Economic equilibrium and optimization problems using GAMS
Notes 3: general-equilibrium modeled as an MCP

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1. Introduction to applied economic- equilibrium (general-equilibrium) modeling

(1) Multiple interacting agents

(2) Individual behavior based on optimization

(3) Most agent interactions are mediated by markets and prices

(4) Equilibrium occurs when endogenous variables (e.g., prices) adjust such that

(a) agents, subject to the constraints they face, cannot do better by altering their behavior

(b) markets (generally, not always) clear so, for example, supply equals demand in each market.

2. Steps in Applied General-Equilibrium Modeling

(1) Specify dimensions of the model.

- Numbers of goods and factors
- Numbers of consumers
- Numbers of countries
- Numbers of active markets

(2) Chose functional forms for production, transformation, and utility functions; specification of side constraints.

- Includes choice of outputs and inputs for each activity
- Includes specification of initially slack activities

(3) Construct micro-consistent data set.

- Data satisfies zero profits for all activities, or if profits are positive, assignment of revenues
- Data satisfies market clearing for all markets

(4) Calibration – parameters are chosen such that functional forms and data are consistent.

- By “consistent”: data represent a solution to the model

(5) Replication – run model to see if it reproduces the input data.

(6) Counter-factual experiments.

Example M3-1a: 2-good, 2-factor closed economy with fixed factor endowments, one representative consumer.

Simply economy, two sectors (X and Y), two factors (L and K), and one representative consumer (utility function W).

L and K are in inelastic (fixed) supply, but can move freely between sectors.

p_x, p_y, p_l, and p_k are the prices of X, Y, L and K, respectively.

CONS is consumer's income and p_w will be used later to denote the price of one unit of W.

(1) $X = F_x(L_x, K_x)$

(2) $Y = F_y(L_y, K_y)$

(3) $L^* = L_x + L_y$

(4) $K^* = K_x + K_y$

(5) $W = W(X, Y)$

(6) $CONS = p_l L^* + p_k K^* = p_x X + p_y Y$

How do we find equilibrium: prices, and factor allocations?

Equilibrium could be solved for by a constrained optimization problem: Max (5) subject to the constraints (1), (2), (3), (4), (6).

The usefulness of this approach breaks down quickly as the model becomes more complicated.

Alternative approach: convert the problem to a system of equations, and solve that system.

Solve the cost minimization problems for producers and consumers: individual optimizing behavior is embedded in the model.

These give the minimum cost of producing a good.

- Cost mimization => unit cost functions for X and Y

$$cx = cx(p_l, p_k),$$
$$cy = cy(p_l, p_k)$$

- Profit maximiztion => competitive pricing equations for X, Y

$$cx(p_l, p_k) \geq p_x$$
$$cy(p_l, p_k) \geq p_y$$

- Shepard's lemma to get demands for K and L in X and Y

$$\frac{\partial cx}{\partial p_l} = cx_{pl} =$$ X demand for labor per unit of output

$$\frac{\partial cx}{\partial p_k} = cx_{pk} =$$ X demand for capital per unit of output

- Consumer maximization of utility => Marshallian demand functions (money income M is denoted CONS in code to follow)

$$X = D_x(p_x, p_y, M) \qquad Y = D_y(p_x, p_y, M)$$

- Income balance => *M* is derived from the equilibrium K and *L* prices and the consumer's "endowment" of *K* and *L*.

$M = p_k K^* + p_l L^*$ *K** and *L** fixed "endowments (supplies)

Big difference from our work to this point?

Prices and income, although taken as exogenous by the individual actors, are now endogenous variables in the economic system.

General-equilibrium formulated as a square system:

(1) Non-positive profits for X	$cx(p_l, p_k) \geq p_x$	
(2) Non-positive profits for Y	$cy(p_l, p_k) \geq p_y$	zero-profit inequalities
(3) Supply $\geq$ Demand for X	$X \geq d_x(p_x, p_y)*CONS$	
(4) Supply $\geq$ Demand for Y	$Y \geq d_y(p_x, p_y)*CONS$	
(5) Supply $\geq$ Demand for L	$L^* \geq cx_{pl}X + cy_{pl}Y$	
(6) Supply $\geq$ Demand for K	$K^* \geq cx_{pk}X + cy_{pk}Y$	
(7) Income balance	$CONS = p_lL^* + p_kK^*$	income balance

These weak inequalities can be solved for the unknowns X, Y, p_x, p_y, p_l, p_k, and CONS.

