Local Voltage Control in Distribution Systems: An Incremental Control Algorithm

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Abstract—Inverter-based local vol/var control forms a closed-loop dynamical system whereby the measured voltage determines the reactive power injection, which in turn affects the voltage. There has been only a limited rigorous treatment of the equilibrium and dynamical properties of such feedback systems. In this paper, we expand on our prior result that reverse-engineers a class of non-incremental voltage control schemes and provides a principled way to rigorously engineer the control to incorporate new design goals and/or achieve better dynamical properties. Specifically, it has been observed in the literature that in practical circumstances the droop-based control scheme, a commonly adopted non-incremental voltage control, can lead to undesirable oscillatory behaviors even in the case of a single inverter unit. This motivates us to forward-engineer the local voltage control and apply the (sub)gradient method to design an incremental voltage control algorithm that demands less restrictive condition for convergence. We provide a sufficient condition to ensure convergence of the proposed control algorithm and evaluate its performance on a real-world distribution feeder in Southern California with multiple large PV generation units through simulations.

I. NOTATION

t time index, \( t \in \mathcal{T} := \{1, 2, \ldots, \infty\} \)
\( \mathcal{N} \) set of buses excluding bus 0, \( \mathcal{N} := \{1, \ldots, n\} \)
\( \mathcal{L} \) set of power lines
\( \mathcal{L}_{ij} \) set of the lines form bus 0 to bus i
\( p_{ij}^r, q_{ij}^r \) real, reactive power consumption at bus i
\( p_{ij}^d, q_{ij}^d \) real, reactive power generation at bus i
\( P_{ij}, Q_{ij} \) real and reactive power flow from i to j
\( r_{ij}, x_{ij} \) resistance and reactance of line (i, j)
\( V_i \) complex voltage at bus i
\( v_i \) \( \substack{v_i := |V_i| \quad \forall i \in \mathcal{N}} \)
\( I_{ij} \) complex current from i to j
\( \ell_{ij} \) \( \substack{\ell_{ij} := |I_{ij}|^2 \quad \forall (i, j) \in \mathcal{L}} \)
\( x^+ \) positive part, \( x^+ = \max\{0, x\} \)
\( [x]_a^b = x + (a - x)^+ - (x - b)^+ \)
\( \lambda_{\max} \) the maximum eigenvalue

A quantity without subscript is usually a vector with appropriate components defined earlier, e.g., \( v := (v_i, i \in \mathcal{N}), q^d := (q_i^d, i \in \mathcal{N}) \).

II. INTRODUCTION

Most developed countries around the globe have set themselves ambitious targets towards a renewable energy future [4]. As the share of intermittent sources such as photovoltaic (PV) and wind generation increases, utility companies may encounter several operational challenges related to voltage regulation in power networks. A large number of recent studies [5]–[10] have explored the possibility of utilizing inverter-based distributed generators (DGs) to control voltage fluctuations in distribution systems with high renewable penetration levels, and recognized it as a viable solution. The basic idea is to have DG inverters to support the network voltage by injecting an appropriate amount of reactive power (vars) during peak demand periods, and absorbing it during surplus power conditions to mitigate the voltage rise problem. But the implementation of this idea requires a departure from the current standard [11] for interconnection of DG units. Indeed, a series of IEEE SCC21 1547 standard development projects [2], [3] are underway to upgrade inverter controls for ancillary services in order to facilitate reliable integration of renewable resources.

The literature on inverter-based vol/var control in distribution systems can be divided into the following three main categories: (i) Approaches that propose a centralized control scheme by solving a global optimal power flow (OPF) problem. These methods implicitly assume an underlying complete two-way communication system between a central computing authority and the controlled nodes [7], [13], [14]; (ii) Distributed message-passing algorithms in which communications are limited to neighboring nodes [8], [12], [15], [17]; (iii) local control methods that require no communications and rely only on local measurements and computations [6], [9], [16]. These include reactive power control based on local real power injection (referred to as Q(P)), power factor control, and the more common voltage based reactive power control (referred to as Q(V)). Although the methods proposed in the first two categories are critical for theoretical analysis and better understanding the impact of renewables on the grid, lack of sufficient telecommunication infrastructure discourages practical implementation of these methods in most practical scenarios.

