

Wage Compression, Learning, and Self-Employment

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Abstract

While the existing literature on the self-employment choice has focussed on individual differences, we combine this with two important institutional features: Wage compression and learning. With wage compression, workers become self-employed for both "carrot" and "stick" reasons. Some prefer self-employment to the low institutionalised wage, while others cannot qualify for this wage. Furthermore, learning induces some workers to start out in wage work before switching to self-employment. Hence, our model can explain two empirical regularities: i) the existence of a group of low-skilled entrepreneurs; and ii) the increasing propensity for self-employment over age groups. Further predictions of the model concerning transition probabilities and earnings distributions are also consistent with existing empirical evidence.

1 Introduction

Self-employment and entrepreneurship are important phenomena for questions ranging from the labour market options and behaviour of individuals to the overall growth dynamics of the economy, see, *e.g.*, Audretsch (2002).¹ It is therefore important to analyse what causes people to choose self-employment.

Existing models explaining the choice of self-employment versus wage employment have focused on individual differences in risk aversion (Kihlstrom and Laffont, 1979), differences in "entrepreneurial ability" (Brock and Evans, 1986, Holmes and Schmitz, 1990, Fonseca et al., 2001), differences in generalised versus specialised skills (Lazear, 2003), and differences in probability assessments (de Meza and Southey, 1996).² Fewer studies have considered the implications of institutional features for the self-employment choice. Kihlstrom and Laffont (1983b) and Kanbur (1981) have analysed the importance of various tax schemes, whereas Fonseca et al. (2001) focus on the role of start-up costs.

The approach taken in this paper is to combine individual differences with two important institutional features, namely on-the-job learning and institutional wage compression. This is partly motivated by the assumed importance of these institutional features, partly by the desire to explain two general findings in the empirical literature: i) the existence of low-skilled/low-income entrepreneurs; and ii) the rising share of self-employed in total employment over age groups.

First, while the idea that some marginalised (low-skilled) individuals are "pushed" into self-employment because of no alternative options has often been raised in the (sociological) literature on entrepreneurship – and seems well documented empirically, see, *e.g.*, Evans and Leighton (1989) – we are not aware of any formal modelling of this phenomenon. Second, the fact that the share of self-employed in total employment is increasing over age groups, see Blanchflower (2000), indicates that some individuals start out in wage work and switch to self-employment at some point in their career. While this could be a result of capital constraints, requiring entrepreneurs to build up savings through an initial period of wage employment, we show that it

¹In this paper – as in much of the existing literature – we use self-employment and entrepreneurship as synonyms, although we recognise that self-employment may include many non-entrepreneurial activities, and *vice versa*.

²Other models have ignored individual differences and have derived the *share* of self-employed as an equilibrium outcome in models with, *e.g.*, uncertain demand (Sheshinski and Drèze, 1976) or uncertain production (Kihlstrom and Laffont, 1983a and 1983b). These models, however, cannot explain *which* individuals choose self-employment over wage work.

could alternatively be a consequence of (faster) human capital building in wage work, which raises the eventual return to self-employment.

With respect to the assumed wage compression, rigid wage structures are still common and enjoy extensive coverage in many European countries. In fact, Boeri and Burda (2004) have documented that there is considerable support among EU citizens for such rigidities in wage setting practices, which imply that wages are set without reference to individual match productivity and local labour market conditions. Furthermore, the empirical evidence supports an egalitarian – or *compressed* – wage structure in which wage differences do not even fully reflect the systematic productivity differences between workers, see Boeri and Burda (2004) and Booth (1995).

While a compressed wage structure is generally believed to reduce labour market efficiency and welfare, it has also been used to explain why firm-sponsored training of employees can arise. By making the firm a residual claimant of productivity increases, a compressed wage structure increases the willingness of firms to finance training of their employees, see, *e.g.*, Acemoglu and Pischke (1999).³ However, a compressed wage structure not only affects the firms' incentives to employ and train workers, it may also affect the workers' incentives to become self-employed, since this is a way of "escaping" the institutionalised compression.

In this paper, we provide a formal analysis of the latter aspect. We set up a partial-equilibrium model in which a continuum of individuals continuously choose between unemployment, self-employment and wage employment. The analysis rests on our two key assumptions. First, we assume that the wage depends linearly on individual productivity according to the compressed wage structure proposed by Boeri and Burda (2004). Second, we assume that productivity depends on innate ability and previous wage work experience. That is, productivity is assumed only to increase during wage work. Admittedly, this assumption is somewhat stylised, but it captures the idea that people learn more by working with and for other people. Knowledge spill-overs are thus assumed to be more important *within* firms than *between* firms.

We find that wage compression can result in workers becoming self-employed in equilibrium for both “carrot” and “stick” reasons. Low-skilled workers may not be offered formal wage employment because the institutional wage is above their productivity. High-skilled workers, on the other hand,

³Acemoglu and Pischke (1999) also show how a compressed wage structure may arise not only as a consequence of institutionalised wage setting, but alternatively as a consequence of search costs or asymmetric information about effort or skills. Wallerstein (1999), however, finds that centralisation of wage setting – either via collective bargaining or government involvement in the wage setting process – is by far the most important factor explaining differences in wage inequality across countries.

may choose self-employment over wage employment because the compressed wage would pay them less than their productivity. Furthermore, learning implies that some workers with intermediate skills may prefer to learn during a period of wage employment before switching to self-employment. Some of these may, however, be forced into self-employment at the outset because firms calculate that the worker will switch too early for the firm to make a profit under the compressed wage structure.

We compare the predictions of our model with other findings on self-employment with respect to observables such as the share of self-employed and transition probabilities by age cohort, the relative frequency distribution of wage earnings and self-employment earnings, and the average earnings of wage workers with respect to work experience. Many of our predictions are consistent with existing empirical evidence.

The basic set-up is presented in Section 2, and the model is solved in Section 3. In Section 4, we consider the equilibrium outcome of the model, while we analyse comparative statics in Section 5. Section 6 derives a number of empirical implications, while Section 7 concludes.

2 The Basic Set-Up

We consider a continuum of individuals indexed by i , who are heterogeneous with respect to their innate ability, α_i , and finitely lived with an exogenous T years of labour market participation. We assume that individuals maximise the net present value (NPV) of their lifetime income by continuously choosing between unemployment, self-employment and wage employment – where the choice of the latter requires that an opportunity for wage employment is offered by a firm. An individual cannot hold more than one job (position) at any point in time. Wages depend on individual productivity – given by innate ability and previous wage-work experience – in an exogenously given manner, which we refer to as the *wage structure*. Income from self-employment also depends on individual productivity, while we assume that an unemployed individual receives a fixed benefit, θ , per unit of time which is independent of her productivity as well as the duration of her unemployment spell.⁴ For simplicity, we neglect any discounting of returns.

A job is offered to a worker if the firm has a positive NPV of employing that worker, *i.e.* if the NPV of worker productivity less wage is positive. A worker may thus be offered a job although the wage structure implies that

⁴In many countries, unemployment benefits depend on both previous work experience (income) and the time since last employment – aspects which would obviously affect the occupational choice in a dynamic setting like the present.

her current wage exceeds her productivity. Because of the exogenous wage structure, workers have no incentives to leave a firm to take up a position in another firm. Hence, in offering a job, the firm does not have to worry about the worker switching to another firm in the future, but only about the worker switching to self-employment.⁵

2.1 Wage Work

Following Boeri and Burda (2004), we assume that the institutionalised wage is linearly related to current productivity. Specifically, the wage to individual i after t years of employment, $w_i(t)$, is given by:

$$w_i(t) = w_m(1 - \tau) + \tau p_i(t) \quad (1)$$

where $p_i(t)$ is the productivity in wage work of individual i after t years of wage employment, and τ and w_m are the parameters of the wage structure. τ gives the degree of wage compression, where $\tau \in [0, 1]$. Without compression, $\tau = 1$, the wage equals productivity, whereas with $\tau < 1$, the wage structure is compressed – or "egalitarian" in the words of Boeri and Burda (2004). $w_m(1 - \tau)$ is the productivity-independent part of the wage structure and can be interpreted as a minimum wage. When $p_i(t)$ is low, the wage may therefore exceed productivity.

Productivity, $p_i(t)$, develops according to:

$$p_i(t) = \alpha_i + \gamma t \quad (2)$$

where α_i is innate ability, t is years of wage employment, and $\gamma \geq 0$ is the "learning rate".

2.2 Self-employment

The income of an entrepreneur with t_i years of wage work experience is given by:

$$e_i(t_i) = \beta_1 p_i(t_i) - \beta_0 \quad (3)$$

where $\beta_0 > 0$ and $\beta_1 > 1$. β_0 is a fixed permanent cost of self-employment compared to wage employment. We interpret β_0 as the resources which an entrepreneur must devote to administration, accounting, etc. β_1 , on the other hand, reflects the better utilisation of individual skills in self-employment,

⁵An exogenous separation rate could be introduced without affecting the qualitative content of our results.

for example, because problems of asymmetric information in the employment relation are avoided. Hence, p_i can be interpreted as the systematic or observable component of individual productivity, which can be exploited in wage work, whereas $\beta_1 p_i$ is the "full" productivity, which can only be realised in self-employment.

The presence of $\beta_0 > 0$ implies that for small t_i and α_i , the return to self-employment might be smaller than the productivity in wage work. High-ability individuals, on the other hand, might have returns to self-employment which are higher than their productivities as wage workers, even with $t_i = 0$.

