

\$TITLE: M10-2.GMS: Infinite horizon dynamic model, MPS/GE formulation

\$ONTEXT

Converts infinite horizon model in to a fixed time horizon MCP

Trick is that there is an outside agent "DEITY" who demands terminal period capital in exchange for the good "HEAVEN"

Price of HEAVEN relative to terminal capital is set by a tax/subsidy so that the steady-state condition is satisfied in the terminal period

\$OFFTEXT

SETS T time periods /1*25/;

PARAMETERS

| | |
|---------------|---|
| DELTA | rate of depreciation |
| RHO | rate of time preference |
| PV | present value from terminal period to first period |
| TERM | number of the terminal period (25) |
| RTERM | present value of terminal values at $t = 1$ |
| INITK | initial capital stock |
| R(T) | present value of period t values at $t = 1$ |
| D(T) | remaining undepreciated portion of initial K at t |
| PVUTIL | present value of the utility path at $t = 1$ |
| TLAST(T) | switch to indicate the terminal (last) period |
| TFIRST(T) | switch to indicate the first time period |
| SOLUTION(T,*) | stores values of the solution path |
| CONSUME(T) | consumption at time t |

```
INVEST(T)      investment at time t
KSTOCK(T)      capital stock at time t;
```

```
RHO = 0.2;
DELTA = 0.1;
INITK = 90;
TERM = CARD(T);
RTERM = (1/(1+RHO))**(CARD(T) - 1);
R(T) = (1/(1+RHO))**(ORD(T)-1);
D(T) = (1-DELTA)**(ORD(T) - 1);
PV = 200*SUM(T, R(T)) + 90*(4*RTERM/(1+RHO));
TLAST(T) = 0;
TLAST('25') = 1;
TFIRST('1') = 1;
```

```
$ONTEXT
```

```
$MODEL: BASIC
```

```
$SECTORS:
```

```
X(T)      ! production at time t
I(T)      ! production of new capital (investment) at time t
K(T)      ! capital stock at time t
U         ! present value of utility
```

\$COMMODITIES:

CX(T) ! present value price of output at *t*
CR(T) ! present value rental rate for capital at time *t*
CK(T) ! present value price of capital (cost of production) at *t*
CL(T) ! present value price of labor at time *t*
CKT ! terminal period present value price of capital
CU ! price of utility (intertemporal consumer price index)
HEAVEN ! price of heaven

\$CONSUMERS:

CONS ! representative consumer
DEITY ! deity who demand terminal capital stock and sells heaven

\$AUXILIARY:

TRANS ! endogenous tax or subsidy achieves steady-state at *TLAST*

\$PROD:K(T)

| | | |
|------------------------|--------------------------|---------------------------|
| <i>O:CK(T+1)</i> | <i>Q:(100*(1-DELTA))</i> | <i>P:(4*R(T+1))</i> |
| <i>O:CKT\$TLAST(T)</i> | <i>Q:(100*(1-DELTA))</i> | <i>P:(4*R(T)/(1+RHO))</i> |
| <i>O:CR(T)</i> | <i>Q:100</i> | <i>P:(R(T))</i> |
| <i>I:CK(T)</i> | <i>Q:100</i> | <i>P:(4*R(T))</i> |

\$PROD:I(T)

| | |
|----------------|-------------|
| <i>O:CK(T)</i> | <i>Q:10</i> |
| <i>I:CL(T)</i> | <i>Q:40</i> |

\$PROD:X(T) s:1

O: CX(T) Q: 200

I: CL(T) Q: 100

I: CR(T) Q: 100

\$PROD:U s:1 a:2

O: CU Q: PV

I: CX(T) Q: 200 P: R(T) a:

I: HEAVEN Q: 90 P: (4*RTerm / (1+RHO)) A: CONS N: TRANS

\$DEMAND:CONS

D: CU Q: PV

E: CL(T) Q: 140

E: CK(T)\$TFIRST(T) Q: INITK

\$DEMAND:DEITY

D: CKT Q: 90

E: HEAVEN Q: 90

\$CONSTRAINT: TRANS

CR('25') = E = (1 - (1-DELTA) / (1+RHO)) * CL('25') * 4;

\$OFFTEXT

\$SYSINCLUDE MPSEGET BASIC

```
TRANS.UP = +INF;
```

```
TRANS.LO = -INF;
```

```
CX.L(T) = R(T);
```

```
CL.L(T) = R(T);
```

```
CR.L(T) = R(T);
```

```
CK.L(T) = 4*R(T);
```

```
CKT.L = 4*R('25')/(1+RHO);
```

```
HEAVEN.L = 4*R('25')/(1+RHO);
```

```
TRANS.L = 0;
```

```
*BASIC.ITERLIM = 0;
```

```
$INCLUDE BASIC.GEN
```

```
SOLVE BASIC USING MCP;
```

```
PVUTIL = SUM(T, X.L(T)*R(T)) + (X.L('25')*R('25'))/RHO;
```

```
DISPLAY PVUTIL;
```

```
CONSUME(T) = X.L(T);
```

```
INVEST(T) = I.L(T);
```

```
KSTOCK(T) = K.L(T);
```

```
SOLUTION(T, "X") = X.L(T);
```

```
SOLUTION(T, "I") = I.L(T);
```

```
SOLUTION(T, "K") = K.L(T);
```

```
$LIBINCLUDE XLDUMP SOLUTION M10.xls SHEET1!A2
```

```
* counterfactual: lower the capital stock below is ss value
```

```
INITK = 30;
```

```
$INCLUDE BASIC.GEN
```

```
SOLVE BASIC USING MCP;
```

```
PVUTIL = SUM(T, X.L(T)*R(T)) + (X.L('25')*R('25'))/RHO;
```

```
DISPLAY PVUTIL;
```

```
CONSUME(T) = X.L(T);
```

```
INVEST(T) = I.L(T);
```

```
KSTOCK(T) = K.L(T);
```

```
SOLUTION(T, "X") = X.L(T);
```

```
SOLUTION(T, "I") = I.L(T);
```

```
SOLUTION(T, "K") = K.L(T);
```

```
$LIBINCLUDE XLDUMP SOLUTION M10.xls SHEET1!F2
```

```
* counterfactual: lower the rate of time preference
```

```
INITK = 90;

RHO = 0.1;
RTERM = (1/(1+RHO))**(CARD(T) - 1);
R(T) = (1/(1+RHO))**(ORD(T)-1);
D(T) = (1-DELTA)**(ORD(T) - 1);
PV = 200*SUM(T, R(T)) + 90*(4*RTERM/(1+RHO));

$INCLUDE BASIC.GEN
SOLVE BASIC USING MCP;

PVUTIL = SUM(T, X.L(T)*R(T)) + (X.L('25')*R('25'))/RHO;

DISPLAY PVUTIL;

CONSUME(T) = X.L(T);
INVEST(T) = I.L(T);
KSTOCK(T) = K.L(T);
SOLUTION(T, "X") = X.L(T);
SOLUTION(T, "I") = I.L(T);
SOLUTION(T, "K") = K.L(T);

$LIBINCLUDE XLDUMP SOLUTION M10.xls SHEET1!K2
```