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Robustness, Optimization, and Architectures

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This paper will review recent progress on developing a unified theory for complex networks from biological systems and physics to engineering and technology. Insights into what the potential universal laws, architecture, and organizational principles are can be drawn from three converging research themes: growing attention to complexity and robustness in systems biology, layering and organization in network technology, and new mathematical frameworks for the study of complex networks. We will illustrate how tools in robust control theory and optimization can be integrated towards such unified theory by focusing on their applications in biology, physics, network design, and electric grid.

Keywords: Robust control theory, complex networks, systems biology, turbulence, layering as optimization, power grid, optimal power flow, glycolysis, wireless networks

1. Introduction

Hard limits on measurement, prediction, communication, computation, decision, and control, as well as the underlying physical energy and material conversion mechanisms necessary to implement these abstract processes are at the heart of modern mathematical theories of systems in engineering and science (often associated with names such as Shannon, Poincaré, Turing, Godel, Bode, Wiener, Heisenberg, and Carnot). They form the foundation for rich and deep subjects that unfortunately remain largely fragmented and incompatible, even as the tradeoffs between these limits are of growing importance in building integrated and sustainable systems. An essential research direction is an integrated theory based on optimization that deals systematically with uncertainty, robustness, and risk in complex systems. For a relatively nontechnical discussion of these issues, see [2] and references therein.

Tools that are commonly used in optimization as well as in systems and control theory may provide a good foundation for moving toward such an integrated theory. The new theory presented herein seeks to build upon and integrate decades of research in pure and applied mathematics with engineering, including robust control theory, dynamical systems, information theory, numerical analysis, operator theory, real algebraic geometry, computational complexity theory, duality and optimization, and semi-definite programming, motivating new interactions between these diverse areas. We illustrate the ways in which these ideas have been used to provide a fresh perspective on problems in a number of diverse areas and demonstrate how this new approach allowed new progress towards long-standing problems in biology, physics, and engineering. We focus on their application in a series of well-studied domain-specific problems such as, glycolytic oscillations in metabolic networks, turbulence in wall-bounded flows, network design, and optimization of power flow in electric grids.

Both engineering and evolution are constrained by tradeoffs between efficiency and robustness, however

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these tradeoffs are rarely formalized in biology. We chose glycolysis as a first case study as it not only has interesting dynamics (oscillations) but also offers a rich history of experimental and theoretical literature [55]. Despite extensive experimental and modeling studies since 1965 [57], the question of whether glycolytic oscillations are beneficial or simply an evolutionary accident remained unresolved. We developed a simple model and used control theoretic analysis to suggest a third alternative: Oscillations are the inevitable consequence of tradeoffs between metabolic overhead and robustness to disturbances, as well as the interplay between feedback control and necessary autocatalysis of network products [12]. Our model is now also the simplest (with only two states) example of a system with a right-half plane zero.

While the components differ and the system processes are far less integrated, advanced technology's complexity is now approaching that of biology and there are striking similarities at the level of organization, architecture, and the role of layering, protocols, and feedback control in structuring complex multi-scale modularity. When examining network problems in a robust optimization framework, it becomes readily apparent that the convergence within/between biology/technology is not accidental, but follows necessarily from their universal system requirements to be fast, efficient, adaptive, evolvable, and robust to perturbations in their environment and component parts, see for example [2, 22, 43, 65]. The cell is highly constrained and massively autocatalytic, yet it maintains very strict layering with specific signaling and regulatory proteins and RNAs. The success of communication networks has largely been a result of adopting this sort of layered architecture.

The layered hourglass architecture is one of the most fundamental structural features of networks such as the internet. Each layer in the protocol stack hides the complexity of the layer below and provides a service to the layer above. While the general principle of layering had long been recognized as one of the key reasons for the enormous success of the Internet, until recently there was little quantitative and systematic understanding of layered network architectures and protocol stacks. Major progress in this direction has been enabled by the application of optimization and control theory explicitly addressing network layering and dynamics. The optimization decomposition framework serves as a top-down approach to systematically carry out the protocol layering process and explicitly trade off design objectives. We will illustrate this idea in cross-layer design of wireless networks. Our current theory integrates three functions: congestion control; routing and scheduling in transport; and network and link layers, into a coherent framework.

