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$TITLE  M6-5.GMS: Public intermediate good with optimal provision
* technique for modeling infrastructure for example

$ONTEXT
            Production Sectors              Consumers

Markets|    X     Y     G    W1         CONS1      GOVT
-----------------------------------------------------------------
  PX    |  100               -100
  PY    |        100         -100
  PG    |              50                           -50
  PL    |  -80  -80   -40                200
  TAX   |  -20  -20   -10                            50
  PW    |                   200         -200
-----------------------------------------------------------------
  X = ALPHA*F(L)  ALPHA = F(G)  ALPHA viewed as exogenous by firms

$OFFTEXT

PARAMETERS
  SHX, SHY, shares of X and Y in consumer's utility
  INFPROD productivity parameter of the public good in X output;

SHX = 0.5;
SHY = 0.5;
INFPROD = 0;
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POSITIVE VARIABLES
    X   Activity level for sector X
    Y   Activity level for sector Y
    W   Activity level for sector W
    G   Activity level for government sector

    PX   Price index for commodity X
    PY   Price index for commodity Y
    PG   Private valuation of the public good
    PL   Price index for primary factor L
    PW   Price index for welfare 1(expenditure function)

    GOVT   Budget restriction for government
    CONS   Income definition for CONS

    TAX   Uniform value-added tax rate
    ALPHA   Public intermediary good multiplier on productivity;

EQUATIONS
    PRF_X   Zero profit for sector X
    PRF_Y   Zero profit for sector Y
    PRF_W   Zero profit for sector W1
    PRF_G   Zero profit in government sector

    MKT_X   Supply-demand balance for commodity X

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    MKT_Y     Supply-demand balance for commodity Y
    MKT_G     Supply-demand balance for commodity G
    MKT_L     Supply-demand balance for primary factor L
    MKT_W     Supply-demand balance for consumer 1

    I_G       Budget restriction for government
    I_CONS    Income definition for CONS

    A_TAX     Auxiliary for government provision
    INFRA     Auxiliary for public intermediate good calculation;
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*        Zero profit conditions:


PRF_X..  80*PL * (1+TAX)/ALPHA =G= 100*PX;


PRF_Y..  80*PL * (1+TAX) =G= 100*PY;


PRF_W .. 200*PX**(SHX) * PY**(SHY) =E= 200*PW;


PRF_G..  40*PL * (1+TAX) =G= 50*PG;


*        Market clearing conditions:


MKT_X..  100*X =G= 200*SHX*W*PW/PX;


MKT_Y..  100*Y =G= 200*SHY*W*PW/PY;
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MKT_G..  50*G =G= GOVT/ PG;

MKT_L..  200 =G= (80*X/ALPHA + 80*Y + 40*G);

MKT_W..  200*W =G= CONS/PW;

*        Income constraints:

I_G..     GOVT =G= PL*(80*X/ALPHA + 80*Y + 40*G )*TAX;

I_CONS.. CONS =E= 200*PL;

*        Auxiliary constraints:

A_TAX..  PG =E= PX*INFPROD;

INFRA..  ALPHA =E= 1 + INFPROD*G;


MODEL PUBINT      /PRF_X.X, PRF_Y.Y, PRF_W.W, PRF_G.G,
                   MKT_X.PX, MKT_Y.PY, MKT_L.PL, MKT_W.PW, MKT_G.PG,
                   I_G.GOVT, I_CONS.CONS,
                   A_TAX.TAX, INFRA.ALPHA /;

X.L     =1;
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Y.L       =1;
W.L       =1;
G.L       =1;
PL.FX     =1;
PX.L      =1;
PY.L      =1;
PG.L      =1;
PW.L      =1;
CONS.L    =200;
GOVT.L    =50;
ALPHA.L = 1;
TAX.L    = .25;


PUBINT.ITERLIM = 0;
SOLVE PUBINT USING MCP;

* with INFPROD = 0 initially, the optimal tax should be zero

PUBINT.ITERLIM = 2000;
SOLVE PUBINT USING MCP;

* now set INFPROD = 2, optimal tax and provision should be positive

INFPROD = 2;
TAX.L = 0.25; G.L = 1;
SOLVE PUBINT USING MCP;
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