma}+\frac{\partial\kappa_3}{\partial\gamma}+\left(\frac{\partial\kappa_2}{\partial\xi_v^x}-\frac{\partial\kappa_3}{\partial\xi_v^x}\right)\frac{\partial\xi_v^x}{\partial\gamma}+\frac{\partial\kappa_3}{\partial\xi_o^x}\frac{\partial\xi_o^x}{\partial\gamma}\right]d\gamma \\
=& \frac{\rho\kappa_1^v/\gamma}{1+\kappa_1^v}-\frac{\rho}{1+\kappa_2^v+\varphi\kappa_3^v}\left[\frac{(1-\beta)[\kappa_2^v+\varphi\kappa_3^v]/\gamma}{[1-\beta+\beta\gamma]}+\frac{\varphi\beta\gamma\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}\frac{\partial\xi_o^x}{\partial\gamma}dG}{1-\beta+\beta\gamma}\right] \\
&+\rho\beta\gamma g(\mathcal{A}_t\xi_v^x)\mathcal{A}_t\frac{\partial\xi_v^x}{\partial\gamma}\Xi
\end{aligned}$$

where $\Xi=\left[\frac{1}{\left[1+\kappa_1^v\right](1-\beta)}+\frac{[\mathcal{A}_t\xi_v^x-\mathcal{A}_t]/\mathcal{A}_t}{\left[1+\kappa_2^v\right](1-\beta+\beta\gamma)}+\frac{\varphi_o[\mathcal{A}_t\xi_v^x-\mathcal{A}_t\xi_o^x]/\mathcal{A}_t}{\left[1+\kappa_2^v\right](1-\beta+\beta\gamma)}\right]$. After some algebra, it is the case that $\Xi=0$, then plugging into the condition

$$\frac{d\ln\theta_e^v}{d\gamma}=\frac{\rho\kappa_1^v/\gamma}{1+\kappa_1^v}-\frac{\rho}{1+\kappa_2^v+\varphi\kappa_3^v}\left[\frac{(1-\beta)[\kappa_2^v+\varphi\kappa_3^v]/\gamma}{[1-\beta+\beta\gamma]}+\frac{\varphi\beta\gamma\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}\frac{\partial\xi_o^x}{\partial\gamma}dG}{1-\beta+\beta\gamma}\right]$$

Then evaluating around the deterministic framework, i.e. $\gamma=0$, $\frac{d\ln\theta_e^v}{d\gamma}>0$ since $\kappa_2^v(e)+\varphi\kappa_3^v(e)<0$ and $(1+\kappa_2^v(e)+\varphi\kappa_3^v(e))>0$. Hence uncertainty delays the decision to integrate.

A.6.2.2 Current Realization

$$\begin{aligned}
\frac{d\ln\theta_e^v}{d\mathcal{A}_t} =& \frac{d\ln\theta_e^{vD}}{d\mathcal{A}_t}+\frac{\rho}{1+\kappa_1^v(e)}\left[\frac{\partial\kappa_1^v(e)}{\partial\mathcal{A}_t}\right] \\
&-\frac{\rho}{1+\kappa_2^v(e)+\kappa_3^v(e)}\left[\frac{\partial\kappa_2^v(e)}{\partial\mathcal{A}_t}+\varphi_o\frac{\partial\kappa_3^v(e)}{\partial\mathcal{A}_t}\right]+ \\
&-\frac{\rho}{1+\kappa_2^v(e)+\kappa_3^v(e)}\varphi_o\frac{\partial\kappa_3^v(e)}{\partial\xi_o^x}\frac{\partial\xi_o^x}{\partial\mathcal{A}_t}
\end{aligned}$$

$$\frac{d\ln\theta_e^v}{d\mathcal{A}_t}-\frac{d\ln\theta_e^{vD}}{d\mathcal{A}_t}=\frac{\rho}{1+\kappa_2^v(e)+\kappa_3^v(e)}\left[\frac{\beta\gamma\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t}\frac{\mathcal{A}}{\mathcal{A}_t^2}dG}{1-\beta+\beta\gamma}+\frac{\varphi_o\beta\gamma\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}\frac{\mathcal{A}}{\mathcal{A}_t^2}dG}{1-\beta+\beta\gamma}-\frac{\varphi_o\beta\gamma\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}\frac{\partial\xi_o^x}{\partial\mathcal{A}_t}dG}{1-\beta+\beta\gamma}\right]$$

Solving for $\frac{\partial \xi_o^x}{\partial \mathcal{A}_t}$,

$$\xi_o^x = \left[\frac{(\psi_v - \psi_o)}{\psi_o}\frac{[1-\beta+\beta\gamma]}{[1-\beta]}\frac{[\theta_e^{vD}]^{\frac{\alpha}{1-\alpha}}}{[\theta_e^v]^{\frac{\alpha}{1-\alpha}}} - \frac{(\psi_v - \psi_o)}{\psi_o} + \frac{\psi_v}{\psi_o}\frac{\mathcal{A}_v^x}{\mathcal{A}_t}\right]$$

$$\begin{aligned}\frac{\partial \xi_o^x}{\partial \mathcal{A}_t} =& \frac{(\psi_v - \psi_o)}{\psi_o}\frac{[1-\beta+\beta\gamma]}{[1-\beta]}\frac{[\theta_e^{vD}]^{\frac{\alpha}{1-\alpha}}}{[\theta_e^v]^{\frac{\alpha}{1-\alpha}}}\frac{1}{\rho}\left[\frac{d\ln\theta_e^{vD}}{d\mathcal{A}_t} - \frac{d\ln\theta_e^v}{d\mathcal{A}_t}\right] \\ &- \frac{\psi_v}{\psi_o}\frac{\mathcal{A}_t\xi_v^x}{\mathcal{A}_t^2}\end{aligned}$$