Inverter-based local vol/var control is a closed-loop dynamical system whereby the measured voltage determines the reactive power injection, which in turn affects the voltage. There has been only a limited theoretical treatment of the equilibrium and dynamical properties of such feedback systems; see, e.g., [1], [18], [19]. In [1], we have reverse-engineered a class of non-incremental local control schemes by showing that they can be seen as distributed algorithms for solving a well-defined optimization problem. The resulting optimization based model not only provides a way to characterize the equilibrium, but also suggests a principled way to rigorously engineer the control to incorporate new design goals such as fairness and economic efficiency and/or achieve different dynamical properties. Specifically, it has been observed in the literature, e.g., [9], that in practical circumstances the droop-based control scheme, a commonly adopted non-incremental voltage control, can lead to undesirable oscillatory behaviors even in the case of a single inverter unit. This motivates us to forward-engineer the local voltage control and apply the (sub)gradient method to design an incremental voltage control algorithm that demands less restrictive condition for convergence. We provide a sufficient condition to ensure convergence of the proposed control algorithm and evaluate its performance on a real-world distribution feeder in Southern California with multiple large PV generation units through simulations.
The rest of this paper is organized as follows. Section III describes the system model and briefly reviews the result on reverse engineering of [1]. Section IV presents an incremental local voltage control algorithm based on the gradient method and its convergence analysis. Section V provides numerical experiments to evaluate the proposed new control algorithm in a real-world distribution circuit, and Section VI concludes the paper.

III. SYSTEM MODEL

A. Power flow model

We adopt the following branch flow model [20], [22] for a radial distribution system:

\begin{align}
P_{ij} &= p_i^f - p_j^f + \sum_{k=1}^{n} P_{jk} + r_{ij}\ell_{ij}, \\
Q_{ij} &= q_i^f - q_j^f + \sum_{k=1}^{n} Q_{jk} + x_{ij}\ell_{ij}, \\
v_i^2 &= v_i^2 - 2\left(r_{ij}P_{ij} + x_{ij}Q_{ij}\right) + \left(r_{ij}^2 + x_{ij}^2\right)\ell_{ij}, \\
\ell_{ij}v_i &= p_{ij}^q + Q_{ij}^q.
\end{align}

(1a) (1b) (1c) (1d)

Following [21], [1], we use a linearized version of the above model by letting \( \ell_{ij} = 0 \) for all \( (i,j) \in \mathcal{L} \) in (1). This approximation neglects the higher order real and reactive power loss terms. Since losses are typically much smaller than power flows \( P_{ij} \) and \( Q_{ij} \), this only introduces a small relative error, typically on the order of 1% [20]. We further assume that \( v_i \approx 1 \) so that we can set \( v_i^2 - v_i^2 = 2(v_i - v_i) \) in equation (1c). This approximation introduces a small relative error of at most 0.25% (1%) if there is a 5% (10%) deviation in voltage magnitude. With the above approximations the power flow model (1) simplifies to the following linear model:

\[ v = \tilde{v}_0 + R(p^c - p^f) + X(q^c - q^f), \]

where \( \tilde{v}_0 = (v_0, \ldots, v_0) \) is an \( n \)-dimensional vector, and resistance matrix \( R = [R_{ij}]_{n \times n} \) and reactance matrix \( X = [X_{ij}]_{n \times n} \) are symmetric matrices with entries

\[ R_{ij} := \sum_{(h,k) \in \mathcal{L} \cap \mathcal{L}_i} r_{hk}, \]

\[ X_{ij} := \sum_{(h,k) \in \mathcal{L} \cap \mathcal{L}_j} x_{hk}. \]

(2)

In this paper we assume that \( \tilde{v}_0, p^c, p^f, q^c \) are given constants. The only variables are (column) vectors \( v := (v_1, \ldots, v_n) \) of voltage magnitudes and \( q^c := (q_1^c, \ldots, q_n^c) \) of reactive powers. Let \( \tilde{v} = \tilde{v}_0 + R(p^c - p^f) - Xq^c \), which is a constant vector. For notational simplicity in the rest of the paper we will ignore the superscript in \( q^c \) and write \( q \) instead. Then the linearized branch flow model reduces to the following simple form:

\[ v = Xq + \tilde{v}. \]

(3)

The following result is important for the rest of this paper.

