Local Voltage Control in Distribution Systems: An Incremental Control Algorithm

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***Abstract*—Inverter-based local volt/var control forms a closed-loop dynamical system whereby the measured voltage determines the reactive power injection, which in turn affects the voltage. There has been only a limited rigorous treatment of the equilibrium and dynamical properties of such feedback systems. In this paper, we expand on our prior result that reverse-engineers a class of non-incremental voltage control schemes and provides a principled way to rigorously engineer the control to incorporate new design goals and/or achieve better dynamical properties. Specifically, it has been observed in the literature that in practical circumstances the droop-based control scheme, a commonly adopted non-incremental voltage control, can lead to undesirable oscillatory behaviors even in the case of a single inverter unit. This motivates us to forward-engineer the local voltage control and apply the (sub)gradient method to design an incremental voltage control algorithm that demands less restrictive condition for convergence. We provide a sufficient condition to ensure convergence of the proposed control algorithm and evaluate its performance on a real-world distribution feeder in Southern California with multiple large PV generation units through simulations.**

I. NOTATION

t	time index, $t \in \mathcal{T} := \{1, 2, \ldots, \infty\}$
$\mathcal{N}$	set of buses excluding bus 0, $\mathcal{N} := \{1, ..., n\}$
$\mathcal{L}$	set of power lines
$\mathcal{L}_i$	set of the lines form bus 0 to bus i
p_i^c, q_i^c	real, reactive power consumption at bus i
p_i^g, q_i^g	real, reactive power generation at bus i
P_{ij}, Q_{ij}	real and reactive power flow from i to j
r_{ij}, x_{ij}	resistance and reactance of line (i, j)
V_i	complex voltage at bus i
v_i	$v_i := \lvert V_i \rvert, \quad i \in \mathcal{N}$
I_{ij}	complex current from i to j
ℓ_{ij}	$\ell_i := \lvert I_{ij} \rvert^2, \quad (i, j) \in \mathcal{L}$
x^+	positive part, $x^+ = max\{0, x\}$
$[x]_a^b$	$[x]_a^b = x + (a - x)^+ - (x - b)^+$
λ_{max}	the maximum eigenvalue

A quantity without subscript is usually a vector with appropriate components defined earlier, e.g., $v := (v_i, i \in \mathcal{N}), q^g := (q_i^g, i \in \mathcal{N})$.

II. INTRODUCTION

Most developed countries around the globe have set themselves ambitious targets towards a renewable energy future [4]. As the share of intermittent sources such as photovoltaic (PV) and wind generation increases, utility companies may encounter several operational challenges related to voltage regulation in power networks. A large number of recent studies [5]–[10] have explored the possibility of utilizing inverter-based distributed generators (DGs) to control voltage fluctuations in distribution systems with high renewable penetration levels, and recognized it as a viable solution. The basic idea is to have DG inverters to support the network voltage by injecting an appropriate amount of reactive power (vars) during peak demand periods, and absorbing it during surplus power conditions to mitigate the voltage rise problem. But the implementation of this idea requires a departure from the current standard [11] for interconnection of DG units. Indeed, a series of IEEE SCC21 1547 standard development projects [2], [3] are underway to upgrade inverter controls for ancillary services in order to facilitate reliable integration of renewable resources.

The literature on inverter-based volt/var control in distribution systems can be divided into the following three main categories: (i) Approaches that propose a centralized control scheme by solving a global optimal power flow (OPF) problem. These methods implicitly assume an underlying complete two-way communication system between a central computing authority and the controlled nodes [7], [13], [14]; (ii) Distributed message-passing algorithms in which communications are limited to neighboring nodes [8], [12], [15], [17]; (iii) local control methods that require no communications and rely only on local measurements and computations [6], [9], [16]. These include reactive power control based on local real power injection (referred to as Q(P)), power factor control, and the more common voltage based reactive power control (referred to as Q(V)). Although the methods proposed in the first two categories are critical for theoretical analysis and better understanding the impact of renewables on the grid, lack of sufficient telecommunication infrastructure discourages practical implementation of these methods in most practical scenarios.

Inverter-based local volt/var control is a closed-loop dynamical system whereby the measured voltage determines the reactive power injection, which in turn affects the voltage. There has been only a limited theoretical treatment of the equilibrium and dynamic properties of such feedback systems; see, e.g., [1], [18], [19]. In [1], we have reverse-engineered a class of non-incremental local control schemes by showing that they can be seen as distributed algorithms for solving a well-defined optimization problem. The resulting optimization based model not only provides a way to characterize the equilibrium, but also suggests a principled way to rigorously engineer the control to incorporate new design goals such as fairness and economic efficiency and/or achieve different dynamical properties. Specifically, it has been observed in the literature, e.g., [9], that in practical circumstances the droop-based control scheme, a commonly adopted non-incremental voltage control, can lead to undesirable oscillatory behaviors even in the case of a single inverter unit. This motivates us to forward-engineer the local voltage control and apply the (sub)gradient method to design an incremental voltage control algorithm that demands less restrictive condition for convergence. We provide a sufficient condition to ensure convergence of the proposed control algorithm and evaluate its performance on a real-world distribution feeder in Southern California with multiple large PV generation units through simulations.

