Slides for Chapter 8: Open Economy Models for Competitive Economies

8.1 Small open economy

Small open economy 2x2 (two goods, two factors) in which the rest of the world is not explicitly modeled.

Know as the "small-country assumption": the country faces fixed world prices

Trading opportunities are summarized by simple functions which allow the economy to transform one good (an export) into another (an import).

The “technology” of these functions represents world prices. We will assume that these technologies or price ratios are fixed in this model.

Country exports X_1 in exchange for X_2 (activities E1, M2)

We will allow for the fact that some policy or endowment change could actually reverse the direction of trade by specifying (initially inactive) functions that transform goods in the opposite direction.

Initial data, in which 50 units of good X_1 are exchanged for 50 units of good X_2 at an implicit price ratio of $p_1/p_2 = 1$.

Markets	Production Sectors X1	X2	E1	M2	W	Consumer CONS
P1	150		-50		-100	
P2		50		50	-100	
PL	-135	-5				140
PK	-15	-45				60
PW					200	-200
PFX			50	-50		

Technology parameters are specified in these functions that allow the modeler to change the terms of trade.

```
PARAMETERS
   PE2     Export price of good 2,
   PM1     Import price of good 1,
   PE1     Export price of good 1,
   PM2     Import price of good 2,
   TM2     Import tariff for good 2;

PE1 = 1;
PM2 = 1;
PE2 = 0.999;
PM1 = 1.001;
```

E1 and M2 production activities that are the initially active trade links.

Small trade costs on inactive links prevent model degeneracy and “round tripping”.

If all prices = 1, then if E1 = M2 = 1 is a solution, then so is E1 = 2, M1 = 1, M2 = 1.

While we could specify this activity as directly transforming X_1 into X_2, in more complicated models it proves useful to define another good which we will call “foreign exchange” and whose price is denoted PFX.

All trade is mediated through the “foreign exchange market”.

Thus activity E1 transforms X_1 into foreign exchange and M2, the import activity for good 2, transforms foreign exchange into imports of good 2.

There are four trade activities as just noted.

```
PRF_E1..  0*P1 =G= 50*PFX*PE1;

PRF_E2..  50*P2 =G= 50*PFX*PE2;

PRF_M1..  50*PFX*PM1 =G= 50*P1;

PRF_M2..  50*PFX*PM2 =G= 50*P2;
```

The equation for “foreign exchange is the trade balance condition:

```
MKT_PFX.. 50*E2*PE2 + 50*E1*PE1 =G= 50*PM2*M2 + 50*PM1*M1;
```

```
$TITLE  M8-1.GMS: Small open economy 2x2
* strips out trade costs and tariffs for simplicity of exposition

$ONTEXT
 CALIBRATION: country exports X1, imports X2
 in free-trade SOE

                Production Sectors                       Consumer

Markets    |      X1       X2       E1       M2       W        CONS
----------------------------------------------------------------
P1         |     150               -50               -100
P2         |              50                50       -100
PL         |    -135      -5                                    140
PK         |     -15     -45                                     60
PW         |                                          200      -200
PFX        |                         50      -50
----------------------------------------------------------------
$OFFTEXT

* the first four parameters allow changes in (exogenous) world prices


PARAMETERS
        PE2     Export price of good 2          /0.999/
        PM1     Import price of good 1          /1.001/
```

```
        PE1     Export price of good 1          /1/
        PM2     Import price of good 2          /1/;

NONNEGATIVE VARIABLES
        X1      Activity level for sector X1
        X2      Activity level for sector X2
        E1      Activity level for sector E1
        E2      Activity level for sector E2
        M1      Activity level for sector M1
        M2      Activity level for sector M2
        W       Activity level for sector W

        P1      Price index for commodity X
        P2      Price index for commodity Y
        PL      Price index for primary factor L
        PK      Price index for primary factor K
        PW      Price index for welfare (consumer price index)
        PFX     Real exchange rate index

        CONS    Income definition for CONS;

EQUATIONS
        PRF_X1  Zero profit for sector X1
        PRF_X2  Zero profit for sector X2
        PRF_E1  Zero profit for sector E1
        PRF_E2  Zero profit for sector E2
```

```
        PRF_M1  Zero profit for sector M1
        PRF_M2  Zero profit for sector M2
        PRF_W   Zero profit for sector W

        MKT_X1  Supply-demand balance for commodity X1
        MKT_X2  Supply-demand balance for commodity X2
        MKT_PFX Supply-demand balance for commodity PFX
        MKT_L   Supply-demand balance for primary factor L
        MKT_K   Supply-demand balance for primary factor L
        MKT_W   Supply-demand balance for aggregate demand

        I_CONS  Income definition for CONS;

*       Zero profit conditions

PRF_X1..    150*PL**(0.9) * PK**(0.1) =G= 150*P1;

PRF_X2..    50*PL**(0.1) * PK**(0.9) =G=  50*P2;

PRF_E1..    50*P1 =G= 50*PFX*PE1;

PRF_E2..    50*P2 =G= 50*PFX*PE2;

PRF_M1..    50*PFX*PM1 =G= 50*P1;

PRF_M2..    50*PFX*PM2 =G= 50*P2;
```

```
PRF_W..       100*P1**0.5 * P2**0.5 =G= 100*PW;

*       Market clearance conditions

MKT_X1..      150*X1 + 50*M1 =G= 50*E1 + 100*W*PW/P1;

MKT_X2..      50*X2  + 50*M2 =G= 50*E2 + 100*W*PW/P2 ;

MKT_PFX..     50*E2*PE2 + 50*E1*PE1 =G= 50*PM2*M2 + 50*PM1*M1;

MKT_W..       200*W =G= CONS / PW;

MKT_L..       140 =G= 135*X1 * P1/PL +  5*X2*P2/PL;

MKT_K..       60 =G=   15*X1*P1/PK + 45*X2*P2/PK;

*       Income balance

I_CONS..      CONS =E= 140*PL + 60*PK;

MODEL SOE1 /PRF_X1.X1, PRF_X2.X2, PRF_E1.E1, PRF_E2.E2,
                 PRF_M1.M1, PRF_M2.M2, PRF_W.W,
                 MKT_X1.P1, MKT_X2.P2, MKT_PFX.PFX, MKT_L.PL,
                 MKT_K.PK, MKT_W.PW, I_CONS.CONS /;
```

```
*       set SOE values:

X1.L    =1;
X2.L    =1;
E2.L    =0;
M1.L    =0;
E1.L    =1;
M2.L    =1;
W.L     =1;

P1.L    =1;
P2.L    =1;
PFX.L   =1;
PK.L    =1;
PL.L    =1;
CONS.L  =200;

* choose the real consumer price index as numeraire

PW.FX   =1;

* check for calibration and starting-value errors

SOE1.ITERLIM = 0;
SOLVE SOE1 USING MCP;
SOE1.ITERLIM = 2000;
```

```
SOLVE SOE1 USING MCP;

* counterfactual: a terms-of-trade improvement

PE1 = 1.2;
PM1 = 1.21;
SOE1.ITERLIM = 2000;
SOLVE SOE1 USING MCP;
```

8.2 Small open economy: tariffs versus trade costs

Zero-profit conditions for trade activities

PFX - price of a unit of "foreign exchange"
TC - gross real trade cost: 1 + the trade-cost rate
TR - gross tariff: 1 + the ad valorem tariff rate

TC = TR = 1: costless trade: tariffs, trade costs *appear* symmetric

```
PRF_E1..    50 * P1 =G= 50 * PFX * PE1;

PRF_E2..    50 * P2 =G= 50 * PFX * PE2;

PRF_M1..    50 * PFX * PM1 * TC * TR =G= 50 * P1;

PRF_M2..    50 * PFX * PM2 * TC * TR =G= 50 * P2;
```

TC and TR appear the same way in the pricing equations. However, they are quite different from one another.

Modeling a tariff that is costlessly collected and redistributed to consumers lump sum.

Recall that TR is defined as the gross tariff (one plus the ad valorem rate), so (TR - 1) is the tariff rate. The tariff revenue from equation PRF_M1 is:

```
[50 * PFX * PM2] * (TR-1) * M2
```

The term in brackets [ ] is the cost of 50 units (the benchmark value) of imports of good 2, (TR - 1) is the tariff rate, and M2 is the activity level, calibrated at one initially.

The assumption that the tariff can be costlessly redistributed means that this amount must be entered in the consumer's budget constraint: Income = factor income + tariff revenue.

```
CONS =E= 140*PL + 60*PK + 50*PFX*PM2*(TR-1)*M2;
```

Modeling an "iceberg" trade cost. Assume that the trade cost is paid in units of the good itself.

This is sometimes referred to as "iceberg trade costs". Some of the good "melts" in transit and less arrives than is shipped.

