1. An ultrafilter is *uniform* if all of its sets have the same size. Show that a regular ultrafilter is uniform.

**Proof.** Let $U$ be an ultrafilter on a set $I$, and $E \subseteq U$ be such that $|E| = |I|$ and every $i \in I$ is a member of finitely many sets in $E$.

Assume towards a contradiction that there is an $A \in U$ such that $|A| < |I|$. Then since $|E| = |I|$, we may index the members of $E$ by the set $I$. Then we note that for each $a \in A$, the set $X_a = \{i \in I : a \in E_i\}$ is finite so that $\cup_{a \in A} X_a \neq I$ by a cardinality argument. But this implies that there exists $j \in I$ such that $a \not\in E_j$ for all $a \in A$, so that $E_j \cap A = \emptyset$. This is a contradiction, as $E_j, A \in U$ and $U$ is closed under finite intersection. Thus we must have that $|A| = |I|$ for all $A \in U$. □