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$TITLE: M2-2.GMS: consumer choice, modeled as an NLP and a MCP
*    maximize utility subject to a linear budget constraint
*    two goods, Cobb-Douglas preferences

$ONTEXT
This program introduces economic students to GAMS and GAMS solvers.
The problem itself is known and loved by all econ students from
  undergraduate intermediate micro economics on up:
Maximizing utility with two goods and a linear budget constraint.

Four versions are considered
 OPTIMIZE: direct constrained optimization using the NLP
           (non-linear programming) solver
 COMPLEM:  uses the first-order conditions (FOC) to create a square
           system of n inequalities in n unknowns, solved using the MCP
           (mixed complementarity problem) solver
 COMPLEM2: instead of the utility function and FOC, uses the expenditure
           function and Marshallian demand functions, solved as an MCP
 COMPLEM3: instead of the utility function and FOC, uses the expenditure
           function and Hicksian demand functions, solved as an MCP
$OFFTEXT

PARAMETERS
 M             Income
 P1, P2        prices of goods X1 and X2
 S1, S2        utility shares of X1 and X2;
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M = 100;
P1 = 1;
P2 = 1;
S1 = 0.5;
S2 = 0.5;

NONNEGATIVE VARIABLES

   X1, X2    Commodity demands
   LAMBDA    Lagrangean multiplier (marginal utility of income);

VARIABLES

   U         Welfare;

EQUATIONS

   UTILITY       Utility
   INCOME        Income-expenditure constraint
   FOC1, FOC2    First-order conditions for X1 and X2;

UTILITY..     U =E= 2*(X1**S1)*(X2**S2);

INCOME..      M =G= P1*X1 + P2*X2;
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FOC1..        LAMBDA*P1 =G= 2*S1*X1**(S1-1)*(X2**S2);

FOC2..        LAMBDA*P2 =G= 2*S2*X2**(S2-1)*(X1**S1);


* set starting values
U.L = 100;
X1.L = 50;
X2.L = 50;
LAMBDA.L = 1;

* modeled as a non-linear programming problem

MODEL OPTIMIZE /UTILITY, INCOME/;
SOLVE OPTIMIZE USING NLP MAXIMIZING U;

* modeled as a complementarity problem

MODEL COMPLEM /UTILITY.U, INCOME.LAMBDA, FOC1.X1, FOC2.X2/;
SOLVE COMPLEM USING MCP;

* counterfactuals

P1 = 2;

SOLVE OPTIMIZE USING NLP MAXIMZING U;
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SOLVE COMPLEM USING MCP;

P1 = 1;
M = 200;

SOLVE OPTIMIZE USING NLP MAXIMZING U;
SOLVE COMPLEM USING MCP;


* now use the expenditure function, giving the minimum cost of buying
* one unit of utility: COSTU = P1**S1 * P2**S2 = PU
* where PU is the "price" of utility: the inverse of lambda
* two versions are presented:
* one using Marshallian (uncompensated) demand: X_i = F_i(P1, P2, M)
* one using Hicksian (compensated) demand: X_i = F_i(P1, P2, U)

P1 = 1;
M = 100;

NONNEGATIVE VARIABLES
   PU        price of utility;

EQUATIONS
   COSTU     expenditure function: cost of producing utility = PU
   DEMANDM1  Marshallian demand for good 1
   DEMANDM2  Marshallian demand for good 2
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   DEMANDH1  Hicksian demand for good 1
   DEMANDH2  Hicksian demand for good 2
   DEMANDU   Demand for utility (indirect utility function);

COSTU..    P1**S1 * P2**S2 =G= PU;

DEMANDM1.. X1 =G= S1*M/P1;

DEMANDM2.. X2 =G= S2*M/P2;

DEMANDH1.. X1 =G= S1*PU*U/P1;

DEMANDH2.. X2 =G= S2*PU*U/P2;

DEMANDU..  U =E= M/PU;

PU.L = 1;

MODEL COMPLEM2 marshall /COSTU.U, DEMANDM1.X1, DEMANDM2.X2, DEMANDU.PU/;
MODEL COMPLEM3 hicks    /COSTU.U, DEMANDH1.X1, DEMANDH2.X2, DEMANDU.PU/;

SOLVE COMPLEM2 USING MCP;
SOLVE COMPLEM3 USING MCP;

* counterfactuals
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```
P1 = 2;

SOLVE COMPLEM2 USING MCP;
SOLVE COMPLEM3 USING MCP;

P1 = 1;
M = 200;

SOLVE COMPLEM2 USING MCP;
SOLVE COMPLEM3 USING MCP;
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