These inequalities are of three types, and this is generally true:

- Zero-profit conditions, inequalities (1)-(2) in the above example.
- Market clearing conditions, inequalities (3)-(6) in the above example
- Income balance, equation (7) in the above example.

Formulating equilibrium as a complementarity problem requires that each inequality is associated with a particular variable.

If a zero profit conditions holds as a strict inequality in equilibrium, profits for that activity are negative, that *activity* will not used.

The complementary variable to a *zero-profit condition* is a *quantity*, the activity level.

If a market-clearing condition holds as a strict inequality, supply exceeds demand for that good or factor in equilibrium so its price must be zero.

Thus the complementary variable to a *market clearing inequality* is the *price* of that good or factor.

The complementary variable to an income balance equation is just the income of that agent.

Inequality		Complementary Variable
(1) Non-positive profits for X	$cx(p_l, p_k) \geq p_x$	X
(2) Non-positive profits for Y	$cy(p_l, p_k) \geq p_y$	Y
(3) Supply $\geq$ Demand for X	$X \geq d_x(p_x, p_y)CONS$	p_x ,
(4) Supply $\geq$ Demand for Y	$Y \geq d_y(p_x, p_y)CONS$	p_y
(5) Supply $\geq$ Demand for L	$L^* \geq cx_{pl}X + cy_{pl}Y$	p_l
(6) Supply $\geq$ Demand for K	$K^* \geq cx_{pk}X + cy_{pk}Y$	p_k
(7) Income balance	$CONS = p_lL^* + p_kK^*$	CONS

3.2 Micro consistent data

A data set is micro consistent when it satisfies the conditions for economic equilibrium (it could be generated as the solution to some model).

Data must satisfy *zero profits*, *market clearing*, and *income balance*.

The above problems can be thought of as consisting of
three production activities, X, Y, and W,
four markets, X, Y, L, and K
income balance

Represent the initial data for this economy by a rectangular matrix. This matrix is related to the concept of a "SAM" – social accounting matrix

There are two types of *columns* in the rectangular matrix, corresponding to *production activities(sectors)* and *consumers*.

In the model outlined above, there are three production sectors (X, Y and W) and a single consumer (CONS).

Rows correspond to *markets*. Complementary variables are prices, so we have listed the price variables on the left to designate rows.

A positive entry signifies a receipt (sale) in a particular market. A negative entry signifies an expenditure (purchase) in a particular market.

Reading down a production column, we then observe a complete list of the transactions associated with that activity.

Marshallian:

	Production Sectors		Consumers	
Markets	X	Y	CONS	Row sum
PX	100		-100	0
PY		100	-100	0
PL	-25	-75	100	0
PK	-75	-25	100	0
Column sum	0	0	0	

A rectangular SAM is balanced or “micro-consistent” when row and column sums are zeros.

Positive numbers represent the value of commodity flows into the economy (sales or factor supplies),

Negative numbers represent the value of commodity flows out of the economy (factor demands or final demands).

A *row sum is zero* if the total amount of commodity flowing into the economy equals the total amount flowing out of the economy.

Row sum = 0 = *market clearance*, and one such condition applies for each commodity in the model.

Columns in this matrix correspond to production sectors or consumers.

A production sector *column sum is zero* if the value of outputs equals the cost of inputs.

A consumer column is balanced if the sum of primary factor sales equals the value of final demands.

Zero column sums thus = *zero profits or "product exhaustion"* in an alternative terminology.

The numbers of the matrix are *values, prices times quantities*. The modeler decides how to interpret these as prices or quantities.

A good practice is to choose units so that as many things initially are equal to one as possible.

Prices can be chosen as one, and “representative quantities” for activities can be chosen such that activity levels are also equal to one (e.g., activity X at level one produces 100 units of good X). More below.

In the case of taxes, both consumer and producer prices cannot equal one of course, a point we will return to in a later section.

3.3 Calibration and replication

Calibration is choosing functional forms and their parameters such that the initial micro-consistent data is a solution to the model.

