# A Game-Theoretic Model for Medium Access Control*

(Invited Paper)

Lijun Chen, Tao Cui, Steven H. Low, and John C. Doyle

Division of Engineering and Applied Science
California Institute of Technology
Pasadena, CA, USA 91125

{chen@cds., taocui@, slow@, doyle@cds.}caltech.edu

## ABSTRACT

In this paper, a game-theoretic model for contention based medium access control (contention control) is proposed. We define a general game-theoretic model, called random access game, to capture the distributed nature of contention control and the interaction among wireless nodes with contention-based medium access. We study the design of random access games, characterize their equilibria, study their dynamics, and propose distributed algorithms to achieve the equilibria. This provides a unique perspective to understand existing MAC protocols and a general framework to guide the design of new ones to improve the system performance. As examples, a series of utility functions is proposed for games achieving the maximum throughput in a network of homogeneous nodes. The convergence of different variants (e.g., asynchronous and stochastic algorithms) of different dynamic algorithms such as gradient play are obtained. An equilibrium selection algorithm is also proposed to guarantee that the dynamic algorithms can actually achieve the desired operating point. Simulation results show that game model based protocols can achieve superior performance over the standard IEEE 802.11 DCF, and comparable performance as existing protocols with the best performance in literature.

## Categories and Subject Descriptors

C.2.5 [**Computer-Communication Networks**]: Local and Wide-Area Networks—*Access schemes*

## General Terms

Algorithms, Performance

*This work is partially supported by NSF through grants CNS-0435520 and CCR-0326554, and Caltech's Lee Center for Advanced Networking.



## Keywords

Medium access control, Game theory, Nash equilibrium, Strategy update mechanism, Fairness, Wireless LANs.

## 1. INTRODUCTION

Wireless channel is a shared medium that is interference-limited. A contention-based medium access control (contention control) is a distributed strategy to access and share a wireless channel among competing wireless nodes. It dynamically adjusts *channel access probability* in response to the amount of contention in the network. Note that the amount of contention itself depends on the channel access probabilities chosen by the wireless nodes. Hence contention control is a distributed, iterative feedback system described mathematically as:

$$p_i(t+1) = \mathcal{F}_i(p_i(t), q_i(t)), \quad q_i(t) = \mathcal{C}_i(\mathbf{p}(t)), \qquad (1)$$

where $p_i(t)$ is the channel access probability of node $i$, $\mathbf{p(t)} = (p_i(t))$ is the corresponding vector, and $q_i(t)$ is certain measure of contention observed by node $i$ that depends on the vector $\mathbf{p}(t)$. The channel access probability $p_i(t)$ is usually implemented either through a backoff algorithm on contention window or as a persistence probability. For example, the standard IEEE 802.11 DCF has a backoff algorithm that induces a channel access probability and can be modeled by some function $\mathcal{F}_i$. The algorithm responds to whether there is a collision, and hence the measure of contention $q_i(t)$ in 802.11 DCF is the probability of collision whose dependence on the channel access probability vector $\mathbf{p}(t)$ can be modelled by some function $\mathcal{C}_i$.

The performance of a MAC, e.g., the throughput, fairness and collision, depends critically on the equilibrium and stability of the dynamical system defined by (1). In this paper, extending from [7] we propose a game-theoretic model to understand the dynamical system (1), use it to design new classes of MAC, and present simulation results that demonstrate its superior performance over 802.11 DCF. Specifically, in Section 3, we propose a general random access game to model MAC protocols. The key idea is to consider each node $i$ to have a utility function $U_i(p_i)$ as a function of its channel access probability $p_i$. The goal of node $i$ is to maximize its payoff function $u_i(\mathbf{p}) := U_i(p_i) - p_i q_i$ given the contention measure $q_i$. Hence, the steady state properties of a MAC can be analyzed or designed through the specification of the utility function $U_i(p_i)$ and the choice of the contention measure $q_i$ (e.g., collision probability, or idle time between channel access, etc). Their specification defines the underlying random access game whose equilibrium determines the steady state properties such as throughput, fairness and collision of MAC. The adaptation of channel access probability can be specified through $(\mathcal{F}, \mathcal{C})$ and corresponds to different strategies to approach the equilibrium of the random access game.

We provide conditions under which equilibrium exists and is unique. Several examples are provided on how to design the utility function and determine the contention measure by reverse engineering from existing protocols and from desired operating points (e.g., in terms of some target throughput and fairness), and by forward engineering from heuristics. Especially, a series of utility functions is proposed for games achieving the maximum throughput in a network of homogeneous nodes. Supermodular game is also considered, which guarantees the existence of Nash equilibrium. Moreover, the best response strategy discussed in Section 4 can converge to a Pareto dominant equilibrium of supermodular random access game. In Section 4, we also consider another two dynamic algorithms to achieve the equilibrium: gradient play and Jacobi play. We show that under mild conditions both algorithms converge to the unique equilibrium. We also consider gradient play under estimation error, and show that it converges to a neighborhood of the equilibrium point. Due to the lack of knowledge of the number of users in the network and the approximation made in utility function design, the dynamic algorithms may not converge to the desired operating point. An equilibrium selection algorithm is thus proposed to make these algorithms actually hit the desired point. Simulation results show that game model based protocols can achieve superior performance over the standard IEEE 802.11 DCF, and comparable performance as existing protocols with the best performance in literature.

# 2. RELATED WORK

There are lots of works on medium access control. Here we only mention a few that are most closely related to this work. Game-theoretic approach has been applied extensively to study medium access, see, e.g., [5–7, 15, 20, 21]. Jin et al. [15] studies noncooperative equilibrium of Aloha networks and their local convergence. Borkar et al. [5] studies distributed scheme for adapting random access. Čagalj et al. [6] studies selfish behavior in CSMA/CA networks and propose a distributed protocol to guide multiple selfish nodes to a Pareto-optimal Nash equilibrium. Lee et. al. [20] reverse-engineers backoff-based MAC protocols using a noncooperative game model. This paper is an extension of earlier work [7]. Related work also includes [11] that proposes an idle sense access method without estimating the number of nodes, which compares the mean number of idle slots between transmission attempts with the optimal value and adopts an additive increase and multiplicative decrease algorithm to dynamically control the contention window in order to improve throughput and short-term fairness.

Finally, a comparison with TCP congestion control is in place. Contention control has striking similarity with congestion control. They need to handle almost the same issues such as congestion or contention measure, load control (e.g., window update) algorithm, and decoupling load control from handling failed transmissions, etc. However, the interaction among wireless nodes is different from that among TCP flows, which means a different model is needed to study contention control. Actually, one of the motivations of this work and earlier work [7] is try to develop a parallel story for contention control to what has been done for TCP congestion control in the utility maximization framework, see, e.g., [16–18].

# 3. GAME-THEORETIC MODEL OF CONTENTION CONTROL

## 3.1 Random Access Game

Consider a set $\mathcal{N}$ of wireless nodes in a wireless LAN with contention-based medium access. In this paper, we only consider *single-cell* wireless LANs, where every wireless node can hear every other node in the network. The analysis in this paper can be extended to general multicell networks. We assume all nodes always have a frame to transmit. The wireless channel is assumed to be error free and packet loss is only due to collision. We will mainly present our theory and analysis in terms of "channel access probability." If a backoff mechanism is implemented, the channel access probability $p$ is related to the contention window $cw$ according to $p = \frac{2}{cw+1}$, which can be derived under the decoupling approximation with constant contention windows, see, e.g., [3].

In practice, it is hard for wireless nodes to learn directly the channel access probabilities of others. Each node infers the contention of the wireless network through observing some contention measure $q_i$, which are functions of the nodes' channel access probabilities. Following [7], we model the interaction among wireless nodes as a non-cooperative game. Formally, we define a random access game as follows.

