Optimization and Control of Networks

Optimization, Dynamics, and Layering in Complex Networked Systems: From the Internet to the Smart Grid

Lijun Chen
03/10/2016
Networked systems

Complexity is ever increasing
- Large in size and scope
- Enormous heterogeneity
- Incomplete information
- Uncertain environments
- Emerging technologies
- New applications
- New design dimensions
- ......

Design (& understanding) is increasingly dominated by
- Efficiency (optimality)
- Manageability
- Reliability & Security
- Economic viability
- Scalability
- Evolvability
- ......

emerging, collective properties
Components

Systems requirements: functional, efficient, robust, secure, evolvable, …

architecture

- Most persistent/shared organizational structure, including abstractions, interfaces, and layering of cyber and physical

- Highly conserved resource allocation, control, and management mechanisms
Components

Systems requirements:
functional, efficient, robust, secure, evolvable, …

architecture

Constraints that deconstrain

- Certain fixed points and structure under which the network can expand/evolve
- Constraining for the issues that the network was originally not designed for

good architecture enables innovation, bad one freezes it
Components

Systems requirements: functional, efficient, robust, secure, evolvable, …

Architecture

Architectural design

- Remains an art, primarily empirical, reasoning-based
- Good architecture easy to recognize in retrospect but elusive to forward-engineer
- No formal theory nor systematic design method
Components

Systems requirements: functional, efficient, robust, secure, evolvable, …

Architecture

- Mathematical underpinning of network architecture
- Systematic methods to develop and evaluate design choices and algorithms

A holy grail
Components

Systems requirements: functional, efficient, robust, secure, evolvable, …

architecture

Approach

- Understand architecture and main mechanisms of existing networks (reverse engineering)
- Design architecture and main mechanisms for emerging networks (forward engineering)
Computer

Software     Diverse

Constraints that deconstrain

OS

Layered architecture (toy model)

Hardware     Diverse
Communication system

Constraints that deconstrain

Source coding

Channel coding

Channel decoding

Source decoding

Layered architecture
Internet

Application    Diverse

TCP/AQM

Highly conserved core mechanisms

IP

MAC

Physical    Diverse

Layered architecture
Internet architecture

- Application: Little quantitative understanding
  - Optimal? In what sense?
- TCP/AQM: Lots of problems
  - Efficiency, security, mobility, accountability, …
- IP
- MAC
- Physical

fixes (middle boxes & overlays & underlays)
Emerging networked systems

future internet

energy-efficient data center

smart grid

architectures are being designed now …
Future Internet architecture

Clean slate Internet design that aims to build in or enable

- security
- mobility
- new communication paradigms
- new computing paradigms
- ......

not clear what the right architecture is and how to best design different components and their interactions
Energy-efficient data center

- How to decompose & coordinate energy management decisions spatially and temporally
- How to interact with other resource allocation algorithms
- How to interconnect servers to balance performance and energy usage

Theories and models are needed to guide architecture and algorithm design.

3-5% of total US energy use.
Smart grid

Power network will go through similar architectural transformation in the next few decades that telephone network has gone through.

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>1876</td>
<td>Bell: telephone</td>
</tr>
<tr>
<td>1888</td>
<td>Tesla: multi-phase AC</td>
</tr>
</tbody>
</table>

- Both started as regulated monopolies
- Both provided a single commodity
- Both were vertically integrated
- Both grew rapidly through two WWs

1980-90s deregulation started

2000s Enron, blackouts

1969: DARPAnet convergence to Internet

Infrastructure (completely?) re-engineered

Industry landscape drastically reshaped
Smart grid

... to become more distributed, more open, more interactive, more autonomous, and with greater user participation

what is an architecture theory to help guide the transformation?

