Given any proper chain of infinite cardinals $\kappa < \lambda < \mu$ we may construct a structure of cardinality $\mu$ which admits a substructure of cardinality $\kappa$ but admits no substructure of cardinality $\lambda$.

Let $\kappa < \lambda < \mu$ be given, let $C$ be a set of cardinality $\mu$. Let $B \subset C$ be a subset of cardinality $\lambda$, and let $A \subset B$ be a subset of cardinality $\kappa$. Then the structure

$$C = \langle C; \{ f_c(x) = \begin{cases} x & x \in A \\ c & c \notin A \end{cases} \mid c \in C \rangle \rangle$$

exhibits the desired property. We note that $A = \langle A; \{ f_c \}_{c \in C} \rangle$ is a substructure of cardinality $\kappa$ but if $B'$ is to be any substructure of cardinality greater than $\kappa$, its domain must contain at least one element not in $A$, and closing that domain under the functions results in $C$, which is of cardinality $\mu$. No intermediate substructures may then exist.