DEFINITION 1. *A random access game $\mathcal{G}$ is defined as a quadruple $\mathcal{G} := \{\mathcal{N}, (\mathcal{S}_i)_{i\in\mathcal{N}}, (u_i)_{i\in\mathcal{N}}, (q_i)_{i\in\mathcal{N}}\}$, where $\mathcal{N}$ is a set of players (wireless nodes), player $i \in \mathcal{N}$ strategy $\mathcal{S}_i := \{p_i | p_i \in [\nu_i, \omega_i]\}$ with $0 \le \nu_i < \omega_i \le 1$, and payoff function $u_i(\mathbf{p}) = U_i(p_i) - p_i q_i$ with utility function $U_i(p_i)$ and given contention measure $q_i = \mathcal{C}_i(\mathbf{p})$.*

The payoff function can be interpreted as the net gain of utility from channel access discounted by the contention "cost". One property of this random access game is that the computation of the payoff function does not require explicit exchange of channel access probabilities between nodes. Thus, this game can be played and implemented distributedly. Random access game is a rather general model for contention control, as the payoff function can be reverse-engineered from (1). The fixed point of (1) defines an implicit relation between channel access probability $p_i$ and contention measure $q_i$,

$$p_i = \mathcal{F}_i(p_i, q_i). \tag{2}$$

If this relation can be written as

$$q_i = F_i(p_i), \tag{3}$$

the utility function of each node $i$ is defined as

$$U_i(p_i) = \int F_i(p_i) \mathrm{d}p_i. \tag{4}$$

Therefore, we can reverse engineer medium access control protocols and study them in game theoretic framework: medium access control can be interpreted as a distributed strategy update algorithm to achieve the equilibrium of the random access game.

We now analyze the equilibrium of random access game. We say a channel access probability vector $\mathbf{p}^*$ is an equilibrium, if for given network contention $(\mathcal{C}_i(\mathbf{p}^*))_{i\in\mathcal{N}}$ no node has an incentive to change.[1]

**Definition** 2. *A channel access probability vector $\mathbf{p}^*$ is said to be an equilibrium of random access game, if for given network contention $(\mathcal{C}_i(\mathbf{p}^*))_{i\in\mathcal{N}}$ no node can improve its payoff by deviating from the equilibrium, i.e., $u_i(\mathbf{p}^*) \geq U_i(p_i) - p_i\mathcal{C}_i(\mathbf{p}^*)$, $\forall p_i \in S_i$. An equilibrium $\mathbf{p}^*$ is a* nontrivial *equilibrium if $p_i^*$ satisfies*

$$\frac{\partial}{\partial p_i} U_i(p_i^*) = \mathcal{C}_i(\mathbf{p}^*),\ \forall \mathbf{i} \in \mathcal{N}. \tag{5}$$

The reason to consider nontrivial equilibrium is to avoid those equilibria in which some player takes strategy at the boundary of the strategy space, which usually results in great unfairness or low payoff. Denote the channel access probability for all nodes but $i$ by $\mathbf{p}_{-i} := (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_{|\mathcal{N}|})$, and write $(p_i, \mathbf{p}_{-i}) := \mathbf{p}$. If a node's contention measure signal is a function only of other nodes' channel access probabilities, i.e., $q_i = \mathcal{C}_i(\mathbf{p_{-i}})$, the equilibrium defined in Definition 2 reduces to the concept of Nash equilibrium [10].

**Definition** 3. *A channel access probability vector $\mathbf{p}^*$ is said to be a Nash equilibrium if no node can improve its payoff by unilaterally deviating from the equilibrium, i.e., $u_i(p_i^*, \mathbf{p}_{-i}^*) \geq u_i(p_i, \mathbf{p}_{-i}^*)$, $\forall p_i \in S_i$. A Nash equilibrium $\mathbf{p}^*$ is a* nontrivial *equilibrium if $p_i^*$ satisfies*

$$\frac{\partial}{\partial p_i} u_i(p_i^*, \mathbf{p}_{-i}^*) = 0,\ \forall i \in \mathcal{N}. \tag{6}$$

Throughout this paper, we will only consider those contention measures that can be described by $q_i = \mathcal{C}_i(\mathbf{p_{-i}})$. So, we will focus on *Nash equilibrium* in the following. To facilitate analysis, we list some assumptions that will be used in this paper.

**A1**: The utility function $U_i(\cdot)$ is twice continuously differentiable, increasing, strictly concave, and with finite curvatures that are bounded away from zero, i.e., there exist some constants $\mu$ and $\chi$ such that $1/\mu \geq -1/U_i''(p_i) \geq 1/\chi > 0$.

**A2**: The inverse function $(U_i')^{-1}(q_i)$ maps any $q_i$ into a point in $\mathcal{S}_i$ for all $i \in \mathcal{N}$.

**A3**: At a nontrivial Nash equilibrium $\mathbf{p}^*$, there exists a function $\Phi_i(p_i)$ for each node $i$ such that $\Phi_i(p_i^*) = \Phi_j(p_j^*)$, $\forall i, j \in \mathcal{N}$ and $\Phi_i(p_i)$ is strictly monotone in $\mathcal{S}_i$, $\forall i \in \mathcal{N}$.

By [10, Theorem 1.2] and Brouwer's fixed point theorem [4], the following two theorems are immediate.

**Theorem** 4. *Under assumption A1, there exists a Nash equilibrium for any random access game $\mathcal{G}$.*

**Theorem** 5. *Suppose A2 holds. Random access game $\mathcal{G}$ has a nontrivial Nash equilibrium.*

[1] If we interpret contention measure as some contention price, under this equilibrium concept wireless nodes can be seen as price-taking agents. For agents with price-anticipating behavior, the appropriate solution concept is that of Nash equilibrium. It would be interesting to compare the performances of MAC under these two kinds of equilibria.

Since the equilibrium determines the operating point of medium access control, it is desired to have a unique nontrivial Nash equilibrium. One way to show this is to use Banach fixed point theorem [4] by showing that $G_i(\mathbf{p}) := (U_i')^{-1}(\mathcal{C}_i(\mathbf{p}))$ is a contraction mapping [2]. However, the conditions obtained using this approach are sometimes restrictive. Another way to show uniqueness is to apply the following theorem.

**Theorem** 6. *Suppose that A1 and A3 hold and random access game $\mathcal{G}$ has a nontrivial Nash equilibrium. If additionally for all $i \in \mathcal{N}$, $\mathcal{C}_i(\mathbf{p})$ is strictly increasing in $\mathbf{p}$, then $\mathcal{G}$ has a unique nontrivial Nash equilibrium.*

Proof. Since $U_i(p_i)$ is a continuously differentiable concave function, $U_i'(p_i)$ is a continuous, decreasing function. Without loss of generality, we consider the case that $\Phi_i(p_i)$ is strictly increasing. Suppose that there are two nontrivial Nash equilibria $\bar{\mathbf{p}}$ and $\hat{\mathbf{p}}$. By A3, there exist $\gamma_1, \gamma_2 > 0$ such that, for all $i$, $\Phi_i(\bar{p}_i) = \gamma_1$, $\Phi_i(\hat{p}_i) = \gamma_2$. Since $\Phi_i(p_i)$ is strictly increasing, $\gamma_1 \neq \gamma_2$. Without loss of generality, assume $\gamma_1 > \gamma_2$. Thus, $\bar{p}_i > \hat{p}_i$ for all $i$. By equation (6), $U_i'(\bar{p}_i) = \mathcal{C}_i(\bar{\mathbf{p}}) > \mathcal{C}_i(\hat{\mathbf{p}}) = U_i'(\hat{p}_i)$, which contradicts the fact that $U_i'(p_i)$ is a decreasing function. Thus, if $\mathcal{G}$ has a nontrivial Nash equilibrium, it is unique. □

## 3.2 Utility Function Design

In the following, we give several examples to show how to interpret existing medium access algorithms within random access game framework and design utility functions to achieve desired equilibrium properties. There are basically three ways to design utility functions.

### 3.2.1 *Reverse Engineering from Existing Protocols*

Take 802.11 DCF as an example. Different from [20], which reverse engineers exponential backoff type of protocols from the dynamic, we reverse engineer 802.11 DCF from the equilibrium point. Let $q_i := 1 - \prod_{j\in\mathcal{N}/\{i\}}(1-p_j)$ be the conditional collision probability of node $i$. It is well established that for a single-cell wireless LAN at steady state, channel access probability $p_i$ relates to conditional collision probability $q_i$ as follows [3]:

$$p_i = \frac{2(1-2q_i)}{(1-2q_i)(a+1) + q_i a(1-(2q_i)^m)}, \tag{7}$$

where $a = CW_{\min}$ is the base contention window and $m$ is the maximum backoff stage. Note that (7) defined an implicit function $q_i = F_i(p_i)$. Following procedures (3)–(4), we can derive a utility function $U_i(p_i)$. When $0 \leq q_i \leq 1$, $m \geq 1$, and $a \geq 1$, it can be verified that $U_i''(p_i) < 0$. Also, it can be readily checked that $F_i^{-1}(q_i)$ maps any $q_i \in [0,1]$ into a point $p_i \in [0,1]$. From Theorem 5, the random access game $\mathcal{G}$ with the derived utility function has a nontrivial Nash equilibrium. To show the uniqueness of equilibrium, we define $\Phi_i(p_i) = (1-p_i)(1-U_i'(p_i))$. At equilibrium $\mathbf{p}^*$, we have $\Phi_i(p_i^*) = \Pi_{i\in\mathcal{N}}(1-p_i^*) = \Phi_j(p_j^*)$, $\forall i, j \in \mathcal{N}$. As $F_i(p_i)$ is an implicit function, we define $\tilde{\Phi}_i(q_i) = (1-F_i^{-1}(q_i))(1-q_i)$, where $F_i^{-1}(q_i)$ is given in (7). It is easy to show that $\tilde{\Phi}_i(q_i)$ is a strictly decreasing function in $q_i$ and $F_i^{-1}(q_i)$ is also a strictly decreasing function in $q_i$. Therefore, $\Phi_i(p_i)$ is a strictly increasing function. Also, $\mathcal{C}_i(\mathbf{p}) = \mathbf{1} - \prod_{\mathbf{j}\in\mathcal{N}/\{\mathbf{i}\}}(\mathbf{1} - \mathbf{p_j})$ is strictly increasing. By Theorem 6, the random access game $\mathcal{G}$ has a unique nontrivial Nash equilibrium.