The rest of this paper is organized as follows. Section III describes the system model and briefly reviews the result on reverse engineering of [1]. Section IV presents an incremental local voltage control algorithm based on the gradient method and its convergence analysis. Section V provides numeral experiments to evaluate the proposed new control algorithm in a real-world distribution circuit, and Section VI concludes the paper.

III. System Model

A. Power flow model

We adopt the following branch flow model [20], [22] for a *radial* distribution system:

$$P_{ij} = p_j^c - p_j^g + \sum_{k:(j,k)\in\mathcal{L}} P_{jk} + r_{ij}\ell_{ij}, \quad (1a)$$

$$Q_{ij} = q_j^c - q_j^g + \sum_{k:(j,k)\in\mathcal{L}} Q_{jk} + x_{ij}\ell_{ij}, \quad (1b)$$

$$v_j^2 = v_i^2 - 2\left(r_{ij}P_{ij} + x_{ij}Q_{ij}\right) + \left(r_{ij}^2 + x_{ij}^2\right)\ell_{ij}, \quad (1c)$$

$$\ell_{ij}v_i = P_{ij}^2 + Q_{ij}^2. \quad (1d)$$

Following [21], [1], we use a linearized version of the above model by letting $\ell_{ij} = 0$ for all $(i, j) \in \mathcal{L}$ in (1). This approximation neglects the higher order real and reactive power loss terms. Since losses are typically much smaller than power flows P_{ij} and Q_{ij}, this only introduces a small relative error, typically on the order of 1% [20]. We further assume that $v_i \approx 1$ so that we can set $v_j^2 - v_i^2 = 2(v_j - v_i)$ in equation (1c).[1] This approximation introduces a small relative error of at most 0.25% (1%) if there is a 5% (10%) deviation in voltage magnitude. With the above approximations the power flow model (1) simplifies to the following linear model:

$$v = \overline{v}_0 + R(p^g - p^c) + X(q^g - q^c),$$

where $\overline{v}_0 = (v_0, \ldots, v_0)$ is an n-dimensional vector, and resistance matrix $R = [R_{ij}]_{n\times n}$ and reactance matrix $X = [X_{ij}]_{n\times n}$ are symmetric matrices with entries

$$R_{ij} := \sum_{(h,k)\in\mathcal{L}_i\cap\mathcal{L}_j} r_{hk},$$

$$X_{ij} := \sum_{(h,k)\in\mathcal{L}_i\cap\mathcal{L}_j} x_{hk}. \quad (2)$$

In this paper we assume that $\overline{v}_0, p^c, p^g, q^c$ are given constants. The only variables are (column) vectors $v := (v_1, \ldots, v_n)$ of voltage magnitudes and $q^g := (q_1^g, \ldots, q_n^g)$ of reactive powers. Let $\tilde{v} = \overline{v}_0 + R(p^g - p^c) - Xq^c$, which is a constant vector. For notational simplicity in the rest of the paper we will ignore the superscript in q^g and write q instead. Then the linearized branch flow model reduces to the following simple form:

$$v = Xq + \tilde{v}. \quad (3)$$

The following result is important for the rest of this paper.

Lemma 1 (Lemma 1 of reference [1])**.** *The matrix X is positive definite.*

[1] Notice that this assumption is not essential and we can also work with v_i^2 instead.

B. Local volt/var control

The goal of volt/var control on a distribution network is to provision reactive power injections $q := (q_1, \ldots, q_n)$ in order to maintain the bus voltages $v := (v_1, \ldots, v_n)$ to within a tight range around their nominal values v_i^{nom}, $i \in \mathcal{N}$. This can be modeled by a feedback dynamical system with state $(v(t), q(t))$ at discrete time t. A general volt/var control algorithm maps the current state $(v(t), q(t))$ to a new reactive power injections $q(t+1)$. The new $q(t+1)$ produces a new voltage magnitudes $v(t+1)$ according to (3). Motivated by the IEEE 1547.8 Standard [2], [3], we have studied in [1] a local volt/var control where each bus i makes an individual decision $q_i(t + 1)$ based only on its own voltage $v_i(t)$.

Definition 1. *A local volt/var control function $f : \mathbb{R}^n \to \Omega$ is a collection of $f_i : \mathbb{R} \to \Omega_i$ functions that map the current voltage $v_i(t)$ to a new local control $q_i(t + 1)$:*

$$q_i(t + 1) = f_i(v_i(t)), \quad \forall i \in \mathcal{N}, \quad (4)$$

where $\Omega = \prod_{i=1}^{n} \Omega_i$, with $\Omega_i = \left\{q_i \mid {q_i}^{\min} \le q_i \le {q_i}^{\max}\right\}$ the set of feasible reactive power injections at each bus $i \in \mathcal{N}$.

The control algorithm (4) is **non-incremental** as the current decision does not depend directly on the decision at the previous time. We obtain the following dynamical system that models the non-incremental local volt/var control of a distribution network:

$$D1 : \left| \begin{array}{lcl} v^{(t)} & = & Xq^{(t)} + \tilde{v}, \\ q^{(t+1)} & = & f(v^{(t)}). \end{array} \right. \quad (5)$$

A fixed point (v^*, q^*) of the above dynamical system represents an equilibrium operating point of the network.

