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$TITLE: M2-3.GMS add a rationing constraint to model M2-2
*    MAXIMIZE UTILITY SUBJECT TO A LINEAR BUDGET CONSTRAINT
*    PLUS RATIONING CONSTRAINT ON X1
*    two goods, Cobb-Douglas preferences

PARAMETERS
 M              Income
 P1, P2         prices of goods X1 and X2
 S1, S2         util shares of X1 and X2
 RATION         rationing constraint on the quantity of X1;

M = 100;
P1 = 1;
P2 = 1;
S1 = 0.5;
S2 = 0.5;
RATION = 100.;

NONNEGATIVE VARIABLES
   X1, X2   Commodity demands
   LAMBDAI  Lagrangean multiplier (marginal utility of income)
   LAMBDAR  Lagrangean mulitplier on rationing constraint;

VARIABLES
   U        Welfare;
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EQUATIONS
   UTILITY       Utility
   INCOME        Income-expenditure constraint
   RATION1       Rationing contraint on good X1
   FOC1, FOC2    First-order conditions for X1 and X2;

UTILITY..    U =E= 2*(X1**S1)*(X2**S2);

INCOME..     M =G= P1*X1 + P2*X2;

RATION1..    RATION =G= X1;

FOC1..       LAMBDAI*P1 + LAMBDAR =G= 2*S1*X1**(S1-1)*(X2**S2);

FOC2..       LAMBDAI*P2 =G= 2*S2*X2**(S2-1)*(X1**S1);

* modeled as a non-linear programming problem
* set starting values

U.L = 100;
X1.L = 50;
X2.L = 50;
LAMBDAI.L = 1;
LAMBDAR.L = 0;

MODEL OPTIMIZE /UTILITY, INCOME, RATION1/;
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SOLVE OPTIMIZE USING NLP MAXIMIZING U;

* modeled as a complementarity problem

MODEL COMPLEM /UTILITY.U, INCOME.LAMBDAI, RATION1.LAMBDAR,
               FOC1.X1, FOC2.X2/;
SOLVE COMPLEM USING MCP;


* try binding rationing constraint at X1 <= RATION = 25;

RATION = 25;
SOLVE OPTIMIZE USING NLP MAXIMIZING U;
SOLVE COMPLEM USING MCP;

* show that shadow price of rationing constraint increases with income
* could lead to a black market in rationing coupons, "scalping" tickets

M = 200;
SOLVE OPTIMIZE USING NLP MAXIMIZING U;
SOLVE COMPLEM USING MCP;


* illustrate the mpec solver
* suppose we want to enforce the rationing contraint via licenses for X1
* consumers are given an allocation of licenses which is RATION
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* PLIC is an endogenous variables whose value is the license price
* the value of the rationing license allocation should be treated as
* part of income


NONNEGATIVE VARIABLES
 PLIC;

EQUATIONS
 INCOMEa
 FOC1a;

M = 100;
RATION = 25;
U.L = 100;
X1.L = 25;
X2.L = 75;
PLIC.L = 0.1;

INCOMEa..    M + (PLIC*RATION) =E= (P1 + PLIC)*X1 + P2*X2 ;

FOC1a..      LAMBDAI*(P1 + PLIC) =G= 2*S1*X1**(S1-1)*(X2**S2);

MODEL MPEC     /UTILITY, INCOMEa.LAMBDAI, FOC1a.X1, FOC2.X2,
                RATION1.PLIC/;
MODEL COMPLEM2 /UTILITY.U, INCOMEa.LAMBDAI, FOC1a.X1, FOC2.X2,
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                RATION1.PLIC/;

OPTION MPEC = nlpec;

SOLVE  MPEC USING MPEC MAXIMIZING U;
SOLVE  COMPLEM2 USING MCP;

M = 200;

SOLVE  MPEC USING MPEC MAXIMIZING U;
SOLVE  COMPLEM2 USING MCP;


* now use the expenditure function, giving the minimum cost of buying
* one unit of utility: COSTU = P1**S1 * P2**S2 = PU
* where PU is the "price" of utility: the inverse of lambda
* two versions are presented:
* one using Marshallian (uncompensated) demand: Xi = F(P1, P2, M)
* one using Hicksian (compensated) demand: Xi = F(P1, P2, U)

RATION = 100;
M = 100;

NONNEGATIVE VARIABLES
   PU        price of utility
   M1        income inclusive of the value of rationing allocation;
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EQUATIONS
   COSTU     expenditure function: cost of producing utility = PU
   DEMANDM1  Marshallian demand for good 1
   DEMANDM2  Marshallian demand for good 2
   DEMANDH1  Hicksian demand for good 1
   DEMANDH2  Hicksian demand for good 2
   DEMANDU   Demand for utility (indirect utility function)
   RATION1b  Rationing constraint (same as before)
   INCOMEb   Income balance equation;

COSTU..    (PLIC+P1)**S1 * P2**S2 =G= PU;

DEMANDM1.. X1 =G= S1*M1/(P1+PLIC);

DEMANDM2.. X2 =G= S2*M1/P2;

DEMANDH1.. X1 =G= S1*PU*U/(P1+PLIC);

DEMANDH2.. X2 =G= S2*PU*U/P2;

DEMANDU..  U =E= M1/PU;

RATION1b.. RATION =G= X1;

INCOMEb..  M1 =E= M + PLIC*RATION;
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PU.L = 1;

MODEL COMPLEM3 /COSTU.U, DEMANDM1.X1, DEMANDM2.X2, DEMANDU.PU,
                RATION1b.PLIC, INCOMEb.M1/;
MODEL COMPLEM4 /COSTU.U, DEMANDH1.X1, DEMANDH2.X2, DEMANDU.PU,
                RATION1b.PLIC, INCOMEb.M1/;

SOLVE COMPLEM3 USING MCP;
SOLVE COMPLEM4 USING MCP;

* counterfactuals

RATION = 25;

SOLVE COMPLEM3 USING MCP;
SOLVE COMPLEM4 USING MCP;

M = 200;

SOLVE COMPLEM3 USING MCP;
SOLVE COMPLEM4 USING MCP;



*$exit
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* scenario generation

SETS I indexes different values of rationing constraint /I1*I10/
     J indexes income levels /J1*J10/;

PARAMETERS
 RLEVEL(I)
 PCINCOME(J)
 LICENSEP(I,J);

U.L = 50;
X1.L = 25;
X2.L = 25;
PLIC.L = 0.;
LAMBDAI.L = 1;

* the following is to prevent solver failure when evaluating X1**(S1-1)
* at X1 = 0 (given S1-1 < 0)
X1.LO = 0.01;
X2.LO = 0.01;

LOOP(I,
LOOP(J,

 RATION = 110 - 10*ORD(I);
 M = 25 + 25*ORD(J);
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SOLVE MPEC USING MPEC MAXIMIZING U;

 RLEVEL(I) = RATION;
 PCINCOME(J) = M;
 LICENSEP(I,J) = PLIC.L;

);
);

DISPLAY RLEVEL, PCINCOME, LICENSEP;

$LIBINCLUDE XLDUMP LICENSEP M2-3.XLS SHEET1!B3
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