We use Cobb-Douglas functions for the three activities. The share parameters for the functions are given in the data matrix above.

Goods in the utility get equal shares 0.5: Marshallian demands are:

$$X = 0.5 * CONS/p_x \qquad Y = 0.5 * CONS/p_y$$

X is capital intensive: a capital share of 0.75, a labor share of 0.25.

$$c_x(p_l, p_k) = p_l^{0.25} p_k^{0.75}$$

Y is labor intensive with the opposite ordering of shares.

$$c_y(p_l, p_k) = p_l^{0.75} p_k^{0.25}$$

Example of Shepard's lemma: demand for labor in *Y*.

$$L_y = \frac{\partial c_y(p_l, p_k)}{\partial p_l} Y = 0.75 p_l^{0.75-1} p_k^{0.25} Y = 0.75 p_l^{0.75} p_k^{0.25} Y / p_l$$

If p_y is the producer price of *Y*, then this can be written as:

$$L_y = \frac{\partial c_y(p_l, p_k)}{\partial p_l} Y = 0.75 p_y / p_l * Y$$

3.4 Choice of units and "reference quantities"

With no taxes initially, we can break the values into prices and quantities in any way we want, subject to being consistent throughout the model!

A good practice is to then pick all prices as = 1.

So we have, for example the 100 units of X in *value* is also 100 units in *quantity*.

However, there is a little trick such that the initial values of *X*, *Y* are also = 1 initially. We use what are called *reference quantities*.

Wherever *X* and *Y* appear in the model, replace them with 100**X* and 100**Y*, and so forth. The 100s are reference quantities, and this re-scales *X* and *Y* so that they are both = 1 in the benchmark.

Advantages: (1) benchmark replication is a nice clean list of 1s. (2) changes in variables are easily, quickly interpreted as proportional changes

"LENDOW" and "KENDOW are multipliers on the initial factor endowment, thus used for comparative statics experiments.

3.5 Indeterminacy of the price level and choice of "numeraire"

Problem: if we find a set of prices p_x, p_y, p_l, p_k, and CONS that solves the model, then any proportional multiple of these prices will also solve the model.

Walras' Law: there is one redundant equation in the model: if (n-1) of the equations hold, the nth holds as well.

Solution: fix one price, referred to as the *numeraire*.

```
                        LOWER      LEVEL      UPPER    MARGINAL

---- VAR X                .        1.000      +INF        .
---- VAR Y                .        1.000      +INF        .
---- VAR PX               .        1.000      +INF        .
---- VAR PY               .        1.000      +INF        .
---- VAR PL             1.000      1.000      1.000      EPS
---- VAR PK               .        1.000      +INF        .
---- VAR CONS             .      200.000      +INF
 .
PARAMETER WELFARE              =        1.000  welfare (utility)
PARAMETER PINDEX               =        1.000  consumer price index
PARAMETER RWAGE                =        1.000  real wage
PARAMETER RRENT                =        1.000  real rental rate
```

In the first counterfactual, we remove the tax, and double the labor endowment of the economy.

```
                          LOWER     LEVEL     UPPER    MARGINAL

---- VAR X                  .       1.189     +INF  -2.642E-8
---- VAR Y                  .       1.682     +INF  -1.868E-8
---- VAR PX                 .       1.682     +INF  -1.652E-8
---- VAR PY                 .       1.189     +INF  1.3624E-8
---- VAR PL               1.000     1.000     1.000 1.4595E-7
---- VAR PK                 .       2.000     +INF  -6.718E-8
---- VAR CONS               .     400.000     +INF       .

PARAMETER WELFARE                =        1.414  welfare (utility)
PARAMETER PINDEX                 =        1.414  consumer price index

PARAMETER RWAGE                  =        0.707  real wage
PARAMETER RRENT                  =        1.414  real rent
PARAMETER RPX                    =        1.189  real price of X
PARAMETER RPY                    =        0.841  real price of Y
```

3.6 General equilibrium II: Hicksian approach, as the solution to a square system of 9 weak inequalities in 9 unknowns.

Treats utility W (welfare) as a produced and "marketed" good. There is a zero-profit inequality for W, and a market-clearing equation for W. The latter is complementary with p_w, the price of a unit of utility, or the true consumer price index.