... while maintaining security & reliability
Research

- Rigorous foundations and new methodologies for understanding & designing architecture and various mechanisms
- Employ and develop techniques in
  - optimization theory/algorithm
  - distributed control
  - game theory
  - systems theory
Approach

Architecture theory
(foundations and design methodologies)

- must be abstract and concrete
- must be foundational and practical

wireless network
network coding
EE data center
smart grid
Research

Architecture theory
(foundations and design methodologies)

networks and protocols & dynamics as distributed decomposition of optimization

wireless network
network coding
EE data center
smart grid
Agenda

- Layering and constrained optimization (communication network)
- Network dynamics as optimization algorithms (power network)
- Look into future
Layered Internet protocol stack

Each layer
- controls a subset of decision variables
- hides the complexity of the layer below
- provides a service to the layer above
- designed independently and evolves asynchronously

<table>
<thead>
<tr>
<th>Layer</th>
<th>Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Application</td>
<td>select user criteria, web layout, utility, ...</td>
</tr>
<tr>
<td>TCP/AQM</td>
<td>select source transmission rates</td>
</tr>
<tr>
<td>IP</td>
<td>select paths from sources to destinations</td>
</tr>
<tr>
<td>Link/MAC</td>
<td>select topology, medium access</td>
</tr>
<tr>
<td>Physical</td>
<td>select coding, power, ...</td>
</tr>
</tbody>
</table>
Optimization and layering

- Each layer is abstracted as an optimization problem
- Operation of a layer is a distributed solution
- Results of one problem (layer) are parameters of others
- Operate at different timescales

![Diagram of layers and optimization goals]

- **Application**: minimize response time, ...
- **TCP/AQM**: maximize utility
- **IP**: minimize path cost
- **Link/MAC**: maximize throughput, ...
- **Physical**: minimize SINR, maximize capacities, ...
Optimization and layering

Networks as optimizers

- integrate various protocol layers, by regarding them as carrying out distributed computation over the network to implicitly solve a certain global optimization problem
- different layers iterate on different subsets of decision variables using local information to achieve individual optimality
- taken together, these local algorithms achieve a global optimality
Protocol decomposition: TCP/AQM

Duality model: TCP/AQM as distributed primal-dual algorithm over network to maximize aggregate utility (Kelly '98, Low '99, '03)

Primal:

$$\max_{x \geq 0} \sum_{s} U_s(x_s)$$

s.t. $$R \leq c$$

Dual:

$$\min_{p \geq 0} \left( \sum_{s} \max_{x_s \geq 0} \left( U_s(x_s) - x_s \sum_{l} R_{ls} p_l \right) + \sum_{l} p_l c_l \right)$$

$$Rx = \begin{pmatrix} 110 \\ 101 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \leq \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$
Protocol decomposition: TCP/AQM

Duality model: TCP/AQM as distributed primal-dual algorithm over the network to maximize aggregate utility (Kelly ’98, Low ’99, ’03)

**Primal:**
\[
\max_{x \geq 0} \sum_s U_s(x_s)
\]
\[
\text{s.t. } Rx \leq c
\]

**Dual:**
\[
\min_{p \geq 0} \left( \sum_s \max_{x_s \geq 0} \left( U_s(x_s) - x_s \sum_l R_{ls} p_l \right) + \sum_l p_l c_l \right)
\]

\[
x_s(t) = U_s^{-1} \left( \sum_l R_{ls} p_l \right)
\]
\[
p_l(t + 1) = [p_l(t) + \gamma(\sum_s R_{ls} x_s(t) - c_l)]^+
\]

horizontal decomposition
Optimization and layering

\[ \max_{x \geq 0} \sum_{s} U_{s}(x_{s}) \quad \text{s.t.} \quad Rx \leq c \]

- Extend to include decision variables and constraints of other layers
- Derive layering from decomposition of extended utility maximization
Generalized utility maximization

- Objective function: user application needs and network cost
- Constraints: restrictions on resource allocation (could be physical or economic)
- Variables: Under the control of this design
- Constants: Beyond the control of this design

\[
\max_{x, R, c, p} \sum_i U_i(x_i) - \lambda^T Rx \\
\text{subj to } Rx \leq c \\
c \in X(p)
\]
Layering as optimization decomposition

- Network: generalized NUM solver
- Layers: sub-problems
- Interface: functions of primal/dual variables
- Layering: decomposition methods

- **Vertical decomposition**: into functional modules of different layers
- **Horizontal decomposition**: into distributed computation and control over geographically disparate network components
Layering as optimization decomposition

- Network: generalized NUM
- Layers: sub-problems
- Interface: functions of primal/dual variables
- Layering: decomposition methods