### 3.2.2 *Reverse Engineering from Desired Operating Points*

In [11], a medium access control method is proposed, which dynamically sets the mean number of idle slots between transmission attempts to an optimal value. Let $T_c$ denote the average collision duration and $T_{\mathrm{SLOT}}$ denote the slot duration. It is derived in [11] that when the number of users in the network $|\mathcal{N}| \to \infty$, the throughput-optimal number of idle slots between two transmission attempts is

$$\bar{n}_{i\infty}^{\mathrm{opt}} = \frac{e^{-\xi}}{1-e^{-\xi}}, \tag{8}$$

where $\xi$ is the solution to $1-\xi = \eta e^{-\xi}$ and $\eta = 1 - T_{\mathrm{SLOT}}/T_c$. Note that $\bar{n}_{i\infty}^{\mathrm{opt}}$ is completely determined by the protocol parameters but not by the number of nodes in the network. Let $q_i := 1 - \prod_{j\in\mathcal{N}/\{i\}}(1-p_j)$. The probability of an idle slot is

$$(1-p_i)(1-q_i) = \frac{\bar{n}_{i\infty}^{\mathrm{opt}}}{\bar{n}_{i\infty}^{\mathrm{opt}}+1} = e^{-\xi}. \tag{9}$$

Applying (4), we obtain the utility function as

$$U_i(p_i) = p_i + e^{-\xi}\log(1-p_i). \tag{10}$$

Note that $U_i(p_i)$ does not satisfy A2 but it is clear that the random access game with the utility (10) has a nontrivial Nash equilibrium. This also shows the limitation of Theorem 5, which only specifies a necessary condition. Utility function (10) does not satisfy the conditions specified in Theorem 6. In fact, there may exist infinite number of equilibria in the game with utility (10). To design a game with unique equilibrium, we note that when $|\mathcal{N}|$ is large the optimal attempt probability that maximizes the throughput is very small. We thus have

$$(1-p_i)^{\alpha}(1-q_i) = (1-p_i)^{\alpha-1}e^{-\xi} \approx e^{-\xi}, \tag{11}$$

where $\alpha > 1$ and the approximation holds when $\alpha$ is not very large. Applying (4), we obtain the utility function as

$$U_i(p_i) = p_i + \frac{e^{-\xi}}{1-\alpha}(1-p_i)^{1-\alpha}. \tag{12}$$

Note that (12) still does not satisfy A2 and we cannot use Theorem 5. But at least one nontrivial Nash equilibrium exists, e.g., $p_i^* = 1 - e^{-\xi/(\alpha+|\mathcal{N}|-1)}$. Define $\Phi_i(p_i) = (1-p_i)(1-U_i'(p_i)) = \frac{e^{-\xi}}{(1-p_i)^{\alpha-1}}$, which is strictly increasing in $p_i$ when $\alpha > 1$. Also $\mathcal{C}_i(\mathbf{p})$ is strictly increasing in $\mathbf{p}$. By Theorem 6, the random access game $\mathcal{G}$ has a unique nontrivial Nash equilibrium. Note that due to the approximation used in (11) the equilibrium point for the game with utility (12) may not achieve the optimal number of idle slots $\bar{n}_{i\infty}^{\mathrm{opt}}$. We will discuss in Section 4.3 how to design equilibrium selection algorithm such that the equilibrium point by using (12) can actually hit $\bar{n}_{i\infty}^{\mathrm{opt}}$.

### 3.2.3 Forward Engineering by Heuristics

Consider a random access game with the following payoff function

$$u_i(\mathbf{p}) := U_i(p_i) - p_i\prod_{j\neq i}(1-p_j) = U_i(p_i) - p_i q_i, \tag{13}$$

where $q_i = \prod_{j\neq i}(1-p_j)$ is the contention measure representing the probability that all nodes except node $i$ do not transmit. This payoff function is motivated by the heuristic that each wireless node should be "charged" by an amount that is proportional to the throughput it achieves.

It turns out that the random access game with payoff (13) is a supermodular game. Supermodularity was introduced into the game theory by Topkis [22]. Supermodular games have many nice properties such as the existence of Nash equilibria and the convergence to the equilibria under different strategy update algorithms. The simplicity of supermodular games makes concavity/convexity and differentiability assumptions unnecessary, though we make such assumptions in this paper. In the setting of random access games, the definition of supermodularity and supermodular game reduces to the following.

**DEFINITION** 7. *The payoff function $u_i(p_i, \mathbf{p}_{-i})$ has increasing differences (supermodularity) in $(p_i, \mathbf{p}_{-i})$ if for all $\mathbf{p}_{-i} \geq \mathbf{p}'_{-i}$ the quantity $u_i(p_i,\mathbf{p}_{-i}) - u_i(p_i,\mathbf{p}'_{-i})$ is increasing in $p_i$. For twice differentiable payoffs, supermodularity is equivalent to $\frac{\partial^2 u_i(\mathbf{p})}{\partial p_i \partial p_j} \geq 0$ for all $j \neq i$.*

**DEFINITION** 8. *A random access game $\mathcal{G}$ is supermodular if for each node $i \in \mathcal{N}$ the payoff function $u_i(p_i, \mathbf{p}_{-i})$ has increasing differences in $(p_i, \mathbf{p}_{-i})$.*

It is easy to verify that $\partial^2 u_i(\mathbf{p})/\partial p_i \partial p_j = \prod_{j'\neq i, j'\neq j}(1-p_{j'}) \geq 0$. The following result is immediate [22].

**THEOREM** 9. *A random access game $\mathcal{G}$ with the payoff function (13) is a supermodular game, and the set of Nash equilibria for $\mathcal{G}$ is nonempty.*

As indicated by Theorem 9, no concavity/convexity assumption on utility function is required to guarantee the existence of Nash equilibria as in non-supermodular games. However, the uniqueness of Nash equilibrium may require stronger condition. By following similar argument as in Theorem 6, we have the following corollary on the uniqueness of equilibrium for supermodular random access games.

**COROLLARY** 10. *Suppose that utility function $U_i(\cdot)$ is twice continuously differentiable, increasing and strictly convex, and the supermodular random access game $\mathcal{G}$ with the payoff (13) has a nontrivial Nash equilibrium. If $\Phi_i(p_i) = (1-p_i)U_i'(p_i)$ is a strictly monotone function in $S_i$, then $\mathcal{G}$ has a unique nontrivial Nash equilibrium.*

As an example, we consider the following utility function given in [7]

$$U_i(p_i) := \frac{1}{a_i}\left(\frac{(a_i-1)b_i}{a_i}\ln(a_i p_i - b_i) - p_i\right), \tag{14}$$

where $0 < b_i < 1, a_i < 1$, and $p_i \in (\frac{b_i}{a_i}, \frac{b_i+\sqrt{b_i^2+a_i(a_ib_i-b_i^2-b_i)}}{a_i})$. It is easy to check that $U_i(p_i)$ is strictly convex and $\Phi_i'(p_i) < 0$ when $p_i < \frac{b_i+\sqrt{b_i^2+a_i(a_ib_i-b_i^2-b_i)}}{a_i}$. From Corollary 10, the supermodular game with utility function (14) has a unique nontrivial Nash equilibrium.

There are many ways to design utility functions and random access games. We only show a few specific examples in this section. The key point of this section is that the random access game model is general enough to include many of existing medium access control algorithms. Most of algorithms can be reverse engineered to be a random access game with specific utility function and contention measure.

