\$TITLE: M10-1.GMS: steady state capital stock, comparative steady-states

## \$ONTEXT

```
    "closure rule": instead of the capital stock being fixed
    (quantity closure), the stock
    adjusts to satisfy the steady-state relationship between the rental
    rate and the price of producing new capital (price closure):
        delta \(=\) depreciation rate
        rho = rate of time preference
    rental rate \(=(1-(1-d e l t a) /(1+r h o))^{*}(p r i c e ~ o f ~ n e w ~ c a p i t a l) ~\)
    this is done via a subsidy to capital use that creates the wedge
    subsidy = (1 - delta)/(1 + rho)
```


## Production Sectors Consumers

| Markets | $x$ | Y | K | W | CONS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $P X$ | 100 |  |  | -100 |  |
| PY |  | 100 |  | -100 |  |
| PW |  |  |  | 200 | -200 |
| PL | - 40 | -60 | -60 |  | 160 |
| PK | -120 | -80 | 60 |  | 140 |
| SUB | 60 | 40 |  |  | -100 |

## \$0FFTEXT

## PARAMETERS

RHO Time preference parameter
DELTA Depreciation rate
TAU Effective capital use subsidy
KTAX Tax on new capital production
NEWCAP New capital stock after counterfactual (= 1 initially);

```
RHO = 0.4;
DELTA = 0.3;
TAU = - (1 - DELTA)/(1 + RHO);
KTAX = 0;
```


## NONNEGATIVE VARIABLES

X Activity level for sector X
Y Activity level for sector Y
W Activity level for sector W (Hicksian welfare index)
K Capital stock index

PX Price index for commodity X
PY Price index for commodity Y
PL Price index for primary factor L
PK Price index for primary factor K
PW Price index for welfare (expenditure function)
$\begin{array}{ll}\text { CONS } & \text { Income definition for coNS } \\ \text { KFORWRD Capital stock from previous period; }\end{array}$

## EQUATIONS

PRF_X Zero profit for sector X
PRF_Y Zero profit for sector Y
PRF_W Zero profit for sector W (Hicksian welfare index)
PRF_K Zero profit for capital index
MKT_X Supply-demand balance for commodity X
MKT_Y Supply-demand balance for commodity Y
MKT_L Supply-demand balance for primary factor L
MKT_K Supply-demand balance for factor K
MKT_W Supply-demand balance for aggregate demand

I_CONS Income definition for CONS
A_KFORWRD Auxiliary equation to determine the carry forward;

Zero profit conditions:
PRF_X.. 100 * PL**0.4 * (PK*(1+TAU)/0.5)**0.6 =G= 100 * PX;
PRF_Y.. 100 * PL**0.6 * (PK* (1+TAU)/0.5)**0.4 =G= 100 * PY;
PRF_W.. 200 * PX**0.5 * PY**0.5 =G= 200 * PW;

```
PRF_K.. 60*PL =G= 60 * PK * (1-KTAX);
    Market clearing conditions:
MKT_X.. 100 * X =G= 100 * W * PW/PX;
MKT_Y.. 100 * Y =G= 100 * W * PW/PY;
MKT_W.. 200 * W =G= CONS / PW;
MKT_L.. 160 =G= 60*K + 40* X*PX/PL + 60*Y*PY/PL;
MKT_K.. 140*KFORWRD + 60*K =G= 120 * X * PX*0.5/(PK*(1+TAU)) +
    80 * Y * PY*0.5/(PK*(1+TAU));
```

* Income constraints:
I_CONS.. CONS =E= 160 * PL + 140*KFORWRD *PK +
PK * TAU * (120 * X * PX*0.5/(PK*(1+TAU)) +
80 * $Y$ * $P Y^{*} 0.5 /\left(\right.$ PK* $\left.\left.^{*}(1+T A U)\right)\right)+$
60* PK * KTAX * K;
Auxiliary constraints:
A_KFORWRD.. 140*KFORWRD $=E=60 * K ~ * ~(1-D E L T A) ~ / ~ D E L T A ; ~$

```
MODEL ALGEBRAIC /PRF_X.X, PRF_Y.Y, PRF_W.W, PRF_K.K,
                                    MKT_X.PX, MKT_Y.PY, MKT_L.PL, MKT_K.PK, MKT_W.PW,
                                    I_CONS.CONS, A_KFORWRD.KFORWRD/;
X.L =1;
Y.L =1;
W.L =1;
K.L =1;
PX.L =1;
PY.L =1;
PK.L =1;
PW.FX =1;
PL.L =1;
CONS.L =200;
KFORWRD.L = 1;
ALGEBRAIC.ITERLIM = 0;
SOLVE ALGEBRAIC USING MCP;
ALGEBRAIC.ITERLIM = 2000;
SOLVE ALGEBRAIC USING MCP;
* Raise the rate of time preference from 0.4 to 0.6:
```

```
RHO = 0.6;
TAU = - (1 - DELTA)/(1 + RHO);
```

SOLVE ALGEBRAIC USING MCP;
NEWCAP = K.L/60;
DISPLAY NEWCAP;

* Set rho back to 0.4, tax new capital at 0.20
RHO = 0.4;
TAU $=-(1-D E L T A) /(1+R H O) ;$
KTAX = 0.20;
SOLVE ALGEBRAIC USING MCP;
NEWCAP = K.L/60;
DISPLAY NEWCAP;