Lemma 1 (Lemma 1 of reference [1]). The matrix \( X \) is positive definite.

B. Local volt/var control

The goal of volt/var control on a distribution network is to provision reactive power injections \( q := (q_1, \ldots, q_n) \) in order to maintain the bus voltages \( v := (v_1, \ldots, v_n) \) to within a tight range around their nominal values \( v^{nom}_i \), \( i \in \mathcal{N} \). This can be modeled by a feedback dynamical system with state \( (v(t), q(t)) \) at discrete time \( t \). A general volt/var control algorithm maps the current state \( (v(t), q(t)) \) to a new reactive power injections \( q(t+1) \). The new \( q(t+1) \) produces a new voltage magnitudes \( v(t+1) \) according to (3). Motivated by the IEEE 1547.8 Standard [2], [3], we have studied in [1] a local volt/var control where each bus \( i \) makes an individual decision \( q_i(t+1) \) based only on its own voltage \( v_i(t) \).

Definition 1. A local volt/var control function \( f : \mathbb{R}^n \to \Omega \) is a collection of \( f_i : \mathbb{R} \to \Omega_i \) functions that map the current voltage \( v_i(t) \) to a new local control \( q_i(t+1) \):

\[ q_i(t+1) = f_i(v_i(t)), \quad \forall i \in \mathcal{N}, \]

where \( \Omega = \prod_{i=1}^{n} \Omega_i \), with \( \Omega_i = \{ q_i \mid q_i^{\text{min}} \leq q_i \leq q_i^{\text{max}} \} \) the set of feasible reactive power injections at each bus \( i \in \mathcal{N} \).

The control algorithm (4) is non-incremental as the current decision does not depend directly on the decision at the previous time. We obtain the following dynamical system that models the non-incremental local volt/var control of a distribution network:

\[ D1: \begin{cases} v^{(t)} &= Xq^{(t)} + \tilde{v}, \\ q^{(t+1)} &= f(v^{(t)}). \end{cases} \]

(5)

A fixed point \( (v^*, q^*) \) of the above dynamical system represents an equilibrium operating point of the network.

Definition 2. \( (v^*, q^*) \) is called an equilibrium point, or a network equilibrium, if it is a fixed point of (5), i.e.,

\[ v^* = Xq^* + \tilde{v}, \\ q^* = f(v^*). \]

(6)

C. Reverse engineering

The local volt/var control functions \( f_i(\cdot) \) are usually decreasing, but are not always strictly monotone because of the deadband in control as well as the bounds on the available reactive power. We assume for each bus \( i \in \mathcal{N} \) a symmetric deadband around the nominal voltage \( \delta_i = v_i^{nom} - \delta_i/2, v_i^{nom} + \delta_i/2 \), with \( \delta_i \geq 0 \). We have shown in [1] that the dynamical system D1 can be seen as a distributed optimization algorithm for solving a well-defined optimization problem under appropriate conditions:

A1: The local volt/var control functions \( f_i(\cdot) \) are nonincreasing over \( \mathbb{R} \) and strictly decreasing and differentiable in \( (v_i, -\delta_i/2) \) and in \( (\delta_i/2, \tilde{v}_i) \).

A2: The derivative of the control function \( f_i(\cdot) \) is bounded, i.e., there exists a finite \( \alpha_i \) such that \( |f_i^\prime(v_i)| \leq \alpha_i \) for all \( v_i \) in the appropriate domain, for all \( i \in \mathcal{N} \).

