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$TITLE  M6-3: Economy with two households and a public good

$ONTEXT
How do we model a public good that is non-excludable and non-rivaled?

              Production Sectors          Consumers

Markets|    X     Y     G    W1    W2        CONS1   CONS2   GOVT
-------------------------------------------------------------------
  PX   |  100                -50   -50
  PY   |        100          -50   -50
  PG   |               50                                    -50
  PL   |  -80   -80   -40                    100     100
  TAX  |  -20   -20   -10                                     50
  PW1  |                     125             -125
  PW2  |                           125               -125
  PG1  |                     -25               25
  PG2  |                           -25                 25
-------------------------------------------------------------------
$OFFTEXT

PARAMETER
    TAX     Value-added tax rate;
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NONNEGATIVE VARIABLES
    X       Activity level for sector X
    Y       Activity level for sector Y
    W1      Activity level for sector W1
    W2      Activity level for sector W2
    G       Activity level for government sector

    PX      Price index for commodity X
    PY      Price index for commodity Y
    PL      Price index for primary factor L
    PW1     Price index for welfare 1(expenditure function)
    PW2     Price index for welfare 2(expenditure function)
    PG1     Private valuation of the public good (consumer 1)
    PG2     Private valuation of the public good (consumer 2)
    PG      Price of (cost of producing) the public good

    GOVT    Budget restriction for government
    CONS1   Income definition for CONS1
    CONS2   Income definition for CONS2

    LGP     Endowment of public good received by each consumer;

EQUATIONS
    PRF_X   Zero profit for sector X
    PRF_Y   Zero profit for sector Y
    PRF_W1  Zero profit for sector W1
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    PRF_W2  Zero profit for sector W2
    PRF_G   Zero profit in government sector

    MKT_X   Supply-demand balance for commodity X
    MKT_Y   Supply-demand balance for commodity Y
    MKT_L   Supply-demand balance for primary factor L
    MKT_W1  Supply-demand balance for consumer 1
    MKT_W2  Supply-demand balance for consumer 2
    MKT_G1  Private valuation of the public good (consumer 1)
    MKT_G2  Private valuation of the public good (consumer 2)
    MKT_G   Supply-demand balance for commodity G

    I_G     Budget restriction for government
    I_CONS1 Income definition for CONS1
    I_CONS2 Income definition for CONS2

    A_LGP   Auxiliary for government provision;

*       Zero profit conditions:

PRF_X..  80*PL * (1+TAX) =G= 100*PX;

PRF_Y..  80*PL * (1+TAX) =G= 100*PY;

PRF_G..  40*PL * (1+TAX) =G= 50*PG;
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PRF_W1.. 125*PX**(50/125) * PY**(50/125) * (PG1/0.5)**(25/125)
            =G= 125*PW1;

PRF_W2.. 125*PX**(50/125) * PY**(50/125) * (PG2/0.5)**(25/125)
            =G= 125*PW2;

*       Market clearing conditions:

MKT_X..  100*X =G= 50*W1*PW1/PX + 50*W2*PW2/PX ;

MKT_Y..  100*Y =G= 50*W1*PW1/PY + 50*W2*PW2/PY;

MKT_L..  200 =G= (80*X + 80*Y + 40*G);

MKT_W1.. 125*W1 =G= CONS1 / PW1;

MKT_W2.. 125*W2 =G= CONS2 / PW2;

MKT_G..  50*G =G= GOVT/ PG;

MKT_G1.. 50*LGP =G= 25 * W1 * PW1/PG1;

MKT_G2.. 50*LGP =G= 25 * W2 * PW2/PG2;

*       Income constraints:
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I_G..      GOVT =G= PL*(80*X + 80*Y + 40*G )*TAX;

I_CONS1.. CONS1 =E= 100*PL + 50*LGP*PG1;

I_CONS2.. CONS2 =E= 100*PL + 50*LGP*PG2;

*       Auxiliary constraints:

A_LGP..    LGP =E= G;

MODEL PUBGOOD /PRF_X.X, PRF_Y.Y, PRF_W1.W1, PRF_W2.W2, PRF_G.G,
                  MKT_X.PX, MKT_Y.PY, MKT_L.PL,
                  MKT_W1.PW1, MKT_W2.PW2,
                  MKT_G.PG, MKT_G1.PG1, MKT_G2.PG2,
                  I_G.GOVT, I_CONS1.CONS1, I_CONS2.CONS2,
                  A_LGP.LGP /;

X.L      =1;
Y.L      =1;
W1.L     =1;
W2.L     =1;
G.L      =1;

PL.FX    =1;
PX.L     =1;
PY.L     =1;
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PG.L    =1;
PW1.L   =1;
PW2.L   =1;
PG1.L   =0.5;
PG2.L   =0.5;

CONS1.L =125;
CONS2.L =125;
GOVT.L  =50;

LGP.L   =1;

TAX     =0.25;

PUBGOOD.ITERLIM = 0;
SOLVE PUBGOOD USING MCP;
PUBGOOD.ITERLIM = 2000;
SOLVE PUBGOOD USING MCP;

*    The following counterfactuals check that the original
*    benchmark is indeed an optimum by
*    raising/lowering the tax

TAX = 0.10;

SOLVE PUBGOOD USING MCP;
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TAX = 0.40;

SOLVE PUBGOOD USING MCP;
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