Definition 2. *(v^*, q^*) is called an* equilibrium point*, or a* network equilibrium*, if it is a fixed point of* (5)*, i.e.,*

$$v^* = Xq^* + \tilde{v},$$
$$q^* = f(v^*). \quad (6)$$

C. Reverse engineering

The local volt/var control functions $f_i(\cdot)$ are usually decreasing, but are not always strictly monotone because of the deadband in control as well as the bounds on the available reactive power. We assume for each bus $i \in \mathcal{N}$ a symmetric deadband around the nominal voltage $(v_i^{\text{nom}} - \delta_i/2, v_i^{\text{nom}} + \delta_i/2)$, with $\delta_i \ge 0$. We have shown in [1] that the dynamical system $D1$ can be seen as a distributed optimization algorithm for solving a well-defined optimization problem under appropriate conditions:

- A1: The local volt/var control functions f_i are nonincreasing over $\mathbb{R}$ and strictly decreasing and differentiable in $(\underline{v}_i, -\delta_i/2)$ and in $(\delta_i/2, \overline{v}_i)$.
- A2: The derivative of the control function f_i is bounded, i.e., there exists a finite α_i such that $|f_i'(v_i)| \le \alpha_i$ for all v_i in the appropriate domain, for all $i \in \mathcal{N}$.

Theorem 1 (Theorem 1 of reference [1])**.** *Suppose A1 holds. Then there exists a unique equilibrium point. Moreover, a point (v^*, q^*) is an equilibrium if and only if q^* is the unique optimal solution of*

$$\min_{q\in\Omega} F(q) := C(q) + \frac{1}{2}q^T Xq + q^T \tilde{v} \quad (7)$$

and $v^* = Xq^* + \tilde{v}$, where $C(q) = \sum_{i\in\mathcal{N}} C_i(q_i)$ with the cost function for each bus $i \in \mathcal{N}$ is defined by:

$$C_i(q_i) \quad := \quad -\int_0^{q_i} f_i^{-1}(q)\,dq.$$

The cost function $C_i(q_i)$ is convex since f_i^{-1} is decreasing.

Theorem 2 (Theorem 2 of reference [1])**.** *Suppose A1–A2 hold. If*

$$\mathrm{diag}\left(\frac{1}{\alpha_i}\right) \succ X, \tag{8}$$

i.e., if the matrix $\mathrm{diag}\left(\alpha_i^{-1}\right) - X$ *is positive definite, then the local volt/var control D1 converges to the unique equilibrium point (v^*, q^*).*

The following result is immediate.

Corollary 1. *if* $\max\{\alpha_i\} < \frac{1}{\lambda_{max}(X)}$ *where λ_{max} denotes the largest eigenvalue, then the local volt/var control D1 converges to the unique equilibrium point (v^*, q^*).*

Proof: If $\max\{\alpha_i\} < \frac{1}{\lambda_{max}(X)}$, we have $\mathrm{diag}\left(\frac{1}{\alpha_i}\right) \succ \lambda_{max}(X)I \succ X$. The result follows from Theorem 2. ■

Notice that α_i can be seen as a metric for the "aggressiveness" of the voltage control: a larger α_i value corresponds to a more aggressive response to the voltage deviation. Theorem 2 (and Corollary 1) implies that, in order to ensure convergence, the voltage control cannot be too aggressive. Intuitively, a too aggressive response will lead to overshoot in the control and thus oscillation.

D. Piecewise linear control function

A particularly interesting example control function is the piecewise linear droop control proposed in the latest draft of the new IEEE 1547.8 Standard [2]:

$$f_i(v_i) := \left[-\alpha_i\left(v_i - v_i^{\mathrm{nom}} - \frac{\delta_i}{2}\right)^+ + \alpha_i\left(-v_i + v_i^{\mathrm{nom}} - \frac{\delta_i}{2}\right)^+\right]_{q_i^{\min}}^{q_i^{\max}}, \tag{9}$$

where (δ_i, α_i) are the local control parameters at each bus.[2] This control function and the corresponding cost function are illustrated in Fig. 1. The numerical examples reported in Section V is based on this control function.

IV. Forward Engineering: An Incremental Voltage Control Algorithm

The optimization based model (7) provides a way to characterize the equilibrium and establishes the convergence of the local volt/var control, as shown in Theorems 1-2. It also suggests a principled way to engineer the control. New design goals such as fairness and economic efficiency can be taken incorporated by engineering the objective function in (7); and new control schemes with better dynamical properties can be designed based on various optimization algorithms, e.g., the gradient algorithm.

In particular, the convergence condition (8) is hard to verify in practice for two reasons. First, it is a computationally demanding problem to verify a linear matrix inequality of potentially very large dimension. Second, matrix X depends on the reactance of every line in the network, which is practically hard to obtain. Moreover, even if you can verify the condition (8), it is rather restrictive in constraining "allowable" control functions, and the existing control schemes may not satisfy this condition. Indeed, as already mentioned in Section II, it has been observed in the literature that in practical circumstances the droop-based control scheme, a commonly adopted non-incremental voltage control, can lead to undesirable oscillatory behaviors even in the case of a single inverter unit. We therefore seek a local var/volt control scheme that demands less restrictive condition for convergence.