General-equilibrium formulated as a square system:

(1) Non-positive profits for X	$cx(p_l, p_k) \geq p_x$	
(2) Non-positive profits for Y	$cy(p_l, p_k) \geq p_y$	zero-profit inequalities
(3) Non-positive "profits" for W	$e(p_x, p_y) \geq p_w$	

(4) Supply $\geq$ Demand for X	$X \geq e_{px}(p_x, p_y)W$	
(5) Supply $\geq$ Demand for Y	$Y \geq e_{py}(p_x, p_y)W$	
(6) Supply $\geq$ Demand for W	$W \geq CONS / p_w$	market-clearing inequalities
(7) Supply $\geq$ Demand for L	$L^* \geq cx_{pl}X + cy_{pl}Y$	
(8) Supply $\geq$ Demand for K	$K^* \geq cx_{pk}X + cy_{pk}Y$	
(9) Income balance	$CONS = p_l L^* + p_k K^*$	income balance

These weak inequalities can be solved for the unknowns X, Y, W, p_x, p_y, p_w, p_l, p_k, and CONS.

Inequality		Complementary Variable
(1) Non-positive profits for X	$cx(p_l, p_k) \geq p_x$	X
(2) Non-positive profits for Y	$cy(p_l, p_k) \geq p_y$	Y
(3) Non-positive "profits" for W	$e(p_x, p_y) \geq p_w$	W
(4) Supply $\geq$ Demand for X	$X \geq e_{px}(p_x, p_y)W$	p_x ,
(5) Supply $\geq$ Demand for Y	$Y \geq e_{py}(p_x, p_y)W$	p_y
(6) Supply $\geq$ Demand for W	$W \geq CONS / p_w$	p_w
(7) Supply $\geq$ Demand for L	$L^* \geq cx_{pl}X + cy_{pl}Y$	p_l
(8) Supply $\geq$ Demand for K	$K^* \geq cx_{pk}X + cy_{pk}Y$	p_k

(9) Income balance $CONS = p_l L^* + p_k K^*$ CONS

Marshall

Markets	X	Y	CONS	Row sum
PX	100		-100	0
PY		100	-100	0
PL	-25	-75	100	0
PK	-75	-25	100	0

Hicks - adds utility treated as a produced, marketed good

Markets	X	Y	W	CONS	Row sum
PX	100		-100		0
PY		100	-100		0
PW			200	-200	0
PL	-25	-75		100	0
PK	-75	-25		100	0

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	1.000	+INF	.
---- VAR Y	.	1.000	+INF	.
---- VAR W	.	1.000	+INF	.
---- VAR PX	.	1.000	+INF	.
---- VAR PY	.	1.000	+INF	.
---- VAR PW	1.000	1.000	1.000	EPS
---- VAR PL	.	1.000	+INF	.
---- VAR PK	.	1.000	+INF	.
---- VAR CONS	.	200.000	+INF	.

Double LENDOW

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	1.189	+INF	.
---- VAR Y	.	1.682	+INF	.
---- VAR W	.	1.414	+INF	.
---- VAR PX	.	1.189	+INF	.
---- VAR PY	.	0.841	+INF	.
---- VAR PW	1.000	1.000	1.000	2.3558E-9
---- VAR PL	.	0.707	+INF	.
---- VAR PK	.	1.414	+INF	.
---- VAR CONS	.	282.843	+INF	.

3.7 Model 3-1c Adds taxes to model 3-1b

There are several important modifications that taxes introduce:

(1) There is no longer “the price” for a good or factor: the price received by a supplier and that paid for by the buyer are different.

(2) The tax base is important:

net basis:	$p = (1+t)mc$	p is consumer price
gross basis:	$p(1-t) = mc$	mc is producer price

(2) Revenue collected must go somewhere for GE adding up to be satisfied.

To consider the effect of purely distortionary taxes, it is typically assumed that the government lump-sum redistributes taxes back to consumers.

Assume that there are commodity taxes on X and Y, on a net basis

$$p_x = (1 + t_x) mc_x \qquad p_y = (1 + t_y) mc_y$$

Then the representative consumer must get a redistribution equal to

$$t_x mc_x X + t_y mc_y Y$$

We can write out the expression for marginal cost or, from (1), note that this is equal to consumer price divided by one plus t.

$$t_x (p_x / (1 + t_x)) X + t_y (p_y / (1 + t_y)) Y$$

The results for our first counterfactual, in which we place a 50% tax on the inputs to X production.