Theorem 1 (Theorem 1 of reference [1]). Suppose A1 holds. Then there exists a unique equilibrium point. Moreover, a point \( (v^*, q^*) \) is an equilibrium if and only if \( q^* \) is the unique optimal solution of

\[ \min_{q \in \Omega} F(q) = C(q) + \frac{1}{2} q^T Xq + q^T \tilde{v} \]
and \( v' = Xq' + \bar{v} \), where \( C(q) = \sum_{i \in N} C_i(q_i) \) with the cost function for each bus \( i \in N \) is defined by:

\[
C_i(q_i) := -\int_0^{q_i} f_i^{-1}(q) dq.
\]

The cost function \( C_i(q_i) \) is convex since \( f_i^{-1} \) is decreasing.

**Theorem 2** (Theorem 2 of reference [1]). Suppose A1–A2 hold. If

\[
\text{diag} \left( \frac{1}{\alpha_i} \right) > X,
\]

i.e., if the matrix \( \text{diag} \left( \frac{1}{\alpha_i} \right) - X \) is positive definite, then the local volt/var control D1 converges to the unique equilibrium point \( (v^*, q^*) \).

The following result is immediate.

**Corollary 1.** if \( \max(\alpha_i) < \frac{1}{\lambda_{\text{max}}(X)} \) where \( \lambda_{\text{max}} \) denotes the largest eigenvalue, then the local volt/var control D1 converges to the unique equilibrium point \( (v^*, q^*) \).

**Proof:** If \( \max(\alpha_i) < \frac{1}{\lambda_{\text{max}}(X)} \), we have \( \text{diag} \left( \frac{1}{\alpha_i} \right) > \lambda_{\text{max}}(X)I > X \). The result follows from Theorem 2.

Notice that \( \alpha_i \) can be seen as a metric for the “aggressiveness” of the voltage control: a larger \( \alpha_i \) value corresponds to a more aggressive response to the voltage deviation. Theorem 2 (and Corollary 1) implies that, in order to ensure convergence, the voltage control cannot be too aggressive. Intuitively, a too aggressive response will lead to overshoot in the control and thus oscillation.

### D. Piecewise linear control function

A particularly interesting example control function is the piecewise linear droop control proposed in the latest draft of the new IEEE 1547.8 Standard [2]:

\[
f_i(v_i) := \left[ -\alpha_i \left( v_i - v_{\text{nom}} - \frac{\delta_i}{2} \right) + \alpha_i \left( -v_i + v_{\text{nom}} - \frac{\delta_i}{2} \right) \right] q_{\text{nom}}^	au.
\]

where \( (\delta_i, \alpha_i) \) are the local control parameters at each bus.\(^2\) This control function and the corresponding cost function are illustrated in Fig. 1. The numerical examples reported in Section V is based on this control function.

### IV. Forward Engineering: An Incremental Voltage Control Algorithm

The optimization based model (7) provides a way to characterize the equilibrium and establishes the convergence of the local volt/var control, as shown in Theorems 1-2. It also suggests a principled way to engineer the control. New design goals such as fairness and economic efficiency can be taken into account and designed using the objective function in (7); and new control schemes with better dynamical properties can be designed based on various optimization algorithms, e.g., the gradient algorithm.

In particular, the convergence condition (8) is hard to verify in practice for two reasons. First, it is a computationally demanding problem to verify a linear matrix inequality of potentially very large dimension. Second, matrix \( X \) depends on the reactivity of every line in the network, which is practically hard to obtain. Moreover, even if you can verify the condition (8), it is a rather restrictive in constraining “allowable” control functions, and the existing control schemes may not satisfy this condition. Indeed, as already mentioned in Section II, it has been observed in the literature that in practical circumstances the droop-based control scheme, a commonly adopted non-incremental voltage control, can lead to undesirable oscillatory behaviors even in the case of a single inverter unit. We therefore seek a local volt/var control scheme that demands less restrictive condition for convergence.