Let p* and M be the price received by the exporter and M be the number of units shipped.

Then the price paid by the importer is (p*TC). The quantity received by the importer is (M/TC): the quantity that arrives unmelted.

Note that revenue received by the exporter and paid by the importer are the same: the payments do and must balance.

$$p^*M = (p^*TC)(M/TC)$$

The trade cost obviously does not generate any revenue and does not become a term in the consumer's budget constraint.

However, it reappears in the model in the supply = demand equations for the two goods: the amount entered is the amount received, not the amount shipped.

The market-clearing conditions for the two goods are production plus imports (received) minus exports equal domestic demand.

```
150*X1 + 50*M1/TC  =G=  50*E1 + 100*W*PW/P1

 50*X2 + 50*M2/TC  =G=  50*E2 + 100*W*PW/P2
```

Finally, there is a market-clearing condition for foreign exchange that determines it price (PFX):

earnings from exports = import costs

```
50*E2*PE2 + 50*E1*PE1 =G= 50*PM2*M2 + 50*PM1*M1;
```

Counter-factual: show the welfare effects of a tariff versus a real trade costs.

Two sets are defined:

I will loop over values of trade costs or tariffs
J will denote the tariff and trade cost scenarios respectively.

Then some two-dimensional parameters are defined in order to extract some results after solving.

The * in RESULTS(I, *) is called a "wild card" and its use should be obvious below.

```
SETS I indexes 25 different gross cost levels      /I1*I25/
     J indexes 2 scenarios: 1 = tar 2 = trade cost /J1*J2/;
```

```
PARAMETERS
    RATE(I)
    WELFARE(I,J)
    IMP2(I,J)
    TRATE(I,J)
    TCOST(I,J)
    RESULTS(I, *);
```

Then there is the double loop, the inner loop over I sets the cost rate, starting at 1.

For J = 1, the rate is a tariff, for J = 2, the rate is a trade costs.

```
LOOP(J,
LOOP(I,

TC = 1; TR = 1;
RATE(I) = 1 + .05*ORD(I) - 0.05;
TR$(ORD(J) EQ 1) = RATE(I);
TC$(ORD(J) EQ 2) = RATE(I);
```

The *left-hand* exception operator $

```
TR$(ORD(J) EQ 1) = RATE(I);
```

is read “set the parameter TR equal to RATE if the ordinal value of J is equal to one, otherwise leave TR unchanged from it’s existing value”.

For future reference, the meaning is different if the exception operator is on *the right-hand* side.

```
TR= RATE(I)$(ORD(J) EQ 1;
```

is read “set the tariff TR equal to RATE if the ordinal value of J is equal to one, otherwise set TR equal to *zero*”. This is something quite different!

The parameter IMP(I, J) extracts the equilibrium value of exports in the two scenarios.

```
SOLVE ALGEBRAIC USING MCP;

WELFARE(I,J) = W.L;
IMP2(I,J) = M2.L/TC;

);
);

RESULTS(I, "RATE") = RATE(I);
RESULTS(I, "WELTR") = WELFARE(I, "J1");
RESULTS(I, "WELTC") = WELFARE(I, "J2");
RESULTS(I, "IMP2TR")= IMP2(I,"J1");
RESULTS(I, "IMP2TC")= IMP2(I,"J2");

DISPLAY RESULTS;
```

```
$TITLE  M8-2.GMS: Small open economy 2x2
*       contrasts tariffs versus (iceberg) trade costs

$ONTEXT
 CALIBRATION: country exports X1, imports X2
 in free-trade benchmark

                Production Sectors                      Consumer

Markets    |      X1       X2       E1       M2       W        CONS
-----------------------------------------------------------------
P1         |      150               -50              -100
P2         |               50                50      -100
PL         |     -135      -5                                  140
PK         |      -15     -45                                   60
PW         |                                          200     -200
PFX        |                         50      -50
-----------------------------------------------------------------
$OFFTEXT

* the first four parameters allow changes in (exogenous) world


PARAMETERS
        PE2     Export price of good 2          /0.999/
        PM1     Import price of good 1          /1.001/
```

```
        PE1     Export price of good 1          /1/
        PM2     Import price of good 2          /1/
        TR      Import tariff (gross basis)     /1/
        TC      Trade costs (gross basis)       /1/
        CONSX1  Consumption of X1
        CONSX2  Consumption of X2;


NONNEGATIVE VARIABLES
        X1      Activity level for sector X1
        X2      Activity level for sector X2
        E1      Activity level for sector E1
        E2      Activity level for sector E2
        M1      Activity level for sector M1
        M2      Activity level for sector M2
        W       Activity level for sector W

        P1      Price index for commodity X
        P2      Price index for commodity Y
        PL      Price index for primary factor L
        PK      Price index for primary factor K
        PW      Price index for welfare (consumer price index)
        PFX     Real exchange rate index

        CONS    Income definition for CONS;
```

```
EQUATIONS
        PRF_X1  Zero profit for sector X1
        PRF_X2  Zero profit for sector X2
        PRF_E1  Zero profit for sector E1
        PRF_E2  Zero profit for sector E2
        PRF_M1  Zero profit for sector M1
        PRF_M2  Zero profit for sector M2
        PRF_W   Zero profit for sector W

        MKT_X1  Supply-demand balance for commodity X1
        MKT_X2  Supply-demand balance for commodity X2
        MKT_PFX Supply-demand balance for commodity PFX
        MKT_L   Supply-demand balance for primary factor L
        MKT_K   Supply-demand balance for primary factor L
        MKT_W   Supply-demand balance for aggregate demand

        I_CONS  Income definition for CONS;

*       Zero profit conditions

PRF_X1..    150*PL**(0.9) * PK**(0.1) =G= 150*P1;

PRF_X2..    50*PL**(0.1) * PK**(0.9) =G=  50*P2;

PRF_E1..    50*P1 =G= 50*PFX*PE1;
```

```
PRF_E2..     50*P2 =G= 50*PFX*PE2;

PRF_M1..     50*PFX*PM1*TC*TR =G= 50*P1;

PRF_M2..     50*PFX*PM2*TC*TR =G= 50*P2;

PRF_W..      100*P1**0.5 * P2**0.5 =G= 100*PW;

*       Market clearance conditions

MKT_X1..     150*X1 + 50*M1/TC =G=  50*E1 + 100*W*PW/P1;

MKT_X2..     50*X2 + 50*M2/TC =G= 50*E2 + 100*W*PW/P2 ;

MKT_PFX..    50*E2*PE2 + 50*E1*PE1 =G= 50*PM2*M2 + 50*PM1*M1;

MKT_W..      200*W =G= CONS / PW;

MKT_L..      140 =G= 135*X1 * P1/PL +  5*X2*P2/PL;

MKT_K..      60 =G=   15*X1*P1/PK + 45*X2*P2/PK;

*       Income balance

I_CONS..     CONS =E= 140*PL + 60*PK + 50*PFX*PM2*(TR-1)* M2;
```

```
MODEL SOE /PRF_X1.X1, PRF_X2.X2, PRF_E1.E1, PRF_E2.E2,
                 PRF_M1.M1, PRF_M2.M2, PRF_W.W,
                 MKT_X1.P1, MKT_X2.P2, MKT_PFX.PFX, MKT_L.PL,
                 MKT_K.PK, MKT_W.PW, I_CONS.CONS /;

*       set benchmark values:

X1.L    =1;
X2.L    =1;
E2.L    =0;
M1.L    =0;
E1.L    =1;
M2.L    =1;
W.L     =1;

P1.L    =1;
P2.L    =1;
PFX.L   =1;
PK.L    =1;
PL.L    =1;
CONS.L  =200;

* choose the real consumer price index as numeraire

PW.FX   =1;
```

```
* check for calibration and starting-value errors

SOE.ITERLIM = 0;
SOLVE SOE USING MCP;

SOE.ITERLIM = 2000;
SOLVE SOE USING MCP;



* SHOW HOW TO DO MULTIPLE SCENARIOS
* SHOW DIFFERENCE BETWEEN TARIFF AND TRADE COST OF EQUAL RATES

SETS I indexes 25 different gross cost levels         /I1*I25/
     J indexes 2 scenarios: 1 = tariff 2 = trade cost /J1*J2/;

PARAMETERS
 RATE(I)
 WELFARE(I,J)
 IMP2(I,J)
 TRATE(I,J)
 TCOST(I,J)
 RESULTS(I, *);

LOOP(J,
LOOP(I,
```

```
TC = 1; TR = 1;
RATE(I) = 1 + .05*ORD(I) - 0.05;
TR$(ORD(J) EQ 1) = RATE(I);
TC$(ORD(J) EQ 2) = RATE(I);

SOLVE SOE USING MCP;

WELFARE(I,J) = W.L;
IMP2(I,J) = M2.L/TC;

);
);

RESULTS(I, "RATE") = RATE(I);
RESULTS(I, "WELTR") = WELFARE(I, "J1");
RESULTS(I, "WELTC") = WELFARE(I, "J2");
RESULTS(I, "IMP2TR")= IMP2(I,"J1");
RESULTS(I, "IMP2TC")= IMP2(I,"J2");

DISPLAY RESULTS;


* Write parameter RESULTS to an Excel file M8.XLS,
* starting in Sheet1
```

```
Execute_Unload 'M8.gdx' RESULTS
execute 'gdxxrw.exe M8.gdx par=RESULTS rng=SHEET1!'

* simpler but less sophisticated dump to excel

$LIBINCLUDE XLDUMP RESULTS M8.XLS SHEET2!
```

8.3 Small open economy: calibrating to tariffs in the benchmark

This model has a 20% tariff in the benchmark data. It is important to keep track of prices and trade balance in this situation.