Plugging back and after some manipulations

$$\left[\frac{d\ln\theta_e^v}{d\mathcal{A}_t} - \frac{d\ln\theta_e^{vD}}{d\mathcal{A}_t}\right] = \frac{\rho\beta\gamma}{\omega_v^e(\mathcal{A}_t)}\frac{\left[\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t}\frac{\mathcal{A}}{\mathcal{A}_t^2}dG + \varphi_o\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}\frac{\mathcal{A}}{\mathcal{A}_t^2}dG + \varphi_v\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}\frac{\mathcal{A}_t\xi_v^x}{\mathcal{A}_t^2}dG\right]}{[1+\kappa_2^v(e)+\kappa_3^v(e)][1-\beta+\beta\gamma]} > 0$$

A.6.3 Exit - Outsourcing

A.6.3.1 Arrival Rate

Computing the total differential with respect to γ

$$\begin{aligned}d\ln\theta_o^x =& \rho\frac{1}{1-\kappa_1^o(x)}\left[\frac{\partial\kappa_1^o(x)}{\partial\gamma} + \frac{\partial\kappa_1^o(x)}{\partial\xi_o^x}\frac{\partial\xi_o^x}{\partial\gamma}\right]d\gamma \\ &- \rho\frac{1}{1+\kappa_2^o(x)}\left[\frac{\partial\kappa_2^o(x)}{\partial\gamma} + \frac{\partial\kappa_2^o(x)}{\partial\xi_o^x}\frac{\partial\xi_o^x}{\partial\gamma}\right]d\gamma \\ \frac{d\ln\theta_o^x}{d\gamma} =& -\frac{\rho}{1-\kappa_1^o(x)}[\kappa_1^o(x)/\gamma] - \frac{\rho}{1+\kappa_2^o(x)}\left[\frac{(1-\beta)\kappa_2^o(x)/\gamma}{[1-\beta+\beta\gamma]}\right] \\ &- \Xi_o^x\beta\gamma g(\mathcal{A}_t\xi_o^x)\mathcal{A}_t\frac{\partial\xi_o^x}{\partial\gamma}\end{aligned}$$

where $\Xi_o^x = \left[\frac{\rho}{1-\kappa_1^o(x)}\frac{[f_e+f_x]}{f_p-[1-\beta]f_x} - \frac{\rho}{1+\kappa_2^o(x)}\frac{[\mathcal{A}_t\xi_o^x-\mathcal{A}_t]/\mathcal{A}_t}{[1-\beta+\beta\gamma]}\right]$

$$\begin{aligned}
\Xi_o^x =& \left[\frac{\rho}{1-\kappa_1^o(x)}\frac{[f_e+f_x]}{f_p-[1-\beta]f_x} - \frac{\rho}{1+\kappa_2^o(x)}\frac{[\mathcal{A}_t\xi_o^x-\mathcal{A}_t]/\mathcal{A}_t}{[1-\beta+\beta\gamma]}\right] \\
=& \left[\frac{\rho}{1-\kappa_2^o(x)}\frac{[\mathcal{A}_t\xi_o^x-\mathcal{A}_t]/\mathcal{A}_t}{(1-\beta+\beta\gamma)} - \frac{\rho}{1+\kappa_2^o(x)}\frac{[\mathcal{A}_t\xi_o^x-\mathcal{A}_t]/\mathcal{A}_t}{[1-\beta+\beta\gamma]}\right] \\
\Xi_o^x =& 0
\end{aligned}$$

Plugging back into the total differential

$$\frac{d\ln\theta_o^x}{d\gamma} = -\rho\frac{\kappa_1^o(x)/\gamma}{1-\kappa_1^o(x)} - \frac{\rho(1-\beta)}{[1-\beta+\beta\gamma]}\frac{\kappa_2^o(x)/\gamma}{1+\kappa_2^o(x)} < 0$$