The tools from optimization have also allowed major progress in the problem of optimal power flow for the electric grid. The Optimal Power Flow (OPF) problem has been extensively studied since the work of Carpentier in 1962 [11], but in general it is not convex and thus not computationally tractable. Some recent work attempted to reformulate the problem into more tractable realizations. For example, references [38, 39] considered radial distribution systems as conic programming problems. The problem was first formulated as a semi-definite program by Bai et al. [4] but they did not provide a relationship between the semi-definite relaxation and the original OPF problem. Significant progress was made when Lavaei et al. in [45, 47] proved a sufficient condition under which there exists a semi-definite convex relaxation that is equivalent to the OPF problem. We discuss how these results and several extensions have not only identified conditions under which the OPF problem can be solved efficiently but also provided insight into the underlying structure of power networks.

Finally, we introduce some results in turbulence, which is a problem in physics that merges the natural with the engineering world in many applications. Wall-turbulence represents a persistent source of inefficiency in many applications from flight and other transportation applications to flow in pipelines. It has remained one of the enduring unsolved problems in physics, but its essential role in the efficiency of engineered systems makes it an important topic of continuing research. In canonical flow examples, such as plane Couette flow, the shapes of the laminar and turbulent velocity profiles are well known (as depicted in Fig 1). However, the underlying mechanisms involved in creating the "S" shaped (blunted) turbulent profile remain unknown. In our work, we have rigorously connected commonly observed flow features to the creation of the turbulent mean velocity profile. We have shown that the so-called 2D/3C model along with a robust control framework captures the blunting of the profile along with other salient features of fully developed turbulent plane Couette flow [30]. Our analysis also illuminated an interesting interaction between energy amplification and the increased velocity gradient at the wall associated with the turbulent profile. Essentially, although the input-output amplification monotonically decreases with increasing forcing amplitude, the velocity profiles become increasingly more blunted. As in the biological system previously discussed, there is likely a tradeoff between the linear amplification mechanisms and non-linear blunting mechanisms that determine important features of the turbulence-like phenomena modeled by the 2D/3C system. This tradeoff appears to have important implications for flow control techniques that target skin friction or the mean profile.

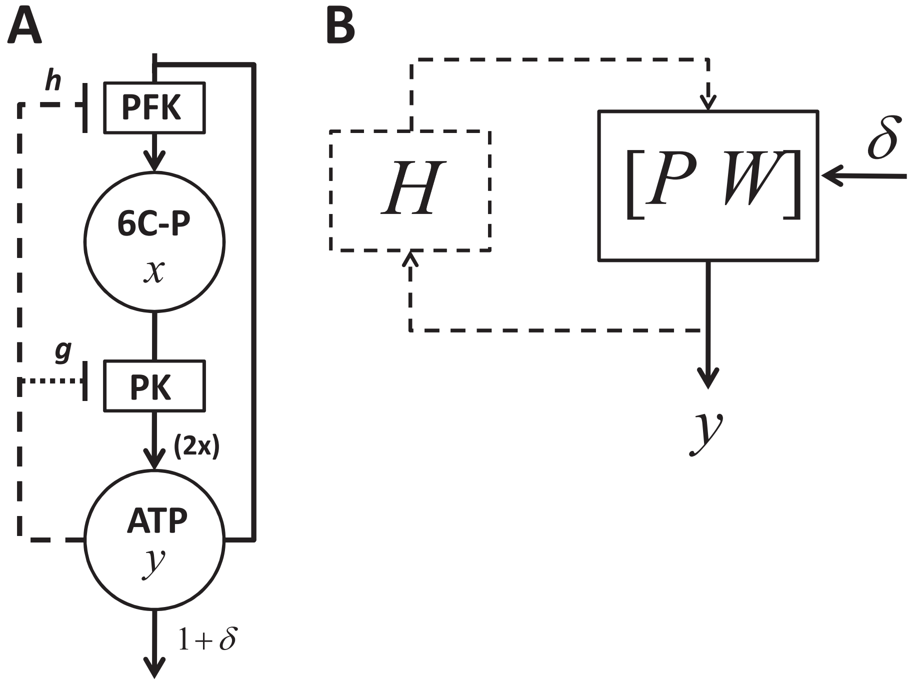

Fig. 1. (A) Diagram of the two state glycolysis model. ATP along with constant glucose input produce a pool of intermediate metabolites, which then produces two ATPs. ATP inhibits both reactions. (B) Control-theoretic diagram of the same system. The system without inhibition/feedback is the Plant while ATP inhibition acts as the Controller.

2. Hard Tradeoffs and Glycolytic Oscillations

In [12], we use a simple two-state model of yeast glycolysis, shown in Fig. 1, to explicitly derive hard tradeoffs between metabolic overhead, network fragility, and oscillations. The goal of this study was not only to explain the existence of oscillations and formalize tradeoffs, but also to introduce and interest experts in biology in the potential applications of control theory.