************* counterfactual: tax on X inputs

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	0.845	+INF	.
---- VAR Y	.	1.147	+INF	.
---- VAR W	.	0.985	+INF	.
---- VAR PX	.	1.165	+INF	.
---- VAR PY	.	0.859	+INF	.
---- VAR PW	1.000	1.000	1.000	-1.602E-12
---- VAR PL	.	0.903	+INF	.
---- VAR PK	.	0.739	+INF	.
---- VAR CONS	.	196.946	+INF	.

X production decreases, Y production increases, and welfare falls due to the distortionary nature of the tax, even though the tax revenue is redistributed back to the consumer.

There is also a redistribution of income between factors.

The relative price of capital, the factor used intensively in X falls, and the relative price of labor rises as resources are shifted to Y production.

3.8 Initially slack activities

As noted in chapter one, an attractive and powerful feature of GAMS is that it solves complementarity problems in which some production activities can be slack for some values of parameters and active for others.

This allows researchers to consider a much wider set of problems that is allowed using software which can only solve systems of equations.

There are many uses of this in practice:

alternative energy technologies that are unprofitable initially

illegal activities that are inefficient relative to legal ones

trade links that are initially inactive due to high trade costs

Model M3-2 presents an example, motivated by tax evasion activities, or a “green” but expensive technology

A third sector, Z, also produces good X but it is 10% less efficient (10% more costly) than the X activity itself. So initially, Z does not operate.

```
PRF_X..  100*PL * (1+TX) =G= 100*PX;

PRF_Z..  100*INEF*PL =G= 100*PX;

MKT_L..  200 =G= 100*X + 100*Y + 100*INEF*Z;
```

We could think of Z as a tax evasion or “informal” activity that is less efficient but can successfully avoid the tax.

But when a tax of 25% is imposed on X, this activity goes slack and Z begins to operate.

Another counterfactual assesses the effect of a 25% tax when the Z technology cannot operate, to assess switching costs.

The command Z.FX fixes the level of Z at its current value.

Fixing a variable involves setting both its upper and lower value. Thus to unfix a variable, both its upper and lower values must be reset.

```
Z.UP = +INF;
Z.LO = 0;
```

Another thing introduced is a trick to find calibration errors

```
SLACK.ITERLIM = 0;
SOLVE SLACK USING MCP;

SLACK.ITERLIM = 2000;
SOlVE SLACK USING MCP;
```

Please see later section for an explanation of this command.

3.9 Quick introduction to sets and scenarios

```
SETS I /I1*I51/;

PARAMETERS
  TAXRATE(I)
  WELFARE1(I), WELFARE2(I)
  RESULTS1(I,*), RESULTS2(I,*);

* compare tax with and without Z allowed
```

```
LOOP(I,

TX = 0.0005*ORD(I)**2 - 0.0005;

  Z.UP = +INF;
  Z.LO = 0;

SOLVE SLACK USING MCP;

TAXRATE(I) = TX;
WELFARE1(I) = W.L;

  Z.FX = 0;

SOLVE SLACK USING MCP;

WELFARE2(I) = W.L;

);
```

```
RESULTS2(I, "TAXRATE") = TAXRATE(I);
RESULTS2(I, "WEFLARE1") = WELFARE1(I);
RESULTS2(I, "WELFARE2") = WELFARE2(I);

$LIBINCLUDE XLDUMP RESULTS2 M3-2.XLS SHEET2!B3
```

3.10 Labor/leisure decision

Often general-equilibrium models used in international trade assume that factors of production, especially labor, are in fixed and inelastic supply.

But designing tax, welfare, and education systems, endogenizing labor supply is a crucial part of the story.

Model M3-3 endogenizes labor supply, allowing labor to chose between leisure and labor supply with leisure entering into the workers utility function.

In our formulation, we introduce an additional activity T, which transforms leisure (price PL) into labor supplied (price PLS).

The consumer is endowed with 200 units of labor/leisure, of which 100 units are supplied to the labor market initially.