A.6.3.2 Current Realization

$$\begin{aligned}
d\ln\theta_o^x =& \frac{d\ln\theta_o^{xD}}{d\mathcal{A}_t}d\mathcal{A}_t + \frac{\rho}{1-\kappa_1^o(x)}\left[\frac{\partial\kappa_1^o(x)}{\partial\mathcal{A}_t} + \frac{\partial\kappa_1^o(x)}{\partial\xi_o^x}\frac{\partial\xi_o^x}{\partial\mathcal{A}_t}\right]d\mathcal{A}_t \\
&- \frac{\rho}{1+\kappa_2^o(x)}\left[\frac{\partial\kappa_2^o(x)}{\partial\mathcal{A}_t} + \frac{\partial\kappa_2^o(x)}{\partial\xi_o^x}\frac{\partial\xi_o^x}{\partial\mathcal{A}_t}\right]d\mathcal{A}_t \\
\frac{d\ln\theta_o^x}{d\mathcal{A}_t} =& \frac{d\ln\theta_o^{xD}}{d\mathcal{A}_t} + \frac{\rho}{1+\kappa_2^o(x)}\left[\frac{\beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi_o^x}\mathcal{A}/\mathcal{A}_t^2 dG}{1-\beta+\beta\gamma}\right] \\
\frac{d\ln\theta_o^x}{d\mathcal{A}_t} - \frac{d\ln\theta_o^{xD}}{d\mathcal{A}_t} =& \frac{\rho}{1+\kappa_2^o(x)}\left[\frac{\beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi_o^x}\mathcal{A}/\mathcal{A}_t^2 dG}{1-\beta+\beta\gamma}\right] > 0
\end{aligned}$$

A.6.4 Exit - Vertical Integration

A.6.4.1 Arrival Rate

$$
\begin{aligned}
d\ln\theta_x^v =& -\frac{\rho}{1-\kappa_1^v(x)}\left[\frac{\kappa_1^v(x)}{\gamma}-\beta\gamma\frac{[f_v+f_e+f_x]}{f_p-[1-\beta]f_x}g(\mathcal{A}_t\xi_v^e)\mathcal{A}_t\frac{\partial\xi_v^e}{\partial\gamma}\right]d\gamma \\
&-\frac{\rho\left[\frac{(1-\beta)}{1-\beta+\beta\gamma}\frac{\kappa_2^v(x)}{\gamma}-\frac{\psi_o}{\psi_v}\frac{(1-\beta)}{1-\beta+\beta\gamma}\frac{\kappa_3^v(x)}{\gamma}\right]}{1+\kappa_2^v(x)-\frac{\psi_o}{\psi_v}\kappa_3^v(x)}d\gamma \\
&-\frac{\rho\left[\frac{[\mathcal{A}_t\xi_v^e-\mathcal{A}_t]}{\mathcal{A}_t}-\frac{\psi_o}{\psi_v}\frac{[\mathcal{A}_t\xi_v^e-\mathcal{A}_t\xi_o^e]}{\mathcal{A}_t}\right]}{1+\kappa_2^v(x)-\frac{\psi_o}{\psi_v}\kappa_3^v(x)}\frac{\beta\gamma}{1-\beta+\beta\gamma}g(\mathcal{A}_t\xi_v^e)\mathcal{A}_t\frac{\partial\xi_v^e}{\partial\gamma}d\gamma \\
&-\frac{\rho}{1+\kappa_2^v(x)-\frac{\psi_o}{\psi_v}\kappa_3^v(x)}\frac{\beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t\xi_v^e}\frac{\partial\xi_o^e}{\partial\gamma}dG}{1-\beta+\beta\gamma}d\gamma \\
\frac{d\ln\theta_x^v}{d\gamma} =& -\frac{\rho\kappa_1^v(x)/\gamma}{1-\kappa_1^v(x)}-\frac{(1-\beta)}{1-\beta+\beta\gamma}\frac{\rho\left[\frac{\kappa_2^v(x)}{\gamma}-\frac{\psi_o}{\psi_v}\frac{\kappa_3^v(x)}{\gamma}\right]}{1+\kappa_2^v(x)-\frac{\psi_o}{\psi_v}\kappa_3^v(x)} \\
&-\frac{\rho}{1+\kappa_2^v(x)-\frac{\psi_o}{\psi_v}\kappa_3^v(x)}\frac{\beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t\xi_v^e}\frac{\partial\xi_o^e}{\partial\gamma}dG}{1-\beta+\beta\gamma}
\end{aligned}
$$

Then evaluating at $\gamma=0$, $\frac{d\ln\theta_x^v}{d\gamma}<0$ since $\kappa_2^v(x)>\kappa_3^v(x)$ because $\psi_v>\psi_o$ and $\xi_o^e>1$ which implies that $\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi_v^e}\frac{[\mathcal{A}-\mathcal{A}_t]}{\mathcal{A}_t}dG>\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t\xi_v^e}\frac{[\mathcal{A}-\mathcal{A}_t\xi_o^e]}{\mathcal{A}_t}dG$. Hence as a first order effect uncertainty delays the decision to stop exporting under integration.