# 4. DYNAMICS OF RANDOM ACCESS GAME

The dynamics of game studies how interacting players could converge to a Nash equilibrium. It is a difficult problem in general. In random access games, wireless nodes can observe the outcome (in terms of some contention measure) of the actions of others, but do not have direct knowledge of other nodes' actions and payoffs. We consider repeated play of random access game, and look for strategy update mechanism in which nodes repeatedly adjust channel access probabilities in response to observations of other players' actions so as to achieve the equilibrium.

## 4.1 Basic Dynamic Algorithms

### 4.1.1 Best Response

The simplest strategy update mechanism is the best response strategy: at each stage, every node chooses the best response to the actions of all the other nodes in the previous stage. Let $\mathbf{p}(0)$ be the largest vector in the strategy space $(\mathcal{S}_i)_{i\in\mathcal{N}}$. At stage $t+1$, node $i \in \mathcal{N}$ chooses a channel access probability

$$p_i(t+1) = B_i(\mathbf{p}(t)) := \max\{\arg\max_{p\in\mathcal{S}_i} u_i\left(p, \mathbf{p}_{-i}(t)\right)\}. \quad (15)$$

At each stage, if more than one probability may be a best response to a given $\mathbf{p}_{-i}(t)$, best response algorithm (15) always chooses the largest probability. Clearly, if the above dynamics reaches a steady state, this state is a Nash equilibrium. As there are no convergence results for general games using this dynamics, we restrict our discussion to supermodular random access game with payoff (13) in this subsection. We have the following result.

**THEOREM** 11. *The best response strategy (15) converges to a Nash equilibrium of random access game $\mathcal{G}$. Moreover, it is the largest equilibrium in the set of Nash equilibria.*

PROOF. The proof basically follows [22, Lemma 4.1]. By supermodularity, the best response is nondecreasing in other player strategies. We can show that $\mathbf{p}(0) \geq \mathbf{p}(1) \geq \cdots \geq \mathbf{p}(t) \geq \cdots$, i.e., $\{\mathbf{p}(t)\}$ is a nonincreasing sequence. As the strategy space $(\mathcal{S}_i)_{i\in\mathcal{N}}$ is compact, $\{\mathbf{p}(t)\}$ has a limit point, which is a Nash equilibrium. Let $\overline{\mathbf{p}} = \lim_{t\to\infty} \mathbf{p}(t)$. Since the best response is nondecreasing in other player strategies, $\overline{\mathbf{p}}$ is the largest Nash equilibrium. □

If we set $\mathbf{p}(0)$ to the smallest vector in the strategy space and always choose the smallest best response probability, the best response strategy will converge to the smallest equilibrium $\underline{\mathbf{p}}$. When there exist multiple equilibria, the following theorem indicates that the equilibrium attained by (15) yields the highest aggregate payoff.

**THEOREM** 12. *The best response strategy (15) converges to a Pareto dominant equilibrium, i.e., $u_i(\overline{\mathbf{p}}) \geq u_i(\mathbf{p})$ for any Nash equilibrium $\mathbf{p}$.*

PROOF. From (13), $u_i$ is an increasing function of $\mathbf{p_{-i}}$ for fixed $p_i$. Since $\overline{\mathbf{p}}$ is the largest equilibrium, for any equilibrium $\mathbf{p}$ we have

$$u_i(p_i, \mathbf{p}_{-i}) \leq u_i(p_i, \overline{\mathbf{p}}_{-i}). \quad (16)$$

On the other hand, by the definition of Nash equilibrium, we have

$$u_i(p_i, \overline{\mathbf{p}}_{-i}) \leq u_i(\overline{p}_i, \overline{\mathbf{p}}_{-i}). \quad (17)$$

Combining (16) and (17), we obtain the theorem. □

The following theorem guarantees that the best response converges to a nontrivial equilibrium.

**THEOREM** 13. *If the best responses to the smallest and largest vectors in the strategy space are within the strategy space, then nontrivial Nash equilibrium exists. Moreover, the best response strategy (15) converges to the largest nontrivial Nash equilibrium.*

PROOF. By supermodularity, the best response is nondecreasing in other player strategies. If the best responses to the smallest and largest vectors in the strategy space are within the strategy space (i.e., not at the boundaries of the strategy space), then Nash equilibrium must be within the strategy space. So, nontrivial Nash equilibrium exists. The second part of the theorem is obvious. □

By using Theorem 13, it is easy to obtain conditions on $a_i$ and $b_i$ in (14) such that the best response strategy converges to a nontrivial equilibrium of the corresponding game. Without using Corollary 10, the uniqueness of nontrivial equilibrium can also be obtained by showing that the best response strategy is a contraction mapping. A condition for convergence of best response strategy is also given in [7] for general random access games, which is hard to verify. Supermodularity greatly simplifies the conditions for the convergence of best response strategy.

### 4.1.2 Gradient Play

An alternative strategy update mechanism is gradient play [9]. Compared to "best response" strategy, gradient play can be viewed as a "better response". In gradient play, every node adjusts its channel access probability gradually in a gradient direction suggested by contention measurements. Mathematically, each node $i \in \mathcal{N}$ updates its strategy according to

$$p_i(t+1) = [p_i(t) + \epsilon_i(t)(U_i'(p_i(t)) - q_i(\mathbf{p}(t)))]^{\mathcal{S}_i}, \quad (18)$$

where the stepsize $\epsilon_i(\cdot) > 0$ is a function in time, $[\cdot]^{\mathcal{S}_i}$ denotes the projection onto node $i$'s strategy space. The gradient play has a nice economic interpretation. If the marginal utility $U_i'(p_i(t))$ is greater than the contention price $q_i(\mathbf{p}(t))$, we increase the access probability, and if the marginal utility is less than the contention price, we decrease the access probability. In the following, we assume that all nodes $\forall i \in \mathcal{N}$ have the same stepsize $\epsilon_i(t) = \epsilon(t)$.

**THEOREM** 14. *Let $\mathcal{C}(\mathbf{p}) = (\mathcal{C}_i(\mathbf{p}))$ and denote by $\mathbf{J}^C = (J_{ij}^C)$ the Jacobian of $\mathcal{C}(\mathbf{p})$. Suppose that the smallest eigenvalue of $\mathbf{J}^C$, $\lambda_{\min}(\mathbf{J}^C)$, satisfies $\mu + \lambda_{\min}(\mathbf{J}^C) > 0$, $\max_j |J_{ij}^C|^2 \leq M$, and the random access game has a unique nontrivial Nash equilibrium $\mathbf{p}^*$. The gradient play (18) converges geometrically to $\mathbf{p}^*$ if the stepsize $\epsilon(t) < \frac{\mu+\lambda_{\min}(\mathbf{J}^C)}{\chi^2+|\mathcal{N}|M}$.*

The proof of Theorem 14 is given in appendix. Theorem 14 also shows the convergence rate of gradient play. As an example of using Theorem 14, we consider the utility function defined in (12). By assuming that all nodes' strategy spaces are identical, i.e., $\mathcal{S} = [\nu, \omega]$. In this case, we have

$$\mu = \frac{\alpha e^{-\xi}}{(1-\nu)^{\alpha+1}}, \chi = \frac{\alpha e^{-\xi}}{(1-\omega)^{\alpha+1}}. \quad (19)$$

To find $\lambda_{\min}(\mathbf{J}^C)$, we note that

$$\mathbf{J}^C(\mathbf{p}) = -(\prod_i (1-p_i))(\text{diag}(\mathbf{x})^2 - \mathbf{x}\mathbf{x}^T), \quad (20)$$

where $\mathbf{x} = \left[\frac{1}{1-p_1}, \ldots, \frac{1}{1-p_{|\mathcal{N}|}}\right]^T$. Note that each entry of $\mathbf{x}$ is less than $\frac{1}{1-\omega}$. By using Rayleigh quotient [12], it is easy to show that the maximum eigenvalue of $\mathrm{diag}(\mathbf{x})^2 - \mathbf{x}\mathbf{x}^T$ is less than $\frac{1}{(1-\omega)^2}$. Thus, Theorem 14 requires that

$$\lambda_{\min}(\mathbf{J}^C) + \mu \geq -\frac{(1-\upsilon)^{|\mathcal{N}|}}{(1-\omega)^2} + \frac{\alpha e^{-\xi}}{(1-\nu)^{\alpha+1}} > 0. \tag{21}$$

Condition (21) is mild. For example, if we take $\omega = 2/33$ and $\alpha = 2$, all $\nu \in [0, 1]$ satisfy (21). We see that a larger $\alpha$ indicates a larger $\mu$, which means a greater convergence rate by (35).