Markets	Production Sectors X1	X2	E1	M2	W	Consumer CONS
P1	150		-50		-100	
P2		40		60	-100	
PL	-100	-20				120
PK	-50	-20				70
PW					200	-200
PFX			50	-50		
Tariff				-10		10

Trade balance holds, since exports of good 1 generate 50 units of foreign exchange and those 50 units are spent on imports.

Consumers spend 60 on imports of which 10 is the tariff, or 20% of the value of imports (10/50).

The convention we adopt here is that all domestic prices are equal to 1 initially. Since the export good X_1 is freely traded, then the international price of X_1 is also equal to 1.

However, the import good has a domestic price equal to (1 + TM2) times the world price, so if the domestic price is equal to 1 then the world price must equal 1/ (1 + TM2) = 1/1.2.

Thus the -50 in the column M2 of the matrix is interpreted as 60 units at a price of 1/1.2.

The counterfactual experiment is free trade.

```
$TITLE: M8-3: Small open economy with a benchmark tariff

$ONTEXT
In this example, units are chosen such that all DOMESTIC prices
equal one initially.  Implied world price of import good
X2: P2 = 1/1.2

                Production Sectors                        Consumer

Markets        |    X1       X2       E1       M2       W      CONS
---------------------------------------------------------------
P1             |   150               -50             -100
P2             |            40                60     -100
PL             |  -100     -20                                120
PK             |   -50     -20                                 70
PW             |                                      200     -200
PFX            |                      50      -50
TARIFF         |                              -10              10
---------------------------------------------------------------
$OFFTEXT

PARAMETERS
        PE2     Export price of good 2
        PM1     Import price of good 1
        PE1     Export price of good 1
        PM2     Import price of good 2
```

```
        TM2     Import tariff for good 2;

PE1     = 1;
PM2     = 1 / (1.2);
PE2     = PM2 * 0.99;
PM1     = 1.01;
TM2     = 0.2;


POSITIVE VARIABLES
        X1      Activity level for sector X1
        X2      Activity level for sector X2
        E1      Activity level for sector E1
        E2      Activity level for sector E2
        M1      Activity level for sector M1
        M2      Activity level for sector M2
        W       Activity level for sector W (Hicksian welfare index)
        P1      Price index for commodity X
        P2      Price index for commodity Y
        PL      Price index for primary factor L
        PK      Price index for primary factor K
        PW      Price index for welfare (expenditure function)
        PFX     Read exchange rate index
        CONS    Income definition for CONS;
```

```
EQUATIONS
        PRF_X1  Zero profit for sector X1
        PRF_X2  Zero profit for sector X2
        PRF_E1  Zero profit for sector E1
        PRF_E2  Zero profit for sector E2
        PRF_M1  Zero profit for sector M1
        PRF_M2  Zero profit for sector M2
        PRF_W   Zero profit for sector W (Hicksian welfare index)

        MKT_X1  Supply-demand balance for commodity X1
        MKT_X2  Supply-demand balance for commodity X2
        MKT_PFX Supply-demand balance for commodity PFX
        MKT_L   Supply-demand balance for primary factor L
        MKT_K   Supply-demand balance for primary factor L
        MKT_W   Supply-demand balance for aggregate demand

        I_CONS  Income definition for CONS;

*       Zero profit conditions

PRF_X1..        150 * PL**(2/3) * PK**(1/3) =G= 150 * P1;

PRF_X2..        40  * PL**(0.5) * PK**(0.5) =G= 40  * P2;

PRF_E1..        50 * P1 =G= 50 * PFX * PE1;
```

```
PRF_E2..         60 * P2 =G= 60 * PFX * PE2;

PRF_M1..         50 * PFX * PM1 =G= 50 * P1;

PRF_M2..         60 * PFX * PM2 *(1+TM2) =G= 60 * P2;

PRF_W..          200 * P1**0.5 * P2**0.5 =G= 200 * PW;

*       Market clearance conditions

MKT_X1..         150 * X1 + 50 * M1 =G=  50 * E1 + 100 * W *PW/P1;

MKT_X2..         40 * X2 + 60 * M2 =G= 60 * E2 + 100 *W *PW/P2 ;

MKT_PFX..        60 * E2 * PE2 + 50 * E1 * PE1 =G=
                 60 * M2 * PM2 + 50 * PM1 * M1;

MKT_W..          200 * W =G= CONS / PW;

MKT_L..          120  =G= 100 * X1 * P1/PL + 20 * X2 * P2/PL;

MKT_K..          70 =G= 50 * X1 * P1/PK + 20 * X2 * P2/PK;

*       Income balance

I_CONS..         CONS =E= 120*PL + 70*PK + 60 * PFX * PM2 * M2 * TM2;
```

```
MODEL SOETARIFF /PRF_X1.X1, PRF_X2.X2, PRF_E1.E1, PRF_E2.E2,
                 PRF_M1.M1, PRF_M2.M2, PRF_W.W,
                 MKT_X1.P1, MKT_X2.P2, MKT_PFX.PFX, MKT_L.PL,
                 MKT_K.PK, MKT_W.PW, I_CONS.CONS /;

*       Check the benchmark (again):

X1.L    =1;
X2.L    =1;
E2.L    =0;
M1.L    =0;
E1.L    =1;
M2.L    =1;
W.L     =1;

P1.L    =1;
P2.L    =1;
PFX.L   =1;
PK.L    =1;
PW.FX   =1;
PL.L    =1;

CONS.L  =200;

SOETARIFF.ITERLIM = 0;
```

```
SOLVE SOETARIFF USING MCP;

SOETARIFF.ITERLIM = 2000;
SOLVE SOETARIFF USING MCP;

* ccounterfactual experiment: free trade

TM2  = 0 ;
SOLVE SOETARIFF USING MCP;
```

8.4a Small open economy: modeling a quota as an endogenous tax equivalent

This model assumes the same benchmark data as the previous two models, but it assumes that there is a quota limiting imports.

The quota generates a gap between the foreign or world supply price and the domestic demand price for the “rationed” good.

This difference, often referred to as a quota rent (it is a form of Ricardian rent), must go to some agent.

In this model, we model this as a “tariff equivalent”.

The size of the tariff equivalent is endogenous, set so that the level of imports is less than or equal to the quota level.

We use an auxiliary variable, PQ, for an endogenous tax rate and a constraint equation to set the value of this “tax”.

The revenue from this tax is assigned to the representative consumer in the block for the import demand for M2.

The constraint equation (A_PQ) says to set the value of the tax PQ such that the activity level for imports (calibrated to be 1 initially) is less than or equal to 1.

```
A_PQ..     1 =G= M2;
```

And “tax” revenue (quota rent) is returned to the consumer.

```
CONS =E= 120*ENDOW*PL + 70*ENDOW*PK
                  + 60*PFX*PM2*M2*PQ;
```

Units are chosen such that all domestic prices equal 1. The counterfactual experiment is to set remove the quota.

This is done with the statement PQ.FX = 0; since PQ is a variable, not a parameter.

An alternative way to do this is to declare a parameter and use it in place of the ‘1’ on the left-hand side of the constraint equation. Then the quota can be set at any level.