A. An incremental control algorithm

As mentioned in the above, for a given optimization problem, there may exist different optimization algorithms. In this subsection, we will apply the (sub)gradient method to the problem (7) to design a new voltage control algorithm:

$$q_i(t+1) \quad = \quad \left[q_i(t) - \gamma\frac{\partial F(q)}{\partial q_i}\right]_{q_i^{min}}^{q_i^{max}}, \tag{10}$$

where $\gamma > 0$ is the stepsize, '$[\ ]_b^a$' denotes the projection onto $[a, b]$, and

$$\frac{\partial F(q)}{\partial q_i} = \begin{cases} C_i'(q_i(t)) + v_i(t) & \text{if } q_i(t) \neq 0 \\ 0 & \text{if } \ q_i(t) = 0\ ,\ \ -\frac{\delta}{2} \le v_i(t) \le \frac{\delta}{2} \\ -\frac{\delta}{2} + v_i(t) & \text{if } \ q_i(t) = 0\ ,\ \ v_i(t) > \frac{\delta}{2} \\ \frac{\delta}{2} + v_i(t) & \text{if } \ q_i(t) = 0\ ,\ \ v_i(t) < -\frac{\delta}{2} \end{cases}. \tag{11}$$

The above control algorithm is **incremental** as at each time the reactive power is "gradually" adjusted upon the provisioning at the previous time. It is also distributed, since the reactive power provisioning decision at each node $i \in \mathcal{N}$ depends only on the current provisioning and voltage at node i.

We thus obtain the following dynamical system:

$$D2: \left| \begin{array}{lcl} v^{(t)} & = & Xq^{(t)} + \tilde{v}, \\ q_i^{(t+1)} & = & \left[q_i(t) - \gamma\frac{\partial F(q)}{\partial q_i}\right]_{q_i^{min}}^{q_i^{max}}. \end{array}\right. \tag{12}$$

The following result is immediate.

Theorem 3. *Suppose A1 holds. Then there exists a unique equilibrium point for the dynamical system D2. Moreover, a point (v^*, q^*) is an equilibrium if and only if q^* is the unique optimal solution of problem (7) and $v^* = Xq^* + \tilde{v}$.*

B. Convergence

We now analyze the convergence of the dynamical system $D2$.

Theorem 4. *Suppose A1 holds. If the stepsize γ satisfies*

$$\gamma < \frac{2}{\lambda_{max}(\nabla^2 C(q) + X)}, \tag{13}$$

where λ_{max} denotes the maximum eigenvalue, then the dynamical system D2 converges to the unique equilibrium.

Proof: Consider first the case when $q_i(t) \neq 0$, $\forall i \in \mathcal{N}$. By the second order Taylor expansion,

$$\begin{aligned} & F(q(t+1)) \\ = \ & F(q(t)) + (\nabla F(q(t)))^T(q(t+1) - q(t)) \\ & + \frac{1}{2}(q(t+1) - q(t))^T(\nabla^2 C(\tilde{q}) + X)(q(t+1) - q(t)), \end{aligned} \tag{14}$$

[2]Here we "reload" notation, and use α_i to also denote the slope of the droop control function. It does not contradict the use of α_i in the condition A2.

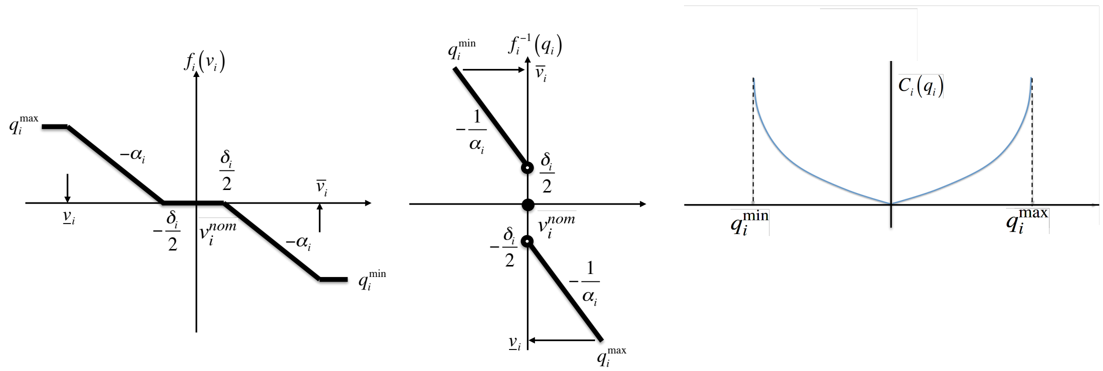


Fig. 1: From left to right: piecewise linear volt/var control curve discussed in the draft of the upcoming IEEE 1547 standard [2], its inverse, and the corresponding reverse-engineered cost function for reactive power injection.

