

\$TITLE M6-2b.GMS: 2x2 Economy with income tax, endogenous tax rate
 * adds equal yield tax reform to model M6-2a

\$ONTEXT

*Illustrates equal yield tax reform to introduce auxiliary
 variable and constraint equaltion
 Distorionary labor tax is lowered and capital tax raised
 endogenously (TXK is now a VARIABLE) to hold revenue constant*

		Production Sectors				Consumers	
Markets	/	X	Y	W	TL	TK	CONS
PX	/	120		-120			
PY	/		120	-120			
PW	/			340			-340
PLS	/	-48	-72		120		
PKS	/	-72	-48			120	
PL	/			-100	-80		180
PK	/					-80	80
TAX	/				-40	-40	80

\$OFFTEXT

SETS S /1*6/;

PARAMETERS

TXL Labor income tax rate
WELFARE(S) Welfare
LABSUP(S) Labor supply
INCOME(S) Money income = consumption of X and Y
CAPTAX(S) Endogenous capital tax for equal yield
TAXREV(S) Tax revenue in terms of purchasing power;

POSITIVE VARIABLES

X Activity level for sector X
Y Activity level for sector Y
TL Supply activity for L
TK Supply activity for K
W Activity level for sector W
PX Price index for commodity X
PY Price index for commodity Y
PL Price index for primary factor L net of tax
PK Price index for primary factor K net of tax
PLS Price index for primary factor L gross of tax
PKS Price index for primary factor K gross of tax
PW Price index for welfare (expenditure function)
CONS Income definition for CONS
TXK Endogenous capital tax from equal yield constraint;

EQUATIONS

PRF_X Zero profit for sector X
 PRF_Y Zero profit for sector Y
 PRF_TL Zero profit for sector TL
 PRF_TK Zero profit for sector TK
 PRF_W Zero profit for sector W

MKT_X Supply-demand balance for commodity X
 MKT_TK Supply-demand balance for commodity TK
 MKT_TL Supply-demand balance for commodity TL
 MKT_Y Supply-demand balance for commodity Y
 MKT_L Supply-demand balance for primary factor L
 MKT_K Supply-demand balance for primary factor K
 MKT_W Supply-demand balance for aggregate demand

I_CONS Income definition for CONS
 A_TXK Auxiliary eq associated with equal yield constraint;

* *Zero profit conditions:*

PRF_X.. $80 * PLS^{**0.4} * PKS^{**0.6} =G= 120 * PX;$

PRF_Y.. $80 * PLS^{**0.6} * PKS^{**0.4} =G= 120 * PY;$

PRF_TL.. $80 * PL * (1 + TXL) =G= 80 * PLS;$

$$\text{PRF_TK}.. 80*PK*(1+TXK) =G= 80*PKS;$$

$$\text{PRF_W}.. 340*(PX)**(12/34) * (PY)**(12/34) * PL**(10/34) \\ =G= 340 * PW;$$

* *Market clearing conditions:*

$$\text{MKT_X}.. 120*X =G= 340*W*PW * (12/34)/PX;$$

$$\text{MKT_Y}.. 120*Y =G= 340*W*PW * (12/34)/PY;$$

$$\text{MKT_W}.. 340*W =G= \text{CONS} / PW;$$

$$\text{MKT_L}.. 180 =G= 80*TL + 340*W*(10/34)*(PW/PL);$$

$$\text{MKT_K}.. 80 =G= 80*TK;$$

$$\text{MKT_TL}.. 80*TL =G= 48*X*PX/PLS + 72*Y*PY/PLS;$$

$$\text{MKT_TK}.. 80*TK =G= 72*Y*PY/PKS + 48*X*PX/PKS;$$

* *Income constraints:*

$$\text{I_CONS}.. \text{CONS} =E= 180*PL + 80*PK + 80*TL*TXL*PL + 80*TK*TXK*PK;$$

```
A_TXK..   TXL*PL*TL*80 + TXK*PK*TK*80
          =E= 80 *(PX**0.5 * PY**0.5);
```

```
MODEL ALGEBRAIC /PRF_X.X, PRF_Y.Y, PRF_TK.TK,PRF_TL.TL,
                 PRF_W.W, MKT_X.PX, MKT_Y.PY, MKT_L.PL,
                 MKT_TK.PKS, MKT_TL.PLS,
                 MKT_K.PK, MKT_W.PW, I_CONS.CONS, A_TXK.TXK /;
```

```
X.L       =1;
Y.L       =1;
TK.L      =1;
TL.L      =1;
W.L       =1;

PL.L      =1;
PX.L      =1;
PY.L      =1;
PLS.L     =1.5;
PKS.L     =1.5;
PK.L      =1;
PW.FX     =1;
CONS.L    =340;

TXL       =0.5;
TXK.L     =0.5;
```

```
ALGEBRAIC.ITERLIM = 0;
```

```
SOLVE ALGEBRAIC USING MCP;
```

```
*      Lets do some counter-factual with taxes shifted to the  
*      factor which is in fixed supply:
```

```
ALGEBRAIC.ITERLIM = 1000;
```

```
SOLVE ALGEBRAIC USING MCP;
```

```
LOOP(S,
```

```
TXL = 0.60 - 0.10*ORD(S);
```

```
SOLVE ALGEBRAIC USING MCP;
```

```
WELFARE(S) = W.L;
```

```
LABSUP(S) = TL.L;
```

```
INCOME(S) = (PX.L*X.L + PY.L*Y.L)  
            / (PX.L**0.5*PY.L**0.5) / 2;
```

```
CAPTAX(S) = TXK.L;
```

```
TAXREV(S) = (TXL*PL.L*TL.L*80 + TXK.L*PK.L*TK.L*80)  
            / (PX.L**0.5*PY.L**0.5);
```

```
DISPLAY WELFARE, LABSUP, INCOME, CAPTAX, TAXREV;
```

TXL = 0;

TXK.FX = 0;

SOLVE ALGEBRAIC USING MCP;