A.6.4.2 Current Realization

$$
\begin{aligned}
d\ln\theta_x^v =& \frac{d\ln\theta_x^{vD}}{d\mathcal{A}_t}d\mathcal{A}_t-\frac{\rho}{1-\kappa_1^v(x)}\left[\frac{\partial\kappa_1^v(x)}{\partial\mathcal{A}_t}\right]d\mathcal{A}_t \\
&-\frac{\rho}{1+\kappa_2^v(x)-\frac{\psi_o}{\psi_v}\kappa_3^v(x)}\left[\frac{\partial\kappa_2^v(x)}{\partial\mathcal{A}_t}-\frac{\psi_o}{\psi_v}\frac{\partial\kappa_3^v(x)}{\partial\mathcal{A}_t}-\frac{\psi_o}{\psi_v}\frac{\partial\kappa_3^v(x)}{\partial\xi_o^e}\frac{\partial\xi_o^e}{\partial\mathcal{A}_t}\right]d\mathcal{A}_t \\
\frac{d\ln\theta_x^v}{d\mathcal{A}_t}-\frac{d\ln\theta_x^{vD}}{d\mathcal{A}_t} =& \frac{\rho\beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi_v^e}[\mathcal{A}_t/\mathcal{A}_t^2]dG+\frac{\psi_o}{\psi_v}\rho\beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t\xi_v^e}[\mathcal{A}_t/\mathcal{A}_t^2]dG-\frac{\psi_o}{\psi_v}\rho\beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t\xi_v^e}\frac{d\xi_o^e}{d\mathcal{A}_t}dG}{[1+\kappa_2^v(x)-\frac{\psi_o}{\psi_v}\kappa_3^v(x)](1-\beta+\beta\gamma)}
\end{aligned}
$$

Solving for $\frac{d\xi_o^e}{d\mathcal{A}_t}$

$$\xi_o^e = \left[\frac{\psi_v}{\psi_o}\frac{[1-\beta+\beta\gamma][f_v+f_e+f_x]}{f_p-[1-\beta]f_x}\frac{[\theta_x^{vD}]^{\frac{1}{\rho}}}{[\theta_v^x]^{\frac{1}{\rho}}} - \frac{(\psi_v-\psi_o)}{\psi_o}\frac{\mathcal{A}_v^e}{\mathcal{A}_t} + \frac{\psi_v}{\psi_o}\right]$$

$$\frac{\partial\xi_o^e}{\partial\mathcal{A}_t} = \frac{\psi_v}{\psi_o}\frac{1}{\rho}\frac{[1-\beta+\beta\gamma][f_v+f_e+f_x]}{f_p-[1-\beta]f_x}\frac{[\theta_x^{vD}]^{\frac{1}{\rho}}}{[\theta_v^x]^{\frac{1}{\rho}}}\left[\frac{d\ln\theta_x^{vD}}{d\mathcal{A}_t} - \frac{d\ln\theta_x^v}{d\mathcal{A}_t}\right] - \frac{(\psi_v-\psi_o)}{\psi_o}\frac{\mathcal{A}_t\xi_v^e}{\mathcal{A}_t^2}$$

Then plugging back into the condition and after some algebra

$$\frac{d\ln\theta_x^v}{d\mathcal{A}_t} - \frac{d\ln\theta_x^{vD}}{d\mathcal{A}_t} = \frac{\rho\beta\gamma}{\left[1+\kappa_2^v(x)-\frac{\psi_o}{\psi_v}\kappa_3^v(x)\right]\omega_v^x}\left[\frac{\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi_v^e}\frac{1}{\mathcal{A}_t}dG}{1-\beta+\beta\gamma} + \frac{\psi_o}{\psi_v}\frac{\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t\xi_v^e}\frac{1}{\mathcal{A}_t}dG}{1-\beta+\beta\gamma} + \frac{(\psi_v-\psi_o)}{\psi_v}\frac{\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t\xi_v^e}\frac{1}{\mathcal{A}_t}dG}{1-\beta+\beta\gamma}\right]$$

where $\frac{d\ln\theta_x^v}{d\mathcal{A}_t} - \frac{d\ln\theta_x^{vD}}{d\mathcal{A}_t} > 0$ since all terms are positive and ω_v^x is a positive weight.

A.6.5 Heterogeneity on The Impact of Demand Changes

A.6.5.1 Entry and Integration

$$\begin{aligned}
\frac{d\ln\theta_e^v}{d\mathcal{A}_t} - \frac{d\ln\theta_e^o}{d\mathcal{A}_t} =& \frac{\rho\beta\gamma}{1+\kappa_2^v(e)+\kappa_3^v(e)}\left[\frac{\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t}\frac{\mathcal{A}}{\mathcal{A}_t^2}dG}{1-\beta+\beta\gamma} + \frac{\varphi_o\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}\frac{\mathcal{A}}{\mathcal{A}_t^2}dG}{1-\beta+\beta\gamma} + \frac{\varphi_v\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}\frac{\mathcal{A}_t\xi_v^x}{\mathcal{A}_t^2}dG}{1-\beta+\beta\gamma}\right] \\
&+ \frac{\beta\gamma}{[1-\beta]}\frac{1}{[1+\kappa_1^v(e)]}\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}\left[\frac{d\ln\theta_e^v}{d\mathcal{A}_t} - \frac{d\ln\theta_e^{vD}}{d\mathcal{A}_t}\right]dG \\
=& \frac{\rho}{1+\kappa_2^v(e)+\kappa_3^v(e)}\frac{\beta\gamma\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t}[\mathcal{A}/\mathcal{A}_t^2]dG}{1-\beta+\beta\gamma} - \frac{\rho}{1+\kappa_2^o(e)}\frac{\beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A}/\mathcal{A}_t^2]dG}{1-\beta+\beta\gamma} \\
&+ \frac{\rho}{1+\kappa_2^v(e)+\kappa_3^v(e)}\left[\frac{\varphi_o\beta\gamma\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}\frac{\mathcal{A}}{\mathcal{A}_t^2}dG}{1-\beta+\beta\gamma} + \frac{\varphi_v\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}\frac{\mathcal{A}_t\xi_v^x}{\mathcal{A}_t^2}dG}{1-\beta+\beta\gamma}\right] \\
&+ \frac{\beta\gamma}{[1-\beta]}\frac{1}{[1+\kappa_1^v(e)]}\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}\left[\frac{d\ln\theta_e^v}{d\mathcal{A}_t} - \frac{d\ln\theta_e^{vD}}{d\mathcal{A}_t}\right]dG \\
\frac{d\ln\theta_v^e}{d\ln\mathcal{A}_t} - \frac{d\ln\theta_o^e}{d\ln\mathcal{A}_t} >& 0
\end{aligned}$$