where $\tilde{q} = \theta q(t) + (1-\theta)q(t+1)$ for certain $\theta \in [0,1]$. By Projection Theorem [23], we have $(\nabla F)^T(q(t+1) - q(t)) \le -\frac{1}{\gamma}\|q(t+1) - q(t)\|^2$, which leads to

$$
\begin{aligned}
& F(q(t+1)) \\
\le\ & F(q(t)) - \frac{1}{\gamma}\|q(t+1) - q(t)\|^2 \\
& + \frac{1}{2}(q(t+1) - q(t))^T(\nabla^2 C(\tilde{q}) + X)(q(t+1) - q(t)) \\
=\ & F(q(t)) \\
& + (q(t+1) - q(t))^T(-\frac{2}{\gamma}I + \nabla^2 C(\tilde{q}) + X)(q(t+1) - q(t)). \quad (15)
\end{aligned}
$$

When the condition (16) holds, $-\frac{2}{\gamma}I + \nabla^2 C(\tilde{q}) + X$ is negative definite, and thus the second term in (15) is strictly negative as long as $q(t+1) \neq q(t)$ and zero only if $q(t+1) = q(t)$. So, $F(q(t+1)) \le F(q(t))$ with the equality if and only if $q(t+1) = q(t)$. Since the equilibrium of the dynamical system $D2$ is unique by Theorem 3, $q(t+1) = q(t)$ can only occur at the unique equilibrium q^* (with $v^* = Xq^* + \tilde{v}$). Thus, $F(q(t+1)) \le F(q(t))$ with the equality if and only if $q(t+1) = q(t) = q^*$. Also, notice that $F(q) \ge F(q^*)$ with equality if and only if $q = q^*$. So, F is a discrete-time Lyapunov function for $D2$, and the Lyapunov stability theorem then implies that q^* is globally asymptotically stable [24].

Consider now the case when $q_i(t) = 0$ and thus $C_i(q_i(t))$ in the function $F(q(t))$ is not differentiable for some $i \in \mathcal{N}$. The complication here is to use well-defined derivatives in the Taylor expansion. We have three sub-cases; see equation (11):

1) $v_i(t) > \delta/2$: The subgradient in $D2$ is chosen as $\frac{\partial F}{\partial q_i} = -\frac{\delta}{2} + v_i(t) > 0$, so $q_i(t+1) = -\gamma\frac{\partial F}{\partial q_i} < 0$. We can use the left derivative $C_i'(0^-)$, which is well-defined, in the Taylor expansion.
2) $v_i(t) < -\delta/2$: The subgradient in $D2$ is chosen as $\frac{\partial F}{\partial q_i} = \frac{\delta}{2} + v_i(t) < 0$, so $q_i(t+1) = -\gamma(\frac{\delta}{2} + v_i(t)) > 0$. We can use the right derivative $C_i'(0^+)$, which is well-defined, in the Taylor expansion.
3) $-\delta/2 \le v_i(t) \le \delta/2$: The subgradient in $D2$ is chosen as $\frac{\partial F}{\partial q_i} = 0$. So, $q_i(t+1) = q_i(t) = 0$. In this case, the Taylor expansion on C_i is not needed, and $F(q(t+1)) \le F(q(t))$ still holds.

With the above choice of the derivatives in the Taylor expansion, we can similarly show that F is a discrete-time Lyapunov function for $D2$ and q^* is globally asymptotically stable. ■

Notice that for any control functions f_i (that satisfies A1), the convergence condition (16) can be always satisfied by a properly chosen stepsize γ. Even though the range of γ depends on the control functions, but the condition (16) does not constrain the allowable control functions. In contrast, the convergence condition (8) for the non-incremental voltage control (4) does constrain the allowable control functions f_i.

For the piecewise linear droop control functions (9), we have the following result on convergence.

Corollary 2. *Suppose A1 holds. If the stepsize γ satisfies*

$$\gamma < \frac{2}{\lambda_{max}(\mathrm{diag}(\frac{1}{\alpha_i}) + X)}, \quad (16)$$

then the dynamical system $D2$ with the piecewise linear droop control functions (9) *converges to the unique equilibrium.*

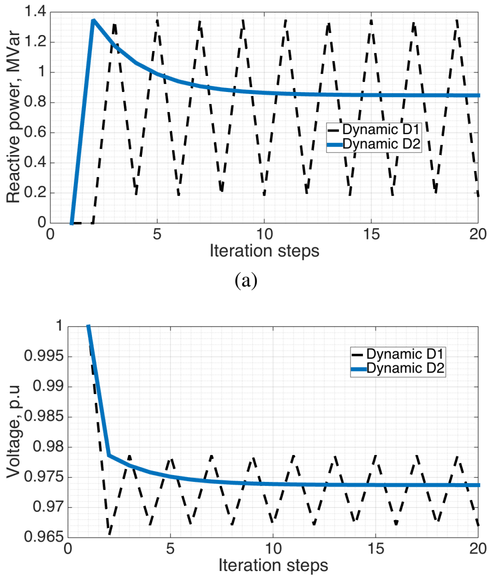
Proof: For the piecewise linear control functions (9), $\nabla^2 C(q) = \mathrm{diag}(\frac{1}{\alpha_i})$. The result follows from Theorem 4. ■

Recall that α_i can be seen as a metric for the "aggressiveness" of the voltage control. Theorem 4 (and Corollary 2) implies that a more aggressive voltage control allows a larger range of the stepsize for the convergence. This is different from the convergence of the non-incremental voltage control (4) where the control cannot be too aggressive. On the other hand, a bound (16) on the "allowable" stepsize means that the control cannot be too aggressive as well.