### 4.1.3 Jacobi Play

Finally, we consider another alternative strategy update mechanism called Jacobi play, whose name comes from Jacobi update scheme, see, e.g., [19]. In Jacobi play, every player adjusts current channel access probability gradually towards the best response strategy. Mathematically, at stage $t+1$, node $i \in \mathcal{N}$ chooses a channel access probability

$$p_i(t+1) = J_i(\mathbf{p}(t)) := [p_i(t) + \epsilon_i(t)\,(B_i(\mathbf{p}(t)) - p_i(t))]^{\mathcal{S}_i}; \tag{22}$$

where the stepsize $\epsilon_i(t) > 0$ and $B_i(\mathbf{p}(t))$ is defined in (15). When $\epsilon_i(t) = 1$, we recover the best response strategy. In case of supermodular game, if $\epsilon_i(t) \leq 1$, it is easy to verify that $\{p_i(t)\}$ is a nonincreasing sequence. Thus, Theorem 13 still applies to Jacobi play. Jacobi play converges slower than best response strategy for supermodular game. Hence, it is not interesting in this case. For general random access games, we can also show the convergence of Jacobi play in a similar way as in gradient play. We omit them for brevity. Details can be found in [8]. We note that in case of nonsupermodular game with unique equilibrium, Jacobi play generally achieves a smoother dynamic than best response does. Sometimes best response does not even work. For example, most of the utility functions in Section 3.2.2 do not satisfy the conditions specified in Theorem 13. So best response does not converge to a nontrivial Nash equilibrium. However, Jacobi play still works in this case.

### 4.1.4 Contention Measure Signal Estimation

The dynamic algorithms require the knowledge of contention measure signals. In practice, contention measure signals can be estimated via the observation of the wireless medium over several time slots. As an example, we consider the contention measure – conditional collision probability used in Section 3.2.2. Let $n$ denote the number of consecutive idle slots between two transmissions. Since $n$ has the geometric distribution with parameter $\gamma(\mathbf{p}) = \prod_{i\in\mathcal{N}}(1-p_i)$, its mean $\bar{n}$ is given by $\bar{n} = \frac{\gamma(\mathbf{p})}{1-\gamma(\mathbf{p})}$, which can be estimated by averaging over $ntrans$ occurrences of this event. At every step, $\bar{n}$ is updated according to $\bar{n} \leftarrow \beta\bar{n} + (1-\beta)isum/ntrans$, where $isum$ is the total number of idle slots during $ntrans$ occurrences. Thus, each node can estimate its conditional collision probability according to

$$q_i = 1 - \frac{\gamma(\mathbf{p})}{1-p_i} = \frac{1-(\bar{n}+1)p_i}{(\bar{n}+1)(1-p_i)}. \tag{23}$$

## 4.2 Dynamic Algorithms under Estimation Error

In practice, due to propagation delay, nodes may not update their channel access probability at the same time. The asynchronous counterparts of the algorithms presented in Section 4.1 can be found in [8]. In this section, we consider dynamic algorithms under estimation error. Due to the use of estimated contention measure signals, the algorithms in Section 4.1 are in fact stochastic algorithms. In the following, we only consider gradient play. The results of Jacobi play can be obtained similarly. We assume that $q_i(\mathbf{p}(t))$ is replaced by $\hat{q}_i(\mathbf{p}(t)) = q_i(\mathbf{p}(t)) + w_i(t)$ in (18), where $w_i(t)$ is the estimation error. Let $\mathcal{F}_t$ be an increasing sequence of $\sigma$-fields. Without loss of generality, we write $w_i(t)$ as $w_i(t) = \bar{w}_i(t) + \tilde{w}_i(t)$, where $\bar{w}_i(t) = E\{w_i(t)|\mathcal{F}_t\}$ can be considered as the deterministic error and $\tilde{w}_i(t) = w_i(t) - \bar{w}_i(t)$ is the stochastic error with zero mean. We further assume that $\lim_{t\to\infty}\bar{w}_i(t) = \bar{w}_i$. The deterministic error may be caused by the bias of signal estimation and carrier sense error due to fading and background noise. For ease of understanding, in the following, we discuss deterministic and stochastic errors separately. The proof of the following theorems can be found in [8].

**THEOREM** 15. *Let $\lambda_{\min}(\mathbf{J}^C)$ denote the smallest eigenvalue of $\mathbf{J}^C$ and $\max_j |J_{ij}^C|^2 \leq M$. Let $\mathbf{p}^*$ denote the equilibrium defined by*

$$U_i'(p_i^*) = q_i(\mathbf{p}^*) + \bar{w}_i. \tag{24}$$

*If $\mathbf{p}^*$ is in the strategy space and it is the unique equilibrium defined by (24), the gradient play converges to $\mathbf{p}^*$ provided $\mu + \lambda_{\min}(\mathbf{J}^C) > 0$ and $\epsilon(t) < \frac{\mu+\lambda_{\min}(\mathbf{J}^C)}{\chi^2+4|\mathcal{N}|M}$.*

The uniqueness of $\mathbf{p}^*$ can be obtained by using Theorem 6. Note that under certain conditions, by implicit function theorem [2], (24) defines an implicit function $\mathbf{p}^*(\bar{\mathbf{w}})$ at the neighborhood of $\bar{\mathbf{w}} = \mathbf{0}$. Therefore, for any $\epsilon > 0$, there exists a $\delta > 0$ such that if $\|\bar{\mathbf{w}}\|_2 < \delta$, $\|\mathbf{p}^*(\bar{\mathbf{w}}) - \mathbf{p}^*(\mathbf{0})\|_2 < \epsilon$. So the gradient play converges to a neighborhood of the equilibrium point without estimation error.

For the stochastic error, we consider gradient play with variable stepsize and constant stepsize, respectively.

**THEOREM** 16. *Let $\lambda_{\min}(\mathbf{J}^C)$ denote the smallest eigenvalue of $\mathbf{J}^C$. Suppose that $E\{w_i(t)|\mathcal{F}_t\} = 0$, $E\{w_i^2(t)|\mathcal{F}_t\} \leq B$, and*

$$\sum_{t=0}^{\infty}\epsilon(t) = \infty, \ \sum_{t=0}^{\infty}\epsilon^2(t) < \infty. \tag{25}$$

*If $\mathbf{p}^*$ is the unique nontrivial Nash equilibrium, the gradient play converges to $\mathbf{p}^*$ with probability 1 provided $\mu + \lambda_{\min}(\mathbf{J}^C) > 0$.*

**THEOREM** 17. *Let $\lambda_{\min}(\mathbf{J}^C)$ denote the smallest eigenvalue of $\mathbf{J}^C$ and $\max_j |J_{ij}^C|^2 \leq M$. Suppose that $E\{w_i(t)|\mathcal{F}_t\} = 0$, $E\{w_i^2(t)|\mathcal{F}_t\} \leq B$, and $\epsilon(t) = \epsilon, \forall t$. If $\mathbf{p}^*$ is the unique nontrivial Nash equilibrium, there exists a constant $D(B,\epsilon) > 0$ such that*

$$\limsup_{t\to\infty}\|\mathbf{p}(t) - \mathbf{p}^*\|_2 \leq D(B,\epsilon) \tag{26}$$

*provided $\mu + \lambda_{\min}(\mathbf{J}^C) > 0$ and $\epsilon < \frac{\mu+\lambda_{\min}(\mathbf{J}^C)}{\chi^2+4|\mathcal{N}|M}$.*

By combining Theorems 15 and 17, we can conclude that with constant stepsize, the stochastic gradient play converges to a neighborhood of the equilibrium.