This is the case since $\kappa_2^v(e) < 0$, $\kappa_3^v(e) < 0\,\kappa_1^o(e) < 0$ and $1 > abs(\kappa_2^v(e)) > abs(\kappa_2^o(e))$, hence $\frac{\rho}{1+\kappa_2^v(e)+\kappa_3^v(e)} > \frac{\rho}{1+\kappa_2^o(e)}$; and $\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t}[\mathcal{A}/\mathcal{A}_t^2]dG > \int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[\mathcal{A}/\mathcal{A}_t^2]dG$. Since $\frac{d\ln\theta^v}{d\ln\mathcal{A}_t} < 0$ then the elasticity with respect to current demand level is higher for outsourcing than integration.

A.6.5.2 Exit across Organizational Forms

$$
\begin{aligned}
\frac{d\ln\theta_v^x}{d\ln\mathcal{A}_t}-\frac{d\ln\theta_o^x}{d\ln\mathcal{A}_t}=&\frac{\rho}{1+\kappa_2^v(x)-\frac{\psi_o}{\psi_v}\kappa_3^v(x)}\left[\frac{\beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi_v^e}\mathcal{A}dG}{1-\beta+\beta\gamma}+\frac{\psi_o}{\psi_v}\frac{\beta\gamma\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t\xi_v^e}\mathcal{A}dG}{1-\beta+\beta\gamma}\right]\\
&-\frac{\rho}{1+\kappa_2^o(x)}\left[\frac{\beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi_o^x}\mathcal{A}dG}{1-\beta+\beta\gamma}\right]\\
=&\frac{\rho\left[\kappa_2^v(x)+\frac{\beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi_v^e}\mathcal{A}_t dG}{1-\beta+\beta\gamma}\right]}{1+\kappa_2^v(x)-\frac{\psi_o}{\psi_v}\kappa_3^v(x)}-\frac{\rho\left[\kappa_2^o(x)+\frac{\beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi_o^x}\mathcal{A}_t dG}{1-\beta+\beta\gamma}\right]}{1+\kappa_2^o(x)}\\
>&0
\end{aligned}
$$

since $\kappa_2^v(x)>\kappa_2^o(x)$ and $\kappa_3^v(x)<1$.

A.6.6 Foreign Demand Uncertainty Distribution

A.6.6.1 Exit with Outsourcing

The exit condition from outsourcing is the following for each firm

$$
(1-\beta)f_x=-\left[\pi_o(\mathcal{A}_o^x)-f_p\right]-\beta\gamma[1-G(\mathcal{A}_o^e)][f_e+f_x]-\beta\gamma\int_{\mathcal{A}_o^x}^{\mathcal{A}_o^e}\frac{\pi_o(\mathcal{A})-\pi_o(\mathcal{A}_o^x)}{1-\beta+\beta\gamma}dG
$$

Integrating by parts

$$
\begin{aligned}
(1-\beta)f_x=&-\left[\pi_o(\mathcal{A}_o^x)-f_p\right]-\beta\gamma[1-G(\mathcal{A}_o^e)][f_e+f_x]\\
&+\beta\gamma\frac{G(\mathcal{A}_o^e)[\pi_o(\mathcal{A}_o^x)-\pi_o(\mathcal{A}_o^e)]}{1-\beta+\beta\gamma}+\beta\gamma[\theta_i]^{\frac{1}{\rho}}\psi_o\frac{\int_{\mathcal{A}_o^x}^{\mathcal{A}_o^e}G(z)dz}{1-\beta+\beta\gamma}\\
=&-\left[\pi_o(\mathcal{A}_o^x)-f_p\right]-\beta\gamma[1-G(\mathcal{A}_o^e)][f_e+f_x]+\beta\gamma G(\mathcal{A}_o^e)[f_e+f_x]\\
&+\beta\gamma[\theta_i]^{\frac{1}{\rho}}\psi_o\frac{\int_{\mathcal{A}_o^x}^{\mathcal{A}_o^e}G(z)dz}{1-\beta+\beta\gamma}\\
(1-\beta)f_x=&-\left[\pi_o(\mathcal{A}_o^x)-f_p\right]-\beta\gamma[f_e+f_x]+\beta\gamma[\theta_i]^{\frac{1}{\rho}}\psi_o\frac{\int_{\mathcal{A}_o^x}^{\mathcal{A}_o^e}G(z)dz}{1-\beta+\beta\gamma}
\end{aligned}
$$

This expression is particularly useful because the distribution function of the stochastic process only shows up in the last term.