Glycolysis is the cell's energy plant, consuming glucose to generate Adenosine Triphosphate (ATP), the energy currency used throughout the cell. It is also autocatalytic as ATP must be consumed to power the early reactions. We propose a minimal system incorporating ATP autocatalysis:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ -q \end{bmatrix} \frac{2y^a}{1+y^{2h}}}_{PFK} + \underbrace{\begin{bmatrix} -1 \\ q+1 \end{bmatrix} \frac{2kx}{1+y^{2g}}}_{PK} + \underbrace{\begin{bmatrix} 0 \\ -1 \end{bmatrix} (1+\delta)}_{Consumption} \quad (1)$$

In the first reaction in (1), the enzyme phosphofructokinase (PFK) consumes q molecules of y (ATP) with inhibition by ATP. We lump the intermediate metabolites into variable x. In the second reaction, pyruvate kinase (PK) produces $q+1$ molecules of y at rate k for a net (normalized) production of 1 unit, which is consumed by the rest of the cell. Some studies suggest that ATP also inhibits PK. We model the feedback strengths on PFK and PK as h and g, respectively, and the cooperativity of the autocatalytic reaction is modeled by a. We first take $g = 0$ as this regulation is typically not modeled with the exception of [62].

Linear stability and steady state analysis revealed a trade-off between minimizing steady state error and maintaining stability. When the feedback gain h is chosen to minimize the steady state error, (1) hits a hard stability limit and the system enters sustained oscillations (supercritical Hopf bifurcation). Thus, our model suggests that oscillations have no direct purpose but are side effects of hard tradeoffs crucial to the functioning of the cell and can be avoided at some expense.

Next we use the sensitivity function to derive more fundamental tradeoffs that capture the transient/dynamic response to disturbances, depend only on very basic properties of autocatalytic and control feedbacks, and are independent of neglected details and model simplifications. The sensitivity function S measures the system's response to disturbances and ideally should be small, but this function has a lower bound. When $q > 0$, $S(s)$ has a right half-plane zero at $z = k/q$. We further show that when $a > 0$, the open loop plant has an unstable pole.

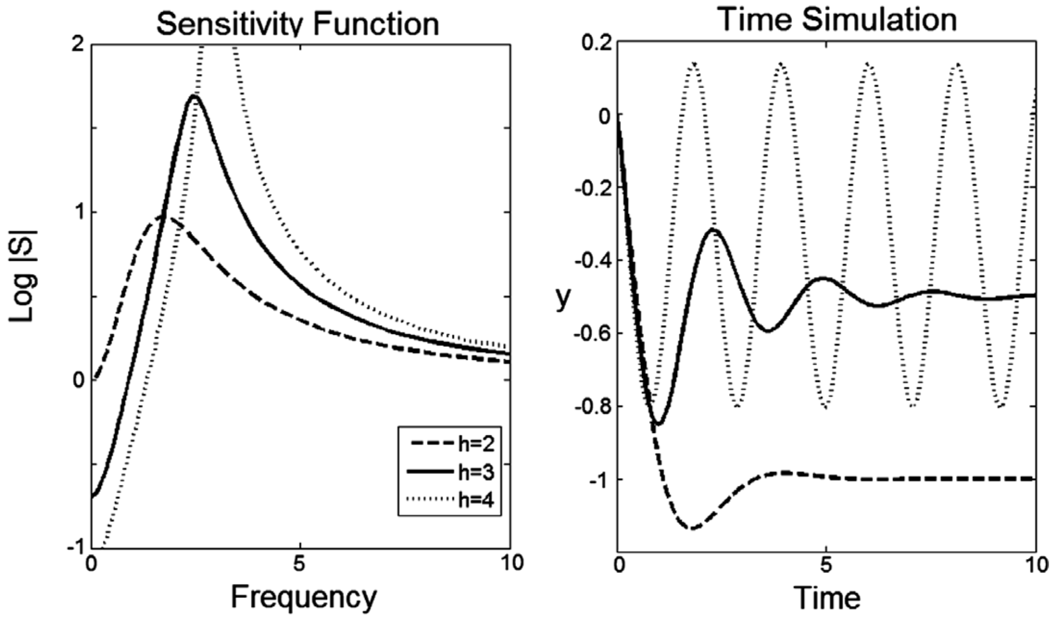

Fig. 2. Log Sensitivity $\ln|S(j\omega)|$ (left) without ATP feedback on PK ($g = 0$) and step response to change in demand δ (right). The integral of $\ln|S(j\omega)|$ is constrained by (2) for all h. Higher h gives better steady state error ($S(0)$) with more oscillatory transient. The system goes into sustained oscillations for large h.