## 4.3 Equilibrium Selection

The equilibrium attained by using the dynamic algorithms in Section 4.1 does not necessarily converge to the desired operating point (that achieves the maximum throughput) when the utility functions in Section 3.2.2 are considered. This is because the approximation used in (11). One approach of equilibrium selection is to estimate the number of users via $\hat{N} = \log(1 - q_i)/\log(1 - p_i)+1$ at equilibrium and to set the channel access probability to be the optimal value computed by using $\hat{N}$. However, as commented in [13], this approach may not converge due to open loop control. The other approach is to use an outer loop iteration and treat the algorithms in Section 4.1 as the inner loop iteration. Take utility function (12) for example. Let $\tau$ denote the counter of outer loop iteration and define the utility function at the $\tau$-th outer iteration as

$$U_i(p_i, \tau) = p_i + \frac{\eta(\tau)}{1-\alpha}(1 - p_i)^{1-\alpha}, \tag{27}$$

where $\eta(0) = e^{-\xi}$. Denote the equilibrium for the game with utility (27) by $\mathbf{p}(\tau)$. To cancel the effect of neglecting $(1 - p_i)^{\alpha-1}$ in (11), we do the outer iteration

$$\eta(\tau + 1) = (1 - p_i(\tau))^{\alpha-1}e^{-\xi}. \tag{28}$$

At equilibrium, all nodes have the same access probability, denoted as $p(\tau)$. By (28), we obtain

$$p(\tau + 1) = 1 - \sqrt[|\mathcal{N}|+\alpha-1]{(1 - p(\tau))^{\alpha-1}e^{-\xi}}. \tag{29}$$

Let $\mathcal{M}(p)$ be the mapping defined by (29). By mean value theorem, it is easy to show

$$|\mathcal{M}(p_1)-\mathcal{M}(p_2)| \le \frac{e^{-\frac{\xi}{|\mathcal{N}|+\alpha-1}}(\alpha - 1)(1 - \omega)^{\frac{\alpha-1}{|\mathcal{N}|+\alpha-1}-1}}{|\mathcal{N}| + \alpha - 1}|p_1-p_2|. \tag{30}$$

Therefore, if $\frac{e^{-\frac{\xi}{|\mathcal{N}|+\alpha-1}}(\alpha-1)(1-\omega)^{\frac{\alpha-1}{|\mathcal{N}|+\alpha-1}-1}}{|\mathcal{N}|+\alpha-1} < 1$, $\mathcal{M}(p)$ is a contraction mapping [2] and (29) converges to the unique fixed point of $\mathcal{M}(p)$, which is the desired operating point. From (30), we can see that a larger $\alpha$ indicates a smaller outer loop convergence rate, while a larger $\alpha$ results in a greater inner loop convergence rate as suggested in Theorem 14. Therefore, there exists an optimal $\alpha$ to achieve the best overall convergence rate. In practice, when exact $p(\tau)$ is not available, we can use the average probability over a long duration. Also, outer loop iteration can be executed without waiting for the convergence of the inner loop iteration.

# 5. EXPERIMENTAL RESULTS

In this section, we run some numerical experiments to compare the performance of different medium access protocols. The system parameters are those specified in the IEEE 802.11b standard with DSSS PHY layer [1], where the values of parameters are summarized in Table 1. We consider a single-cell network with perfect wireless channel, i.e., there is no corrupted frame. In all simulations, the initial channel access probability is set to be $2/33$, which corresponds to $CW_{\min} = 32$ in 802.11b DCF. For our game based protocols, we set $ntrans = 5$ and $\beta = 0.8$ (see Subsection 4.1.4) for contention measure estimation unless specifically stated. Throughput and fairness are obtained after $10^6$ transmissions. The throughput in this section is the aggregate throughput of all nodes. Except in Section 5.3, equilibrium selection is not applied to the game based design as without equilibrium selection its throughput is already close to the optimal value.

**Table 1: Parameters used in simulations**

| | |
|---|---|
| Slot Time ($T_{\mathrm{SLOT}}$) | 20 $\mu s$ |
| SIFS | 10 $\mu s$ |
| DIFS | 50 $\mu s$ |
| Basic Rate | 1 Mbps |
| Data Rate | 11 Mbps |
| Propagation Delay | 1 $\mu s$ |
| PHY Header | 192 bits |
| MAC Header | 272 bits |
| ACK | 112 bits |
| Packet Payload ($s_d$) | 12000 bits |

## 5.1 Comparison of Dynamic Algorithms

We consider a system of homogeneous users, and compare the dynamics of different strategy update algorithms for supermodular game with utility function (14). To compare the performance of our game based design with that of 802.11 DCF on the same ground, we choose $a, b$ such that $b/a = CW_{\min}$ and $\frac{b+\sqrt{b^2+a(ab-b^2-b)}}{a} = 2^m CW_{\min} = CW_{\max}$, corresponding to a maximum backoff stage $m$.

Figures 1.(a), (b), and (c) show the evolution of channel access probability under best response (15), gradient play (18) and Jacobi play (22), respectively, where $|\mathcal{N}| = 20$, $CW_{\min} = 32$ and $CW_{\max} = 256$, and the stepsize is chosen to be $\epsilon_i(t) = 0.0016$ for the gradient play and $\epsilon_i(t) = 0.5$ for the Jocobi play. We see that best response converges close to the equilibrium only after 2 iterations with perfect contention measure estimation. Even with estimation error, best response converges to a neighborhood of the equilibrium after 5 iterations. Gradient play requires at least 15 iterations to converge to the equilibrium. The convergence rate of Jacobi play is between that of best response and that of gradient play. From Figure 1, we also observe that the estimation error does not affect the dynamic too much in supermodular games.

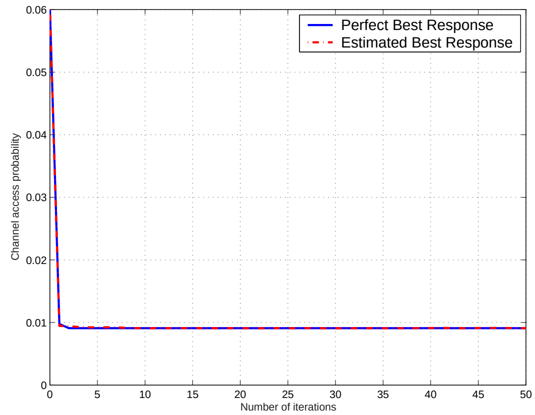
Figure 2 compares the throughput between the game based design according to gradient play and 802.11b DCF with $CW_{\min} = 32$ and $CW_{\max} = 256$. We see that for a small number of wireless nodes, DCF provides a higher throughput. But when the number of nodes is greater than 7, our method achieves a much higher throughput. We also find that by using estimated signal no noticeable performance loss is incurred.

## 5.2 Game Reverse-Engineered from the Desired Operating Point

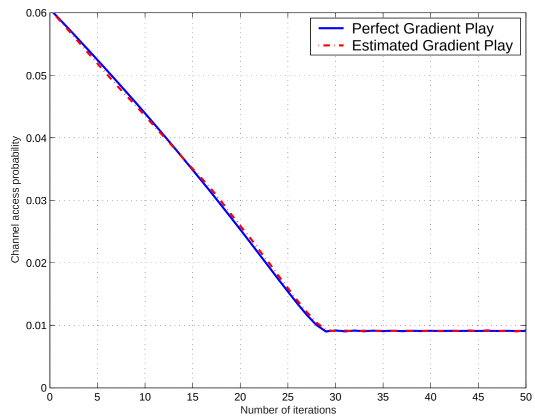
We then consider the game model with utility function (12) derived in Subsection 3.2.2, and compare the performance of MAC based on this game model with that of idle sense protocol in [11]. We choose $\xi = 0.1622$ and $\alpha = 2$. The parameters in idle sense are set as those in [11].

Figure 3 compares the dynamics of idle sense and gradient play (18) of the game with utility (12) in a network of 20 nodes, where the stepsize is chosen to be $\epsilon_i(t) = 0.02$. We see even with perfect knowledge of expected number of idle slots, idle sense oscillates around the optimal value. On the other hand, game model achieves a smoother dynamic in both cases with perfect signal and estimated signal. Both algorithms have roughly the same convergence rate. We can clearly see the geometric convergence rate predicted by Theorem 14. The equilibrium by our method is close to the optimal value but not equal due to the approximation used in (11).