The cost of switching to self-employment is given by $C > 0$, which is interpreted as the one-and-for-all cost of setting up a business.

2.3 Firms

A firm will offer a job to worker i if it has a positive NPV of employing that worker, *i.e.* if the NPV of worker productivity less wage is positive. If firms have no discounting, the profit from employing a worker of type i with no previous experience, $t = 0$, until experience reaches $t = t_i$ is given by:

$$\pi(t_i) = \int_0^{t_i} (p_i(t) - w_i(t)) dt = (1 - \tau) [\alpha_i - w_m] t_i + \gamma (1 - \tau) \frac{1}{2} t_i^2 \quad (4)$$

Without wage compression, $\tau = 1$, profit is always zero, and workers will therefore always be offered employment. Without learning, $\gamma = 0$, profit is non-negative if and only if $\alpha_i \geq w_m$, independently of the length of the employment period.

3 Solving the Model

In this Section, we derive the optimal choices by individuals and firms. However, to fully appreciate the role of wage compression and learning in explaining self-employment, we will first consider the outcome in the absence of these.

Without learning and wage compression ($\gamma = 0$ and $\tau = 1$), each wage worker receives a constant wage equal to her marginal product. All individuals will therefore be offered employment by the firms. Furthermore, there will be no incentive to switch occupation during your working life. The individual simply chooses the occupation that yields the highest life-time return. As a consequence, this scenario can generate at most three different types of workers: i) low-ability unemployed who prefer unemployment to both self-employment and wage work; ii) medium-ability wage workers

who prefer wage work to self-employment, and wage work to unemployment; and iii) high-ability entrepreneurs who prefer self-employment to both wage work and unemployment. Figure 1 illustrates a case where all three types are present.

[Insert Figure 1]

With wage compression, not everybody will be offered employment. Hence, wage compression can generate a class of low-skilled entrepreneurs. Furthermore, learning creates an incentive to stay longer in wage work or to start out in wage work before switching to self-employment, thereby creating a class of "wage-worker entrepreneurs". However, learning also makes firms more willing to hire workers in cases where the current wage exceeds productivity, although they must also take into account that workers may quit at some point.

As a consequence, we must derive the optimal occupational choice by an individual both when she is offered a job and when she is *not* offered a job. Then analysing when a job will in fact be offered by a firm allows us to characterise the equilibrium distribution of workers on different occupations in Section 4.

3.1 The Individual Decision Problem

Assuming that θ is unemployment benefits per time unit, the life-time return from being unemployed is given by:

$$V_i^u = \int_0^T \theta dt = \theta T \quad (5)$$

which is independent of α_i . The life-time utility from wage work for individual i is given by:

$$V_i^w = \int_0^T w_i(t) dt = w_m(1 - \tau)T + \tau\alpha_i T + \frac{\tau\gamma}{2}T^2 \quad (6)$$

whereas utility from a life as self-employed is:

$$V_i^e = \int_0^T e_i(0) dt - C = (\beta_1\alpha_i - \beta_0)T - C \quad (7)$$

With learning in wage work, some workers may also choose to switch occupation during their career. The utility from starting in wage work and

switching to self-employment at time t_i is given by:

$$V_i^{we}(t_i) = \int_0^{t_i} w_i(t) dt + \int_{t_i}^T e_i(t_i) dt - C = \\ (w_m(1-\tau) + \tau\alpha_i)t_i + \frac{\tau\gamma}{2}t_i^2 + (\beta_1(\alpha_i + \gamma t_i) - \beta_0)(T - t_i) - C \quad (8)$$

Note that $V_i^{we}(0) = V_i^e$. Since t_i must lie between 0 and T , we can define the *maximum* life-time utility from starting as an employee and switching to self-employment as:

$$V_i^{we} = \max_{t_i \in [0, T]} V_i^{we}(t_i) \quad (9)$$

To determine the optimal occupation of an individual, we must compare her utilities from the different career possibilities. First, assume that if individual i is not offered a job at the outset, she will never be offered a job, since her productivity only evolves in wage work.⁶ Hence, her life-time utility is given by:

$$V_i^{out} = \max \{V_i^u, V_i^e\} \quad (10)$$

In other words, she must choose between a life as unemployed and a life as entrepreneur.

On the other hand, if individual i is offered a job, life-time utility is given by:

$$V_i^{in} = \max \{V_i^u, V_i^w, V_i^{we}\} \quad (11)$$

That is, the individual chooses between unemployment, a life as a wage worker, and a career where she switches to self-employment at some (optimal) point in time – possibly already at the outset.

Consider the first situation in (10) where the individual is not offered a job. Define α_{eu} as the value of α_i at which $V_i^e = V_i^u$. Since only V_i^e is increasing in α_i , an individual who is not offered a job will choose self-employment if and only if $\alpha_i \geq \alpha_{eu}$, whereas she will choose unemployment if $\alpha_i < \alpha_{eu}$.⁷ From (5) and (7), it follows that the critical value, α_{eu} , is given by:

$$\alpha_{eu} = \frac{\theta + \frac{C}{T} + \beta_0}{\beta_1} \quad (12)$$

⁶Below, we argue that in some situations an individual who is not offered a job at the outset may actually in a few situations be offered a job after a period of unemployment or self-employment. In these situations, a third (and fourth) option enters (10). Introducing this at present would, however, only complicate matters unnecessarily.

⁷We assume that individuals who are indifferent between unemployment and self-employment, *i.e.* $\alpha_i = \alpha_{eu}$, will choose self-employment.

Consider then the second situation in (11). First, define α_{wu} as the value of α_i where $V_i^w = V_i^u$. Then for $\alpha_i > \alpha_{wu}$, individual i prefers wage employment to unemployment, and *vice versa*.⁸ From (5) and (6), it follows that:

$$\alpha_{wu} = \frac{\theta - w_m(1 - \tau)}{\tau} - \frac{\gamma T}{2} \quad (13)$$

The optimal date for a worker to switch from wage work to self-employment is not a simple programming problem for two reasons. First, the switching date (the choice variable) has both a lower bound of zero and an upper bound of T . Second, the optimised value of V *assuming* switching occurs must be compared to the value of V from lifetime wage work. The worker will switch only if the "optimal" switching date yields a lifetime value greater than lifetime wage work. The latter is analogous to an individual rationality or participation constraint in game theory and arises here due to the fixed costs of switching. Ignore the second problem for the moment and consider the maximisation problem in (9). Maximising $V_i^{we}(t_i)$ with respect to t_i yields the following first-order condition for an *interior* solution for t_i :⁹

$$w_i(t_i) + \int_{t_i}^T \frac{de_i(t_i)}{dt_i} dt = e_i(t_i) \quad (14)$$

The left-hand side is the marginal benefit of continuing in wage work, which consists of two terms: i) the wage rate evaluated at the current experience level; and ii) the effect on the entrepreneurial return of more experience, which affects the return throughout the self-employment period. The right-hand side is the marginal cost of continuing in wage work given by the foregone entrepreneurial return evaluated at the current experience level. Using the expressions for $w_i(t)$ and $e_i(t_i)$, the first-order condition in (14) implies the following value of t_i :

$$\hat{t}_i = \frac{\beta_0 + \beta_1 \gamma T + w_m(1 - \tau) - (\beta_1 - \tau) \alpha_i}{\gamma(2\beta_1 - \tau)} \quad (15)$$

Now, consider the upper and lower bounds on t_i – the first problem alluded to above. Since \hat{t}_i is decreasing in α_i , $\hat{t}_i \leq T$ if and only if $\alpha_i \geq \alpha_T$, where α_T is the ability level that gives an optimal switching time of $\hat{t}_i = T$:

$$\alpha_T = \frac{\beta_0 + w_m(1 - \tau)}{\beta_1 - \tau} - T\gamma \quad (16)$$

⁸Individuals who are indifferent between unemployment and wage work, $\alpha_i = \alpha_{wu}$, are assumed to choose the latter occupation.

⁹This condition is only relevant when $\gamma > 0$. Without learning, $\gamma = 0$, the solution to the maximisation problem in (9) is always given by a corner solution: $t_i = 0$ or $t_i = T$.

Similarly, $\hat{t}_i \geq 0$ if and only if $\alpha_i \leq \alpha_e$, where:

$$\alpha_e = \frac{\beta_0 + \beta_1 \gamma T + w_m (1 - \tau)}{\beta_1 - \tau} \quad (17)$$

Now, since $V_i^{we}(t_i)$ is globally concave in t_i , $t_i = T$ must be the best feasible choice for $\alpha_i \leq \alpha_T$, while $t_i = 0$ is the best choice when $\alpha_i \geq \alpha_e$. The optimal switching date for an individual – assuming that switching must occur – can therefore be summarised as:

$$t_i^* = \begin{cases} T & , \quad \alpha_i \leq \alpha_T \\ \hat{t}_i & , \quad \alpha_i \in [\alpha_T, \alpha_e] \\ 0 & , \quad \alpha_i \geq \alpha_e \end{cases} \quad (18)$$

Now we have to check the rationality constraint, $V_i^{we} \geq V_i^w$, i.e. whether or not the lifetime return to switching at the "optimal" time, t_i^* , is greater than the value of lifetime wage work – the second problem alluded to above. Switching at T must always be inferior to staying in wage work all life since switching costs are strictly positive, $C > 0$. Hence, $V_i^{we} - V_i^w = -C$ for all $\alpha_i \leq \alpha_T$. Furthermore, differentiating $V_i^{we} - V_i^w$ with respect to α_i yields $(\beta_1 - \tau)(T - t_i^*)$ by the envelope theorem. Hence, $V_i^{we} - V_i^w$ is increasing in α_i for $\alpha_i > \alpha_T$. As a consequence, there exists a unique $\alpha_{we} > \alpha_T$ given by the value of α_i where $V_i^{we} = V_i^w$, and such that $V_i^{we} < V_i^w$ for $\alpha_i < \alpha_{we}$, and $V_i^{we} > V_i^w$ for $\alpha_i > \alpha_{we}$. In other words, individuals with α_i less than α_{we} prefer lifetime wage work to a career where they switch, whereas individuals with α_i above α_{we} prefer the career where they switch to self-employment.¹⁰

Similarly, $V_i^{we} - V_i^u$ is strictly increasing in α_i . Hence, there exists a unique α_{weu} such that $V_i^{we} = V_i^u$ at $\alpha_i = \alpha_{weu}$, and such that $V_i^{we} < V_i^u$ for $\alpha_i < \alpha_{weu}$, and $V_i^{we} > V_i^u$ for $\alpha_i > \alpha_{weu}$. Note that since $V^{we} \geq V^e$, then $\alpha_{weu} \leq \alpha_{eu}$. A closed form solutions for α_{we} is derived in the appendix.