For the marginal firm that is exiting from outsourcing with productivity θ_o^x and $\mathcal{A}_o^x = \mathcal{A}_t$ then

$$
\begin{aligned}
(1-\beta)f_x &= -([\theta_o^x]^{\frac{1}{\rho}}\psi_o\mathcal{A}_t - f_p) - \beta\gamma[f_e + f_x] + \beta\gamma[\theta_o^x]^{\frac{1}{\rho}}\psi_o\frac{\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi} G(z)dz}{1-\beta+\beta\gamma} \\
[\theta_o^x]^{\frac{1}{\rho}} &= [\theta_o^{xD}]^{\frac{1}{\rho}}(1 - \frac{\beta\gamma[f_e+f_x]}{f_p-(1-\beta)f_x})/\left[\frac{1-\beta+\beta\gamma-\beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi} G(z)dz/\mathcal{A}_t}{1-\beta+\beta\gamma}\right] \\
[\theta_o^x]^{\frac{1}{\rho}} &= [\theta_o^{xD}]^{\frac{1}{\rho}}(1 - \frac{\beta\gamma[f_e+f_x]}{f_p-(1-\beta)f_x})\frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma\omega(\mathcal{A}_t)}
\end{aligned}
$$

Then consider $G(z)$ and $H(z)$ and the objective is to compare the exit productivity threshold between the two demand distribution: θ_o^x and $\theta_o^{x\prime}$. In order to compare the cutoff I compute the ratio

$$
\begin{aligned}
\left[\frac{\theta_o^x}{\theta_o^{x\prime}}\right]^{\frac{1}{\rho}} &= \frac{\left[1-\beta+\beta\gamma-\beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi} H(z)dz/\mathcal{A}_t\right]}{\left[1-\beta+\beta\gamma-\beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi} G(z)dz/\mathcal{A}_t\right]} \\
\left[\frac{\theta_o^x}{\theta_o^{x\prime}}\right]^{\frac{1}{\rho}} &= \frac{[1-\beta+\beta\gamma\omega'(\mathcal{A}_t)]}{[1-\beta+\beta\gamma\omega(\mathcal{A}_t)]}
\end{aligned}
$$

Hence by comparing $\omega(\mathcal{A}_t)$ and $\omega'(\mathcal{A}_t)$ the productivity cutoff can be ranked. Note thatif $\theta_o^x < \theta_o^{x\prime}$ then it should be the case that $[1-\beta+\beta\gamma\omega(\mathcal{A}_t)] > [1-\beta+\beta\gamma\omega'(\mathcal{A}_t)]$ since $\rho > 0$.

$$
\begin{aligned}
\omega(\mathcal{A}_t) =& 1 - \frac{\beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi} G(z)dz}{\mathcal{A}_t} \\
\omega'(\mathcal{A}_t) =& 1 - \frac{\beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi} H(z)dz}{\mathcal{A}_t} \\
\Delta\omega(\mathcal{A}_t) =& \frac{\beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi}[H(z)-G(z)]dz}{\mathcal{A}_t} \\
\Delta\omega(\mathcal{A}_t) =& \frac{\beta\gamma}{\mathcal{A}_t}\left\{\int_0^{\mathcal{A}_t\xi}[H(z)-G(z)]dz - \int_0^{\mathcal{A}_t}[H(z)-G(z)]dz\right\}
\end{aligned}
$$

If $G(z)$ FOSD $H(z)$if $G(z) \leq H(z)$ for all z with strict inequality for at least one z, then $\Delta\omega(\mathcal{A}_t) \geq 0$ with strict inequality when it is the case that $G(z) < H(z)$ and $\theta_o^x < \theta_o^{x\prime}$.

Now suppose that the distribution of demand $H(\mathcal{A})$ is a mean-preserving spread of $G(\mathcal{A})$ such that $G(\mathcal{A})$ and $H(\mathcal{A})$ cross only once at $\tilde{\mathcal{A}}$. Then for a current realization $\mathcal{A}_t < \tilde{\mathcal{A}}/\xi$ it is the case that $\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi}[H(z)-G(z)]dz > 0$ and $\Delta\omega(\mathcal{A}_t) > 0$ which in turns implies that $\theta_o^x < \theta_o^{x\prime}$.

In the case that the current realization $\mathcal{A}_t > \tilde{\mathcal{A}}$, $\int_{\mathcal{A}_t}^{\mathcal{A}_t \xi}[H(z) - G(z)]dz < 0$ and $\Delta\omega(\mathcal{A}_t) < 0$ which in turns implies that $\theta_o^x > \theta_o^{x\prime}$.