Therefore, when there is autocatalysis ($q > 0, a > 0$), we can show that:

$$\frac{1}{\pi}\int_0^\infty \ln|S(j\omega)|\frac{z}{z^2+\omega^2}d\omega \geq \max\{0, \ln|\frac{z+p}{z-p}|\} \tag{2}$$

with finite $z > 0$ and $p > 0$ as defined above (this is a variant of Bode's Integral Formula that applies for degree < 2. One particular interest for control theorists is that our model happens to be the simplest system we know of that has a right-half plane zero). Like energy and materials, robustness can be gratuitously wasted when the inequality is large, is at best conserved, and must trade off with metabolic efficiency. As shown in Fig. 2, higher h reduces steady state error (corresponding to $S(0)$) but with more transient fluctuations (corresponding to higher peak $||S(j\omega)||_\infty$). Eventually, the system oscillates at the frequency where $S(j\omega) \to \infty$. With no autocatalysis ($q = 0$), the tradeoff disappears and the bound in (2)$\to 0$. Zero steady state error with stability is then possible by taking $h \to \infty$.

The low pass filter $|\frac{z+p}{z-p}|$ constrains the waterbed effect to frequencies below $\omega = z$. Since $z = k/q$, high k and low q are desirable. Low k should worsen transient performance and we confirmed this experimentally. We also provide evidence that ATP feedback on PK (g) plays an important role in stabilization against noise in enzyme levels. For experimentally observed values of a, q, and h, $g > 0$ is necessary to simultaneously maintain acceptable steady state error and stability for all $k > 0$. This allows the cell to fine tune the performance by tuning k via transcriptional or translational control of enzyme levels, at the same time allowing robustness to noise in gene expression. From an engineering perspective, this is a remarkably clever control architecture, and the presence of $g > 0$ suggests that at least in this case evolution favors higher complexity in exchange for this kind of flexibility and robustness.

3. Network Design and Layering as Optimization

The last decade has witnessed the development of a framework that views the network as solving an appropriately defined optimization problem, ranging from the classical network flow problems often formulated as linear programs [1] to the recent and more general Network Utility Maximization (NUM) problem [49, 42]. By setting up an abstract framework of optimizing a global objective function subject to all the physical and resource constraints in the network, network layering can be understood as a decomposition of the problem into decentralized subproblems, and various protocol layers are regarded as carrying out asynchronous, distributed computation to implicitly solve this global optimization problem. Different layers iterate on different subsets of the decision variables using local information to achieve individual optimality. Taken together, these local algorithms attempt to achieve a global objective.

Such a theory facilitates both understanding and design of network architectures. In reverse engineering a given network, identifying an underlying optimization problem being solved will expose the interconnection between protocol layers and can be used to rigorously study performance tradeoffs in protocol layering as different ways to distribute a centralized computation. In the context of design or forward engineering, this framework formalizes the common practice of breaking down the desired system into simpler modules and allows us to systematically carry out the layering process by providing guidance on how to structure and modularize different functions, suggesting interfaces among these functions and the information that must be exchanged among them, and making transparent the interactions among different components and their global behavior.

To illustrate the idea of protocol layering as an optimization decomposition, we briefly discuss cross-layer design in wireless networks [14, 48]. Consider an ad hoc wireless network with a set N of nodes and a set L of logical links. We assume some form of power control so that each logical link l has a fixed capacity c_l when it is active. The feasible rate region at the link layer is the convex hull of the corresponding rate vectors of independent sets of the conflict graph. Let Π denote the feasible rate region. Let x_i^k be the flow rate generated at node i for destination k. We assume there is a queue for each destination k at each link (i,j). Let f_{ij}^k be the amount of capacity of link (i,j) allocated to the flows on that link for final destination k. Consider the following generalized NUM in variables $x_s \geq 0, f_{ij}^k \geq 0$:

$$\begin{array}{ll} \text{maximize} & \sum_s U_s(x_s) \\ \text{subject to} & x_i^k \leq \sum_{j:(i,j)\in L} f_{ij}^k - \sum_{j:(j,i)\in L} f_{ji}^k, \forall i,j,k \\ & f \in \Pi \end{array} \quad (3)$$

where x_s is a shorthand for x_i^k. The first constraint is flow balance equation: the flow originated from node i for final destination k plus total capacity allocated for transit flows through node i for final destination k should be no more than the total capacity going out of node i for final destination k. The second constraint is on schedulability. The dual problem of (3) decomposes into the following two subproblems:

$$\max_{\lambda \geq 0} D_1(\lambda) := \max_{x_s \geq 0} \sum_s (U_s(x_s) - x_s \lambda_s) \quad (4)$$

$$\max_{\lambda \geq 0} D_2(\lambda) := \max_{f_{ij}^k \geq 0} \sum_{i,k} \lambda_i^k \sum_j (f_{ij}^k - f_{ji}^k) \text{ s. t. } f \in \Pi \quad (5)$$