The value of PQ can be thought of as the tariff equivalent of the quota or the “shadow tariff”. To reintroduce the quota, we must free up PQ with two statements:

```
PQ.L = 0;
PQ.UP = +INF;
```

The second counterfactual is to triple the size of the economy.

```
$TITLE  M8-4a.GMS: Small open economy with a benchmark quota
* modeled as an endogenous tax rate

$ONTEXT
In this example, units are chosen such that all DOMESTIC prices
equal one initially.  Implied world price of import good
X2: P2 = 1/1.2

                Production Sectors                       Consumer

Markets       |    X1       X2       E1       M2       W      CONS
---------------------------------------------------------------
P1            |   150               -50              -100
P2            |             40                60     -100
PL            |  -100      -20                                 120
PK            |   -50      -20                                  70
PW            |                                       200     -200
PFX           |                      50      -50
Q (quota rent)|                              -10                10
---------------------------------------------------------------
$OFFTEXT

PARAMETERS
        PE2     Export price of good 2
        PM1     Import price of good 1
        PE1     Export price of good 1
```

```
        PM2     Import price of good 2
        ENDOW   Endowment multiplier (size of the economy);

PE1     = 1;
PM2     = 1 / (1.2);
PE2     = PM2 * 0.99;
PM1     = 1.01;
ENDOW   = 1;

NONNEGATIVE VARIABLES
    X1      Activity level for sector X1,
    X2      Activity level for sector X2,
    E1      Activity level for sector E1,
    E2      Activity level for sector E2,
    M1      Activity level for sector M1,
    M2      Activity level for sector M2,
    W       Activity level for sector W,
    P1      Price index for commodity X,
    P2      Price index for commodity Y,
    PL      Price index for primary factor L,
    PK      Price index for primary factor K,
    PW      Price index for welfare (expenditure function),
    PFX     Read exchange rate index,
    CONS    Income definition for CONS
    PQ      Quota shadow price (ad valorem tariff equivalent);
```

```
EQUATIONS
    PRF_X1  Zero profit for sector X1
    PRF_X2  Zero profit for sector X2
    PRF_E1  Zero profit for sector E1
    PRF_E2  Zero profit for sector E2
    PRF_M1  Zero profit for sector M1
    PRF_M2  Zero profit for sector M2
    PRF_W   Zero profit for sector W

    MKT_X1  Supply-demand balance for commodity X1
    MKT_X2  Supply-demand balance for commodity X2
    MKT_PFX Supply-demand balance for commodity PFX
    MKT_L   Supply-demand balance for primary factor L
    MKT_K   Supply-demand balance for primary factor L
    MKT_W   Supply-demand balance for aggregate demand

    I_CONS  Income definition for CONS
    A_PQ    Quota auxiliary (sets endogenous shadow tax PQ);

*       Zero profit conditions

PRF_X1..  150 * PL**(2/3) * PK**(1/3) =G= 150*P1;

PRF_X2..  40  * PL**(0.5) * PK**(0.5) =G= 40*P2;

PRF_E1..  50 * P1 =G= 50 * PFX * PE1;
```

```
PRF_E2..  60 * P2 =G= 60 * PFX * PE2;

PRF_M1..  50 * PFX * PM1 =G= 50 * P1;

PRF_M2..  60 * PFX * PM2 *(1+PQ) =G= 60*P2;

PRF_W..   200 * P1**0.5 * P2**0.5 =G= 200*PW;

*       Market clearance conditions

MKT_X1..  150*X1 + 50*M1 =G=  50*E1 + 100*W*PW/P1;

MKT_X2..  40*X2 + 60*M2 =G= 60*E2 + 100*W*PW/P2 ;

MKT_PFX.. 60*E2*PE2 + 50*E1*PE1 =G= 60*M2*PM2 + 50*PM1*M1;

MKT_W..   200*W =G= CONS / PW;

MKT_L..   120*ENDOW  =G= 100*X1*P1/PL + 20*X2*P2/PL;

MKT_K..   70*ENDOW   =G=  50*X1*P1/PK + 20*X2*P2/PK;

*       Income balance

I_CONS..  CONS =E= 120*ENDOW*PL + 70*ENDOW*PK + 60*PFX*PM2*M2*PQ;
```

```
A_PQ..    1 =G= M2;

MODEL SOE2 /PRF_X1.X1, PRF_X2.X2, PRF_E1.E1, PRF_E2.E2,
                 PRF_M1.M1, PRF_M2.M2,
                 PRF_W.W, MKT_X1.P1, MKT_X2.P2, MKT_PFX.PFX,
                 MKT_L.PL, MKT_K.PK, MKT_W.PW,
                 I_CONS.CONS, A_PQ.PQ /;

*       Check the benchmark (again):

X1.L    =1;
X2.L    =1;
E2.L    =0;
M1.L    =0;
E1.L    =1;
M2.L    =1;
W.L     =1;

P1.L    =1;
P2.L    =1;
PFX.L   =1;
PK.L    =1;
PW.FX   =1;
PL.L    =1;
```

```
CONS.L  =200;

PQ.L = 0.20;

SOE2.ITERLIM = 0;
SOLVE SOE2 USING MCP;

SOE2.ITERLIM = 2000;
SOLVE SOE2 USING MCP;

* counterfactual: fixed PQ = 0 to calculate free trade

PQ.FX = 0;
SOLVE SOE2 USING MCP;

* show that the quota becomes more restrictive
* as the economy grows

PQ.LO = 0;
PQ.UP = +INF;

ENDOW = 3;
SOLVE SOE2 USING MCP;

* show what would happen if there had been a fixed 0.20 tariff
* instead of the quota
```

```
PQ.FX = 0.20;
SOLVE SOE2 USING MCP;


PQ.LO = 0;
PQ.UP = +INF;

* show what would happen if the economy were smaller than the benchmark

ENDOW = 0.25;
SOLVE SOE2 USING MCP;
```

8.4b Small open economy: modeling a quota as a supply/demand for licenses

This is the same exercise as the previous M8.4a, except is shows an alternative way of modeling a quota or indeed any quantitative restriction.

This is to assume that the quota is enforced by licences sold to importers.

The government might auction them off, for example (which does happen in practice for certain licenses and permits).

We will assume that the government returns the receipt lump sum to the representative consumer.

Now, instead of an auxiliary variable and constraint equation, we have an added variable PLIC, the price of a license,

and a market clearing equation MKT_LIC which equates the exogenous supply of licences (SLIC) to the demand for them.

```
PARAMETER
   SLIC    Supply of import licenses for X2;

NON-NEGATIVE VARIABLE
   PLIC    Price of a license;

EQUATION
   MKT_LIC Market for import licenses;


PRF_M2..  60*PLIC + 60*PFX*PM2 =G= 60*P2;

MKT_LIC.. 60*SLIC =G= 60*M2;

I_CONS..  CONS =E= 120*ENDOW*PL + 70*ENDOW*PK + 60*PLIC*SLIC;
```

```
$TITLE  M8-4b: Small open economy with a benchmark quota
* modeled as supply/demand for import licenses

$ONTEXT
In this example, units are chosen such that all DOMESTIC
 prices equal one initially.
Implied world price of import good X2 P2 = 1/1.2


                Production Sectors                    Consumer

Markets       |     X1      X2      E1      M2       W      CONS
----------------------------------------------------------------
P1            |    150             -50             -100
P2            |             40              60     -100
PL            |   -100     -20                               120
PK            |    -50     -20                                70
PW            |                                     200     -200
PFX           |                     50     -50
PLIC          |                            -10                10
----------------------------------------------------------------

$OFFTEXT

PARAMETERS
        PE2     Export price of good 2
```

```
        PM1     Import price of good 1
        PE1     Export price of good 1
        PM2     Import price of good 2
        SLIC    Supply of import licenses for X2 (M2)
        ENDOW   Endowment multiplier (size of the economy);

PE1     = 1;
PM2     = 1 / (1.2);
PE2     = PM2 * 0.99;
PM1     = 1.01;
SLIC    = 1;
ENDOW   = 1;

NONNEGATIVE VARIABLES
    X1      Activity level for sector X1,
    X2      Activity level for sector X2,
    E1      Activity level for sector E1,
    E2      Activity level for sector E2,
    M1      Activity level for sector M1,
    M2      Activity level for sector M2,
    W       Activity level for sector W,
    P1      Price index for commodity X,
    P2      Price index for commodity Y,
    PL      Price index for primary factor L,
    PK      Price index for primary factor K,
    PW      Price index for welfare (expenditure function),
```

```
    PFX     Read exchange rate index,
    CONS    Income definition for CONS
    PLIC    Price of a license (ad valorem tariff equivalent);

EQUATIONS
    PRF_X1  Zero profit for sector X1
    PRF_X2  Zero profit for sector X2
    PRF_E1  Zero profit for sector E1
    PRF_E2  Zero profit for sector E2
    PRF_M1  Zero profit for sector M1
    PRF_M2  Zero profit for sector M2
    PRF_W   Zero profit for sector W

    MKT_X1  Supply-demand balance for commodity X1
    MKT_X2  Supply-demand balance for commodity X2
    MKT_PFX Supply-demand balance for commodity PFX
    MKT_L   Supply-demand balance for primary factor L
    MKT_K   Supply-demand balance for primary factor L
    MKT_W   Supply-demand balance for aggregate demand

    I_CONS  Income definition for CONS
    MKT_LIC Market for import licenses;

*       Zero profit conditions

PRF_X1..    150* PL**(2/3) * PK**(1/3) =G= 150*P1;
```

```
PRF_X2..    40* PL**(0.5) * PK**(0.5) =G= 40*P2;

PRF_E1..    50*P1 =G= 50*PFX*PE1;

PRF_E2..    60*P2 =G= 60*PFX*PE2;

PRF_M1..    50*PFX * PM1 =G= 50*P1;

PRF_M2..    60*PLIC + 60*PFX*PM2 =G= 60*P2;

PRF_W..     200*P1**0.5 * P2**0.5 =G= 200*PW;

*       Market clearance conditions

MKT_X1..    150*X1 + 50*M1 =G=  50*E1 + 100*W *PW/P1;

MKT_X2..    40*X2 + 60*M2 =G= 60*E2 + 100*W*PW/P2 ;

MKT_PFX..   60*E2*PE2 + 50*E1*PE1 =G= 60*M2*PM2 + 50*PM1*M1;

MKT_W..     200*W =G= CONS / PW;

MKT_L..     120*ENDOW  =G= 100*X1*P1/PL + 20*X2*P2/PL;

MKT_K..     70*ENDOW   =G=  50*X1*P1/PK + 20*X2*P2/PK;
```

```
MKT_LIC..   60*SLIC =G= 60*M2;

*       Income balance

I_CONS..    CONS =E= 120*ENDOW*PL + 70*ENDOW*PK + 60*PLIC*SLIC;


MODEL ALGEBRAIC /PRF_X1.X1, PRF_X2.X2, PRF_E1.E1, PRF_E2.E2,
                 PRF_M1.M1, PRF_M2.M2,
                 PRF_W.W, MKT_X1.P1, MKT_X2.P2, MKT_PFX.PFX,
                 MKT_L.PL, MKT_K.PK, MKT_W.PW,
                 MKT_LIC.PLIC, I_CONS.CONS/;

*       Check the benchmark:

X1.L    =1;
X2.L    =1;
E2.L    =0;
M1.L    =0;
E1.L    =1;
M2.L    =1;
W.L     =1;

P1.L    =1;
P2.L    =1;
```

```
PFX.L   =1;
PK.L    =1;
PW.FX   =1;
PL.L    =1;

CONS.L  =200;

PLIC.L = 1/6;

ALGEBRAIC.ITERLIM = 0;
SOLVE ALGEBRAIC USING MCP;

ALGEBRAIC.ITERLIM = 2000;
SOLVE ALGEBRAIC USING MCP;

* show what happens if the supply of licenses is greatly expanded

SLIC = 5;
SOLVE ALGEBRAIC USING MCP;

* show that the quota becomes more restrictive as
* the economy grows

SLIC = 1;
ENDOW = 3;
SOLVE ALGEBRAIC USING MCP;
```