In sum, in the second situation in (11), unemployment is preferred when $\alpha_i < \min(\alpha_{wu}, \alpha_{weu})$, whereas wage employment is preferred when $\alpha_{wu} \leq \alpha_i < \alpha_{we}$. Finally, a career involving a switch is preferred in situations where $\alpha_i \geq \max(\alpha_{weu}, \alpha_{we})$ where the optimal switching date is given by (18).

Table 1 below summarises the critical values.

¹⁰We assume that individuals who are indifferent between the two careers, i.e. $\alpha_i = \alpha_{we}$, will prefer the one involving a switch to self-employment.

3.2 Firms

Using (4), the "break-even" condition for the firm, $\pi(t_i) = 0$, can be expressed as a required employment time, t_i^f , given by:

$$t_i^f = \frac{2[w_m - \alpha_i]}{\gamma} \quad (19)$$

We can think of two situations in which a job will not be offered. First, if $t_i^f > T$, the firm can never break even on a worker before she retires. This is the case if $\alpha_i < \alpha_f$, where α_f , the ability level such that firms break even at $t_i = T$, is given by:¹¹

$$\alpha_f = w_m - \frac{T\gamma}{2} \quad (20)$$

Second, those workers who find it optimal to switch to self-employment after an initial period of wage employment, *i.e.* those with $\alpha_{we} \leq \alpha_i < \alpha_e$, may want to do this too early from the firm's perspective. This is the case if t_i^f exceeds \hat{t}_i for these workers. Setting t_i^f equal to \hat{t}_i yields a critical value of α_i given by:

$$\alpha_0 = \frac{w_m(4\beta_1 - \tau - 1) - \beta_0 - \beta_1\gamma T}{3\beta_1 - \tau} \quad (21)$$

such that if $\alpha_{we} \leq \alpha_i < \alpha_e$ and $\alpha_i < \alpha_0$, worker i will not be offered a job initially, because of her incentive to switch to self-employment.

Note that since a reduction in T increases α_{we} , a worker who does not qualify for employment at the outset may qualify after a period as unemployed or entrepreneur, simply because the shorter remaining working life removes her incentive to switch to self-employment later on. We choose to ignore this possibility in the following, treating it only in footnotes.

[Insert Table 1]

4 Equilibrium

Having derived optimal worker and firm behaviour given the choice of the "opponent", this section puts the pieces together. The realised occupation of an individual depends on the interaction of worker and firm behaviour. In

¹¹Note for future reference that α_f is independent of compression. The reason lies in our definition of compression, the "pivoting" of the wage profile: A decrease in compression requires more skilled workers to be paid more but less skilled workers are paid less. The pivot point (the level of α_i where the wage is independent of τ) turns out to be exactly the same as the α_i that allows the firm to break even at $t_i = T$, namely α_f .

Section 4.1, this leads us to identify 8 different worker types that can arise in equilibrium. However, not all types can be present at the same time leading to different "regimes" (Section 4.2). An illustration of what we term the basic regime is provided in Section 4.3.

4.1 Worker Types

If a worker is not offered a job, she must choose between self-employment and unemployment. If offered a job, she can choose between unemployment, self-employment, wage employment, and a career where she switches. This allows us to identify eight different types of workers in equilibrium:

First, among those who cannot qualify for lifetime employment, $\alpha_i < \alpha_f$, we find two types:

- Type 1, the *low-skilled unemployed*, cannot qualify for a job and prefers unemployment to self-employment: $\alpha_i < \min(\alpha_f, \alpha_{eu})$.
- Type 2, the *low-skilled entrepreneur*, who cannot get a job either, but prefers self-employment to unemployment: $\alpha_{eu} \leq \alpha_i < \alpha_f$.

Second, among those who qualify for lifetime employment, $\alpha_i \geq \alpha_f$, we find:

- Type 3, the *voluntarily unemployed*, is sufficiently productive to qualify for a job, but prefers unemployment, i.e. $\alpha_f \leq \alpha_i < \min(\alpha_{wu}, \alpha_{weu})$.
- Type 4, the *lifetime employee*. Her ability is sufficient to qualify for a job, but not sufficient to warrant self-employment at some point in time: $\max(\alpha_f, \alpha_{wu}) \leq \alpha_i < \alpha_{we}$.
- Type 5, the *wage-worker entrepreneur*. She starts out as a wage worker but switches to self-employment later on: $\max(\alpha_f, \alpha_{weu}, \alpha_{we}, \alpha_0) \leq \alpha_i < \alpha_e$
- Type 6, the *high-skilled entrepreneur*, prefers to be self-employed from the outset: $\max(\alpha_f, \alpha_{weu}, \alpha_{we}, \alpha_e) \leq \alpha_i$.
- Type 7, the *medium-skilled entrepreneur*, is forced into self-employment because she would switch too early if employed: $\max(\alpha_f, \alpha_{weu}, \alpha_{we}, \alpha_{eu}) \leq \alpha_i < \min(\alpha_e, \alpha_0)$.

- Type 8, the *medium-skilled unemployed*, is forced into unemployment because she would switch too early if employed: $\max(\alpha_f, \alpha_{weu}, \alpha_{we}) \leq \alpha_i < \min(\alpha_e, \alpha_0, \alpha_{eu})$.¹²

4.2 Equilibrium Regimes

Consider a distribution of workers over the skill parameter, α . We will use the term "regime" to refer to the set of worker types observed in equilibrium, ordering the observed types from those observed at low α 's to those observed at high α 's. Note that not all types can exist at the same time. For example, the low-skilled entrepreneur (type 2) and the medium-skilled unemployed (type 8) are mutually exclusive.

The relative locations of the critical values determine which types will actually be present in equilibrium. As an example, consider the case where:

$$(\alpha_{wu}, \alpha_{eu}) < \alpha_f < \alpha_{we} < \alpha_e \quad \text{and} \quad \alpha_0 < \alpha_{we}$$

and remember that $\alpha_{weu} \leq \alpha_{eu}$. The resulting distribution of workers then follows directly from the previous section. It is illustrated in Figure 2.

[Insert Figure 2]

We will refer to this regime as our basic regime. Hence, our basic regime is capable of generating both the low-skilled unemployed and the wage-worker entrepreneurs, but does not contain types 3, 7 and 8.

4.3 An Illustration

In this section, we present simulation results which we hope will help develop some intuition behind the analytical results. We will concentrate on our basic regime. The parameter values used are as follows: $C = 90$, $T = 30$, $w_m = 20$, $\gamma = 0.5$, $\beta_0 = 10$, $\beta_1 = 1.4$, $\tau = 0.7$, and $\theta = 0$. These exogenous parameter values imply the following values for our critical ability levels: $\alpha_{wu} = -16.071$, $\alpha_{eu} = 9.286$, $\alpha_f = 12.500$, $\alpha_0 = 13.429$, $\alpha_{we} = 27.497$, and $\alpha_e = 52.857$. As $\alpha_{weu} \leq \alpha_{eu}$, the exact size of α_{weu} does not matter.

In the basic regime, all worker types, except types 3, 7, and 8, are present. The curve ZPR (zero profits) in Figure 3 gives the employment time that is necessary for the firm to break even, t_i^f . As the terminal time is $T = 30$ in our example, α_f is the ability level at which the curve ZPR intersects

¹²If we would allow for rematching with firms after an initial period of unemployment or self-employment, two extra types could be identified in relation to types 7 and 8.

$t = T = 30$. The curve $TSRO$ gives the optimal time for the worker to switch, t_i^* . This curve is only relevant for $\alpha_i \geq \alpha_{we}$, because below the latter ability level, switching at t_i^* is inferior to staying employed for life. Recall that $\alpha_{we} = 27.5$ in our example, so only points to the right of this value on $TSRO$ are relevant.