Provides a top-down approach to design protocol stack

- explicitly tradeoff design objective
- explicitly model constraints and effects of, e.g., new technologies
- provide guidance on how to structure and modularize different functions
- make transparent the interactions among different components and their global behaviors
Optimization and layering

- Yield a much deeper understanding of existing protocols/algorithms and help design better/new ones
  - though many unresolved issues remain

- This series of work (starting with Kelly-Low model) had rekindled an interest in theory-based network design
  - several important players: CU (Chen), Caltech (Low, Doyle), Princeton (Chiang), UIUC (Srikant), Purdue/OSU (Shroff, Lin)
  - work on cross-layer design (Chen-Low-Doyle ‘05, Chen-Low-Chiang-Doyle ‘06) was among the first
  - see survey articles Chiang et al ‘07 and Chandra-Gayme-Chen-Doyle ‘11, and Chen-Low-Doyle ‘11
Cross-layer design in ad hoc wireless networks

Network performance can be improved if network layers are jointly designed.

Most works:
- design based on intuition, evaluated by simulations
- unintended consequences
Cross-layer design in ad hoc wireless networks

A holistic/principled approach

- Capture global structure of the problem
  - design objective
  - constraints
- Derive the design from the distributed decomposition of certain optimization problem
Cross-layer design/optimization

Each source $s$: sending rate $x_s$ and utility $U_s(x_s)$

Network model

Routing of service requirement $H(x)$
Allocation of service capacity $A(f)$

$c_{l \in L}, \Pi$
$f_{l \in L}; f \in \Pi$
Problem formulation

Network resource allocation:

\[
\begin{align*}
\max_{x,f} & \quad \sum U_s(x_s) \\
\text{s.t.} & \quad H(x) \leq A(f) \\
& \quad f \in \Pi
\end{align*}
\]
Protocol decomposition

Primal: \( \max_{x,f} \sum_s U_s(x_s) \quad \text{s.t.} \quad H(x) \leq A(f), \ f \in \Pi \)

Dual: \( \min_{p \geq 0} \left\{ \max_x \left( \sum_s U_s(x_s) - p^T H(x) \right) \right\} + \max_{f \in \Pi} p^T A(f) \)

Rate control
Routing
Scheduling

Rate constraint
Schedulability constraint
Cross-layer implementation

Dual: \[ \min_{p \geq 0} \left\{ \max_x \left( \sum_s U_s(x_s) - p^T H(x) \right) + \max_{f \in \Pi} p^T A(f) \right\} \]

- **Rate control:**
  \[ x(t) = x(p(t)) = \arg \max_x \left( \sum_s U_s(x_s) - p^T (t) H(x) \right) \]

- **Routing:**
  solved with rate control or scheduling

- **Scheduling:**
  \[ f(t) = f(p(t)) = \arg \max_{f \in \Pi} p^T (t) A(f) \]

- **Congestion update:**
  \[ p(t + 1) = \left[ p(t) - \gamma_t \{ A(f(p(t))) - H(x(p(t))) \} \right]^+ \]

---

**Application**

- Rate control:

**Transport**

- **Routing:** solved with rate control or scheduling

**Network**

**Link/MAC**

**Physical**

vertical decomposition
Extension to time-varying channel

- **Application**: channel state \( h \) : i.i.d. finite state process with distribution \( q(h) \)

- **Transport**
  - Rate control:
    \[
    x(t) = x(p(t)) = \arg \max_x \sum_s U_s(x_s) - p^T(t)H(x)
    \]
  - Routing:
    solved with rate control or scheduling

- **Network**
  - Scheduling:
    \[
    f(t) = f(p(t)) = \arg \max_{f \in \Pi(h(t))} p^T(t)A(f)
    \]

- **Link/MAC**
  - Congestion update
    \[
    p(t+1) = \left[ p(t) - \gamma_t \{ A(f(p(t))) - H(x(p(t))) \} \right]^+
    \]

- **Physical**
  - random
Stability and optimality

**Theorem** (Chen-Low-Chiang-Doyle ’06, ‘11): The Markov chain is stable. Moreover, the cross-layer algorithm solve the following optimization problem

\[
\max_{x, f} \sum_s U_s(x_s) \\
\text{s.t.} \quad H(x) \leq A(f) \\
f \in \Pi \\
\bar{\Pi} = \{ \bar{r} : \bar{r} = \sum_h q(h)r(h), r(h) \in \Pi(h) \} 
\]

- Applicable to any queueing network with interdependent, time-varying, parallel servers
  - optimality holds even with time-varying topologies
  - throughput-optimal when flow-level dynamics is considered
- A Wi-Fi implementation by Rhee’s group at NCSU shows significantly better performance than existing system
Outline

- Layering and constrained optimization (communication network)
- Network dynamics as optimization algorithms (power network)
- Look into future
Smart grid

Power network will go through a similar architectural transformation in the next few decades that telephone network has gone through.