V. Numerical Examples

Focusing on the piecewise linear droop control functions (9), we evaluate the proposed incremental var/volt control algorithm (10) and compare it against the existing non-incremental algorithm (4) on a distribution feeder of South California Edison with a high penetration of photovoltaic (PV) generation.[3] Fig. 5 shows a 42-bus model of this feeder, where bus 1 is the substation and five photovoltaic generators are integrated at buses 2, 12, 26, 29, and 31. As we aim to study the volt/var control through PV inverters, all shunt capacitors are assumed to be off. Table I contains the network data including the line impedance, the peak MVA demand of loads, and the capacity of the PV generators. It is important to note that all studies are run with a full AC power flow model (not

[3]This feeder is a modified version of the original feeder published in [2], where zero impedance lines have been removed, and also the location and capacity of the PV generators have changed for demonstration purpose.



(a)

(b)

Fig. 2: Dynamics in reactive power injection and voltage magnitude for the case of a single inverter.

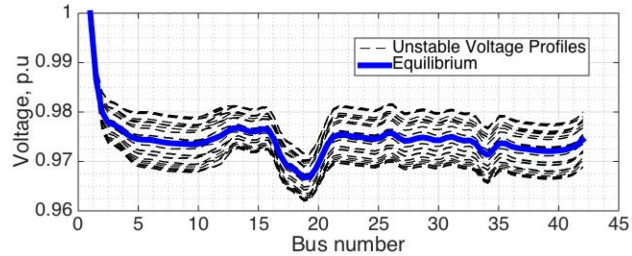
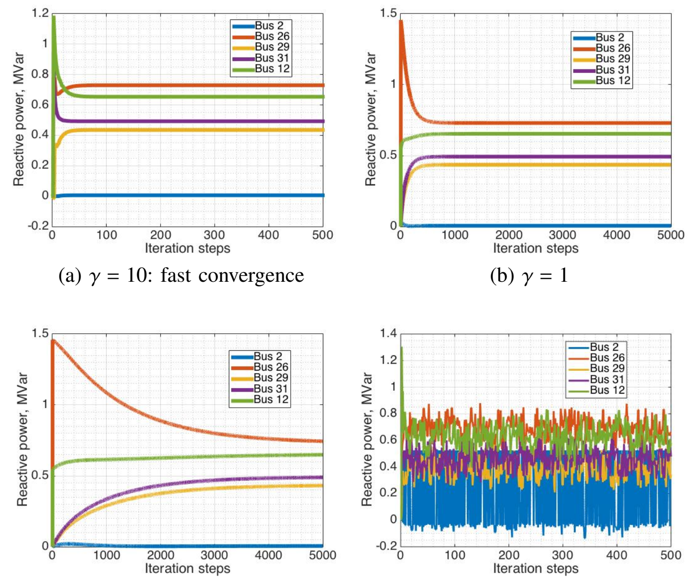


Fig. 3: Oscillation in voltage profile when all inverters operate.



(a) $\gamma = 10$: fast convergence

(b) $\gamma = 1$

(c) $\gamma = 0.1$: slow convergence

(d) $\gamma = 50$: divergence

Fig. 4: Convergence of the proposed incremental voltage control with different stepsizes.

the linearized model). Droop parameters at voltage controlling nodes are such that the deadband is from $0.98^{p.u.}$ to $1.02^{p.u.}$, and the hard voltage thresholds are $\bar{v}_i = 0.97^{p.u.}, \underline{v_i} = 0.97^{p.u.}$ on all inverters.

A. *Case of a single inverter*

We first provide a simple example to illustrate the potential instability of the non-incremental voltage control scheme (4). In the feeder in Fig. 5, assume that all loads are at 80% of their peak value with a constant 0.9 Power Factor (PF), i.e., a total demand of 8.24^{MW} and 3.99^{MVar}. We further assume that all five PV generators are running at 60% of their nameplate capacity with PF=1, except for the generator at bus 12 which is enabled to inject/absorb reactive power within a range of PF $\in [0.8, 1]$, corresponding to $q_{12}^{max} = 1.35^{MVar}$. In this setup, it is observed that the reactive power output of the inverter at bus 12 oscillates between 0.18 and 1.35^{Mvar} (dash line in Fig. 2a), corresponding to a voltage oscillation between 0.967 and $0.979^{p.u.}$ (dash line in Fig. 2b). In contrast, when the proposed incremental control algorithm (10) with $\gamma = 20$ is applied, there is no oscillation and the system converges very quickly to the equilibrium point of 0.85^{MVar}, $0.974^{p.u.}$ at bus 12 (solid lines in Fig. 2a, 2b).

