\$TITLE M6-2a.GMS: 2x2 Economy with labor supply and income tax \$ONTEXT

Production Sectors
Consumers

| Markets | $X$ | $Y$ | W | TL | TK | CONS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PX | 120 |  | - 120 |  |  |  |
| PY |  | 120 | -120 |  |  |  |
| PW |  |  | 340 |  |  | -340 |
| PLS | -48 | -72 |  | 120 |  |  |
| PKS | - 72 | -48 |  |  | 120 |  |
| PL |  |  | -100 | -80 |  | 180 |
| PK |  |  |  |  | -80 | 80 |
| TAX |  |  |  | -40 | -40 | 80 |

## \$OFFTEXT

SETS S /1*6/;
PARAMETERS
TXL Labor income tax rate,
TXK Capital income tax rate,
WELFARE(S) Welfare,
LABSUP(S) Labor supply
INCOME(S) Money income $=$ consumption of $X$ and $Y$

CAPTAX(S) The level of the capital tax
TAXREV(S) Tax revenue generated;

## POSITIVE VARIABLES

| X | Activity level for sector $X$ |
| :--- | :--- |
| Y | Activity level for sector $Y$ |
| TL | Supply activity for $L$ |
| TK | Supply activity for K |
| W | Activity level for sector $W$ |
| PX | Price index for commodity X |
| PY | Price index for commodity Y |
| PL | Price index for primary factor L net of tax |
| PK | Price index for primary factor K net of tax |
| PLS | Price index for primary factor L gross of tax |
| PKS | Price index for primary factor K gross of tax |
| PW | Price index for welfare (expenditure function) |
| CONS | Income definition for conS; |

## EQUATIONS

```
PRF_X Zero profit for sector X
PRF_Y Zero profit for sector Y
PRF_TL Zero profit for sector TL
```

```
PRF_TK Zero profit for sector TK
PRF_W Zero profit for sector W
MKT_X Supply-demand balance for commodity X
MKT_TK Supply-demand balance for commodity TK
MKT_TL Supply-demand balance for commodity TL
MKT_Y Supply-demand balance for commodity Y
MKT_L Supply-demand balance for primary factor L
MKT_K Supply-demand balance for primary factor K
MKT_W Supply-demand balance for aggregate demand
I_CONS Income definition for CONS;
```

Zero profit conditions:
PRF_X.. 80*PLS**0.4 * PKS**0.6 =G= 120*PX;
PRF_Y.. 80*PLS**0.6 * PKS**0.4 =G= 120*PY;
PRF_TL.. 80*PL*(1+TXL) =G= 80*PLS;
PRF_TK.. 80*PK*(1+TXK) =G= 80*PKS;
PRF_W.. 340*(PX)**(12/34) * (PY)**(12/34) * PL**(10/34)
$=G=340$ * PW;

## * Market clearing conditions:

```
MKT_X.. 120*X =G= 340*W*PW * (12/34)/PX;
MKT_Y.. 120*Y =G= 340*W*PW * (12/34)/PY;
MKT_W.. 340*W =G= CONS / PW;
MKT_L.. 180 =G= 80*TL + 340*W*(10/34)*(PW/PL);
MKT_K.. 80 =G= 80*TK;
MKT_TL.. 80*TL =G= 48*X*PX/PLS + 72*Y*PY/PLS;
MKT_TK.. 80*TK =G= 72*Y*PY/PKS + 48*X*PX/PKS;
* Income constraints:
```

I_CONS.. CONS =E= 180*PL + 80*PK + 80*TL*TXL*PL + 80*TK*TXK*PK;
MODEL INCOMETAX /PRF_X.X, PRF_Y.Y, PRF_TK.TK,PRF_TL.TL,
PRF_W.W, MKT_X.PX, MKT_Y.PY, MKT_L.PL,
MKT_TK.PKS, MKT_TL.PLS,
MKT_K.PK, MKT_W.PW, I_CONS.CONS /;
X.L $=1$;

| Y.L | $=1 ;$ |
| :--- | :--- |
| TK.L | $=1 ;$ |
| TL.L | $=1 ;$ |
| W.L | $=1 ;$ |
| PL.L | $=1 ;$ |
| PX.L | $=1 ;$ |
| PY.L | $=1 ;$ |
| PLS.L | $=1.5 ;$ |
| PKS.L | $=1.5 ;$ |
| PK.L | $=1 ;$ |
| PW.FX | $=1 ;$ |
| CONS.L | $=340 ;$ |
|  |  |
| TXL | $=0.5 ;$ |
| TXK | $=0.5 ;$ |

INCOMETAX.ITERLIM = 0; SOLVE INCOMETAX USING MCP;

* Lets do some counter-factual with taxes shifted to the
* factor which is in fixed supply:

INCOMETAX.ITERLIM = 1000;
SOLVE INCOMETAX USING MCP;

## LOOP (S,

```
TXL = 0.60 - 0.10*ORD(S);
TXK = 0.40 + 0.10*ORD(S);
```

SOLVE INCOMETAX USING MCP;

```
WELFARE(S) = W.L;
```

$\operatorname{LABSUP}(S)=T L . L ;$
$\operatorname{INCOME}(S)=((P X . L / 1.5) * X . L+(P Y . L / 1.5) * Y . L)$
/((PX.L/1.5)**0.5*(PY.L/1.5)**0.5)/2;
CAPTAX(S) = TXK;
TAXREV(S) $=(T X L * P L . L * T L . L * 80+T X K * P K . L * T K . L * 80) ~$
/((PX.L/1.5)**0.5*(PY.L/1.5)**0.5);
);
DISPLAY WELFARE, LABSUP, INCOME, CAPTAX, TAXREV;

```
PARAMETER
    RESULTS(S, *);
```

RESULTS(S, "WELFARE") = WELFARE(S);
RESULTS(S, "LABSUP") = LABSUP(S);
RESULTS(S, "TAXREV") = TAXREV(S);

## DISPLAY RESULTS;

TXL = 0;
TXK = 0;
SOLVE INCOMETAX USING MCP;