<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1876</td>
<td>Bell: telephone</td>
<td>Infrastructure dramatically re-engineered. Industry landscape drastically reshaped.</td>
</tr>
<tr>
<td>1888</td>
<td>Tesla: multi-phase AC</td>
<td>Both started as regulated monopolies. Both provided a single commodity. Both were vertically integrated. Both grew rapidly through two WWs.</td>
</tr>
<tr>
<td>1969</td>
<td>DARPA net</td>
<td>Convergence to Internet.</td>
</tr>
<tr>
<td>1980-90s</td>
<td>Deregulation started</td>
<td></td>
</tr>
<tr>
<td>2000s</td>
<td></td>
<td>Enron, blackouts.</td>
</tr>
</tbody>
</table>
Emerging trends

- Proliferation of renewable and distributed generation
- Electrification of transportation
- Participation of end users

- Advances in power electronics
- Deployment of sensing, communication, and computation infrastructure
Global renewable power capacities, excluding hydro

Proliferation of renewables

Real opportunity for sustainability

Regional growth (US) in non-hydro renewable generation (billion KWH), 2011-2040

Sources: REN21, Renewables global status report (2006-2012); DOE/EIA-0383 (2013)
Random/rapid fluctuations

Source: Rosa Yang, EPRI
Migration to distributed architecture

• Small CHP (combined heat & power)
• Large CHP (combined heat & power)
• Wind

- 2-3x generation efficiency
- relieve demand on grid capacity
- also in control and management

Denmark’s experience
Large-scale network of DERs

Distributed energy resources (DERs): Photovoltaics (PVs), wind turbines, smart loads, inverters, storages, electric vehicles (EVs)

millions of active endpoints that may generate, sense, compute, communicate, and control

Source: ITERES
Large-scale network of DERs

- **Challenge**: an interconnected system of millions of DERs introducing rapid, large, and random fluctuations in supply, demand, voltage, and frequency

- **Opportunity**: increased capability to coordinate and optimize their operation for unprecedented efficiency and robustness
Current control paradigm

- Hierarchical control structure spanning multiple timescales from subseconds to hours and up

- Frequency control as an example

- Dynamic model: e.g. swing eqns
- Power flow model: e.g. DC/AC power flow

- Sec, Min, 5 min, 60 min, Day, Year
Current control paradigm

- Centralized, open-loop, worst-case preventive, and often human-in-the-loop at slow timescales
  - cope with slow/predictable but often large variations
  - economic efficiency and system security are the key (optimization model)

- Local and automatic at fast timescales
  - cope with fast but relatively small variations
  - (local) stability is the key (dynamical model)
  - oblivious of system-wide properties or global perspective

- Sufficient for today’s power system
  - relatively low uncertainties, few active assets, mainly to match controllable supply to passive load
  - the lack of ubiquitous sensing, control and communication
Current control paradigm

- Centralized, open-loop, worst-case preventive, and often human-in-the-loop at slow timescales
  - cope with slow/predictable but often large variations
  - economic efficiency and system security are the key (optimization model)

- Local and automatic at fast timescales
  - cope with fast but relatively small variations
  - (local) stability is the key (dynamical model)
  - oblivious of system-wide properties or global perspective

- Sufficient for today’s power system
  - relatively low uncertainties, few active assets, mainly to match controllable supply and passive load
  - the lack of ubiquitous sensing, control and communication

All are changing
Future control needs

- **Real-time and close-loop**
  - with rapid, large, and random fluctuations, feedback control based on real-time information is needed

- **Distributed to ensure scalability**
  - with large number of control points, information must be decentralized and decisions must be made locally

- **Fast/local controls actively and globally coordinated**
  - local controls must be bridged with the global situation, to ensure system-wide efficiency and robustness

- **Enabled by the deployment of sensing, control, and communication infrastructure and the advances in power electronics**
New control paradigm

- **Autonomous DERs for distributed real-time control**
  - Each DER made autonomous through local sensing, computing, communication, and control.
  - Intelligence embedded everywhere.