[Insert Figure 3]

Points labelled $TSRA$ are the actual times (if at all) at which the workers become self-employed. There are three distinct segments to this curve or rather locus of points. The first is the (disjoint) segment for Type 2 workers: workers who are not offered a job because they are not sufficiently productive even if they work for life. These workers become self-employed at $t = 0$. Next, there is the segment of Type 5 workers, who switch at their optimal time t_i^* , since they have ability levels greater than α_{we} . This segment is contiguous with the segment of Type 6 workers, who become entrepreneurs at $t = 0$ by choice, not because they cannot get a job. The other two regions are the Type 1 workers, the low-ability workers who cannot get a job and prefer unemployment to being an entrepreneur, and Type 4 workers who are employees for life. The latter are workers for whom switching at the optimal switch time t_i^* is inferior to staying employed for life: $\alpha_i < \alpha_{we}$.

While we feel that presenting all possible regimes would be rather dull and have little value added, we will briefly present one additional case in detail which supports six worker types in equilibrium. We found this case by raising C from 90 to 150, β_1 from 1.4 to 2.0, and θ from 0 to 5. The fixed cost of becoming an entrepreneur is larger, but the marginal effect of ability on entrepreneurial productivity, β_1 , is much larger. The effect of these changes is to reverse the ordering of α_{we} and α_0 so that we now have $\alpha_{we} < \alpha_0$. This will mean that some Type 7 workers will exist in equilibrium; workers that the firm would like to hire except that the workers will switch too early. These workers seem to fit the term “overqualified” as noted above.

Results corresponding to Figure 3 are shown in Figure 4. We now see that there are three sets of workers who are entrepreneurs for life. Moving from low ability to high, we have the Type 2 workers as before, who are not offered a job because they are not productive enough even if they worked for life. Then we have our Type 7 workers just mentioned who the firm would like to hire but knows the workers would switch too early for the firm to earn a profit. Finally, we have the high ability Type 6 workers who will want to be entrepreneurs from the start.

[Insert Figure 4]

5 Wage Compression and Learning

Having shown how learning and wage compression can generate both low-skilled entrepreneurs and wage-worker entrepreneurs, this section analyses the effects of wage compression and learning in more detail.

The comparative statics of a parameter change can be analysed at three different levels: i) the effects on critical values and the resulting occupational distribution, *i.e.* the resulting intervals of workers; ii) the effects on the prevailing regime; and iii) the effects on aggregate statistics such as the wage distribution and the share of entrepreneurs in the economy.

While the isolated effect of a parameter change on a critical value may not be too interesting in its own right, the implications for the resulting occupational distribution of individuals are of more relevance. These, however, depend on i) the initial regime, as, *e.g.*, an effect on α_0 only matters when $\alpha_0 \geq \alpha_{we}$; and ii) the effects of the parameter change on *all* critical values, as typically more than one is affected. Hence, in the following we focus on a situation where the economy is initially in our basic regime.

While marginal changes in parameter values can be relied upon to consider the effects on critical values and intervals of workers, a regime change can only come about as a consequence of a discrete change in a parameter, provided that the economy is not initially at the border between two regimes. Discrete changes are, however, more difficult to handle analytically. This calls for a numerical approach when it comes to analysing possible regime changes.

In Section 5.1, we analyse the effects of wage compression, and in Section 5.2 the effects of learning. In both cases, we first consider the effects on the occupational distribution of a marginal change in the parameter, and then possible regime changes of discrete changes in the parameters.

5.1 Wage Compression

The following Proposition characterises the effects of a change in wage compression, τ , provided that the economy is initially in the basic regime:

Proposition 1 *Starting in the basic regime, a marginal increase in τ (a reduction in compression):*

- *reduces the set of high-skilled entrepreneurs (type 6);*
- *shifts the set of wage-entrepreneurs (type 5) to the right, while postponing their switching date;*
- *expands the set of life-time wage workers (type 4) from above; and*

- leaves the set of low-skilled entrepreneurs (type 2) and the set of low-skilled unemployed (type 1) unaffected.

Proof. Proof in Appendix. ■

Intuitively, less compression makes wage work relatively more attractive at the upper end of the productivity scale, but less attractive at the lower end. The wage-worker entrepreneurs and high-skilled entrepreneurs can be shown to belong to the former group at the time when they switch and therefore choose to switch at a later point in time. This causes the least able of them to stay in wage work all life, while some of those who started out in entrepreneurship will now prefer an initial period of wage employment.¹³ As a consequence, the set of wage-entrepreneurs shifts to the right, expanding the set of life-time wage workers and reducing the set of high-skilled entrepreneurs.

Much of this likely seems intuitive with the exception of the result that the set of low-ability entrepreneurs does not change, since that would seem to be due to compression in the first place. This result is due to the fact that α_{eu} , the lower bound of the set of type 2 workers, is not affected by compression at all. It is the willingness to choose self-employment over unemployment. Second, it is due to the fact that α_f , the upper bound, is independent of τ . The intuition here is that a reduction in compression forces the firm to pay workers more later in their careers but allows the firm to pay less to the worker earlier in his or her career. This exactly balances for the marginal individual in our formulation.

Turning now to discrete changes in τ , a *reduction* in compression need not lead to a regime shift and does not do so for our base-case parameter values. But for alternative values, basically a high value of C , a reduction in compression can eliminate wage-worker entrepreneurs, as lifetime wage work becomes preferable for them. Thus, when compression is low and switching costly, workers (of sufficient skill) are either lifetime wage workers or enter directly as high-ability entrepreneurs at $t = 0$.¹⁴

Depending on the initial parameter values chosen to support the basic regime, an *increase* in wage compression may also lead to a regime shift. Specifically, by making switching more attractive, an increase in compression

¹³Note that while the latter has only a marginal effect on the entrepreneurial career, the change in α_{we} causes some individuals to go from a situation with no entrepreneurial activity to a strictly positive time period as entrepreneurs.

¹⁴It can be shown that although α_{wu} and α_{weu} are affected by changes in τ , they always remain below α_f and α_{eu} , respectively. Hence they do not affect the regime. Similarly, α_0 may increase as τ increases, but can be shown to always remain below α_{we} .

may introduce type 7 workers. These are individuals who would like to begin in wage work and later switch, but the firm calculates that they would switch too early for the firm to make a profit and so these individuals become self-employed for life. Further increases in compression can even eliminate lifetime wage workers completely.

This ambiguous response of the equilibrium regime to an increase in compression and its dependence on the initial parameter values used to support the basic regime can be explained as follows. Individuals want to switch earlier with increased compression, so α_{we} falls and the set of lifetime wage workers is reduced. The firm is also willing to hire some workers who will switch earlier with increased compression since the firm benefits by underpaying those workers more in the periods before they switch. Specifically, the break-even ability level, α_0 , for workers who switch falls. However, for a much lower value of C than in our central case (e.g., 30 instead of 90), α_{we} falls faster than α_0 and passes (falls below) α_0 at a strictly positive value of τ , in which case medium-skilled entrepreneurs are introduced (type 7). This is the six-worker-type regime given in Figure 4. Then α_{we} may further fall below α_f , which we noted was independent of compression, eliminating the lifetime wage workers altogether.¹⁵

5.2 The Learning Rate

The following Proposition characterises the effects of a change in the learning rate, γ , provided that the economy is initially in the basic regime:

Proposition 2 *Starting in the basic regime, a marginal increase in γ (the learning rate):*

- *reduces the set of high-skilled entrepreneurs (type 6);*
- *expands the interval of wage-entrepreneurs (type 5) from above, but reduces or expands it from below;*
- *postpones the switching date for the most able wage-entrepreneurs, but postpones or advances it for the least able;*
- *expands the set of lifetime workers (type 4) from below, but expands or reduces it from above;*

¹⁵As above, effects via α_{wu} and α_{weu} cannot affect the regime. Analytically, we have not been able to rule out a possible case where an increase in compression eliminates wage-worker entrepreneurs if α_e falls faster than α_{we} as τ decreases. Extensive parameter searches did not produce a case where this occurs as an actual outcome.

- *reduces the set of low-skilled entrepreneurs (type 2) from above; and*
- *leaves the set of low-skilled unemployed (type 1) unaffected.*

Proof. Proof in Appendix. ■

The learning rate, γ , affects productivity in both wage work and entrepreneurship, but only through accumulated experience. This has some important implications. First, the relative return from lifetime unemployment versus lifetime entrepreneurship remains unaffected, leaving the set of unemployed unaffected. Second, the increase in the learning rate makes it profitable for firms to hire individuals at lower ability levels, thereby reducing the set of low-skilled entrepreneurs who are forced into self-employment. Third, the interval of high-skilled entrepreneurs shrinks as the return from this activity is unaffected by the higher learning rate, whereas the return to wage-entrepreneurs increases. This increases the set of wage-entrepreneurs from above.

Fourth, the effect on the switching date is ambiguous. It is postponed for the most able wage-entrepreneurs, while the opposite might be true for the least able wage-entrepreneurs. To see this, consider the first-order equation in (14). There are two forces at work. On the one hand, an increase in γ raises the current entrepreneurial return, $e_i(t_i)$, by more than it raises the current wage, $w_i(t_i)$. This tends to advance the switching date. On the other hand, a higher learning rate means that the marginal effect of wage-work experience on future entrepreneurial earnings increase. This tends to postpone the switching date. The size of the first effect increases with the switching date, while the second effect is most important for those who switch early and therefore spend a long period as entrepreneurs. As a result, high-ability individuals (who switch early) postpone their switching date, while less able individuals may advance it.

If the least able wage-worker entrepreneurs advance their switching date, then the set of wage-worker entrepreneurs also expands from below. The earlier optimal switching date makes it easier to cover the cost of switching. However, even when the switching date is postponed for the least able wage-entrepreneurs, the higher γ in itself makes it easier to cover the cost of switching and may thus cause the set of wage-entrepreneurs to expand from below.