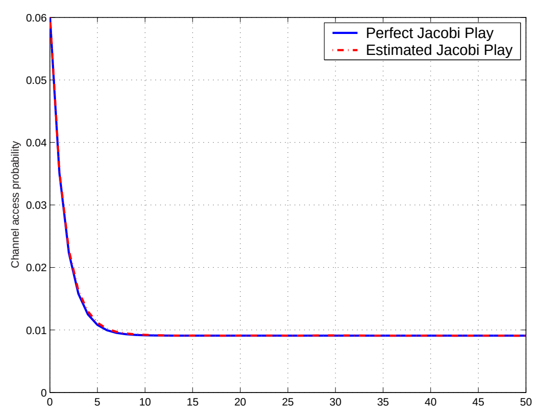
Figure 4 compares the throughput of idle sense, game based dsign, and DCF with the same parameters as in Figure 3. We use estimated signals in both idle sense and game based design. When




(a) Best response




(b) Gradient play




(c) Jacobi play

**Figure 1: The evolution of channel access probability under different strategy update algorithms for supermodular random access game with utility function (14) in a network of 20 wireless nodes.**

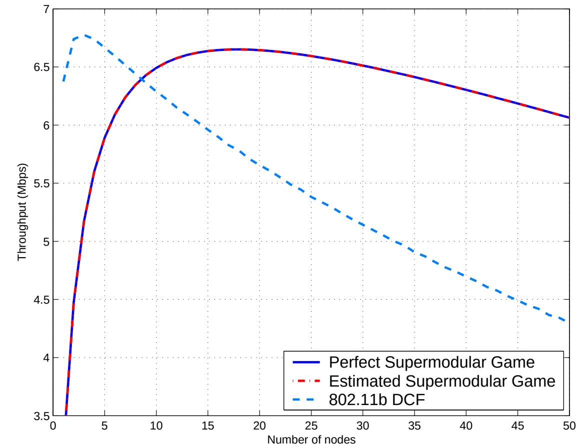



**Figure 2: Throughput comparison between supermodular game based design and 802.11b DCF with $CW_{\min} = 32$ and $CW_{\max} = 256$.**

the number of nodes in the network is small, idle sense achieves a higher throughput than both the game based design and 802.11b DCF. Game based design performs worse in this case because the approximation used in (11) is not accurate when the number of node is small. The performance of game based design can be improved by using equilibrium selection algorithm. As the number of users increases, both idle sense and game based design perform fairly close to the optimal throughput. They achieve a much higher throughput than DCF. This also indicates that when the number of users is large, equilibrium selection is not necessary as the achieved throughput by game based design is already very close to the optimal throughput.

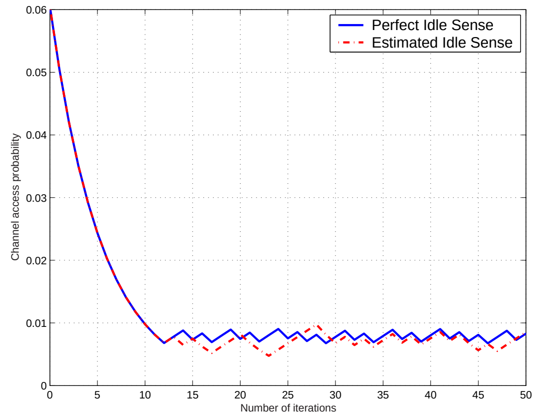
Figure 5 compares the short-term fairness of different protocols using Jain fairness index [14] for normalized window sizes that are multiples of the number of wireless nodes. All parameters are the same as in Figure 3. We see that both idle sense and game based design provide much better short-term fairness than 802.11b as in both protocols wireless nodes have roughly the same contention window size.

## 5.3 Equilibrium Selection

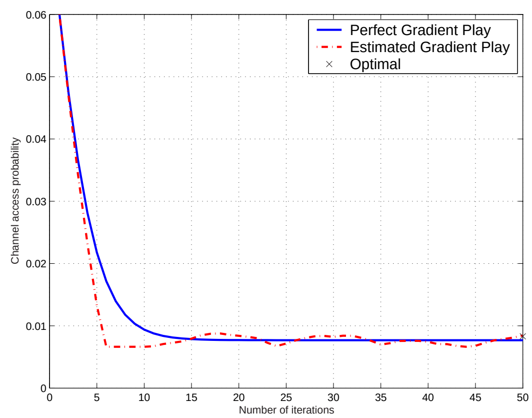
Finally, we check the equilibrium selection algorithm described in Section 4.3. We consider a network of 5 nodes. The gradient play (18) for the game model with utility (12) is simulated, where $\xi = 0.1622$ and the stepsize $\epsilon_i(t) = 0.02$. We assume perfect contention measure signals and we decide that the inner loop convergence is attained if $\|\mathbf{p}(t+1) - \mathbf{p}(t)\|_2 \leq 3 \times 10^{-4}$. Figure 6 compares the dynamics with different $\alpha$ values in utility (12). We see that the inner loop convergence rate increases by increasing $\alpha$, while the outer loop convergence rate decreases by increasing $\alpha$.

# 6. CONCLUSIONS

We have presented a game-theoretic model to capture the distributed nature of contention control and the contention/interaction among wireless nodes with contention-based medium access. This presents a unique perspective to understand existing medium access protocols, and a systematic design methodology for medium access control. Several examples have been given on how to design random access games from reverse-engineering and forward engi-




(a) Idle sense




(b) Game model

**Figure 3: Dynamics of idle sense and the game with utility function (12) with gradient play (18) in a network of 20 nodes, where $\xi = 0.1622$ and $\alpha = 2$, and the stepsize is chosen to be $\epsilon_i(t) = 0.02$.**

neering. Simulation results have shown that, with appropriately designed game models, game based protocols can achieve superior performance over the IEEE 802.11 DCF, and comparable performance as existing protocols with the best performance in literature.

## 7. REFERENCES


[1] Wireless LAN media access control (MAC) and physical layer (PHY) specifications. IEEE Standard 802.11, 1999.

[2] R. Abraham, J. E. Marsden, and T. Ratiu. *Manifolds, Tensors, Analysis, and Applications*. Springer-Verlag, 2 edition, 1988.

[3] G. Bianchi. Performance analysis of the IEEE 802.11 distributed coordination function. *IEEE J. Select. Areas Commun.*, 18(3):535–547, Mar. 2000.

[4] K. C. Border. *Fixed Point Theorems with Applications to Economics and Game Theory*. Cambridge University Press, 1985.

[5] V. S. Borkar and A. A. Kherani, Random access in wireless

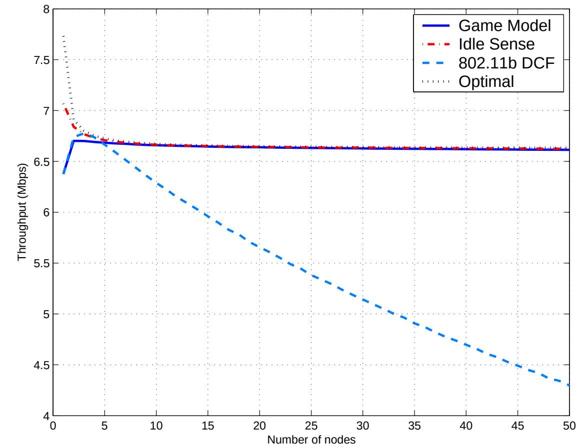




**Figure 4: Throughput comparison between idle sense and game based design with utility function (12) in a network of 20 nodes.**

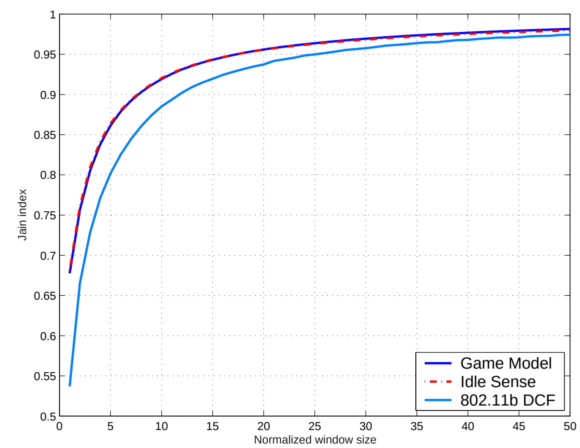



**Figure 5: Fairness comparison between idle sense and game based design with utility function (12) in a network of 20 nodes.**

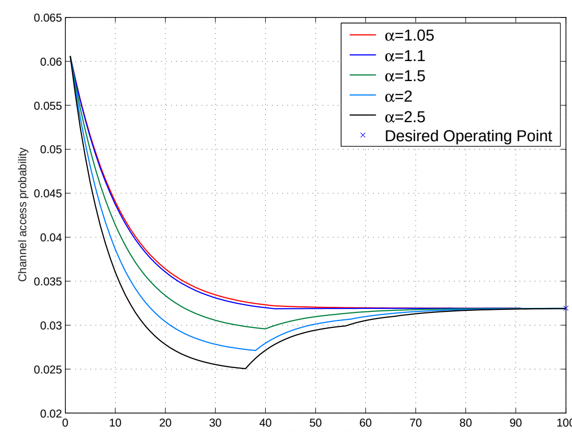



**Figure 6: Dynamics of the game with utility function (12) using equilibrium selection in a network of 5 nodes, where $\xi = 0.1622$ and the stepsize $\epsilon_i(t) = 0.02$. Different $\alpha$ in (12) are compared.**

ad hoc networks as a distributed game, *Proceedings of WiOpt*, March 2004.