The use of the labor supply activity is going to imply two separate prices in the present of the labor tax.

PL is the household (consumer) price of leisure and the take-home wage of labor supplied to the market. PLS is going to be the producer's cost of labor.

PLS = PL(1+TL)

There is only one final consumption good, denoted *X.*

Model M3-3a is a partial-equilibrium version. Prices for *X* and *L* are exogenous, as is the consumer's income.

Preferences are Cobb-Douglas between X and Leisure.

The trick I use here is to define "income" as the total value of the consumer's endowment of labor/leisure = 200 units.

In the benchmark, 100 units are supplied to the labor market to produce X and 100 units are retained as leisure.

	Production Sectors			Consumers
Markets	X	T	LEISURE	CONS
PX	100			-100
PLS	-100	100		
PLE			100	-100
PL		-100	-100	200

```
PARAMETERS
 PXF fixed price of X
 PLF fixed price of labor and leisure;

PXF = 1; PLF = 1;
TL = 0;

NONNEGATIVE VARIABLES
     XPE     X in partial equilibrium
     LPE     Leisure consumption in partial equilibrium
     CONSPE  Consumption in PE inclusive of value of leisure;

EQUATIONS
 MKT_XPE   demand for X
 MKT_LPE   demand for Leisure
 I_CONSPE  income;


MKT_XPE..     100*XPE =G= 0.5*CONSPE/PXF;

MKT_LPE..     100*LPE =G= 0.5*CONSPE/PLF;

I_CONSPE..    CONSPE =E= 200*PLF;
```

Counter Factual experiment is to think of a tax on labor input of 50%, which raises the price of X to 1.5 (there are no other inputs in X).

Curiously, there is no effect on leisure demand, which is due to a property of Cobb-Douglas that I'll explain on the board.

The same would be true if reduced the wage by half: income falls, but the cost of leisure (the opportunity cost of not working) also falls by half. CONSPE/PLF doesn't change.

But what it true in partial equilibrium cannot be true for the economy as a whole: Consumption of X falls, but labor supply doesn't, and labor is only used to produce X.

```
NONNEGATIVE VARIABLES
     X        Activity level for sector X
     T        Labor supply (transforms leisure to labor)
     PX       Price of X
     PL       Price of L (household price)
     PLS      Price of labor supply (producer cost)
     CONS     Income definition for CONS;

EQUATIONS
     PRF_X   Zero profit for sector X
     PRF_T   Zero profit for sector T

     MKT_X   Supply-demand balance for commodity X
     MKT_L   Supply-demand balance for primary factor L
     MKT_LS  Supply-demand balance for Leisure

     I_CONS  Income definition for CONS;
```

```
*        Zero profit conditions:

PRF_X..  100*PLS =G= 100*PX;

PRF_T..  100*(PL *(1+TL)) =G= 100 * PLS;


*        Market clearing conditions:

MKT_X..  100*X =G= 0.5*CONS/PX;

MKT_L..  200 =G= 100*T + 0.5*CONS/PL;

MKT_LS.. 100*T =G= 100*X;


*        Income constraints:

I_CONS.. CONS =E= 200*PL + TL*100*T*PL;


MODEL LABLEIS /PRF_X.X, PRF_T.T, MKT_X.PX, MKT_L.PL,
              MKT_LS.PLS,I_CONS.CONS /;
```

3.11 Two households with different preferences and endowments

Questions of interest to trade and public finance economists involve issues of distribution rather than or in addition to issues of aggregate welfare.

Households may differ in their preferences and more importantly in their sources of income (or their factor endowments).

Adding multiple household types is a straightforward extension of our earlier models. M3-4 assumes two households.

Household A is relatively well endowed with labor, and also has a preference for good Y, which is the labor-intensive good.

Household B is relatively well endowed with capital and has a relative preference for the capital intensive good.

	Production Sectors				Consumers	
Markets	X	Y	WA	WB	A	B
PX	100		-40	-60		
PY		100	-60	-40		
PWA			100		-100	
PWB				100		-100
PL	-25	-75			100	
PK	-75	-25				100

Our counterfactual experiment is to place a tax on the factor inputs to X, assigning half the revenue to each consumer. This lowers the welfare of household B.

However, household A is better off.