Our simulations are especially valuable in light of the ambiguity regarding the switching date. For our base-case values used to produce Figure 3, a marginal increase in γ (base value 0.5) reduces α_{we} , thus expanding the set of wage-worker entrepreneurs from both above and below, and reducing the set

of lifetime wage workers from above. But this relationship is non-monotonic, and for low values of γ , increases in γ increase α_{we} .¹⁶

A discrete *increase* in learning may eliminate low-skilled entrepreneurs (type 2) as firms become more willing to hire low-skilled individuals.¹⁷ It also reduces the set of high-skilled entrepreneurs. Thus the number of all workers who enter directly as entrepreneurs is reduced with a higher learning rate. It also shows that while wage compression can generate a class of low-skilled entrepreneurs, learning can remove this class again.

A discrete *decrease* in γ may have several possible consequences for the regime. First, it makes firms less willing to hire (α_f and α_0 increase). The increase in α_f may eventually eliminate lifetime employees (type 4), whereas the increase in α_0 may introduce the medium-skilled entrepreneur (type 7) and eventually eliminate the wage-worker entrepreneurs (type 5). Second, it makes it less attractive for the most skilled individuals to start in wage work (α_e drops). This may eventually eliminate type 5 workers if α_e falls below α_{we} .

6 Empirical Implications

Despite its simplicity, the model has implications for observed behaviour which can be contrasted with existing findings. We have already seen that the model is capable of generating a class of low-skilled entrepreneurs, a general finding in the empirical literature, see Evans and Leighton (1989).

Another general finding is that the share of self-employment in total employment is increasing over age groups, see, *e.g.*, Blanchflower (2000). Figure 5 shows how this share depends on working years (age) in our model, using the base case parameter values and assuming a uniform distribution of individuals across ability (between 0 and 70) and working years (between 0 and T). Although not entirely realistic, the uniform distribution allows us to separate the aggregate implications caused by the mechanics of the model from any aggregate implications caused by a non-uniform density across ability (and working years).

¹⁶Note that this non-monotonicity of α_{we} , first rising and then falling, is consistent with the derivative of α_{we} with respect to γ in appendix A.2. The positive term in that derivative involves $1/\gamma$, and thus the positive term shrinks with higher γ while the negative term is independent of γ .

¹⁷Furthermore, because of the ambiguous effect on α_{we} (the lower bound of wage-worker entrepreneurs) we cannot (analytically) rule out the possibilities that: i) type 4 is eliminated and/or type 7 (and even type 2) is introduced if α_{we} falls sufficiently fast; and ii) type 5 is eliminated if α_{we} increases (above α_e). Numerically, we have not seen examples of this, however.

[insert Figures 5 and 6]

The self-employment share first increases with working years and then becomes constant around 17 years.¹⁸ Empirically, a positive but diminishing effect of workforce experience has been found by Borjas (1986), while several papers find a similar effect of age, see Le (1999).

Obviously, this relationship, including the location of the "kink", is sensitive to the values of the parameters used. Less compression in the wage structure would cause less switching and hence shift both the strictly increasing part and the horizontal part of the line downwards. An increase in learning would also shift the strictly increasing part of the line downwards, but only because people would switch later. The eventual stock of self-employed and hence the horizontal part would be almost unaffected.

The transition probability from wage work into self-employment has also been the object of many empirical studies, see Le (1999). Figure 6 shows how this probability depends on working years in our model. The transition probability is seen to increase up to a certain age after which it drops (to zero). The increase in transition probability occurs despite the fact that we have assumed a uniform distribution of individuals across working years and ability and the fact that the optimal switching date is linear in ability. The reason is that as the number of working years increases, the stock of wage workers decreases, causing the transition probability to increase although the observed number of transitions per period remains constant. In other words, the transition probabilities in Figure 6 are conditional probabilities; conditional on not having switched previously. Or more generally, conditional on being a wage worker in the last period. The unconditional probability of switching at age t , on the other hand, is constant over age groups (for age ≤ 16).

This illustrates an important point, namely that finding a positive effect of age or working years on the (conditional) transition probability, as has been done in several empirical studies, cannot necessarily be interpreted as a positive effect of age *per se*, but may instead reflect an ongoing selection in the population.

Figures 7 and 8 show the earnings distributions of wage workers and self-employed, respectively. The hump-shaped and right-skewed distribution in Figure 7 arises despite the fact that we have a uniform distribution across both ability and age. Let us try to provide some intuition for this result. At the lower end of the distribution, we have the least skilled lifetime wage workers at the earliest stage of their careers. Between 14 and 25, the frequency

¹⁸The relatively large stock of self-employed – 75 per cent in the oldest generations – is a consequence of the uniform distribution across innate ability.

rises because observations from more able lifetime employees are constantly added – in addition to the observations from the less skilled later in their careers. Above 25, we start to “lose” observations from the lowest skilled lifetime employees, but observations from young wage-worker entrepreneurs are added in the same proportion. Hence, the frequency stays constant. However, above 32, we also start to “lose” observations from the lowest skilled wage-worker entrepreneurs when they switch, and the frequency therefore drops. Above 36, we stop losing observations from lifetime employees, who have all left the sample and the frequency therefore drops at a smaller rate.

As a consequence, the distribution becomes right skewed. This happens because of the wage-worker entrepreneurs. While the wage distribution over life-time wage workers would be symmetric, the observed distribution over life-time wage workers *and* wage-worker entrepreneurs becomes skewed to the right. The right skewness is in fact an attractive outcome of the model, as most empirical studies of earnings distributions tend to find this, see, *e.g.*, Hamilton (2000).

Note also that an increase in compression will clearly induce less wage dispersion, but for two reasons. First, because more compression reduces the wage differential at given productivity differences. Second, because more compression induces wage-worker entrepreneurs to switch earlier, thereby shifting the set of wage-worker entrepreneurs to the left. This implies less dispersion in *observed* productivities among wage workers and hence observed wages.

[Insert Figures 7 and 8]

In Figure 8, we see that the distribution of earnings in self-employment contains two parts. To the left, we have the low-skilled entrepreneurs. To the right, we first have the wage-worker entrepreneurs (earnings less than 65) and then the high-skilled entrepreneurs. The increasing density among the wage-worker entrepreneurs occurs because those with lowest innate abilities choose to enjoy more training. At a given productivity difference, the observed earnings differential for wage-worker entrepreneurs thus becomes smaller than for high-skilled entrepreneurs. In short, the exogenous compressed wage structure for wage workers induces an endogenous earnings compression among the self-employed!

Furthermore, Figure 8 predicts that earnings are generally more dispersed among self-employed. This is also fully consistent with the existing empirical evidence, see, *e.g.*, Hamilton (2000).

In Figure 9, we have shown average earnings in wage work as a function of experience in wage work. Despite the assumed positive effect of experience

on wages, the relationship is U-shaped. This is a consequence of a negative selection among the wage workers. As we move to the right, the most able wage-worker entrepreneurs switch to self-employment imposing a negative effect on the observed average wage. At some point (around 17), all wage-worker entrepreneurs have left the sample, leaving only the positive effect of experience on earnings.

[Insert Figure 9]

In general, the empirical implications of this simple model seem broadly consistent with observed empirical findings. Furthermore, they suggest some simple testable hypotheses for further empirical analyses.

7 Conclusion

We have developed a model that permits us to analyse the effects of institutional wage compression on self-employment. Individuals have a finite working life and learn while (and if) employed, a productivity improvement that carries over if they switch to self-employment.

We show that three types of lifetime self-employed can arise in equilibrium, along with individuals who may switch to self-employment after a period of wage employment. First, there are low-ability individuals who are forced into self-employment because they do not receive offers of employment since their productivity is less than the mandated wage. Second, there are high-ability individuals who will choose self-employment at the beginning of their work life because the mandated wage for their ability level is too low. Third, there may exist medium-ability individuals who would like to work and learn for a period of time before switching to self-employment, but the firm calculates that they would switch too early for the firm to make a profit. The latter are individuals who would commonly be referred to as “over qualified”. Individuals who are employed and then switch in equilibrium are relatively but not extremely high skilled individuals, placed in the ability distribution between lifetime wage workers and lifetime high-skilled entrepreneurs.

The model provides a number of empirical predictions. Some of these are consistent with existing findings, such as the existence of a group of marginalised low-skilled entrepreneurs; the higher self-employment rates for older workers; the right-skewed wage distribution; and the higher dispersion in earnings among self-employed. Other predictions provide testable hypotheses for further empirical work.

First, to the extent that it is possible to measure or proxy ability, the model predicts bi-modal or even tri-modal self-employment rates across the ability distribution. Second, with panel data on workers, we should observe workers who switch to self-employment to be in the middle to upper middle of the ability distribution. Third, any instances of labour-market reform should lower self-employment rates for at least the low and medium-skilled self-employed if these reforms can be interpreted as reducing wage compression or minimum wage rates. Fourth, policies that encourage or make it cheaper for firms to train workers could be crudely interpreted in our model as increasing γ , the learning rate. This has some ambiguous implications in our model, but it does predict that there will be some shifting of the low-skilled self-employed to wage work.

A Appendix

The first part of this appendix derives a closed-form solution for α_{we} . The second part contains proofs of the propositions in Section 5.