- **Local algorithms with global perspective**
  - Algorithm design starts with global objectives, which will be decomposed into local algorithms.

- **Layered architecture**
  - Control as a service: the network should provide a set of common control services to various applications.
  - Applications call and synthesize the control services to meet performance specifications.

**What are fundamental challenges?**
Comparison with the Internet

- Partially motivated by the Internet

  the precedence on the Internet lends hope to a much larger scale and more dynamic and distributed control architecture

- The physics of electricity cuts through all power system functionalities and operations
  - nonconvexity of power flow
  - dynamics cannot be “designed”
Fundamental challenges

Convexification of power flow
- for fast computation for real-time optimization
- for distributed algorithm

Distributed decomposition under dynamics constraints
- for distributed real-time control with global perspective
- exploit or implemented as power system dynamics

Integrating sensing, communication, and control
- fundamental limits on control performance arising from sensing constraints and communication constraints
- communication/networking for distributed control

Architecture and layering
- mathematical underpinning of smart grid architecture
- systematic methods to develop/evaluate design choices
Fundamental challenges

Convexification of power flow
- convex relaxation

Distributed decomposition under dynamics constraints
- network dynamics as optimization algorithms

Integrating sensing, communication, and control
- fundamental limits on control performance arising from sensing constraints and communication constraints
- communication/networking for distributed control

Architecture and layering
- mathematical underpinning of smart grid architecture
- systematic methods to develop/evaluate design choices
Convexification of OPF

- Optimal power flow (OPF) problem
  - a fundamental problem underlying power system controls and operations
  - huge literature since first formulated in 1962, focusing on approximate algorithms and solutions

- Convexity critical to the development of efficient, distributed, and robust algorithms
  - for real-time computation at scale
  - for distributed algorithms
  - for efficient market, as foundation for pricing schemes such as LMP
  - for global optimality, required for new/enhanced application
Branch flow model

**Power flow constraints**

\[
V_i - V_j = z_{ij} I_{ij}
\]

\[
S_{ij} = V_i I_{ij}^*
\]

\[
\sum_{i \rightarrow j} \left( S_{ij} - z_{ij} |I_{ij}|^2 \right) - \sum_{j \rightarrow k} S_{jk} = s_j
\]

Kirchhoff law

power definition

power balance
Optimal power flow

OPF:

\[
\begin{align*}
\max & \quad \sum_i U_i(p_i) - C\left(\sum_{(0,j)} P_{0,j}\right) - r_{i,j} |I_{i,j}|^2 \\
\text{over} & \quad x := (P, Q, I, V, p, q) \\
\text{s. t.} & \quad x \in \text{PFC}(x), (v_i, p_i, q_i) \in \text{OC}(x)
\end{align*}
\]

social welfare
power flow constraints
operation constraints, e.g., in safe range
Convexity structure

**OPF:**

\[
\begin{align*}
\text{max} & \quad \sum_i U_i(p_i) - C\left(\sum_{(0,j)} P_{0,j}\right) - r_{i,j} |I_{i,j}|^2 \\
\text{over} & \quad x := (P, Q, I, V, p, q) \\
\text{s. t.} & \quad x \in PFC(x) \quad \text{nonconvex} \\
& \quad (v_i, p_i, q_i) \in OC(x)
\end{align*}
\]
Convexity relaxation

\[ \begin{align*}
\text{OPF:} & \quad \max \sum_i U_i(p_i) - C \left( \sum_{(0,j)} P_{0,j} \right) - r_{i,j} |I_{i,j}|^2 \\
& \quad \text{over} \quad x := (P, Q, I, V, p, q) \\
& \quad \text{s. t.} \quad x \in \text{PFC}(x) \quad \text{and} \quad (v_i, p_i, q_i) \in \text{OC}(x)
\end{align*} \]