B. *Multiple inverter interactions*

As demonstrated above, the non-incremental voltage control (4) can potentially be unstable even with just a single inverter. With multiple inverters operating simultaneously in a distribution feeder, instability is even more of a serious concern. To see this, suppose that all five PV units of the feeder in Fig. 5 are active in controlling their inverters. Now let all spot loads be at their peak value with a constant 0.9 PF, and let the PV units be running at 70% of their capacity all enabled to control their reactive power output within a range of PF $\in [0.8, 1]$. Again, as shown in Fig. 3, it is observed that in this case the non-incremental control scheme fails to converge, causing the voltage profile of the feeder to oscillate around the equilibrium (dashed blue line). Also, notice that, when the control at an inverter oscillates, it causes oscillation at all buses except for the substation bus.

In contrast, with the incremental voltage control algorithm (10) there is no oscillation and the system converges with appropriate stepsizes, as shown in Fig. 4. We see that with "small" enough stepsize γ, the proposed incremental voltage control scheme converges to the equilibrium; and the larger the stepsize, the faster the convergence, which is a typical characteristics of the gradient algorithm. Also notice that, as shown in Fig. 4(d), if the stepsize is too large, the system will oscillate. In practice, we can start with an analytical estimate of the bound on the stepsize (16), and then run some numerical experiments around the bound to choose a stepsize that achieves a good tradeoff between convergence speed and robustness.

VI. Conclusion

Motivated by the oscillatory behavior of the existing non-incremental local var/volt control schemes, we have applied the reverse-engineering result in our prior work to design an incremental voltage control algorithm based on the gradient

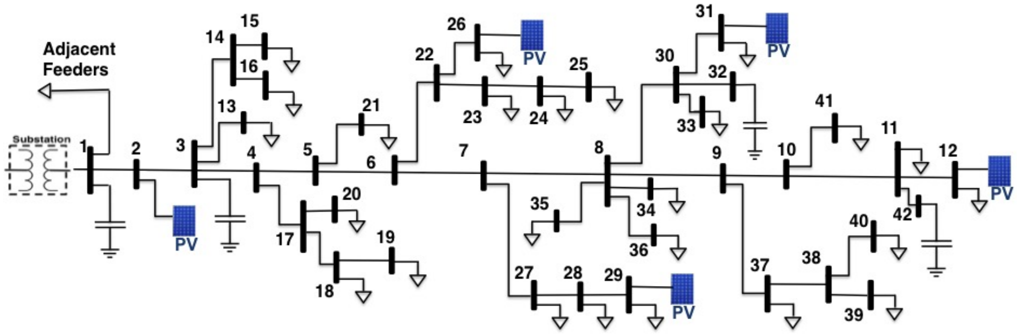


Fig. 5: Circuit diagram for SCE distribution system.

TABLE I: Network of Fig. 5: Line impedances, peak spot load KVA, Capacitors and PV generation's nameplate ratings.

Network Data																			
Line Data				Line Data				Line Data				Load Data		Load Data		PV Generators	
From Bus.	To Bus.	R (Ω)	X (Ω)	From Bus.	To Bus.	R (Ω)	X (Ω)	From Bus.	To Bus.	R (Ω)	X (Ω)	Bus No.	Peak MVA	Bus No.	Peak MVA	Bus No.	Capacity MW
1	2	0.259	0.808	8	34	0.244	0.046	18	19	0.198	0.046	11	0.67	28	0.27		
2	3	0.031	0.092	8	36	0.107	0.031	22	26	0.046	0.015	12	0.45	29	0.2	2	1
3	4	0.046	0.092	8	30	0.076	0.015	22	23	0.107	0.031	13	0.89	31	0.27	26	2
3	13	0.092	0.031	8	9	0.031	0.031	23	24	0.107	0.031	15	0.07	33	0.45	29	1.8
3	14	0.214	0.046	9	10	0.015	0.015	24	25	0.061	0.015	16	0.67	34	1.34	31	2.5
4	17	0.336	0.061	9	37	0.153	0.046	27	28	0.046	0.015	18	0.45	35	0.13	12	3
4	5	0.107	0.183	10	11	0.107	0.076	28	29	0.031	0	19	1.23	36	0.67		
5	21	0.061	0.015	10	41	0.229	0.122	30	31	0.076	0.015	20	0.45	37	0.13		
5	6	0.015	0.031	11	42	0.031	0.015	30	32	0.076	0.046	21	0.2	39	0.45		
6	22	0.168	0.061	11	12	0.076	0.046	38	39	0.107	0.015	23	0.13	40	0.2		
6	7	0.031	0.046	14	16	0.046	0.015	38	40	0.061	0.015	24	0.13	41	0.45		
7	27	0.076	0.015	14	15	0.107	0.015	43	44	0.061	0.015	25	0.2	V_{base} = 12.35 KV			
7	8	0.015	0.015	17	18	0.122	0.092	43	45	0.061	0.015	26	0.07	S_{base} = 1000 KVA			
8	35	0.046	0.015	17	20	0.214	0.046					27	0.13	Z_{base} = 152.52 Ω			

method that demands less restrictive condition for convergence. We provide a sufficient condition to ensure convergence of the proposed control algorithm and evaluate its performance on a real-world distribution feeder in Southern California.