The first subproblem is congestion control where λ_s is the congestion price locally at source $s = (i,k)$. The second subproblem corresponds to a joint problem of multi-path routing and allocation of link capacities. Thus, by dual decomposition, the flow optimization problem decomposes into separate local optimization problems of transport, network and physical layers which interact through congestion prices. This suggests a system architecture for a wireless ad hoc network that uses hop-by-hop routing and the back pressure flow control first studied by Tassiulas and Ephremides [61]. A Wi-Fi implementation of this architecture has been done in [64] and performs significantly better than the current system.

4. Convexification of the OPF Problem

For a given grid configuration (interconnection topology), with supply and demand nodes, the basic OPF problem is concerned with determining how much power each generator should supply to minimize a cost function in the face of system constraints on physical phenomena such as generator capacities and transmission line losses, see e.g. [34, 53] for a survey of OPF problems and solution methods. The non-linear coupling between power and voltage (magnitude) makes the general OPF problem non-convex and non-deterministic polynomial time hard [45, 66]. Given the practical importance of the problem there has been a lot of research into efficient solution algorithms, and historically the most common solution techniques have relied on linear programming techniques [60, 3]. Researchers have also proposed a number of relaxations to make the OPF problem more tractable. Jabr made progress toward convexifying the problem using a conic quadratic model of radial distribution systems [38] and meshed networks [40] and demonstrated an efficient solution method to these problems using an interior point method for convex conic quadratic programming. However, the meshed network generalization included some non-convex constraints. The problem has also been formulated as a semi-definite program (SDP) by Bai et al. [4, 5].

Lavaei et al. made significant progress toward this difficult problem again using tools from optimization and systems theory. They showed that the OPF is equivalent to a semi-definite program with a rank-constraint and provided a sufficient condition that makes a convex rank relaxation exact [44, 46]. The procedure is as follows: the voltage and power constraints in the OPF problem, which are quadratic in nature, are transformed into linear matrix inequalities (LMIs). This yields the rank constrained reformulation. The solution of the convex SDP is obtained as the Lagrangian dual to a rank relaxation of this equivalent reformulation provides a lower bound to the OPF solution. In general, the lower bound may not be tight (nonzero duality gap). However, a sufficient condition that guarantees zero duality gap and optimality of the resulting OPF solution was derived by [44]. Specifically, this dual is a convex SDP that can be solved efficiently and the duality gap is zero if and only if the solution of a certain LMI inequality in the dual problem has specific properties. The sufficient condition guarantees that a globally optimal solution of OPF is recoverable from the dual problem's optimal solution. Further study showed that the sufficient condition always holds for resistive power networks and provided strong evidence that the method works for most practical circuits [46]. Specifically, an exact (globally optimal) solution was obtained for all of the IEEE benchmark systems archived at [52]. The formulation is very general and has been extended to OPF problems with arbitrary convex cost functions [59].

Methods from systems theory and optimization have also been used to develop a computationally efficient solution for the problem through the addition of multiple controllable phase shifters (additional power electronics that allow phase adjustments along the power lines). These devices simplify the verification of the duality gap. In fact, if the load is allowed to be over-satisfied and a sufficient number of phase shifters are added to the network, the duality gap can be eliminated altogether. The implication of this result is that any power transmission network, regardless of its topology, can be augmented using phase shifters to guarantee the polynomial time solvability of OPF over the space of all possible values of loads, physical limits and convex cost functions [59].

The formulation can also be extended to integrate simple charge/discharge dynamics of energy storage distributed over the network [27]. The inclusion of these energy storage dynamics leads to a finite-horizon optimal control problem that enables optimization of (dynamic) power allocation over time in addition to the static allocation over the network. This framework allows the analysis of systems with time-varying energy sources that can be used to model renewable energy sources such as solar and wind power. It is widely accepted that storage technologies will be an integral part of systems with a high penetration of renewable resources [7, 63].