8.5 Large economy and the optimal tariff

For many countries, or for a small number of commodities for one country, the small-country assumption of fixed world prices may not be appropriate.

One alternative is to model the whole world.

But it may be convenient and appropriate for the modeler to stick with the country and question and continue to represent the rest of the world with trade transformation functions.

We show how to do this, allowing for prices to change with quantities supplied and demanded.

We declare and auxiliary variable, TOT for terms of trade, the relative price of the country’s export good (good 1).

Then there is a constraint equation T_TOT, which specifies that the TOT falls as the country exports more of good 1.

```
T_TOT  =G= E1**(-0.3)
```

The TOT will also appear two other places in the model: in the pricing equation for exports (the higher the TOT, the more foreign exchange (FX) is earned from a unit of good one exported (E1)

```
PRF_E1..   50*P1 =G= 50*PFX*TOT;
```

TOT also appears in the equation for trade balance which determines the price of foreign exchange.

```
MKT_PFX.. 50*0.99*E2 + (50*TOT)*E1 =G= 50*M2 + 50*1.01*M1;
```

Assumed there are many consumers/producers inside the country: each views themselves as a price taker even though the country as a whole has influence over world prices.

	Production Sectors					Consumer
Markets	X1	X2	E1	M2	W	CONS
P1	150		-50		-100	
P2		50		50	-100	
PL	-100	-20				120
PK	-50	-30				80
PW					200	-200
PFX			50	-50		

We have specified this as a Cobb-Douglas function, so we are assuming that it is of the form:

$$TOT = E_1^{-0.3} \qquad \frac{E_1}{TOT}\frac{\partial TOT}{\partial E_1} = -0.3$$

The elasticity of the relative export price with respect to export quantity is -0.3. Think of export earnings as: $TOT * E1 = E_1^{0.7}$

Then if you do the algebra, the elasticity of export earnings with respect to export quantity is 0.7. This is a strong term-of-trade effect, and will lead to the usual optimal tariff argument for protection.

Counterfactual is to impose a 0.20 tariff, which improves welfare.

Optimal tariff solved as an MPEC

First, add one variable (which is just welfare) and one equation.

```
VARIABLES
 WELOPT;

EQUATIONS
 WELFOPT;

WELFOPT..  WELOPT =E= W;
```

ADD the objective function to the model and DELETE the dummy equation TARIFF.

The tariff TM2 now does not appear as a matched variable but will be solved for in the optimization.

```
$TITLE:  M8-5.GMS: Large open economy model
* modeled as an MCP and then as an MPEC to solve for the optimal tariff

$ONTEXT
This is similar to model M8-1.gms, but the terms of trade
now depend on the level of exports: high exports, lower prices
here this is modeled with an auxiliary variable TOT
where TOT is the relative price of exports to imports
TOT is set by the constraint equation T_TOT:  TOT = E1**(-0.3)

                Production Sectors                   Consumer

Markets  |   X1      X2      E1      M2       W    CONSH
-----------------------------------------------------------------
P1       |  150             -50             -100
P2       |           50              50     -100
PL       | -100     -10                             110
PK       |  -50     -40                              90
PW       |                                   200    -200
PFX      |                   50     -50
-----------------------------------------------------------------
$OFFTEXT

PARAMETERS
  FIXT     Fixed values of the tariff
  OPTIMALT Optimal tariff in the MPEC;
```

```
FIXT = 0;

NONNEGATIVE VARIABLES
   X1       Activity level for sector X1,
   X2       Activity level for sector X2,
   E1       Activity level for sector E1,
   E2       Activity level for sector E2,
   M1       Activity level for sector M1,
   M2       Activity level for sector M2,
   W        Activity level for sector W,
   P1       Price index for commodity X,
   P2       Price index for commodity Y,
   PL       Price index for primary factor L,
   PK       Price index for primary factor K,
   PW       Price index for welfare (expenditure function),
   PFX      Read exchange rate index,

   CONSH    Income definition for home agent
   TOT      Terms of trade: world price of export good 1
   TM2      Tariff - initially held fixed;

EQUATIONS
   PRF_X1  Zero profit for sector X1
   PRF_X2  Zero profit for sector X2
   PRF_E1  Zero profit for sector E1
```

```
   PRF_E2  Zero profit for sector E2
   PRF_M1  Zero profit for sector M1
   PRF_M2  Zero profit for sector M2
   PRF_W   Zero profit for sector W

   MKT_X1  Supply-demand balance for commodity X1
   MKT_X2  Supply-demand balance for commodity X2
   MKT_PFX Supply-demand balance for commodity PFX
   MKT_L   Supply-demand balance for primary factor L
   MKT_K   Supply-demand balance for primary factor L
   MKT_W   Supply-demand balance for aggregate demand

   I_CONSH Income definition for CONSH
   T_TOT   Equation for TOT
   TARIFF  Dummy equation to fix the tariff as if a parameter;

*       Zero profit conditions

PRF_X1..   150*PL**(2/3) * PK**(1/3) =G= 150*P1;

PRF_X2..   50*PL**(1/4) * PK**(3/4)  =G= 50*P2;

PRF_E1..   50*P1 =G= 50*PFX*TOT;

PRF_E2..   50*P2 =G= 50*0.99*PFX;
```

```
PRF_M1..   50*PFX*TOT*1.01 =G= 50*P1;

PRF_M2..   50*PFX*(1+TM2) =G= 50*P2;

PRF_W..    200*P1**0.5*P2**0.5 =G= 200*PW;

*       Market clearance conditions

MKT_X1..   150*X1 + 50*M1 =G= 50*E1 + 100*W*PW/P1;

MKT_X2..   50*X2  + 50*M2 =G= 50*E2 + 100*W*PW/P2 ;

MKT_PFX..  50*0.99*E2 + (50*TOT)*E1 =G= 50*M2  + 50*1.01*M1;

MKT_W..    200*W =G= CONSH / PW;

MKT_L..    110  =G= 100*X1*P1/PL + 10*X2*P2/PL;

MKT_K..    90 =G= 50*X1*P1/PK + 40*X2*P2/PK;

*       Income balance, auxiliary equation

I_CONSH..  CONSH =E= 110*PL + 90*PK + 50*M2*PFX*TM2;

T_TOT..    TOT =G= E1**(-0.3);
```

```
TARIFF..   TM2 =E= FIXT;

MODEL LOE /PRF_X1.X1, PRF_X2.X2, PRF_E1.E1, PRF_E2.E2,
                 PRF_W.W, PRF_M1.M1, PRF_M2.M2,
                 MKT_X1.P1, MKT_X2.P2, MKT_PFX.PFX, MKT_L.PL,
                 MKT_K.PK, MKT_W.PW,
                 I_CONSH.CONSH, T_TOT.TOT, TARIFF.TM2 /;

*       Check the benchmark:

X1.L    =1;
X2.L    =1;
E2.L    =0;
E1.L    =1;
M2.L    =1;
M1.L    =0;
W.L     =1;
P1.L    =1;
P2.L    =1;
PFX.L   =1;
PK.L    =1;
PW.FX   =1;
PL.L    =1;
CONSH.L =200;
TOT.L   = 1;
TM2.L  = 0;
```

```
LOE.ITERLIM = 0;
SOLVE LOE USING MCP;

LOE.ITERLIM = 10000;
SOLVE LOE USING MCP;

*   Apply a tariff which improves the terms of trade and home
*   welfare:

FIXT  = 0.2;

SOLVE LOE USING MCP;


* now let's reformulate the problem as an MPEC to solve for the
* optimal tariff

VARIABLES
 WELOPT;

EQUATIONS
 WELFOPT;

WELFOPT..  WELOPT =E= W;
```

```
* ADD the objective function and DELETE the dummy equation TARIFF

MODEL OPTTARIFF /WELFOPT, PRF_X1.X1, PRF_X2.X2, PRF_E1.E1, PRF_E2.E2,
                 PRF_W.W, PRF_M1.M1, PRF_M2.M2,
                 MKT_X1.P1, MKT_X2.P2, MKT_PFX.PFX, MKT_L.PL,
                 MKT_K.PK, MKT_W.PW,
                 I_CONSH.CONSH, T_TOT.TOT/;

OPTION MPEC = nlpec;
SOLVE OPTTARIFF USING MPEC MAXIMIZING WELOPT;

OPTIMALT = TM2.L;
DISPLAY OPTIMALT;



* go back to the orignal MCP and "brute force" search for the optimal
* tariff and see if it matches the MPEC value

SETS I /I1*I51/;

PARAMETERS
 IMTARIFF(I)
 WELFARE(I);
```

```
LOOP(I,

FIXT = 0.005*ORD(I)**2 - 0.005;

SOLVE LOE USING MCP;

IMTARIFF(I) = TM2.L;
WELFARE(I) = W.L;

);

DISPLAY IMTARIFF, WELFARE;
```

