

\$TITLE: M6-7.GMS: two households with different preferences, endowments  
 \* adaptation of model M3-7:  
 \* modeled as an MPEC: find the optimal tax maximizing social welfare

\$ONTEXT

Two household: differ in preferences and in endowments

Household A: well endowed with labor,  
 preference for labor-int good Y

Household B: well endowed with capital,  
 preference for capital-int good X

Allows for tax to be redistributed unevenly between households

Markets	Production Sectors				Consumers	
	X	Y	WA	WB	A	B
PX	100		-40	-60		
PY		100	-60	-40		
PWA			100		-100	
PWB				100		-100
PL	-25	-75			90	10
PK	-75	-25			10	90

The tax redistribution or sharing rule can also be interpreted as the relative number of households in each group, with all households getting an equal share of tax receipts

\$OFFTEXT

**PARAMETERS**

WEIGHTA weight of consumer A in social welfare  
WEIGHTB weight of consumer B in social welfare  
SHA share of tax redistributed to consumer A  
SHB share of tax redistributed to consumer B;

WEIGHTA = 0.5;  
WEIGHTB = 0.5;  
SHA = 0.5;  
SHB = 0.5;

**VARIABLES**

WS social welfare  
TAX endogenous tax rate on X;

**NONNEGATIVE VARIABLES**

X Activity level for sector X,  
Y Activity level for sector Y,  
WA Activity level for welfare for consumer A  
WB Activity level for welfare for consumer B  
PX Price index for commodity X,  
PY Price index for commodity Y,  
PK Price index for primary factor K,

PL Price index for primary factor L,  
 PWA Price index for welfare A(expenditure function),  
 PWB Price index for welfare B(expenditure function),  
  
 CONSA Income definition for CONSA,  
 CONSB Income definition for CONSB;

## EQUATIONS

OBJ Social welfare function  
 PRF\_X Zero profit for sector X  
 PRF\_Y Zero profit for sector Y  
 PRF\_WA Zero profit for sector WA (Hicksian welfare index)  
 PRF\_WB Zero profit for sector WB (Hicksian welfare index)  
  
 MKT\_X Supply-demand balance for commodity X  
 MKT\_Y Supply-demand balance for commodity Y  
 MKT\_L Supply-demand balance for primary factor L  
 MKT\_K Supply-demand balance for primary factor K  
 MKT\_WA Supply-demand balance for aggregate demand consumer A  
 MKT\_WB Supply-demand balance for aggregate demand consumer B  
  
 I\_CONSA Income definition for CONSA  
 I\_CONSB Income definition for CONSB;

\* *Objective function (social welfare function) to be maximized*

OBJ..                     $WS =E= (WA**WEIGHTA) * (WB**WEIGHTB);$

\*                    *Zero profit conditions:*

PRF\_X..                 $100 * (PL**0.25 * PK**0.75) * (1+TAX) =E= 100 * PX;$

PRF\_Y..                 $100 * (PL**0.75 * PK**0.25) =E= 100 * PY;$

PRF\_WA..               $100 * PX**0.4 * PY**0.6 =E= 100 * PWA;$

PRF\_WB..               $100 * PX**0.6 * PY**0.4 =E= 100 * PWB;$

\*                    *Market clearing conditions:*

MKT\_X..                 $100 * X =E= 40*WA*PWA/PX + 60*WB*PWB/PX;$

MKT\_Y..                 $100 * Y =E= 60*WA*PWA/PY + 40*WB*PWB/PY;$

MKT\_WA..               $100 * WA =E= CONSA / PWA;$

MKT\_WB..               $100 * WB =E= CONSB / PWB;$

MKT\_L..                 $90 + 10 =E= 25*X*(PX/(1+TAX))/PL + 75*Y*PY/PL;$

MKT\_K..                 $10 + 90 =E= 75*X*(PX/(1+TAX))/PK + 25*Y*PY/PK;$



PK.L =1;  
PWB.L =1;  
PWA.L =1;

CONSA.L =100;  
CONSB.L =100;

TAX.L =0.;

PWA.FX = 1;

**SOLVE** MPEC USING MPEC MAXIMIZING WS;

WEIGHTA = 0.7;  
WEIGHTB = 0.3;

**SOLVE** MPEC USING MPEC MAXIMIZING WS;

WEIGHTA = 0.7;  
WEIGHTB = 0.3;  
SHA = 0.75; SHB = 0.25;

**SOLVE** MPEC USING MPEC MAXIMIZING WS;

WEIGHTA = 0.3;  
WEIGHTB = 0.7;

SHA = 0.5; SHB = 0.5;

TAX.L = -0.2;

**SOLVE** MPEC USING MPEC MAXIMIZING WS;

*\* this one is interesting: a subsidy is optimal but consumer B  
\* must finance 75% of the subsidy. Result is a very small subsidy*

WEIGHTA = 0.3;

WEIGHTB = 0.7;

SHA = 0.25; SHB = 0.75;

**SOLVE** MPEC USING MPEC MAXIMIZING WS;