A.6.6.2 Entry with Outsourcing

Reexpressing the entry condition, I obtain:

$$\begin{aligned}
[1-\beta]f_e =& [\pi_o(\mathcal{A}_o^e) - f_p] - \beta\gamma G(\mathcal{A}_o^x)[f_x + f_e] \\
& - \frac{[G(\mathcal{A}_o^e) - G(\mathcal{A}_o^x)]\pi_o(\mathcal{A}_e^o)}{1-\beta+\beta\gamma} + \frac{\beta\gamma \int_{A_x^o}^{A_e^o} \pi_o(A)dG}{[1-\beta(1-\gamma)]} \\
1-\beta]f_e =& [\pi_o(\mathcal{A}_o^e) - f_p] - \frac{\beta\gamma[\theta_i]^{\frac{1}{\rho}}\psi_o \int_{A_x^o}^{A_e^o} G(z)dG}{1-\beta+\beta\gamma}
\end{aligned}$$

Parametrizing for $\mathcal{A}_t = \mathcal{A}_o^e$

$$\begin{aligned}
[1-\beta]f_e =& -f_p + [\theta_o^e]^{\frac{1}{\rho}}\left[\psi_o\mathcal{A}_t - \frac{\beta\gamma\psi_o \int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t} G(z)dG}{1-\beta+\beta\gamma}\right] \\
\psi_o\mathcal{A}_t[\theta_o^{eD}]^{\frac{1}{\rho}} =& [\theta_o^e]^{\frac{1}{\rho}}\left[\psi_o\mathcal{A}_t - \frac{\beta\gamma\psi_o \int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t} G(z)dG}{1-\beta+\beta\gamma}\right] \\
[\theta_o^e]^{\frac{1}{\rho}} =& [\theta_o^{eD}]^{\frac{1}{\rho}} / [1 - \frac{\beta\gamma \int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t} G(z)dG/\mathcal{A}_t}{1-\beta+\beta\gamma}] \\
[\theta_o^e]^{\frac{1}{\rho}} =& [\theta_o^{eD}]^{\frac{1}{\rho}} \frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma\omega(\mathcal{A}_t)}
\end{aligned}$$

Then

$$\left[\frac{\theta_o^e}{\theta_o^{e\prime}}\right]^{\frac{1}{\rho}} = \frac{[1-\beta+\beta\gamma\omega'(\mathcal{A}_t)]}{[1-\beta+\beta\gamma\omega(\mathcal{A}_t)]}$$

since $\omega'(\mathcal{A}_t) < \omega(\mathcal{A}_t)$ if $G(z)$ FOSD $H(z)$ then $\theta_o^e < \theta_o^{e\prime}$.

Now suppose that the distribution of demand $H(\mathcal{A})$ is a mean-preserving spread of $G(\mathcal{A})$ such that $G(\mathcal{A})$ and $H(\mathcal{A})$ cross only once at $\tilde{\mathcal{A}}$. Then for a current realization $\mathcal{A}_t < \tilde{\mathcal{A}}$ it is the case that $\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[H(z) - G(z)]dz > 0$ and $\Delta\omega(\mathcal{A}_t) > 0$ which in turns implies that $\theta_o^e < \theta_o^{e\prime}$. In the case that the current realization $\mathcal{A}_t > \tilde{\mathcal{A}}/\xi_o^e$, $\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t}[H(z) - G(z)]dz < 0$ and $\Delta\omega(\mathcal{A}_t) < 0$ which in turns implies that $\theta_o^e > \theta_o^{e\prime}$.

A.6.6.3 Integration

Rearranging the integration condition, integrating by parts and after the algebra

$$[1-\beta]f_v = (\psi_v-\psi_o)\theta^{\frac{\alpha}{1-\alpha}}\mathcal{A}_e^v - \frac{\beta\gamma(\psi_v-\psi_o)\theta^{\frac{\alpha}{1-\alpha}}\int_{\mathcal{A}_x^o}^{\mathcal{A}_e^v}G(z)dz}{1-\beta+\beta\gamma} - \frac{\beta\gamma\psi_v\theta^{\frac{\alpha}{1-\alpha}}\int_{\mathcal{A}_v^x}^{\mathcal{A}_x^o}G(z)dz}{1-\beta+\beta\gamma}$$

Parametrizing this expression for the marginal integrated firm $\mathcal{A}_e^v = \mathcal{A}_t$, I obtain

$$[\theta_e^v]^{\frac{\alpha}{1-\alpha}} = [\theta_e^{vD}]^{\frac{\alpha}{1-\alpha}}\frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma\omega^v(\mathcal{A}_t)}$$

where $\omega^v(\mathcal{A}_t) = 1-\int_{\mathcal{A}_t\xi_o^x}^{\mathcal{A}_t}G(z)dz/\mathcal{A}_t - \varphi_v\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}G(z)dz/\mathcal{A}_t$.