### A. An incremental control algorithm

As mentioned in the above, for a given optimization problem, there may exist different optimization algorithms. In this subsection, we will apply the (sub)gradient method to the problem (7) to design a new voltage control algorithm:

\[
q_i(t + 1) = q_i(t) - \gamma \frac{\partial F(q_i)}{\partial q_i} \Bigg|_{q_i^{\text{nom}}}, \quad \text{(10)}
\]

where \( \gamma > 0 \) is the stepsize, \( [\: ] \) denotes the projection onto \([a, b]\), and

\[
\frac{\partial F(q)}{\partial q_i} = \begin{cases} 
C_i'(q_i(t)) + v_i(t) & \text{if } q_i(t) \neq 0, \\
0 & \text{if } q_i(t) = 0, \: v_i(t) > \frac{\delta_i}{2} \\
-\frac{\delta_i}{2} + v_i(t) & \text{if } q_i(t) = 0, \: v_i(t) < -\frac{\delta_i}{2}.
\end{cases}
\]

The above control algorithm is incremental as at each time the reactive power is “gradually” adjusted upon the provisioning at the previous time. It is also distributed, since the reactive power provisioning decision at each node \( i \in N \) depends only on the current provisioning and voltage at node \( i \).

We thus obtain the following dynamical system:

\[
D_2 : \begin{bmatrix} v_i(t) & q_i(t+1) \end{bmatrix} = \begin{bmatrix} Xq^{0\text{t}} + \bar{v} & q_i^{(t+1)} \end{bmatrix} = \begin{bmatrix} Xq(t) + \gamma \frac{\partial F(q_i)}{\partial q_i} \Bigg|_{q_i^{\text{nom}}}. \quad \text{(12)}
\end{bmatrix}
\]

The following result is immediate.

**Theorem 3.** Suppose A1 holds. Then there exists a unique equilibrium point for the dynamical system D2. Moreover, a point \( (v^*, q^*) \) is an equilibrium if and only if \( q^* \) is the unique optimal solution of problem (7) and \( v^* = Xq^* + \bar{v} \).

### B. Convergence

We now analyze the convergence of the dynamical system D2.

**Theorem 4.** Suppose A1 holds. If the stepsize \( \gamma \) satisfies

\[
\gamma < \frac{2}{\lambda_{\text{max}}(\nabla^2 C(q) + X)}, \quad \text{(13)}
\]

where \( \lambda_{\text{max}} \) denotes the maximum eigenvalue, then the dynamical system D2 converges to the unique equilibrium.

**Proof:** Consider first the case when \( q_i(t) \neq 0 \), \( \forall i \in N \). By the second order Taylor expansion,

\[
F(q(t+1)) = F(q(t)) + (\nabla F(q(t)))^T (q(t+1) - q(t)) + \frac{1}{2} (q(t+1) - q(t))^T (\nabla^2 C(q)(q(t+1) \text{ and } q(t), q(t+1) \text{ and } q(t)), \quad \text{(14)}
\]

\(2\)Here we “reload” notation, and use \( \alpha_i \) to also denote the slope of the droop control function. It does not contradict the use of \( \alpha_i \) in the condition A2.
where $\tilde{q} = \theta q(t) + (1 - \theta)q(t + 1)$ for certain $\theta \in [0, 1]$. By Projection Theorem [23], we have $(\nabla F)^T(q(t+1) - q(t)) \leq -\frac{1}{\gamma}||q(t+1) - q(t)||^2$, which leads to

$$F(q(t+1)) \leq F(q(t)) - \frac{1}{\gamma}||q(t+1) - q(t)||^2$$

$$+ \frac{1}{2}(q(t+1) - q(t))^T(\nabla^2 C(\tilde{q}) + X)(q(t+1) - q(t))$$

$$= F(q(t)) + (q(t+1) - q(t))^T(-\frac{2}{\gamma}I + \nabla^2 C(\tilde{q}) + X)(q(t+1) - q(t)). \quad (15)$$

