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$TITLE: M5-2.GMS, example of non-linear, constrainted least squares

$ONTEXT
there are I observations on two variables (set J), a dependent
variables Y and an independent variable J.

the objective is to estimate a log linear relationship vias OLS
minimizing the sum of squared deviations

imagine that this is production data (two inputs), added constraint
equation imposes constant returns to scale: sum of slope coeff = 1

Y = output, X1 = capital, X2 = labor
$OFFTEXT

SETS  I    observations                 /I1*I6/
      J    dep and ind var              /J1*J3/
      K(J) set of independent variables /J2*J3/;



PARAMETERS
      Y0(I)
      X0(I,K);

TABLE BENCH(I,J)
```

```
     J1    J2    J3
I1    4     2     2
I2    3     4     1
I3   10     6     5
I4   14    11     3
I5   18    13     4
I6   22    14     6;

DISPLAY BENCH;

Y0(I)    = BENCH(I, "J1");
X0(I,K)  = BENCH(I, K);

DISPLAY Y0, X0;

VARIABLES
  ALPHA      intercept
  BETA(K)    slope coefficients (elasticities since estimated in logs)
  DEV        sum of squared deviations
  YHAT(I)    fitted values of the dependent variable;

EQUATIONS
  OBJECTIVE  objective function = sum of squared residuals
  EYHAT(I)   equation for the fitted values of Y (log linear)
  CRS        constraint constant returns: sum of slope coefficients = 1;
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OBJECTIVE..  DEV =E= SUM(I, (YHAT(I) - Y0(I))*(YHAT(I) - Y0(I)));

EYHAT(I)..   LOG(YHAT(I)) =E= ALPHA + SUM(K, BETA(K)*LOG(X0(I,K)));

CRS..        SUM(K, BETA(K)) =E= 1;


* model OLS: unconstrainted OLS

MODEL OLS /OBJECTIVE, EYHAT/;

ALPHA.L   = 1;
BETA.L(K) = 1;
YHAT.L(I) = 2;

SOLVE OLS USING NLP MINIMIZING DEV;


* model OLSC: constrainted least squares, imposes CRS

MODEL OLSC /ALL/;

SOLVE OLSC USING NLP MINIMIZING DEV;
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* process output to get observed and fitted values of Y


PARAMETER
RESULTS(I,*);

RESULTS(I, "YHAT") = YHAT.L(I);
RESULTS(I, "Y0") = Y0(I);


DISPLAY RESULTS;

$LIBINCLUDE XLDUMP RESULTS M5.XLS SHEET1!B3
```