PROBLEMS

1. (Berg, Jack) Let \( \sigma \) be a signature with one binary relation symbol and no other symbols. Show that there are exactly \( 2^\kappa \) nonisomorphic \( \sigma \)-structures of cardinality \( \kappa \) for each infinite \( \kappa \).

2. (Lessard, Lotfi) Show that if \( \alpha: A \to B \) is an isomorphism, \( \varphi \) is a formula, and \( v \) is a valuation, then \( A \models \varphi[v] \) iff \( B \models \varphi[\alpha(v)] \).

3. (Shriner, Sparks, Tanksalvala) Show that the following pairs of abelian groups are not elementarily equivalent.
   (a) \( \mathbb{Z} \) and \( \mathbb{Q} \)
   (b) \( \mathbb{Z} \) and \( \mathbb{Z} \times \mathbb{Z} \)

4. (Berg, Jack) Show that the class of simple groups is not elementary. (Use the language of groups.)

5. (Lessard, Lotfi) Let \( \kappa < \lambda < \mu \) be infinite cardinals. Give an example of a structure of cardinality \( \mu \) that has a substructure of cardinality \( \kappa \) but no substructure of cardinality \( \lambda \).

6. (Shriner, Sparks, Tanksalvala) Show that the class of well ordered sets is not elementary. (Use the language of partially ordered sets.)

7. (Shriner, Sparks, Tanksalvala) If \( A \) is a structure, then a subset \( U \subseteq A \) is definable if there is some formula \( \varphi(x) \) such that \( u \in U \) iff \( A \models \varphi[u] \). Explain why the following sets are definable in \( \mathbb{R} = \langle \text{reals}; \cdot, + \rangle \).
   (a) The unit interval.
   (b) \( \{n\} \) for any integer \( n \).
   (c) \( \{x\} \) for any real algebraic number \( x \).