[6] M. Čagalj, S. Ganeriwal, I. Aad and J. P. Hubaux, On selfish behavior in CSMA/CA networks, *Proceedings of IEEE Infocom*, March 2005.

[7] L. Chen, S. H. Low, and J. C. Doyle. Random access game and medium access control design. Caltech CDS, Tech. Rep., Mar. 2006. `http://www.cds.caltech.edu/~chen/papers/ramac.pdf`.

[8] T. Cui, L. Chen, S. H. Low, and J. C. Doyle. Game-theoretic framework for medium access control. Caltech CDS, Tech. Rep., Aug. 2007. `http://www.its.caltech.edu/~taocui/rag.pdf`.

[9] S. D. Flam. Equilibrium, evolutionary stability and gradient dynamics. *International Game Theory Review*, 4(4):357–370, Dec. 2002.

[10] D. Fudenburg and J. Tirole. *Game Theory*. MIT Press, 1991.

[11] M. Heusse, F. Rousseau, R. Guillier, and A. Dula. Idle sense: An optimal access method for high throughput and fairness in rate diverse wireless LANs. In *Proc. of ACM Sigcomm*, pages 121–132, Aug. 2005.

[12] R. A. Horn and C. R. Johnson. *Matrix Analysis*. Cambridge University Press, 1985.

[13] C. Hu and J. C. Hou. A novel approach to contention control in IEEE 802.11e-operated WLANs. In *Proc. of IEEE Infocom*, pages 1190–1198, May 2007.

[14] R. Jain, D. Chiu, and W. Hawe. A quantitative measure of fairness and discrimination for resource allocation in shared computer systems. DEC Research Report TR-301, 1984.

[15] Y. Jin and G. Kesidis, Equilibria of a noncooperative game for heterogeneous users of an Aloha networks, *IEEE Communication Letters*, 6(7):282-284, July 2002.

[16] F. P. Kelly, A. K. Maulloo and D. K. H. Tan, Rate control for communication networks: Shadow prices, proportional fairness and stability, *Journal of Operations Research Society*, 49(3):237-252, March 1998.

[17] S. H. Low and D. E. Lapsley, Optimal flow control, I: Basic algorithm and convergence, *IEEE/ACM Transactions on Networking*, 7(6):861-874, December 1999.

[18] S. Kunniyur and R. Srikant, End-to-end congestion control schemes: Utility functions, random losses and ECN marks, *IEEE/ACM Trans. on Networking*, 11(5):689-702, 2003.

[19] R. La and V. Anantharam. Utility based rate control in the internet for elastic traffic. *IEEE/ACM Trans. Networking*, 10(2):271–286, Apr 2002.

[20] J. W. Lee, A. Tang, J. Huang, M. Chiang, and A. R. Calderbank. Reverse-engineering MAC: A non-cooperative game model. *IEEE J. Select. Areas Commun.*, 25(6):1135–1147, Aug. 2007.

[21] A. B. MacKenzie and S. B. Wicker, Stability of multipacket slotted Aloha with selfish users and perfect information, *Proceedings of IEEE Infocom*, April 2003.

[22] D. M. Topkis. Equilibrium points in nonzero-sum n-person submodular games. *SIAM J. of Contr. and Optim.*, 17(6):773–787, 1979.

# APPENDIX

## A. PROOF OF THEOREM 14

PROOF. By equation (18), we have

$$\begin{aligned}
&\|\mathbf{p}(t+1)-\mathbf{p}^*\|_2^2\\
=&\sum_{i\in\mathcal{N}}|[p_i(t)+\epsilon(t)(U_i'(p_i(t))-C_i(\mathbf{q}_i(\mathbf{p}(t))))]^{\mathcal{S}_i}-p_i^*|^2\\
\le&\sum_{i\in\mathcal{N}}|p_i(t)+\epsilon(t)(U_i'(p_i(t))-C_i(\mathbf{p}(t)))-p_i^*|^2\\
\le&\|\mathbf{p}(t)-\mathbf{p}^*\|_2^2+2\epsilon(t)\sum_i(p_i(t)-p_i^*)(U_i'(p_i(t))-C_i(\mathbf{p}(t)))\\
&+\epsilon^2(t)\sum_i(U_i'(p_i(t))-C_i(\mathbf{p}(t)))^2\\
\overset{(a)}{\le}&\|\mathbf{p}(t)-\mathbf{p}^*\|_2^2+2\epsilon(t)\sum_i(p_i(t)-p_i^*)(U_i'(p_i(t))-U_i'(p_i^*))\\
&-2\epsilon(t)\sum_i(p_i(t)-p_i^*)(C_i(\mathbf{p}(t))-C_i(\mathbf{p}^*))\\
&+\epsilon^2(t)\sum_i(U_i'(p_i(t))-C_i(\mathbf{p}(t)))^2,
\end{aligned} \tag{31}$$

where we have used $C_i(\mathbf{p}(t))$ to denote $C_i(\mathbf{q}_i(\mathbf{p}(t)))$. In (a), we use the fact that $U_i'(p_i^*)=C_i(\mathbf{p}^*)$ at the nontrivial Nash equilibrium. By mean value theorem, we find

$$\begin{aligned}
&\sum_i(p_i(t)-p_i^*)(U_i'(p_i(t))-U_i'(p_i^*))\\
=&\sum_i U_i'(\tilde{p}_i)(p_i(t)-p_i^*)^2\le-\mu\|\mathbf{p}(t)-\mathbf{p}^*\|_2^2,
\end{aligned} \tag{32}$$

where $\tilde{p}_i\in\{p_i|p_i=\gamma p_i(t)+(1-\gamma)p_i^*,\ \gamma\in[0,1]\}$. Define a scalar function $f(\mathbf{p})=(\mathbf{p}(t)-\mathbf{p}^*)^TC(\mathbf{p})$. By mean value theorem, we have

$$\begin{aligned}
f(\mathbf{p}(t))-f(\mathbf{p}^*)=&(\mathbf{p}(t)-\mathbf{p}^*)^T\mathbf{J}^C(\tilde{\mathbf{p}})(\mathbf{p}(t)-\mathbf{p}^*)\\
\ge&\lambda_{\min}(\mathbf{J}^C)\|\mathbf{p}(t)-\mathbf{p}^*\|_2^2.
\end{aligned} \tag{33}$$

We also have

$$\begin{aligned}
&\sum_i(U_i'(p_i(t))-C_i(\mathbf{p}(t)))^2\\
=&\sum_i(U_i'(p_i(t))-U_i'(p_i^*)+C_i(\mathbf{p}^*)-C_i(\mathbf{p}(t)))^2\\
\le&2\sum_i(U_i'(p_i(t))-U_i'(p_i^*))^2+2\sum_i(C_i(\mathbf{p}(t))-C_i(\mathbf{p}^*))^2\\
\overset{(a)}{\le}&2\chi^2\|\mathbf{p}(t)-\mathbf{p}^*\|_2^2+2\sum_i(\mathbf{J}_i^C(\tilde{\mathbf{p}}^i)(\mathbf{p}(t)-\mathbf{p}^*))^2\\
\le&2\chi^2\|\mathbf{p}(t)-\mathbf{p}^*\|_2^2+2(\sum_i\max_j\ J_{ij}^C(\tilde{\mathbf{p}}^i)^{\ 2})\|\mathbf{p}(t)-\mathbf{p}^*\|_2^2\\
\le&2(\chi^2+|\mathcal{N}|M)\|\mathbf{p}(t)-\mathbf{p}^*\|_2^2,
\end{aligned} \tag{34}$$

where $(a)$ comes from mean value theorem. Substituting (32)-(34) into (31), we obtain

$$\begin{aligned}
&\|\mathbf{p}(t+1)-\mathbf{p}^*\|_2^2\le\\
&(1-2\epsilon(t)(\mu+\lambda_{\min}(\mathbf{J}^C)-\epsilon(t)(\chi^2+|\mathcal{N}|M)))\|\mathbf{p}(t)-\mathbf{p}^*\|_2^2.
\end{aligned} \tag{35}$$

Therefore, if $\mu+\lambda_{\min}(\mathbf{J}^C)>0$ and $\epsilon(t)<\frac{\mu+\lambda_{\min}(\mathbf{J}^C)}{\chi^2+|\mathcal{N}|M}$, $\mathbf{p}(t)$ converges to $\mathbf{p}^*$ geometrically. □