This welfare gain for A is a combination of

(a) a lowering of the relative consumer price of Y, the good favored by A

(b) an increase in the real return to labor, due to the shift toward the labor-intensive sector (Stolper-Samuelson)

(c) neutral redistribution of tax revenue.

3.12 Identifying and correcting calibration errors.

A calibration error occurs when parameters are set at incorrect values such that the model does not reproduce the benchmark data when run.

Run model M3-5 is our earlier model M3-1, with a couple of parameters added to produce errors.

The problem is that you will have no idea (in general) where the mistake occurs.

In the model, parameter M2 creates an inconsistency between the cost function for producing good Y and the demand for capital in the Y sector unless M2 = 1.

If you run the model with M2 = 1.5, you will get the following.

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	0.947	+INF	.
---- VAR Y	.	1.073	+INF	.
---- VAR PX	.	1.207	+INF	.
---- VAR PY	.	1.065	+INF	.
---- VAR PL	1.000	1.000	1.000	-14.286
---- VAR PK	.	1.286	+INF	.
---- VAR CONS	.	228.571	+INF	.

Not only does this not reproduce the benchmak, there is an error as indicated by the marginal on PL, the numeraire.

When a numeraire is required, the solver drops the equation associated with that variable, and the error (inconsistency) in the model is typically loaded onto that sector.

But nothing here tells us where that error is.

Solution is to (at first) not allow the model to solve, returning the initial imbalances in the model: syntax <MODELNAME>.ITERLIM = 0;

```
TWOxTWO.ITERLIM = 0;
SOLVE TWOxTWOa USING MCP;
```

The solver will simply do a function evaluation but then stop their and report back a set of variable values. In our case, we get

```
                       LOWER     LEVEL     UPPER    MARGINAL

---- VAR X               .        1.000     +INF        .
---- VAR Y               .        1.000     +INF        .
---- VAR PX              .        1.000     +INF        .
---- VAR PY              .        1.000     +INF        .
---- VAR PL            1.000      1.000     1.000      EPS
---- VAR PK              .        1.000     +INF    -12.500
---- VAR CONS            .      200.000     +INF        .
```

This indicates that the error is in the equation associated with PK, which is equation MKT_K. Look for the error here.

However, this can create a problem if there is not actually an error in setting parameters, but in setting the starting values.

Try running M3-5 with PX.L = 2 and iterlim = 0, Here is what you will get:

```
                          LOWER      LEVEL      UPPER     MARGINAL

---- VAR X                  .         1.000      +INF   -100.000
---- VAR Y                  .         1.000      +INF       .
---- VAR PX                 .         2.000      +INF     50.000
---- VAR PY                 .         1.000      +INF       .
---- VAR PL                1.000      1.000      1.000   -25.000
---- VAR PK                 .         1.000      +INF    -75.000
---- VAR CONS               .       200.000      +INF       .
```

This is a big mess. But there is actually no error in the model, just in the starting values. To check for this, always run the model twice as

```
TWOxTWO.ITERLIM = 0;
SOLVE TWOxTWO USING MCP;

TWOxTWO.ITERLIM = 1000;
SOLVE TWOxTWO USING MCP;
```

A true calibration error means non-zero marginals in the first, and the second does not reproduce the benchmark.

A starting value error reproduces the benchmark in the second solve.

But there are some errors that will not show up, typically because something such as a tax is coded incorrectly, but the model works fine if initially T = 0. An error only show up when T is changed to be positive.

There is a tax parameter T on the production of X in M3-5. M1 is a parameter that makes the model correct if M1 = 1.

But if M1 = 0, then we are neglecting to account for the tax revenue, specifically here it is not returned lump sum to the consumer.

With M1 = 0 and T = 0, the model works fine, but when T1 = 0.5, the solver returns the solution.

```
---- VAR X                    .         0.782     +INF        .
---- VAR Y                    .         0.909     +INF        .
---- VAR PX                   .         1.023     +INF        .
---- VAR PY                   .         0.880     +INF        .
---- VAR PL                 1.000       1.000     1.000    26.667
---- VAR PK                   .         0.600     +INF        .
---- VAR CONS                 .       160.000     +INF        .
```

The positive marginal indicates that this is not a valid solution (note also that the output of both X and Y fall, contradicting theory).