When the condition (16) holds, $-\frac{2}{\gamma}I + \nabla^2 C(\tilde{q}) + X$ is negative definite, and thus the second term in (15) is strictly negative as long as $q(t+1) \neq q(t)$ and zero only if $q(t+1) = q(t)$. So, $F(q(t+1)) \leq F(q(t))$ with the equality if and only if $q(t+1) = q(t)$. Since the equilibrium of the dynamical system $D_2$ is unique by Theorem 3, $q(t+1) = q(t)$ can only occur at the unique equilibrium $q^*$ (with $v^* = Xq^* + \tilde{v}$). Thus, $F(q(t+1)) \leq F(q(t))$ with the equality if and only if $q(t+1) = q(t) = q^*$. Also, notice that $F(q) \geq F(q^*)$ with equality if and only if $q = q^*$. So, $F$ is a discrete-time Lyapunov function for $D_2$, and the Lyapunov stability theorem then implies that $q^*$ is globally asymptotically stable [24].

Consider now the case when $q(t) = 0$ and thus $C_i(q(t))$ in the function $F(q(t))$ is not differentiable for some $i \in N$. The complication here is to use well-defined derivatives in the Taylor expansion. We have three sub-cases; see equation (11):

1) $v_i(t) > \frac{\delta}{2}$: The subgradient in $D_2$ is chosen as $\frac{\partial V}{\partial v_i} = -\frac{\lambda}{\alpha}v_i(t) > 0$, so $q_i(t+1) = -\gamma \frac{\partial F}{\partial q_i} < 0$. We can use the left derivative $C_i^L(0^+)\gamma$, which is well-defined, in the Taylor expansion.

2) $v_i(t) < -\frac{\delta}{2}$: The subgradient in $D_2$ is chosen as $\frac{\partial V}{\partial v_i} = \frac{\lambda}{\alpha}v_i(t) < 0$, so $q_i(t+1) = -\gamma \frac{\partial F}{\partial q_i} > 0$. We can use the right derivative $C_i^R(0^-)\gamma$, which is well-defined, in the Taylor expansion.

3) $-\frac{\delta}{2} \leq v_i(t) \leq \frac{\delta}{2}$: The subgradient in $D_2$ is chosen as $\frac{\partial V}{\partial v_i} = 0$. So, $q_i(t+1) = q_i(t) = 0$. In this case, the Taylor expansion on $C_i$ is not needed, and $F(q(t+1)) \leq F(q(t))$ still holds.

With the above choice of the derivatives in the Taylor expansion, we can similarly show that $F$ is a discrete-time Lyapunov function for $D_2$ and $q^*$ is globally asymptotically stable.

Notice that for any control functions $f_i$ (that satisfies A1), the convergence condition (16) can be always satisfied by a properly chosen stepsizes $\gamma$. Even though the range of $\gamma$ depends on the control functions, the condition (16) does not constrain the allowable control functions. In contrast, the convergence condition (8) for the non-incremental voltage control (4) does constrain the allowable control functions $f_i$.

For the piecewise linear droop control functions (9), we have the following result on convergence.

**Corollary 2.** Suppose A1 holds. If the stepsize $\gamma$ satisfies

$$\gamma < \frac{2}{\lambda_{\text{max}}(\text{diag}(\frac{1}{\alpha}) + X)} \quad \text{(16)}$$

then the dynamical system $D_2$ with the piecewise linear droop control functions (9) converges to the unique equilibrium.

**Proof:** For the piecewise linear control functions (9), $\nabla^2 C(q) = \text{diag}(\frac{1}{\alpha})$. The result follows from Theorem 4.

Recall that $\alpha_i$ can be seen as a metric for the “aggressiveness” of the voltage control. Theorem 4 (and Corollary 2) implies that a more aggressive voltage control allows a larger range of the stepsizes for the convergence. This is different from the convergence of the non-incremental voltage control (4) where the control cannot be too aggressive. On the other hand, a bound (16) on the “allowable” stepsizes means that the control cannot be too aggressive as well.