A.1 Closed form solution for α_{we}

First, to derive a closed form solution for α_{we} , we can insert t_i^* in $V_i^{we}(t_i)$ to get an expression for $V_i^{we} - V_i^w$. As the relevant expression for t_i^* in (18) depends on the value of α_i , so will the expression for $V_i^{we} - V_i^w$.

If $\alpha_{we} < \alpha_e$, the relevant expression for t_i^* is the second line in (18) which holds for $\alpha_i \in [\alpha_T, \alpha_e]$, as we know that $\alpha_{we} > \alpha_T$. This results in:

$$\begin{aligned} V_i^{we} - V_i^w = & \\ & \frac{(\beta_1 - \tau)^2}{2\gamma(2\beta_1 - \tau)} \alpha_i^2 + \frac{(\beta_1 - \tau)}{\gamma(2\beta_1 - \tau)} [\gamma T (\beta_1 - \tau) - \beta_0 - w_m (1 - \tau)] \alpha_i - \\ & \left(w_m (1 - \tau) T + \frac{\tau\gamma}{2} T^2 + C + \beta_0 T \right) + \frac{[\beta_0 + \beta_1 \gamma T + w_m (1 - \tau)]^2}{2\gamma(2\beta_1 - \tau)} \end{aligned} \quad (22)$$

for $\alpha_i \in [\alpha_T, \alpha_e]$. This expression is quadratic in α_i with a positive coefficient on α_i^2 , and since $V_i^{we} - V_i^w$ is increasing for $\alpha_i > \alpha_T$, it follows that α_{we} must be given by the largest root of $V_i^{we} - V_i^w = 0$. After some manipulations, this results in the following expression for α_{we} :

$$\alpha_{we} = \frac{-\gamma T (\beta_1 - \tau) + \beta_0 + w_m (1 - \tau) + 2\sqrt{\gamma C (\beta_1 - \frac{\tau}{2})}}{\beta_1 - \tau} \quad (23)$$

If $\alpha_{we} \geq \alpha_e$, the relevant expression for t_i^* is the last line in (18), which gives:

$$V_i^{we} - V_i^w = \alpha_i (\beta_1 - \tau) T - \beta_0 T - C - w_m (1 - \tau) T - \frac{\tau\gamma}{2} T^2$$

In this case, the value of α_{we} becomes:

$$\alpha_{we} = \frac{\beta_0 + w_m (1 - \tau) + \frac{\tau\gamma}{2} T + \frac{C}{T}}{(\beta_1 - \tau)}$$

A closed form solution for α_{weu} can be found in a similar way.

A.2 Proofs of Propositions

Proof of Proposition 1. From (12) and (20), it follows that $d\alpha_{eu}/d\tau = d\alpha_f/d\tau = 0$. Furthermore, from (23), (17), and (15), we get:

$$\begin{aligned}\frac{d\alpha_e}{d\tau} &= \frac{\alpha_e - w_m}{\beta_1 - \tau} > 0 \\ \frac{d\alpha_{we}}{d\tau} &= \frac{\alpha_{we} - w_m - \frac{1}{2}\sqrt{\gamma C / (\beta_1 - \frac{\tau}{2})} + \gamma T}{\beta_1 - \tau} > 0 \\ \frac{d\hat{t}_i}{d\tau} &= \frac{\alpha_i - w_m}{\gamma(2\beta_1 - \tau)} + \frac{\hat{t}_i}{(2\beta_1 - \tau)} > 0 \quad \text{for } \alpha_i \geq \alpha_{we}\end{aligned}$$

To signs of the derivatives can be proved as follows. First, when $\alpha_0 \leq \alpha_{we}$ as in Regime 1, then $t_i^f \leq \hat{t}_i$ at $\alpha_i = \alpha_{we}$. Since $dt_i^f/d\alpha_i < d\hat{t}_i/d\alpha_i < 0$, we get $t_i^f \leq \hat{t}_i$ for all $\alpha_i \geq \alpha_{we}$. Using (19), this implies that:

$$w_m \leq \alpha_i + \frac{\gamma \hat{t}_i}{2}$$

for $\alpha_i \geq \alpha_{we}$, which shows that $d\hat{t}_i/d\tau > 0$ for $\alpha_i \geq \alpha_{we}$. Second, setting $\alpha_i = \alpha_e$ implies:

$$w_m < \alpha_e$$

since $\hat{t}_i = 0$ when $\alpha_i = \alpha_e$. Finally, from differentiation of $V_i^{we} - V_i^w$ with respect to τ we get:

$$\frac{d(V_i^{we} - V_i^w)}{d\tau} = \left(\alpha_i + \frac{\gamma t_i^*}{2} - w_m \right) (t_i^* - T)$$

for $\alpha_i > \alpha_T$, which is negative when evaluated at $\alpha_i = \alpha_{we}$. Hence, an individual with $\alpha_i = \alpha_{we}$ will prefer lifetime wage work. This implies that $d\alpha_{we}/d\tau$ must be positive.

Together with Proposition 2, this gives us the results of Proposition 7. ■

Proof of Proposition 2. From (12), it follows that $d\alpha_{eu}/d\gamma = 0$. Furthermore, from (20), (23), (17), and (15), we get:

$$\begin{aligned}\frac{d\alpha_f}{d\gamma} &= -\frac{T}{2} < 0 \\ \frac{d\alpha_{we}}{d\gamma} &= \frac{-T(\beta_1 - \tau) + \sqrt{\frac{C}{\gamma}(\beta_1 - \frac{\tau}{2})}}{\beta_1 - \tau} \leq 0 \\ \frac{d\alpha_e}{d\gamma} &= \frac{\beta_1 T}{\beta_1 - \tau} > 0 \\ \frac{d\hat{t}_i}{d\gamma} &= \frac{\beta_1 T - (2\beta_1 - \tau)\hat{t}_i}{\gamma(2\beta_1 - \tau)} = \frac{\beta_1(T - \hat{t}_i) - (\beta_1 - \tau)\hat{t}_i}{\gamma(2\beta_1 - \tau)}\end{aligned}$$

$d\hat{t}_i/d\gamma$ is positive if and only if:

$$\begin{aligned} \hat{t}_i < \frac{\beta_1}{(2\beta_1 - \tau)}T &\iff \alpha_i > \frac{\beta_0 + w_m(1 - \tau)}{\beta_1 - \tau} \\ &\iff \beta_1\alpha_i - \beta_0 > w_m(1 - \tau) + \tau\alpha_i \end{aligned}$$

which is the same as $e_i(0) > w_i(0)$. The condition is always satisfied for $\alpha_i = \alpha_e$, since:

$$\alpha_e = \frac{\beta_0 + w_m(1 - \tau) + \beta_1\gamma T}{\beta_1 - \tau}$$

However, it need not be satisfied for $\alpha_i = \alpha_{we}$. Specifically, $d\hat{t}_i/d\gamma$ is negative at $\alpha_i = \alpha_{we}$ if:

$$2\sqrt{\gamma C\left(\beta_1 - \frac{\tau}{2}\right)} - \gamma T(\beta_1 - \tau) < 0$$

It follows that if $\frac{d\hat{t}_i}{d\gamma} < 0$ at $\alpha_i = \alpha_{we}$, then $\frac{d\alpha_{we}}{d\gamma} < 0$.

Together with Proposition 2, this gives us the results of Proposition 2. ■

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TABLE 1: CRITICAL VALUES

Defined for workers without reference to firm behaviour:

- α_{wu} : worker indifferent between lifetime wage work and lifetime unemployment
- α_{eu} : worker indifferent between lifetime entrepreneurship and lifetime unemployment
- α_{we} : worker indifferent between lifetime wage work and switching at t_i^*
- α_{weu} : worker indifferent between lifetime unemployment and switching at t_i^*
- α_e : worker has optimal switching date: $t_i^* = 0$.

Defined for firms without reference to worker behaviour:

- α_f : firm breaks even on lifetime employment.

Defined for firms with reference to worker behaviour:

