

# Economic equilibrium and optimization problems using GAMS

## Chapter 2

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### Tools of Economic Analysis

- (1) Analytical theory models
- (2) Econometric estimation and testing.
- (3) Simulation modeling - complement to (1) and (2)
  - (A) greatly extends the reach of theory to problems that are analytically intractable
  - (B) extends the economic usefulness of econometrics allowing counter-factuals using parameter estimates to calibrate simulation models

- (4) Two ways of formulating economic models
  - (A) as an constrained optimization problem
  - (B) as an economic equilibrium problem: square system of equations/inequalities and unknowns
  
- (5) Limitations of analytical theory

“Many branches of both pure and applied mathematics are in great need of computing instruments to break the present stalemate created by the failure of the purely analytical approach to nonlinear problems”

--- John Von Neumann, 1945

## Analytical methods quickly become intractable

- (1) functions or equation systems have no closed-form solution
- (2) large dimensionality (# of equations and unknowns)
- (3) correct model consists of non-linear weak inequalities
- (4) large changes in parameter values

## (6) Responses to difficulties

- (1) stick to analytics, eliminate difficulties by restrictive assumptions  
(can eliminate the most interesting parts of the problem)
- (2) simulate the model you really want to solve
- (3) make an analytical model-of-the-model (e.g., partial equilibrium)  
and then simulate the richer model

## (7) Structure of a typical model

Economic models are based on the assumption of optimizing agents: consumers, firms, governments

But, generally the model itself cannot be written as a simple constrained optimization problem

Example 1: two households with different preferences/incomes

Example 2: two-firm duopoly model

An economic model typically embodies optimization at the level of the agent, the model becomes an  $n \times n$  equilibrium problem

## (8) Dimensionality, inequalities, bounded variables

Dimensionality, non-linearity, and simultaneity make models hard to solve analytically past 2 equations in 2 unknowns

Example: 2 factor, 2 good, 2 country Heckscher-Ohlin model

Economics variables are typically bounded (e.g., prices and quantities are non-negative) and economic equilibrium conditions are weak inequalities.

Example: what goods produced, technologies used?

Example: what trade links are active in equilibrium?

Example: do emissions permits have positive or zero prices?

Economics is often sacrificed to ensure a strictly interior solution.

(9) Complementarity: analytically hard, computationally “easy”

Equilibrium conditions are weak inequalities

Each inequality is associated with a particular variable, called the complementary variable.

If the equation holds as an equality in equilibrium, then the complementary variable is generally strictly positive.

If the equation holds as a strict inequality in equilibrium, the complementary variable is zero.

- (10) GAMS solvers: two ways of formulating economic models
  - (A) NLP: non-linear programming  
constrained optimization
  - (B) MCP: mixed complementarity problem  
square system of equations/inequalities and unknowns  
matched inequalities and variables
  - (C) MPEC: mathematical programming with equilibrium  
constraints: NLP + MCP constraint set

Matching of equations/inequalities and the direction of the inequalities must come from the modeler in accordance with economic theory.

(11) Simple supply-demand problem illustrating complementarity

Supply and demand model of a single market. Two equations: supply and demand. Two variables: price and quantity.

Economic equilibrium problems are represented as a system of  $n$  equations/inequalities in  $n$  matched unknowns.

Supply of good  $X$  with price  $P$ . The supply curve exploits the firm's optimization decision,  $P = MC$ .

$MC \geq P$  with the complementarity condition that  $X \geq 0$

The *price* equation is complementary with a *quantity* variable.



Suppose that  $\text{COST} = aX + (b/2)X^2$ .

Marginal cost is then given by  $MC = a + bX$ .

$a + bX \geq P$  complementary with  $X \geq 0$ .

Optimizing consumer utility for a given income and prices will yield a demand function of the form  $X = D(P, M)$  where  $M$  is income.

$X \geq D(P, M)$  with the complementary condition that  $P \geq 0$ .

The *quantity* equation is complementary with a *price* variable.

We will suppress income and assume a simple function:

$$D(P) = c - dP \quad \text{where } c > 0, d > 0.^1$$

$$X \geq c - dP \quad \text{complementary with } P \geq 0.$$

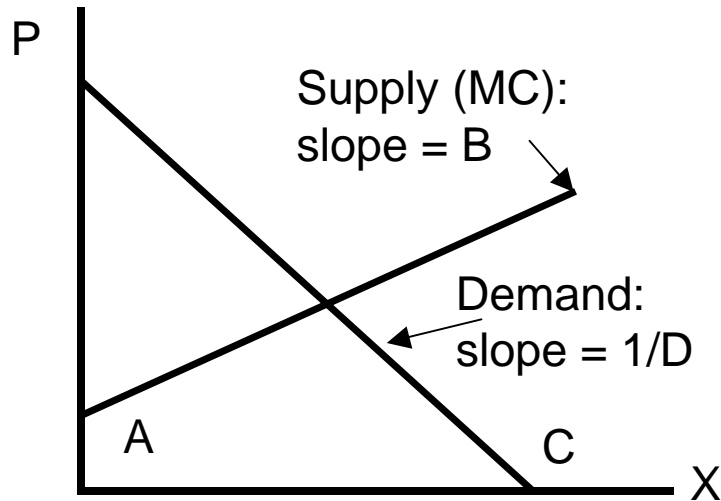
How do we know which inequality is associate with which variable and the direction of the inequality?

*Economic theory* tells you which variable must be associated with which inequality and which way the inequality goes.