\[ \begin{align*}
\text{ROPF:} & \quad \max \sum_i U_i(p_i) - C \left( \sum_{(0,j)} P_{0,j} \right) - r_{i,j} |I_{i,j}|^2 \\
& \quad \text{over} \quad x := (P, Q, I, V, p, q) \\
& \quad \text{s. t.} \quad x \in \text{RPFC}(x) \quad \text{and} \quad (v_i, p_i, q_i) \in \text{OC}(x)
\end{align*} \]
Theorem (Li-Chen-Low ‘12a): Convex relaxation is exact provided that for any $i$, $v_i^{\text{lin}}(p, q) < \bar{v}_i$ and for any link $(k, l)$ in the network and $(i, j)$ on the path from 0 to $k$,

If $\frac{r_{k,l}}{x_{k,l}} - \frac{X_k}{R_k} > 0$, then $v_i + 2P_i^{\text{lin}} \left( \frac{r_{k,l}}{x_{k,l}} X_k - R_{i,k} \right) + 2Q_i^{\text{lin}} X_i > 0$

Otherwise, $v_i + 2Q_i^{\text{lin}} \left( \frac{x_{k,l}}{r_{k,l}} R_k - X_{i,k} \right) + 2P_i^{\text{lin}} R_i > 0$

- if only load buses, relaxation is always exact
- relaxation is always exact for real systems where
  \[ v \sim 1, r, x < < 1, P, Q < 1 \]
  - IEEE distribution test systems
  - Southern California Edison circuits
- many decomposition approaches (thus distributed algorithms) apply (Li-Chen-Low ‘12b, ‘12c)
Convexification of OPF

- Hidden convexity for efficient, distributed computation
  - tremendous progress since Lavaei and Low ‘11; see survey article Low ‘14
  - effectiveness depends on graph properties of underlying physical and/or communication networks
  - not always possible, and conditions may violate operation constraints

- Convex approximation?
  - geometry of power flow and its dependence on operation constraints and graph properties
  - systematic approach to construct convex approximation, to trade off tractability and optimality
Distributed decomposition under dynamics constraints

- Power network is a physical system
  - cannot be “re-set” arbitrarily, but has to evolve from one state to another
  - algorithms must be “consistent” with system dynamics

- Reverse engineering
  - can we bridge existing local control with system-wide property?

- Forward engineering
  - engineer the model from reverse engineering to guide systematic design of new algorithms
Frequency control

$P_i^M, P_i^R \rightarrow P_{ij} \rightarrow P_i^S + P_i^I \rightarrow j$

- mechanical or renewable power
- branch power flow
- frequency-sensitive load
- uncontrolled load
Synchronous generator: \( P_i^M = F_i(\omega_i) \)
- decreasing; e.g., \( F_i(\omega_i) = -S_i \omega_i \)

Renewable generator: \( P_i^R = H_i(\omega_i) \)
- deceasing

Frequency sensitive load: \( P_i^S = G_i(\omega_i) \)
- increasing; e.g., \( G_i(\omega_i) = D_i \omega_i \)
Dynamics

Synchronous generator bus:

\[ M_i \dot{\omega}_i = F_i(\omega_i) - G_i(\omega_i) - P_i^L - \sum_{j:i \sim j} P_{ij} \]

Renewable generator bus:

\[ 0 = H_i(\omega_i) - G_i(\omega_i) - P_i^L - \sum_{j:i \sim j} P_{ij} \]

Load bus (no generator):

\[ 0 = G_i(\omega_i) + P_i^L + \sum_{j:i \sim j} P_{ij} \]

Real branch power flow:

\[ \dot{P}_{ij} = b_{ij} \left( \omega_i - \omega_j \right) \]
Cost/disutility functions

Control functions define relations between equilibrium frequency and equilibrium generation and load

- synchronous generator: \[ C_i^M (P_i^M) = -\int_0^{P_i^M} F_i^{-1} (P) dP \]
- renewable generator: \[ C_i^P (P_i^R) = -\int_0^{P_i^M} H_i^{-1} (P) dP \]
- frequency sensitive load: \[ C_i^S (P_i^S) = \int_0^{P_i^M} G_i^{-1} (P) dP \]