5. A Tractable Model for Wall-Bounded Turbulent Flows

In contrast to many of the problems in natural and engineered systems, it is a lack of robustness that plays a fundamental role in wall turbulence. This characteristic is related to the long accepted potential of wall-bounded shear flows to produce large amplification of disturbances. Even stable linear operators associated with these systems, the so-called linearized Navier Stokes equations (LNS), experience temporal growth which exceeds the size of the initial disturbance by large factors (e.g., $O(R^2)$ where R is the Reynolds number) [10, 33, 54]. They have an input-output response that grows as a function of the Reynolds number e.g., $O(R^{\frac{3}{2}})$ or $O(R^3)$ depending on the nature of the input [6, 26].

The control theoretic interpretation of this behavior is that transition is not a stability problem but rather a robustness issue. The large growth/amplification is merely a "high gain", which is common in systems such as these because their underlying linear operators are non-normal. The small gain theorem provides mathematical measure of the system's potential growth (in a normed sense) and the amount of permissible "uncertainty", such as modeling errors or external disturbances, before a system is unable to maintain stability or performance. The bound on the uncertainty is inversely proportional to the maximal response of the system. Therefore, as the upper bound on the amplification increases, the amount of uncertainty at the frequencies corresponding to the maximal response must be reduced. In a wall-bounded shear flow, an increase in the Reynolds number decreases the amount of uncertainty required for the system's performance to degrade from that of streamlined laminar flow to turbulent flow decreases because of the aforementioned $O(R^{\frac{3}{2}})-O(R^3)$ disturbance amplification. In these terms, the main driving factor in the transition to turbulence can be viewed as a robustness issue in which the stability/performance of the laminar flow is not robust to disturbances (uncertain parameters). The observation that transition can be delayed in experiments with extremely carefully controlled conditions (and equivalently numerical accuracy in simulations) comes directly from the fact that the magnitude of the system norm (and the associated transient energy growth and/or input-output amplification) increases with Reynolds number.

This purely linear analysis can illuminate many issues, however the one fundamental flow feature that linear models are unable to capture is the change in the mean velocity profile as the flow transitions from laminar to turbulent. A non-linear model is required to capture this momentum transfer. Unfortunately, the full Navier Stokes (NS) equations for incompressible flows are known to be analytically intractable [37]. So, we selected a simple, mathematically tractable non-linear representation with modeling assumptions based on experimental observations and linear analysis, in particular the dominance of streamwise infinitely elongated modes (as depicted in Fig. 3). This idea is supported by a growing body of work that points to characterization of wall-bounded shear flows in terms of dynamically significant coherent structures, the most common of which show streamwise and quasi-streamwise alignment, see for example [32, 35, 51]. Linear analysis reinforces this notion in that streamwise constant perturbations to the LNS also produce the largest input-output response [6, 41, 21]. Also, streaks of streamwise velocity naturally arise from the set of initial conditions that produce the largest energy growth [10, 25], namely streamwise vortices.

Our work uses this streamwise constant projection of the NS, the so-called 2D/3C model, to rigorously connect

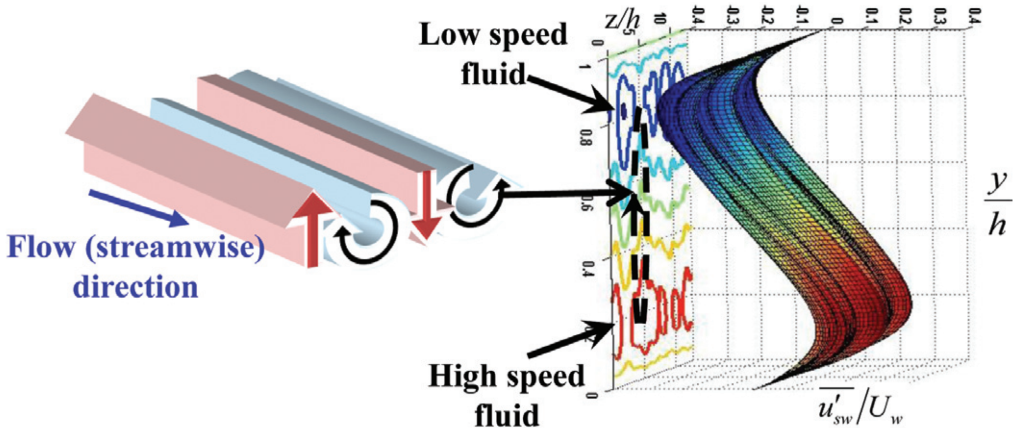

Fig. 3. Cartoon of streamwise elongated structures with surface plot of 2D/3C simulation results.