V. **Numerical Examples**

Focusing on the piecewise linear droop control functions (9), we evaluate the proposed incremental var/volt control algorithm (10) and compare it against the existing non-incremental algorithm (4) on a distribution feeder of South California Edison with a high penetration of photovoltaic (PV) generation. Fig. 5 shows a 42-bus model of this feeder, where bus 1 is the substation and five photovoltaic generators are integrated at buses 2, 12, 26, 29, and 31. As we aim to study the volt/var control through PV inverters, all shunt capacitors are assumed to be off. Table I contains the network data including the line impedance, the peak MVA demand of loads, and the capacity of the PV generators. It is important to note that all studies are run with a full AC power flow model (not

![Figure 1: From left to right: piecewise linear volt/var control curve discussed in the draft of the upcoming IEEE 1547 standard [2], its inverse, and the corresponding reverse-engineered cost function for reactive power injection.](image-url)
the linearized model). Droop parameters at voltage controlling nodes are such that the deadband is from \(0.98p.u.\) to \(1.02p.u.\), and the hard voltage thresholds are \(\bar{v}_i = 0.97p.u., v_i = 0.97p.u.\) on all inverters.

**A. Case of a single inverter**

We first provide a simple example to illustrate the potential instability of the non-incremental voltage control scheme (4). In the feeder in Fig. 5, assume that all loads are at 80% of their peak value with a constant 0.9 Power Factor (PF), i.e., a total demand of \(8.24\text{MW}\) and \(3.99\text{MVar}\). We further assume that all five PV generators are running at 60% of their nameplate capacity with \(PF = 1\), except for the generator at bus 12 which is enabled to inject/absorb reactive power within a range of \(PF \in [0.8, 1]\), corresponding to \(q_{\text{max}}^{12} = 1.35\text{MVar}\). In this setup, it is observed that the reactive power output of the inverter at bus 12 oscillates between \(0.16\text{MVar}\) and \(1.35\text{MVar}\) (dash line in Fig. 2a), corresponding to a voltage oscillation between \(0.967p.u.\) and \(0.979p.u.\) (dash line in Fig. 2b). In contrast, when the proposed incremental control algorithm (10) with \(\gamma = 20\) is applied, there is no oscillation and the system converges very quickly to the equilibrium point of \(0.98p.u., 0.97p.u.\) at bus 12 (solid lines in Fig. 2a, 2b).

**B. Multiple inverter interactions**

As demonstrated above, the non-incremental voltage control \(\text{(4)}\) can potentially be unstable even with just a single inverter. With multiple inverters operating simultaneously in a distribution feeder, instability is even more of a serious concern. To see this, suppose that all five PV units of the feeder in Fig. 5 are active in controlling their inverters. Now let all spot loads be at their peak value with a constant 0.9 PF, and let the PV units be running at 70% of their capacity all enabled to control their reactive power output within a range of \(PF \in [0.8, 1]\). Again, as shown in Fig. 3, it is observed that in this case the non-incremental control scheme fails to converge, causing the voltage profile of the feeder to oscillate around the equilibrium (dashed blue line). Also, notice that, when the control at an inverter oscillates, it causes oscillation at all buses except for the substation bus.

In contrast, with the incremental voltage control algorithm (10) there is no oscillation and the system converges with appropriate stepsizes, as shown in Fig. 4. We see that with “small” enough stepsize \(\gamma\), the proposed incremental voltage control scheme converges to the equilibrium; and the larger the stepsize, the faster the convergence, which is a typical characteristics of the gradient algorithm. Also notice that, as shown in Fig. 4(d), if the stepsize is too large, the system will oscillate. In practice, we can start with an analytical estimate of the bound on the stepsize (16), and then run some numerical experiments around the bound to choose a stepsize that achieves a good tradeoff between convergence speed and robustness.

**VI. Conclusion**

Motivated by the oscillatory behavior of the existing non-incremental local var/volt control schemes, we have applied the reverse-engineering result in our prior work to design an incremental voltage control algorithm based on the gradient
method that demands less restrictive condition for convergence. We provide a sufficient condition to ensure convergence of the proposed control algorithm and evaluate its performance on a real-world distribution feeder in Southern California.

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