The equivalence of control and decision problem

\[ P_i^M = \arg \min_P C_i^M (P) + P \omega_i \]

depend only on the control function but is independent of how the feedback signal is updated.
Theorem (You-Chen ‘14a): Power system dynamics is a distributed primal-dual gradient algorithm to solve

$$\min \sum_{i \in N^M} C_i^M (P_i^M) + \sum_{i \in N^R} C_i^R (P_i^R) + \sum_{i \in N} C_i^S (P_i^S)$$

s.t.  
$$P_i^S + P_i^L + \sum_j P_{ij} = P_i^M, \quad i \in N^M$$

$$P_i^S + P_i^L + \sum_j P_{ij} = P_i^R, \quad i \in N^R$$

$$P_i^S + P_i^L + \sum_j P_{ij} = 0, \quad i \in N^L$$

and the dual variables are frequencies and equal.

network dynamics as optimization algorithms (network as optimizer)
Network dynamics as optimization algorithms

- A new perspective to understand collective behavior arising from interaction between local controls
  - structural properties of the equilibrium point
  - efficiency and tradeoffs, etc

- Suggests a Lyapunov function for global stability or convergence analysis
  - important both theoretically and practically

- Suggests a principled way to systematically design new algorithms and control schemes
Forward engineering

Suggests a principled way to systematically design new algorithms and control schemes

- new design goals (e.g., frequency recovery, fairness, and economic efficiency) incorporated by engineering the global objective function and the constraints

- new control schemes with different dynamical properties and complexities based on various optimization algorithms

- insights from reverse engineering can guide particular way to engineer the model and derive the algorithm
Nominal frequency recovery

- **Key observation:** \( \omega = 0 \) can be ensured if \( \sum_{i \in N} G_i(\omega) = 0 \) at equilibrium.

\[
\sum_{i \in N} P_i^I = \sum_{i \in N^M} P_i^M + \sum_{i \in N^R} P_i^R
\]

- Impose the above indirectly by imposing decoupling constraints:

\[
\begin{align*}
P_i^I + \sum_j Q_{ij} &= P_i^M, \; i \in N^M \\
P_i^I + \sum_j Q_{ij} &= P_i^R, \; i \in N^R \\
P_i^I + \sum_j Q_{ij} &= 0, \; i \in N^L \\
Q_{ij} &= Q_{ji}
\end{align*}
\]
Nominal frequency recovery

A new optimization problem (You-Chen ‘14a):

\[
\begin{align*}
\text{max} \quad & \sum_{i \in N^M} C_i^M(P_i^M) + \sum_{i \in N^R} C_i^R(P_i^R) + \sum_{i \in N} C_i^S(P_i^S) \\
\text{s.t.} \quad & P_i^S + P_i^I + \sum_{j} P_{ij} = P_i^M, \quad i \in N^M \\
& P_i^S + P_i^I + \sum_{j} P_{ij} = P_i^R, \quad i \in N^R \\
& P_i^S + P_i^I + \sum_{j} P_{ij} = 0, \quad i \in N^L \\
& \quad P_i^I + \sum_{j} Q_{ij} = P_i^M, \quad i \in N^M \\
& \quad P_i^I + \sum_{j} Q_{ij} = P_i^R, \quad i \in N^R \\
& \quad P_i^I + \sum_{j} Q_{ij} = 0, \quad i \in N^L
\end{align*}
\]

all are physical variables

\( Q \) is not physical
Nominal frequency recovery

New control scheme (You-Chen ’14a):

\[ P_i^M = F_i(2\omega_i - \nu_i), \quad i \in \mathbb{N}_M \]
\[ P_i^R = H_i(\omega_i + \mu_i), \quad i \in \mathbb{N}_R \]
\[ \dot{\nu}_i = -\left( G_i(\omega_i) + \sum_j (P_{ij} - Q_{ij}) \right) / M_i, \quad i \in \mathbb{N}_M \]
\[ \dot{\mu}_i = \xi_i(G_i(\omega_i) + \sum_j (P_{ij} - Q_{ij})), \quad i \in \mathbb{N}_R \cup \mathbb{N}_L \]
\[ \dot{Q}_{ij} = \epsilon_{ij}(\mu_i - \mu_j) \]