References

[1] M. Farivar, L. Chen, and S. Low, "Equilibrium and dynamics of local voltage control in distribution systems," 52nd IEEE Annual Conference on Decision and Control (CDC), pp. 4329–4334, 2013.

[2] Standards Coordinating Committee 21 of Institute of Electrical and Electronics Engineers, Inc., IEEE Standard P1547.8™/D8, "Recommended Practice for Establishing Methods and Procedures that Provide Supplemental Support for Implementation Strategies for Expanded Use of IEEE Standard 1547", IEEE ballot document, Aug 2014.

[3] Institute of Electrical and Electronics Engineers, Inc., "IEEE Standard 1547a™(2014) Standard for Interconnecting Distributed Resources with Electric Power Systems Amendment 1", May 2014.

[4] J. L. Sawin, F. Sverrisson, "Renewables 2014: Global Status Report", Paris: REN21 Secretariat REN21, 2014.

[5] K. Turitsyn, P. Sulc, S. Backhaus, and M. Chertkov, "Options for control of reactive power by distributed photovoltaic generators, Proceedings of the IEEE, 99(6): 1063-1073, 2011.

[6] J. Smith, W. Sunderman, R. Dugan, and B. Seal, "Smart inverter volt/var control functions for high penetration of pv on distribution systems, *IEEE Power Systems Conference and Exposition (PSCE)*, pp. 1-6, 2011.

[7] M. Farivar, C. R. Clarke, S. H. Low, and K. M. Chandy. "Inverter VAR control for distribution systems with renewables," IEEE SmartGrid-Comm, pp. 457–462, 2011.

[8] B. A. Robbins, C. N. Hadjicostis, and A. D. Dominguez-Garcia, "A two-stage distributed architecture for voltage control in power distribution systems,", IEEE Trans. on Power Systems, 28(2): 1470-1482, 2013.

[9] P. Jahangiri, D. C. Aliprantis, "Distributed Volt/VAr Control by PV Inverters," Power Systems, IEEE Transactions on , vol.28, no.3, pp.3429-3439, 2013

[10] J. Neely, S. Gonzalez, M. Ropp, D. Schutz, "Accelerating Development of Advanced Inverters: Evaluation of Anti-Islanding Schemes with Grid Support Functions and Preliminary Laboratory Demonstration," Sandia National Laboratories Technical Report SAND2013-10231, 2013.

[11] T. Basso, "IEEE Standard for Interrconnecting Distributed Resources With the Electric Power System," IEEE PES Meeting, p. 1, 2004.

[12] P. Sulc, S. Backhaus, M. Chertkov, "Optimal Distributed Control of Reactive Power Via the Alternating Direction Method of Multipliers," IEEE Transactions on Energy Conversion, 29(4): 968-977, 2014

[13] R. A. Jabr, "Radial distribution load flow using conic programming," IEEE Transactions on Power Systems, 21(3):1458-1459, 2006

[14] J. Lavaei, S. H. Low, "Zero Duality Gap in Optimal Power Flow Problem," IEEE Transaction on Power Systems, 27(1): 92-107, 2012.

[15] M. Kraning, E. Chu, J. Lavaei, S. Boyd, "Dynamic Network Energy Management via Proximal Message Passing", Foundations and Trends in Optimization, 1(2): 73-126, 2013.

[16] J. Shihadeh, S. You, L. Chen, "Signal-anticipating in local voltage control in distribution systems," IEEE SmartGridComm, pp. 212-217, 2014.

[17] N. Li, G. Qu, M. Dahleh, "Real-time decentralized voltage control in distribution networks," 52nd Annual Allerton Conference on Communication, Control, and Computing, pp. 582-588, 2014.

[18] A. Kam, J. Simonelli, "Stability of Distributed, Asynchronous VAR-based Closed-loop Voltage Control Systems", IEEE PES General Meeting Conference & Exposition, 2014.

[19] F. Andren, B. Bletterie, S. Kadam, P. Kotsampopoulos, C. Bucher, "On the Stability of local Voltage Control in Distribution Networks with a High Penetration of Inverter-Based Generation, IEEE Transaction on Industrial Electronics, 62(4): 2519-2529, 2015

[20] M. E. Baran, F. F Wu, "Optimal Capacitor Placement on radial distribution systems", IEEE Trans. Power Delivery, 4(1):725-734, 1989.

[21] M. E. Baran, F.F. Wu, "Network reconfiguration in distribution systems for loss reduction and load balancing", IEEE Transaction on Power Delivery, 4(2): 401-1407, 1989.

[22] M. Farivar and S. H. Low. "Branch flow model: relaxations and convexification (parts I, II)," *Power Systems, IEEE Transactions on*, 28(3):2554-2572, 2013.

[23] D. P. Bertsekas, J. N. Tsitsiklis, "Parallel and distributed computation: numerical methods", Prentice hall Englewood Cliffs, NJ, 1989.

[24] H. K. Khalil and J. W. Grizzle, "Nonlinear systems", 3rd Edition, Prentice hall, 200.