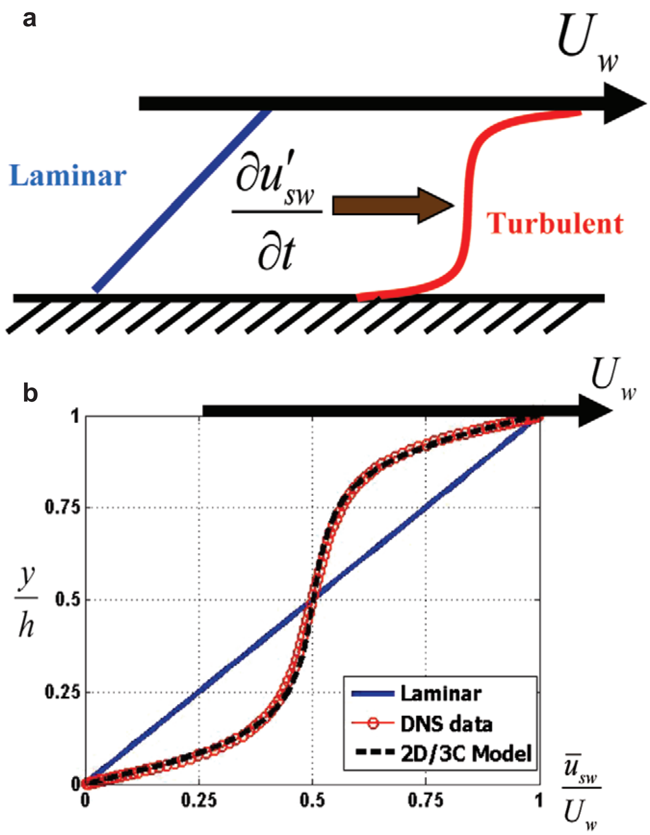

Fig. 4. (a) The laminar and blunted or "S" shaped turbulent velocities profile for plane Couette flow driven by motion (at velocity U_W) of the top plate. (b) The 2D/3C simulation captures the change in profile from a laminar initial condition to a turbulent mean velocity profile.

the observed flow features to the creation of the turbulent mean velocity profile. A two-dimensional rendering of the shapes of the linear laminar profile and turbulent profiles are illustrated in Fig. 4(a). We show that the 2D/3C model along with a robust control framework captures the blunting of the profile along with other salient features of fully developed turbulent plane Couette flow [30]. The robust control framework employs small amplitude Gaussian noise forcing to simulate the 2D/3C model's response in the presence of disturbances, uncertainty and modeling errors. Fig. 4(b) shows the simulation results compared to experimentally verified direct numerical simulation data of a fully turbulent Couette flow. The model captures the change in mean velocity profile from the nominal laminar to the characteristic "S" shaped turbulent profile [28, 29]. A surface plot of the deviation from the laminar flow is shown in the left side of Fig. 3, where the direct relationship between the streamwise structures and the momentum transfer that leads to the "S" shape of the turbulent profile is illustrated through the dashed line on the surface plot.

The laminar flow solution is globally stable [8], which indicates that the flow state should always return to the streamlined laminar flow condition. The fact that the 2D/3C model is able to generate "turbulent-like" behavior under small-amplitude stochastic noise indicates that transition to turbulence in this model is likely a consequence of the laminar flow solution's lack of robustness (inability to maintain the maintain this flow condition) that is directly related to the large amplification discussed above.

A simple cross-stream model of large-scale streamwise elongated structures non-linearly coupled through a steady-state 2D/3C streamwise momentum equation isolates important mechanisms involved in determining the shape of the turbulent velocity profile. We use cross-stream components (i.e. those representing a cross-section of the three-dimensional flow) to create a model of structures consistent with the experimentally and numerically observed flow features. This is used as input to develop a forced 2D/3C streamwise velocity equation, i.e. we study a model of the velocity component that describes the shape of the mean profile. This input-output response from a spanwise/wall-normal (z-y) plane stream function to the streamwise velocity component illuminates a strong relationship between 2D/3C nonlinear coupling in the streamwise velocity evolution equation and the momentum redistribution that produces features consistent with the mean characteristics of fully developed turbulence [27]. Isolating these momentum transfer mechanisms represents an important step in the development of flow control strategies because delaying the onset of turbulence and turbulence suppression are common goals in flow control applications.

The input-output analysis in this framework also illuminates an interesting interaction between energy amplification and the increased velocity gradient at the wall associated with the turbulent profile. Essentially, although the input-output amplification monotonically decreases with increasing forcing amplitude, the velocity profiles become increasingly more blunted. Thus, there is likely a tradeoff between the linear amplification mechanisms and non-linear blunting mechanisms that determine important features of the turbulence-like phenomena modeled by the $2D/3C$ model. This tradeoff may have important implications for flow control techniques that target skin friction or the mean profile.