Consider two distribution $G(z)$ and $H(z)$ with respective θ_v^e and $\theta_v^{e\prime}$ integration productivity cutoffs. Computing the ratio for two distribution

$$\left[\frac{\theta_v^e}{\theta_e^{v\prime}}\right]^{\frac{1}{\rho}} = \frac{1-\beta+\beta\gamma\omega^{v\prime}(\mathcal{A}_t)}{1-\beta+\beta\gamma\omega^v(\mathcal{A}_t)}$$

Hence $\theta_v^{e\prime} > \theta_v^e$ if $\omega^{v\prime}(\mathcal{A}_t) < \omega^v(\mathcal{A}_t)$ or $\Delta\omega^v > 0$. Then

$$\Delta\omega^v = \int_{\mathcal{A}_t\xi_o^x}^{\mathcal{A}_t}[H(z)-G(z)]dz + \varphi_v\beta\gamma\int_{\mathcal{A}_t\xi_v^x}^{\mathcal{A}_t\xi_o^x}[H(z)-G(z)dz$$

and $\Delta\omega^v > 0$ if $G(z)$ FOSD $H(z)$ since $H(z) \geq G(z)$ for all z with strict inequality for at least one z. Hence $\theta_v^e < \theta_v^e$ if $G(z)$ FOSD $H(z)$.

Now suppose that the distribution of demand $H(\mathcal{A})$ is a mean-preserving spread of $G(\mathcal{A})$ such that $G(\mathcal{A})$ and $H(\mathcal{A})$ cross only once at $\tilde{\mathcal{A}}$. Then for a current realization $\mathcal{A}_t<\tilde{\mathcal{A}}$ it is the case that $\Delta\omega^v > 0$ which in turns implies that $\theta_v^e < \theta_v^{e\prime}$. In the case that the current realization $\mathcal{A}_t>\tilde{\mathcal{A}}/\xi_v^x$, $\Delta\omega^v > 0 < 0$ which in turns implies that $\theta_v^e > \theta_v^{e\prime}$.

A.6.6.4 Exit with Vertical Integration

$$\begin{aligned}[1-\beta]f_x = &-[\psi_v[\theta]^{\frac{1}{\rho}}\mathcal{A}_v^x - f_p] - \beta\gamma[f_v+f_e+f_x] \\ &+ \frac{\beta\gamma\psi_v[\theta]^{\frac{1}{\rho}}\int_{\mathcal{A}_v^x}^{\mathcal{A}_v^e}G(z)dz}{1-\beta+\beta\gamma} - \frac{\beta\gamma\psi_o[\theta]^{\frac{1}{\rho}}\int_{\mathcal{A}_o^e}^{\mathcal{A}_v^e}G(z)dz}{1-\beta+\beta\gamma}\end{aligned}$$

Parametrizing for the marginal exit integrated exporter and after some algebra

$$[\theta_x^v]^{\frac{1}{\rho}} = [\theta_x^{vD}]^{\frac{1}{\rho}} \left[1 - \frac{\beta\gamma[f_v + f_e + f_x]}{f_p - [1-\beta]f_x}\right] \left[\frac{1-\beta+\beta\gamma}{1-\beta+\beta\gamma\omega_v^x(\mathcal{A}_t)}\right]$$

where $\omega_v^x(\mathcal{A}_t) = 1 - \int_{\mathcal{A}_t}^{\mathcal{A}_t\xi_v^e} \frac{G(z)}{\mathcal{A}_t} dz + \varphi_o \int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t\xi_v^e} \frac{G(z)}{\mathcal{A}_t} dz$.

Consider two distribution $G(z)$ and $H(z)$ with respective θ_v^x and $\theta_v^{x\prime}$ exit from integration productivity cutoffs. Computing the ratio for two distribution

$$\left[\frac{\theta_x^v}{\theta_x^{v\prime}}\right]^{\frac{1}{\rho}} = \frac{1-\beta+\beta\gamma\omega_v^{x\prime}(\mathcal{A}_t)}{1-\beta+\beta\gamma\omega_v^x(\mathcal{A}_t)}$$

and $\theta_v^{x\prime} > \theta_v^x$ if $\Delta\omega_v^x > 0$. Computing $\Delta\omega_v^x$ I get

$$\Delta\omega_v^x = \beta\gamma(1-\varphi_o)\int_{\mathcal{A}_t\xi_o^e}^{\mathcal{A}_t\xi_v^e} [H(z) - G(z)]dz + \beta\gamma\int_{\mathcal{A}_t}^{\mathcal{A}_t\xi_o^e} [H(z) - G(z)]dz$$

If $G(z)$ FOSD $H(z)$ then $H(z) \geq G(z)$ for all z with strict inequality for at least one z, then $\Delta\omega_v^x > 0$ and $\theta_v^x < \theta_v^{x\prime}$. Now suppose that the distribution of demand $H(\mathcal{A})$ is a mean-preserving spread of $G(\mathcal{A})$ such that $G(\mathcal{A})$ and $H(\mathcal{A})$ cross only once at $\tilde{\mathcal{A}}$. Then for a current realization $\mathcal{A}_t < \tilde{\mathcal{A}}/\xi_v^e$ it is the case that $\Delta\omega_v^x > 0$ which in turns implies that $\theta_v^x < \theta_v^{x\prime}$. In the case that the current realization $\mathcal{A}_t > \tilde{\mathcal{A}}$, $\Delta\omega(\mathcal{A}_t) < 0$ which in turns implies that $\theta_v^x > \theta_v^{x\prime}$.