The optimal tariff in this scenario is 0.480. How does this compare to the optimal tariff from economic theory?

Let t be the import tariff and let τ (tao) be an export tax with the world price as the base, world price denote with stars

$$p_1 = p_1^*(1 - \tau)$$

Economic theory show that the optimal value of τ should be given by an inverse elasticity rule:

τ should equal the inverse of the foreign elasticity of demand (defined as positive) for our export good.

Let this be given by η. Then $\tau = 1/\eta$. Note that this is just saying that marginal cost of exports (domestic price) should equal marginal revenue.

$mc = p_1 = p_1^*(1 - 1/\eta)$ = marginal revenue on export sales

In our formulation above, 0.3 is the inverse elasticity, and hence the optimum export tax is $\tau = 0.3$.

Does this hold in our numerical example? The relationship between an import tariff and export tax as defined above can be solved from

$$\frac{p_1}{p_2} = \frac{p_1^*}{p_2^*(1+t)} \qquad \frac{p_1}{p_2} = \frac{p_1(1-\tau)}{p_2^*} \qquad 1-\tau = \frac{1}{1+t} \qquad \tau = \frac{t}{1+t}$$

In our example, t = 0.480 and so τ = 0.324. This is close to but not equal to 0.30 so I am not quite sure what is going on.

Model M8-5b reformulates the problem with an export tax and arrives at the same result: the optimal export tax is 0.324.

8.6a Two-country Heckscher-Ohlin model: Nash tariffs as an iterative MPEC

This is the classic Heckscher-Ohlin model of trade: two goods, two factors of production, and two countries.

	XHH	YHH	XHF	YHF	XFF	YFF	XFH	YFH	WH	WF	CONSH	CONSF
PXH	150		-50						-100			
PYH		50						50	-100			
PXF			50		50					-100		
PYF						150		-50		-100		
PWH									200		-200	
PWF										200		-200
PLH	-120	-10									130	
PKH	-30	-40									70	
PLF					-40	-30						70
PKF					-10	-120						130