6. Summary and Discussion

In this paper, we have summarized our research that has branched out substantially from mainstream control theory into other areas of engineering, biology and physics, but still maintains strong thematic contact with controls. We introduced the formalization of tradeoffs between efficiency and robustness in biology. The steady state tradeoff reveals that the observed oscillation in glycolysis is neither directly purposeful nor an evolutionary accident but a necessary consequence of autocatalysis and hard tradeoffs between fragility, efficiency, and complexity. We showed that nature has evolved a feedback structure that effectively manages these tradeoffs with flexibility to adapt to changes in supply and demand and robustness to noisy gene expression, at the cost of higher enzyme complexity. Consistent with engineering, complexity in biology is primarily driven by robustness, not minimal functionality [18].

The integral tradeoff in equation (2) is deeper and captures system dynamics. However, z and p still depend on phenomenological assumptions about autocatalysis (q and a) and enzyme efficiencies (k). This motivates further unification of control theory with thermodynamics and statistical mechanics and recent progress is encouraging [56]. It also motivates rethinking how biology overcomes the causality limit with various mechanisms that exploit predictable environmental fluctuations (e.g. circadian rhythms) or provide remote sensing (e.g. vision, hearing), both of which can greatly mitigate hard limits such as (2) [50].

Determining what is essential about the network-level convergence within biology and with technology requires a deeper understanding of protocols and their architecture. Our research on optimization decomposition as layering facilitates provides a promising framework to understand not just *what* works but *why* it works. It has also led to new theories of the internet and related networking technologies (e.g. [17]), and to new protocols that have been tested and deployed. We are expanding this framework to cleaner integration of congestion control, routing, scheduling, power control, and network coding [13–16, 19–20], and to more explicitly treat dynamics [45].

The discussion above and the extensive list of references in [17] have overwhelmingly demonstrated the value and promise of such an optimization based framework. However, fundamental issues remain to be addressed before this theory-based approach matures into a full-fledged framework for network design. One of the main research topics is developing a common analytical framework and language that integrates computation, communication, and control in complex network or networked systems across all protocol layers from physical layer to application layer and to dynamics over the network. The most exciting opportunity for use of these methods, however, is in fundamentally redesigning network architectures to clean slate architectures, where control and dynamical systems theory plays an integral role at the outset, rather than patch a leaky architecture when problems (e.g. congestion collapse) arise.

Optimization tools have also enabled significant progress in the optimal power flow (OPF) problem, which has been studied for about half a century and is notorious for its high nonconvexity. By formulating the OPF problem as a semi-definite program, we were able to solve the problem efficiently (under certain conditions) by solving the Lagrangian dual to a rank relaxation of an equivalent formulation for the OPF problem. We provided a sufficient condition for zero-duality gap between OPF and this convex dual. The sufficient condition might hold widely in practice based on the fact that physical quantities such as resistance, capacitance, and inductance, are all positive. Further work has demonstrated this zero-duality gap for a variety of test cases and a rich class of network topologies [9, 66, 46]. Our current results focused on cases with no uncertainties. Integration of uncertainties due to either intermittency in generation or fluctuations in demand is a subject of ongoing study [58].

Last but not least, turbulence can be viewed using a robust control framework. We have described how tools from robust control theory [23, 24] can be combined with a streamwise constant model to provide a framework for understanding some of the salient features of fully developed turbulence in plane Couette flow. The $2D/3C$ nominal model includes non-linear effects that are stressed by some researchers, while maintaining the mathematical properties associated with linear disturbance amplification. This framework also reveals an important tradeoff between linear and non-linear phenomena. Linear models provide important information about energy amplification and structural features in turbulent flows whereas a nonlinear model is required to capture the momentum transfer that produces a turbulent velocity profile. These types of tradeoffs are very common in engineering systems and understanding them provides important information in designing systems that are both safe and able to meet advanced performance requirements. Further understanding of this particular tradeoff may also provide important insight into the mechanisms associated with both transition and fully turbulent flow.

The universal hard limits on systems and their components have until recently been studied separately in fragmented domains of physics, chemistry, biology, communications, computation, and control, but a unified theory is needed and appears feasible. We have illustrated the power of robustness and optimization as an underlying mathematical framework to clarify biological and physical phenomena using classical case studies in turbulence and glycolysis. Conversely, these fields can motivate new theoretical directions [36]. The control theory and optimization framework also allows a coherent framework that integrates congestion control, routing and scheduling in network design and also reveals tractability of real-world problems such as the optimization of power grid.

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