- α_0 : firm breaks even on a worker who switches at $t_i^* \in (0, T)$

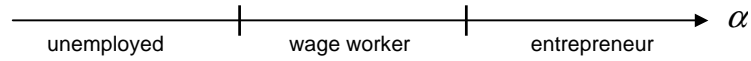


Figure 1: Occupational distribution without learning and wage compression

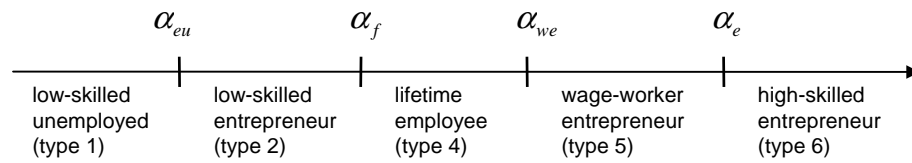


Figure 2: Occupational distribution in the basic regime

Figure 3: Worker types as a function of ability

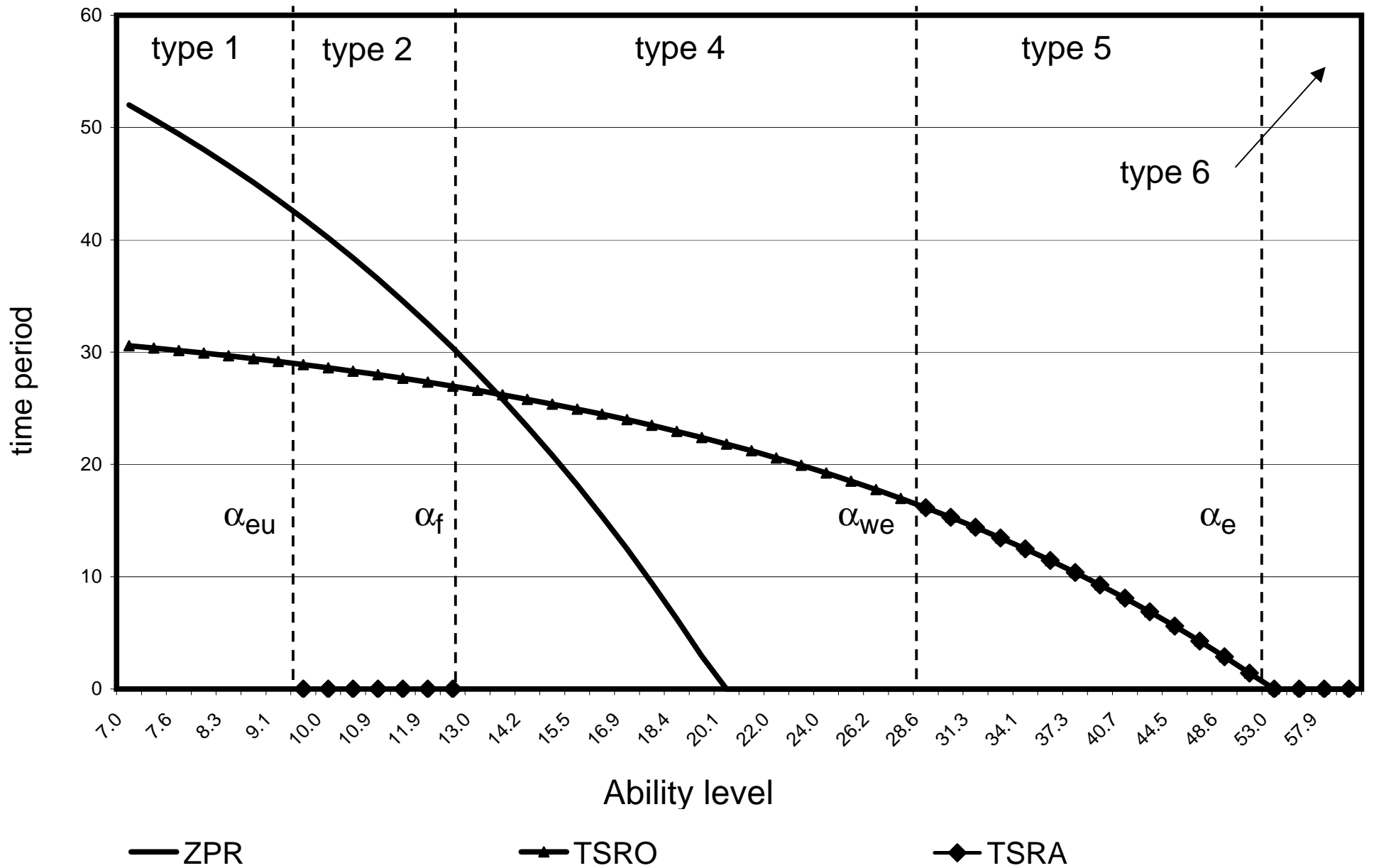


Figure 4: Worker types as a function of ability

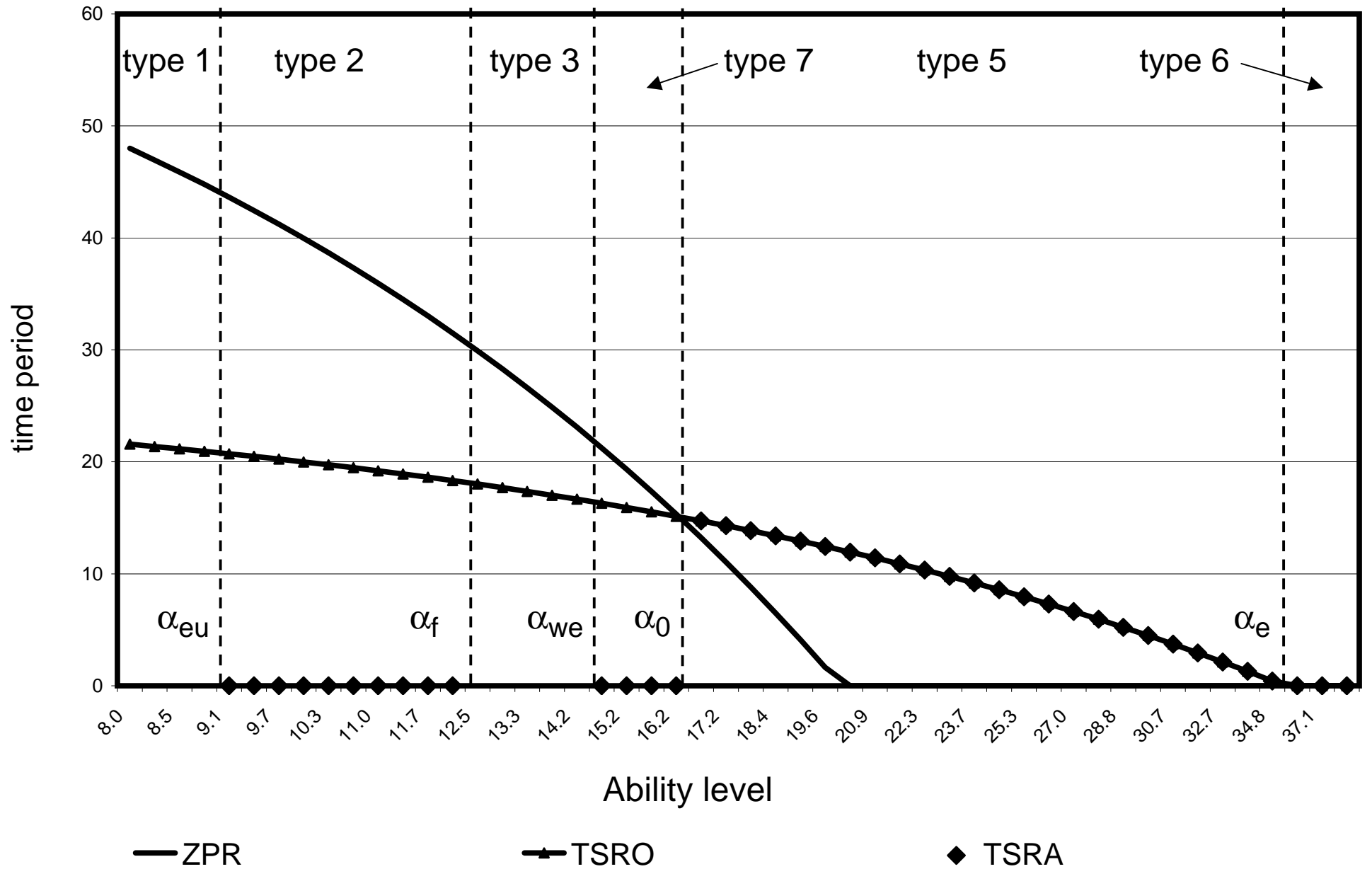