Countries are symmetric, mirror images of each other.

```
$TITLE:  M8-6a.GMS: Full two-country Heckscher-Ohlin model
* formulation as an MPEC allows solutions for joint welfare max
* and non-cooperative Nash equilibrium in tariffs

$ONTEXT

     XHH  YHH  XHF  YHF  XFF  YFF  XFH  YFH   WH   WF   CONSH CONSF

PXH  150     -50                                -100
PYH        50                               50  -100
PXF             50          50                        -100
PYF                              150      -50         -100
PWH                                              200        -200
PWF                                                    200        -200
PLH  -120 -10                                                130
PKH   -30 -40                                                 70
PLF                         -40  -30                                70
PKF                         -10 -120                               130

$OFFTEXT

PARAMETERS
  SIZEH, SIZEF  scales the endowments of countries h and f up and down
  WELBH, WELBF  store free trade (B for benchmark) welfare level shares
  CASE          denotes which case (of size differences) is displayed
  RESULTS(*, *), RESULTS2(*,*,*);
```

```
SIZEH = 1; SIZEF = 1;
WELBH = 0.5; WELBF = 0.5;

VARIABLES
  JWELMAX       joint welfare maximization
  WELHMAX       welfare of country h
  WELFMAX       welfare of country f
  TARH          tariff of country h
  TARF          tariff of country f;

POSITIVE VARIABLES
  XHH     X home prododuction - home consumption
  YHH     Y home prododuction - home consumption
  XHF     X home prododuction - foreign consumption
  YHF     Y home prododuction - foreign consumption
  XFF     X foreign prododuction - foreign consumption
  YFF     Y foreign prododuction - foreign consumption
  XFH     X foreign prododuction - home consumption
  YFH     Y foreign prododuction - home consumption
  WH      Activity level for sector WH (Hicksian domestic welfare index)
  WF      Activity level for sector WF (Hicksian foreign welfare index)
  PXH     Price index for commodity home X
  PYH     Price index for commodity home Y
  PXF     Price index for commodity foreign X
  PYF     Price index for commodity foreign X
```

```
  PWH     Price index for welfare domestic (expenditure function)
  PWF     Price index for welfare foerign (expenditure function)
  PLH     home price labor
  PKH     home price capital
  PLF     foreign price labor
  PKF     foreign price capital
  CONSH   Income definition for home agent
  CONSF   Income definition for foreign agent;

EQUATIONS
  OBJJ    objective function for maximizing joint welfare
  OBJH    objective function of country h (maximize WH)
  OBJF    objective function of country f (maximize WF)
  PRF_XHH Zero profit for sector XHH
  PRF_YHH Zero profit for sector YHH
  PRF_XHF Zero profit for sector XHF
  PRF_YHF Zero profit for sector YHF
  PRF_XFF Zero profit for sector XFF
  PRF_YFF Zero profit for sector YFF
  PRF_XFH Zero profit for sector XFH
  PRF_YFH Zero profit for sector YFH
  PRF_WH  Zero profit for sector WH (Hicksian home welfare index)
  PRF_WF  Zero profit for sector WF (Hicksian foreign welfare index)

  MKT_XH  Supply-demand balance for commodity XH
  MKT_XF  Supply-demand balance for commodity XF
```

```
  MKT_YH  Supply-demand balance for commodity YH
  MKT_YF  Supply-demand balance for commodity YF
  MKT_LH  Supply-demand balance for primary factor LH
  MKT_KH  Supply-demand balance for primary factor KH
  MKT_LF  Supply-demand balance for primary factor LF
  MKT_KF  Supply-demand balance for primary factor KF
  MKT_WH  Supply-demand balance for aggregate home demand
  MKT_WF  Supply-demand balance for aggregate foreign demand

  I_CONSF  Income definition for CONSF
  I_CONSH  Income definition for CONSH;

*       Zero profit conditions

OBJJ..        JWELMAX =E= WH**WELBH*WF**WELBF;
OBJH..        WELHMAX =E= WH;
OBJF..        WELFMAX =E= WF;

PRF_XHH..     150 * PLH**(4/5) * PKH**(1/5) =G= 150 * PXH;

PRF_YHH..     50  * PLH**(1/5) * PKH**(4/5) =G= 50 * PYH;

PRF_XFF..     50  * PLF**(4/5) * PKF**(1/5) =G= 50 * PXF;

PRF_YFF..     150 * PLF**(1/5) * PKF**(4/5) =G= 150 * PYF;
```

```
PRF_XHF..    50 * (1+TARF) * PXH =G= 50 * PXF;

PRF_YHF..    50.1 * PYH =G= 50 * PYF;

PRF_XFH..    50.1 * PXF =G= 50 * PXH;

PRF_YFH..    50 * (1+TARH) * PYF =G= 50 * PYH;

PRF_WH..     200 * PXH**0.5 * PYH**0.5 =G= 200 * PWH;

PRF_WF..     200 * PXF**0.5 * PYF**0.5 =G= 200 * PWF;

*       Market clearance conditions

MKT_XH..     150 * XHH + 50 * XFH =G= 50 * XHF  + 100 * WH * PWH /PXH;

MKT_XF..     50 * XFF + 50 * XHF =G= 50.01 * XFH + 100 * WF * PWF/PXF;

MKT_YH..     50 * YHH + 50 * YFH =G= 50.01 * YHF +100 * WH * PWH/PYH;

MKT_YF..     150 * YFF + 50 * YHF =G= 50 * YFH + 100 * WF * PWF/PYF;

MKT_LH..     130*SIZEH =G= 120 * XHH * PXH/PLH + 10 * YHH * PYH/PLH;

MKT_KH..     70*SIZEH  =G=  30 * XHH * PXH/PKH + 40 * YHH * PYH/PKH;
```

```
MKT_LF..      70*SIZEF  =G=  40 * XFF * PXF/PLF + 30 * YFF * PYF/PLF;

MKT_KF..      130*SIZEF =G= 120 * YFF * PYF/PKF + 10 * XFF * PXF/PKF;

MKT_WH..      200 * WH =G= CONSH/PWH;

MKT_WF..      200 * WF =G= CONSF/PWF;

*       Income balance states

I_CONSH..     CONSH =E= 130*SIZEH*PLH + 70*SIZEH*PKH + 50*YFH*PYF*TARH;

I_CONSF..     CONSF =E= 70*SIZEF*PLF + 130*SIZEF*PKF + 50*XHF*PXH*TARF;

OPTION MPEC=nlpec;

MODEL HO /
OBJJ,
OBJH,
OBJF,
PRF_XHH.XHH,
PRF_YHH.YHH,
PRF_XHF.XHF,
PRF_YHF.YHF,
PRF_XFF.XFF,
PRF_YFF.YFF,
```

```
PRF_XFH.XFH,
PRF_YFH.YFH,
PRF_WH.WH,
PRF_WF.WF,

MKT_XH.PXH,
MKT_XF.PXF,
MKT_YH.PYH,
MKT_YF.PYF,
MKT_LH.PLH,
MKT_KH.PKH,
MKT_LF.PLF,
MKT_KF.PKF,
MKT_WH.PWH,
MKT_WF.PWF,

I_CONSF.CONSF,
I_CONSH.CONSH
/;

*       Check the benchmark

XHH.L   =1;
YHH.L   =1;
XHF.L   =1;
YHF.L   =0;
```

```
XFF.L   =1;
YFF.L   =1;
XFH.L   =0;
YFH.L   =1;
WH.L    =1;
WF.L    =1;

PXH.L   =1;
PXF.L   =1;
PYH.L   =1;
PYF.L   =1;
PLH.L   =1;
PKH.L   =1;
PLF.L   =1;
PKF.L   =1;
PWH.FX  =1;
PWF.L   =1;

* "benchmark" with countries symmetric and free trade:
* all activities, prices = 1

HO.ITERLIM = 1000;
SOLVE HO USING MPEC MAXIMIZING JWELMAX;

* allow for size differences (scaling endowments up and down)
```

```
SIZEH = 1;
SIZEF = 1;

TARH.FX = 0;
TARF.FX = 0;

SOLVE HO USING MPEC MAXIMIZING JWELMAX;

* store free trade welfare levels, used to compute JWELMAX

WELBH = WH.L/(WH.L + WF.L);
WELBF = WF.L/(WH.L + WF.L);
DISPLAY WELBH, WELBF;

SOLVE HO USING MPEC MAXIMIZING JWELMAX;

RESULTS("SIZEH", "FREETR") = SIZEH;
RESULTS("WELJ", "FREETR") = JWELMAX.L;
RESULTS("WELH", "FREETR") = WH.L;
RESULTS("WELF", "FREETR") = WF.L;
RESULTS("TARIFFH", "FREETR") = TARH.L;
RESULTS("TARIFFF", "FREETR") = TARF.L;

* solve for the (unilateral by h) non-cooperative Nash equilibrium

TARH.LO = -INF;
```

```
TARH.UP = +INF;

SOLVE HO USING MPEC MAXIMIZING WELHMAX;

RESULTS("SIZEH", "UNIH") = SIZEH;
RESULTS("WELJ", "UNIH") = JWELMAX.L;
RESULTS("WELH", "UNIH") = WH.L;
RESULTS("WELF", "UNIH") = WF.L;
RESULTS("TARIFFH", "UNIH") = TARH.L;
RESULTS("TARIFFF", "UNIH") = TARF.L;

* solve for the joint welfare max

TARH.LO = -INF;
TARH.UP = +INF;
TARF.LO = -INF;
TARF.UP = +INF;

SOLVE HO USING MPEC MAXIMIZING JWELMAX;

RESULTS("SIZEH", "JMAX") = SIZEH;
RESULTS("WELJ", "JMAX") = JWELMAX.L;
RESULTS("WELH", "JMAX") = WH.L;
RESULTS("WELF", "JMAX") = WF.L;
RESULTS("TARIFFH", "JMAX") = TARH.L;
RESULTS("TARIFFF", "JMAX") = TARF.L;
```

```
DISPLAY RESULTS;

* now let's look at a non-cooperative outcome in tariffs
* iterative procedure:
*  max WH subject to TARF fixed
*  hold TARH at it's solution level and free up TARF
*  max WF solve model for fixed TARF
*  repeat 10 time

SETS J size difference for the countries /J1*J9/;
SETS I iterative procedure to determine Nash tariffs /I1*I10/;

LOOP(J,
 SIZEH = 0.9 + 0.1*ORD(J);
 SIZEF = 1.1 - 0.1*ORD(J);

TARH.FX = 0;
TARF.FX = 0;

SOLVE HO USING MPEC MAXIMIZING JWELMAX;

* store free trade welfare levels in order to get
* correct value of JWELMAX

WELBH = WH.L/(WH.L + WF.L);
```

```
WELBF = WF.L/(WH.L + WF.L);
SOLVE HO USING MPEC MAXIMIZING JWELMAX;


RESULTS2(J, "SIZEH", "FREETR") = SIZEH;
RESULTS2(J, "WELJ", "FREETR") = JWELMAX.L;
RESULTS2(J, "WELH", "FREETR") = WH.L;
RESULTS2(J, "WELF", "FREETR") = WF.L;
RESULTS2(J, "TARIFFH", "FREETR") = TARH.L;
RESULTS2(J, "TARIFFF", "FREETR") = TARF.L;

* loop to compute non-cooperative Nash equilibrium in tariffs
* fist solve for TARH given TARF, then TARF given the existing
* value of TARH.  Loop 10 times to get best-response Nash eq.

LOOP(I,

TARH.LO = -INF;
TARH.UP = +INF;
TARF.FX = TARF.L;

SOLVE HO USING MPEC MAXIMIZING WELHMAX;

TARF.LO = -INF;
TARF.UP = +INF;
TARH.FX = TARH.L;
```

```
SOLVE HO USING MPEC MAXIMIZING WELFMAX;

);

RESULTS2(J, "SIZEH", "NONCOOP") = SIZEH;
RESULTS2(J, "WELJ", "NONCOOP") = JWELMAX.L;
RESULTS2(J, "WELH", "NONCOOP") = WH.L;
RESULTS2(J, "WELF", "NONCOOP") = WF.L;
RESULTS2(J, "TARIFFH", "NONCOOP") = TARH.L;
RESULTS2(J, "TARIFFF", "NONCOOP") = TARF.L;

);

DISPLAY RESULTS, RESULTS2;
```

8.6b Two-country Heckscher-Ohlin model: Nash tariffs as an iterative MPEC, using set notation

This is the same model, but illustrating the use of sets in problems like this where countries, goods and factors can be entered more compactly as sets.

```
SETS I goods     /1, 2/
     J factors   /1, 2/
     C countries /H, F/;
```

Data is read in as a table, and then values such as representative quantities and Cobb-Douglas share parameters in production and utility are calculated.