distributed control
Economic dispatch

Theorem (You-Chen ‘14a): Power system dynamics with the new control scheme solves economic dispatch problem

\[
\begin{align*}
\min & \quad \sum_{i \in N^M} C_i^M(P_i^M) + \sum_{i \in N^R} C_i^R(P_i^R) + \sum_{i \in N} C_i^S(P_i^S) \\
\text{s.t.} & \quad P_i^I + \sum_j P_{ij} = P_i^M, \quad i \in N^M \\
& \quad P_i^I + \sum_j P_{ij} = P_i^R, \quad i \in N^R \\
& \quad P_i^I + \sum_j P_{ij} = 0, \quad i \in N^L
\end{align*}
\]

- real-time frequency control recovering frequency and achieving economic efficiency at the same time
- different from current approach achieving these objectives at different timescales and with centralized control
- needed for future smart grid to cope with rapid/large fluctuations and manage a huge number of control points
Network dynamics as optimization algorithms

- Natural system dynamics exploited for simplicity, scalability, and robustness
  - desired for distributed real-time control

- Lots progress
  - local volt/var control (Farivar-Chen-Low ‘13); automatic generation control (Li-Chen-Zhao-Low ‘14); load side frequency control (Zhao et al ‘12, ‘14, Mallada et al ‘14); distributed frequency control in microgrids (Dorfler et al ‘14)

- More work needed
  - remove approximations
  - integrate frequency and voltage control
  - distributed decomposition of AC OPF problem
Outline

- Layering and constrained optimization (communication network)
- Network dynamics as optimization algorithms (power network)
- Look into the future
Layering and optimization

- Derive the layering structure and modularity of various mechanisms

- Make transparent the interactions among different components and their global behaviors

**Architecture**
(foundations and design methodologies)

- networks and protocols & dynamics as distributed decomposition of optimization
Mathematical underpinning of network architecture

Architecture
(foundations and design methodologies)

- Networks and protocols & dynamics as distributed decomposition of optimization

- A common analytical framework and language
  - Handle and integrate sensing, computation, communication, control, and incentives
  - Allow rigorous analysis and systematic design

- Close the gap between theory and practice
  - Implementation and verification
  - Theory and implementation inform each other
Approach

Architecture (foundations and design methodologies)

- must be both abstract and concrete
- must be both foundational and practical

future Internet
EE data center
smart grid
SDN controller architecture

- Software-defined networking (SDN)
  - decouple control and data planes
  - dynamic and active networking
  - network virtualization and controller as network OS

- Aim to develop models/theories to guide the analysis and design
  - distributed controller architecture
  - dynamic resource (re-)allocation and management algorithms
Energy-efficient data center

- A new branch of research with its own rich structures and unique challenges
- Aim to develop models/theories to guide the analysis and design of practical algorithms for energy efficient data centers
  - our initial step on some of these issues (Chen-Li-Low ‘10, Chen-Li ‘13, Chen-Andrew-Wierman ‘14)
  - lots of work existed already

3-5% of total US energy use
Smart grid

- Nonconvexity of power flows
  - convex approximation
- Network dynamics as optimization algorithms
  - distributed decomposition of AC OFP
- Integrating sensing, communication, control
  - fundamental limits on control performance under sensing/communication constraints (You-Chen '14b, Shihadeh-You-Chen '14)
  - communication/networking for distributed control
- Architecture and layering
  - mathematical underpinning & systematic methods
  - framework to reason about architectural questions
    - design goals
    - design principles: layering, division of functionality, placement of intelligence, …
Research agenda

Communication networks

Core Theory challenges (comm., comp., contr., sensing; optimization, game, systems, ……)

Power networks

Learning, inference, sparse sampling, parsimonious solutions
Thank you!

Electricity: 1800…

… (most) architecture today

Electricity: today…

\[
\begin{align*}
\int \vec{E} \cdot d\vec{A} &= \frac{q}{\varepsilon_0} \\
\int \vec{B} \cdot d\vec{A} &= 0 \\
\int \vec{E} \cdot d\vec{s} &= -\frac{d\Phi_B}{dt} \\
\int \vec{B} \cdot d\vec{s} &= \mu_0 i + \frac{1}{c^2} \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{A}
\end{align*}
\]

… and our goal here

methodological transformation