Figure 5: Share of self-employed as a function of working years

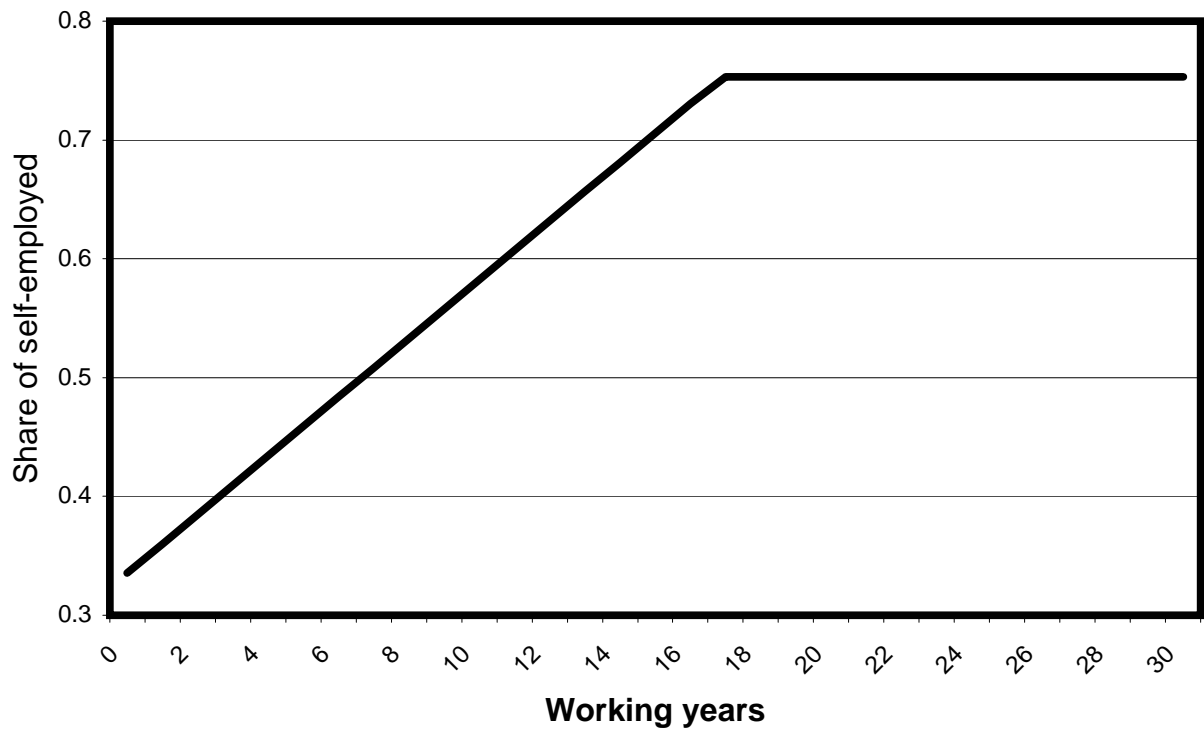


Figure 6: Transition probabilities as a function of working years

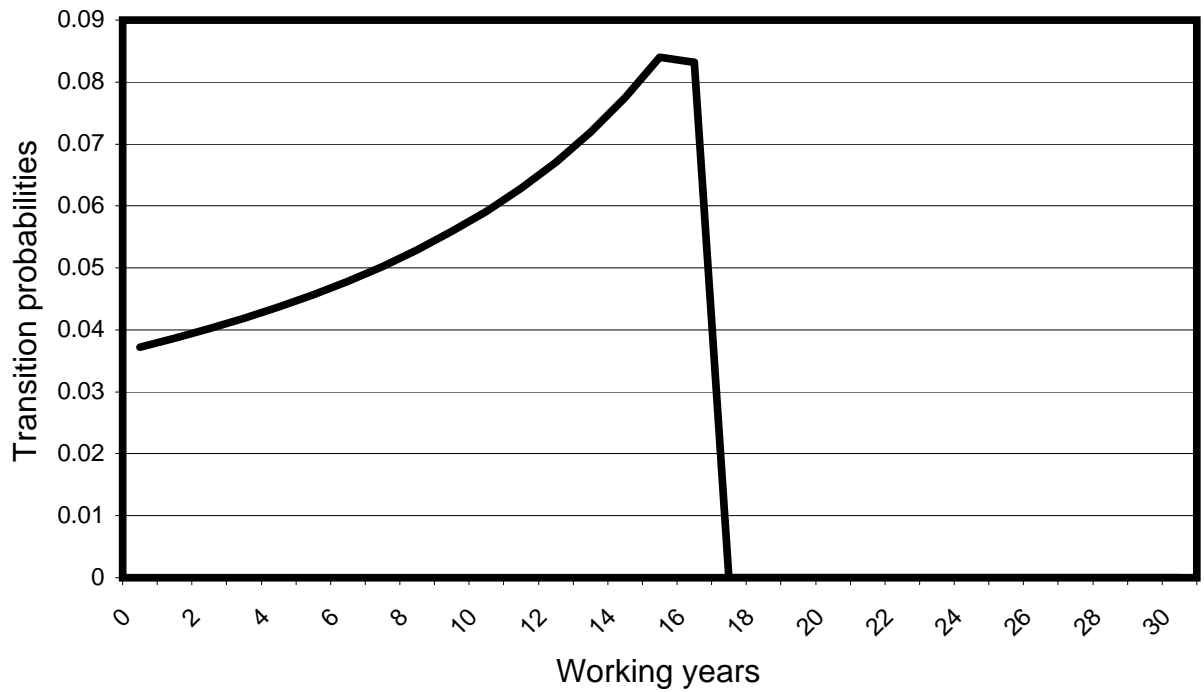


Figure 7: Distribution of wage earners

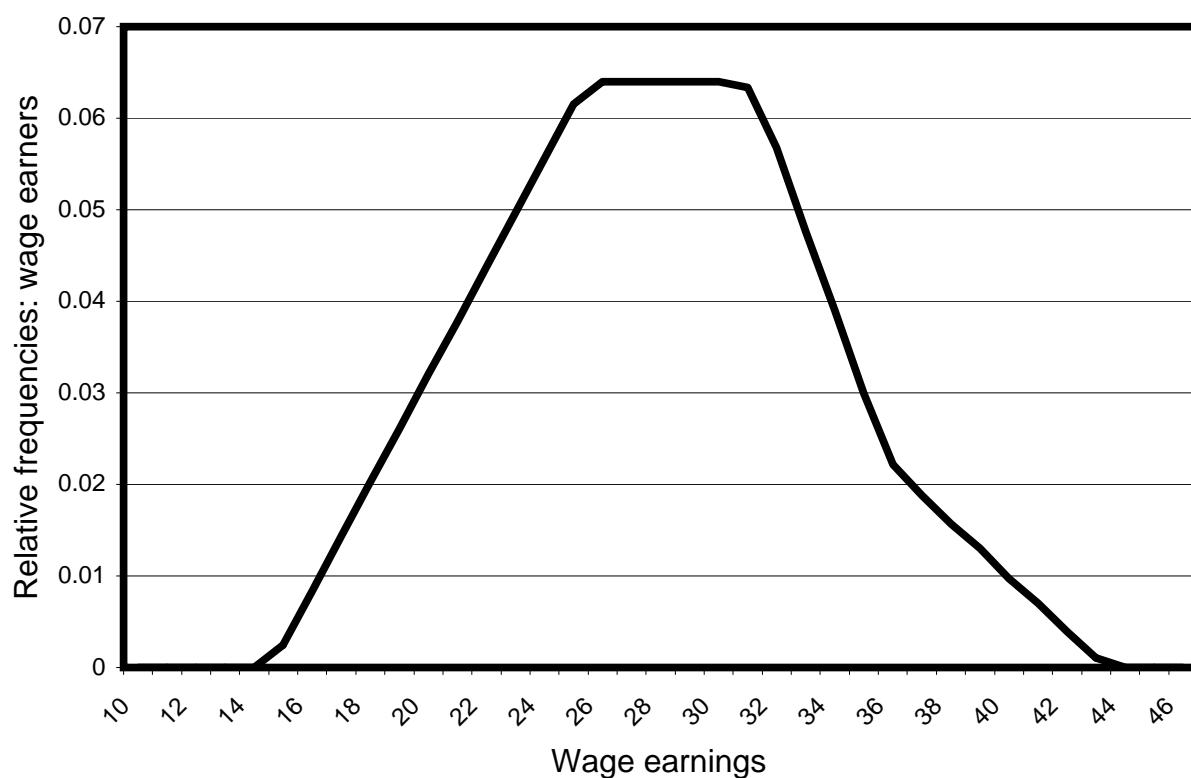


Figure 8: Distribution of entrepreneur earnings

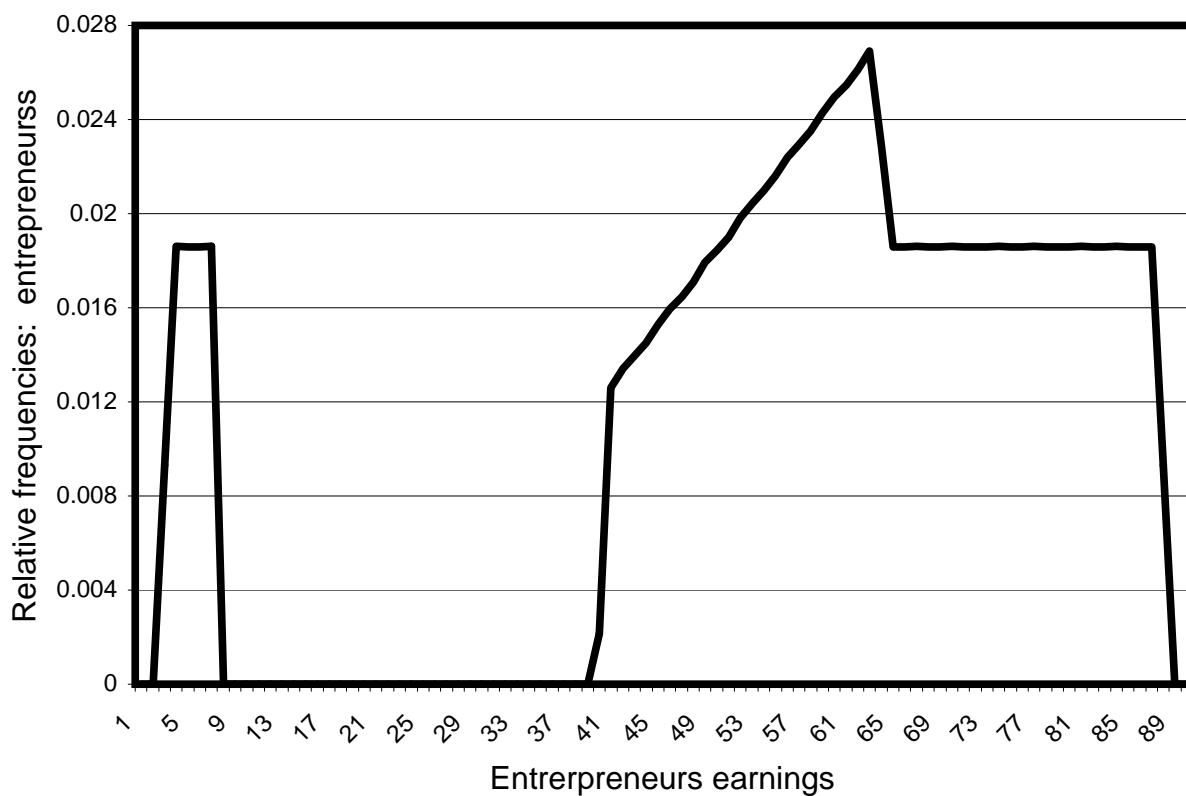


Figure 9: Average wage earnings as a function of experience