Not that no number appear in the equations of the model.

```
$TITLE  M8-6b.GMS: 2x2x2 of M8-6a in set notation

TABLE BENCH(*,*)
     XHH  YHH  XHF  YHF  XFF  YFF  XFH  YFH   UH   UF   CONSH CONSF
PXH  150      -50                              -100
PYH        50                              50  -100
PXF             50         50                        -100
PYF                             150      -50         -100
PUH                                              200        -200
PUF                                                   200         -200
PLH  -120 -10                                                130
PKH   -30 -40                                                 70
PLF                        -40  -30                                 70
PKF                        -10 -120                                130;

DISPLAY BENCH;

SETS I goods     /1, 2/
     J factors   /1, 2/
     C countries /H, F/;

ALIAS (J,JJ), (I,II);

PARAMETERS

 VS(J,C)       endowment of factor j in country c
```

```
 TC(I)          trade cost of importing good i for country c
 BETA(I,J)      share of factor j in the production of good i
 GAMMA(I)       share of good i in the utility function
 WELW(C)        welfare weight of country c in world welfare
 RESULTS(*,*)  assemble the results;

VS("1", "H") = BENCH("PLH", "CONSH");
VS("2", "H") = BENCH("PKH", "CONSH");
VS("1", "F") = BENCH("PLF", "CONSF");
VS("2", "F") = BENCH("PKF", "CONSF");

BETA("1","1") = -BENCH("PLH", "XHH")/ BENCH("PXH", "XHH");
BETA("1","2") = -BENCH("PKH", "XHH")/ BENCH("PXH", "XHH");
BETA("2","1") = -BENCH("PLH", "YHH")/ BENCH("PYH", "YHH");
BETA("2","2") = -BENCH("PKH", "YHH")/ BENCH("PYH", "YHH");

GAMMA("1")    = -BENCH("PXH", "UH")/ BENCH("PUH", "UH");
GAMMA("2")    = -BENCH("PYH", "UH")/ BENCH("PUH", "UH");

WELW("H") = BENCH("PUH", "UH")/(BENCH("PUH", "UH")+BENCH("PUF", "UF"));
WELW("F") = BENCH("PUF", "UF")/(BENCH("PUH", "UH")+BENCH("PUF", "UF"));

DISPLAY VS, BETA, GAMMA, WELW;

TC(I)    = 1.0001;
```

```
VARIABLES
  JWELMAX         joint welfare maximization
  WELMAXH         welfare of country H
  WELMAXF         welfare of country F
  TR(I,C)         tariff of country C on good I;

POSITIVE VARIABLES
        X(I,C)   Activity level for prduction of I by C
        M(I,C)   Activity level for imports of I by C
        E(I,C)   Activity level for exports of I by C
        U(C)     Activity level for sector U in country C

        PX(I,C)  Price of commodity I in country C
        PW(I)    World price of commodity I
        PV(J,C)  Price of factor J in country C
        PU(C)    Price of welfare (expenditure function) in country C

        CONS(C)  Aggregate income;

EQUATIONS
        OBJJ
        OBJH, OBJF
        PRX(I,C)   Zero profit for sector I in country J
        PRM(I,C)   Zero profit for imports of I by country j
        PRE(I,C)   Zero profit for exports of I by country j
        PRU(C)     Zero profit for sector U
```

```
        MKX(I,C)   Supply-demand balance for commodity I
        MKW(I)     Supply-demand balance for imports and exports of I
        MKV(J,C)   Supply-demand balance for primary factor V1
        MKU(C)     Supply-demand balance for welfare (aggregate demand)

        ICONS(C)  Income definition for CONS;

OBJJ..      JWELMAX =E= PROD(C, U(C)**WELW(C));

OBJH..      WELMAXH =E= U("H");
OBJF..      WELMAXF =E= U("F");

*       Zero profit inequalities

PRX(I,C)..  PROD(J, PV(J,C)**BETA(I,J))  =G=  PX(I,C);

PRM(I,C)..  PW(I)*TC(I)*TR(I,C)  =G=  PX(I,C);

PRE(I,C)..  PX(I,C)  =G=  PW(I);

PRU(C)..    PROD(I, PX(I,C)**GAMMA(I)) =G=  PU(C);

*       Market clearance inequalities

MKX(I,C)..  X(I,C)- E(I,C) + M(I,C)/TC(I) =G=
```

```
            PROD(II, PX(II,C)**GAMMA(II))*(GAMMA(I)/PX(I,C))*U(C);

MKW(I)..    SUM(C, E(I,C) - M(I,C)) =G= 0;

MKV(J,C)..  VS(J,C)  =G=  SUM(I,
            PROD(JJ, PV(JJ,C)**BETA(I,JJ))*(BETA(I,J)/PV(J,C))*X(I,C));

MKU(C)..    U(C) =E= CONS(C) / PU(C);

*       Income balance equations

ICONS(C)..  CONS(C) =E= SUM(J, VS(J,C)*PV(J,C)) +
                        SUM(I, PW(I)*(TR(I,C) - 1)*M(I,C));

MODEL MCP /PRX.X, PRM.M, PRE.E, PRU.U,
                 MKX.PX, MKW.PW, MKV.PV, MKU.PU,
                 ICONS.CONS/;

MODEL MPEC /OBJJ, OBJH, OBJF, PRX.X, PRM.M, PRE.E, PRU.U,
                 MKX.PX, MKW.PW, MKV.PV, MKU.PU,
                 ICONS.CONS/;

PU.L(C) = 1;
PU.FX("H") = 1;

*       Set initial values of variables:
```

```
X.L(I,C)       =100;
M.L(I,C)       =100;
E.L(I,C)       =100;
U.L(C)         =200;
PX.L(I,C)      =1;
PW.L(I)        =1;
PV.L(J,C)      =1;
PV.L(J,C)      =1;
CONS.L(C)      =200;
WELMAXH.L      =U.L("H");
WELMAXF.L      =U.L("F");
JWELMAX.L      = 1;

* fix tariffs at zero (meaning TR = 1) to solve for free trade

TR.FX(I,C) = 1;

OPTION MPEC = nlpec;
SOLVE MPEC USING MPEC MAXMIZING JWELMAX;

RESULTS("WELJ", "FREETR") = JWELMAX.L/200;
RESULTS("WELH", "FREETR") = U.L("H")/200;
RESULTS("WELF", "FREETR") = U.L("F")/200;
RESULTS("TARIFFH", "FREETR") = TR.L("2","H")-1;
RESULTS("TARIFFF", "FREETR") = TR.L("1","F")-1;
```

```
DISPLAY RESULTS;

* show that this can be done with the mcp version when TR variables
* are fixed

SOLVE MCP USING MCP;
TR.FX(I,"H") = 1.2;
SOLVE MCP USING MCP;

* solve for the optimal tariff for country h when TRF is still fixed
* at zero.

TR.UP("2", "H") = +INF;
TR.LO("2", "H") = -INF;

SOLVE MPEC USING MPEC MAXIMIZING WELMAXH;

RESULTS("WELJ", "UNIH") = JWELMAX.L/200;
RESULTS("WELH", "UNIH") = U.L("H")/200;
RESULTS("WELF", "UNIH") = U.L("F")/200;
RESULTS("TARIFFH", "UNIH") = TR.L("2","H")-1;
RESULTS("TARIFFF", "UNIH") = TR.L("1","F")-1;
DISPLAY RESULTS;

*$EXIT
```

```
SETS K iterative procedure to determine Nash tariffs /K1*K10/;

TR.FX(I,C) = 1;

LOOP(K,

TR.UP("2", "H") = +INF;
TR.LO("2", "H") = -INF;
TR.FX("1", "F") = TR.L("1", "F");

SOLVE MPEC USING MPEC MAXIMIZING WELMAXH;

TR.UP("1", "F") = +INF;
TR.LO("1", "F") = -INF;
TR.FX("2", "H") = TR.L("2", "H");

SOLVE MPEC USING MPEC MAXIMIZING WELMAXF;

);

RESULTS("WELJ", "NASH") = JWELMAX.L/200;
RESULTS("WELH", "NASH") = U.L("H")/200;
RESULTS("WELF", "NASH") = U.L("F")/200;
RESULTS("TARIFFH", "NASH") = TR.L("2","H")-1;
RESULTS("TARIFFF", "NASH") = TR.L("1","F")-1;
DISPLAY RESULTS;
```

```
$LIBINCLUDE XLDUMP RESULTS M8.XLS SHEET2!A3
```