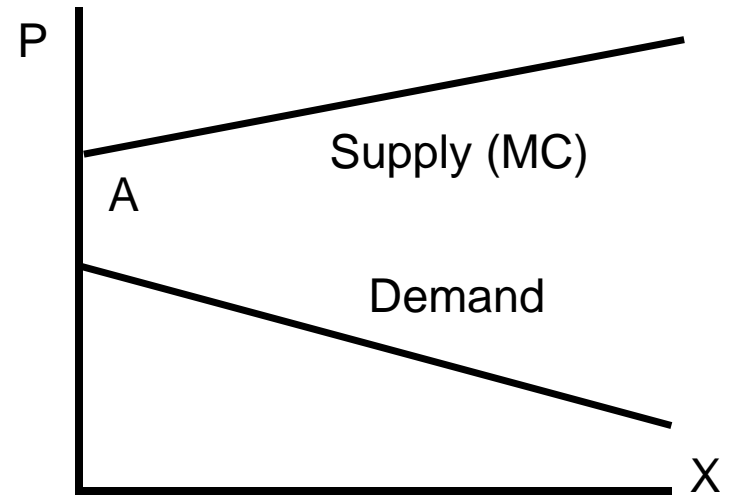
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<sup>1</sup>This demand function can be derived as the solution to a constrained optimization problem in which the consumer has a quasi-linear utility function of the form  $U = \alpha X - \beta X^2 + Y$  and budget constraint  $M = p_x X + p_y Y$

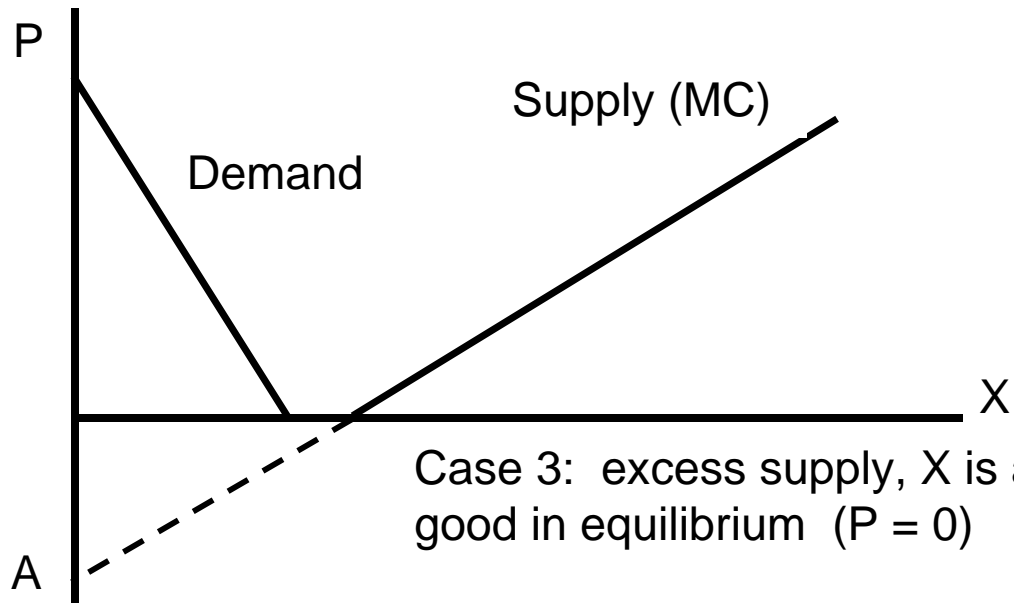
Figure 1: Three outcomes of partial equilibrium example



Case 1: interior solution



Case 2: X is too expensive, not produce in equilibrium ( $X = 0$ )



Case 3: excess supply, X is a free good in equilibrium ( $P = 0$ )

## (12) Coding an economic equilibrium problem in GAMS

First, comment statements can be used at the beginning of the code, preceded with a \*, in the *first column* of a line.

```
$TITLE: M2-1.GMS  introductory model using MCP and MPEC
* simple supply and demand model
```

Begin a series of declaration and assignment statements.

### PARAMETERS

```
A      intercept of supply on the P axis (MC at Q = 0)
B      slope of supply:  this is dP over dQ
C      demand on the Q axis (demand at P = 0)
D      (inverse) slope of demand, dQ over dP;
```

Parameters must be assigned values before the model is solved

A = 2; C = 6; B = 1; D = 1;

Declare a list of variables. They are restricted to be positive to make any economic sense, so declaring them as “nonnegative variables” tells GAMS to set lower bounds of zero.

#### NONNEGATIVE VARIABLES

P price of good X  
X quantity of good X;

Now we similarly declare a list of equations. Name not otherwise in use or, of course, a keyword.

#### EQUATIONS

SUPPLY supply relationship (mc cost ge price)  
DEMAND quantity demanded as a function of price;

Specify equations. Format: equation name followed by two periods  
 The equation is written with =G= for “greater than or equal to”.

```
SUPPLY..      A + B*X   =G=   P ;
```

```
DEMAND..     X   =G=   C + D*P ;
```

Declare a model. Keyword *model*, followed by a model name.

Then a “/” followed by a list of the equation names, each ends with a period followed by the name of the complementary variable.

```
MODEL EQUIL /SUPPLY.X, DEMAND.P/ ;
```

Tell GAMS to solve the model and what solver is needed.

```
SOLVE EQUIL USING MCP ;
```

This example uses parameter values which generate an “interior solution”, meaning that both  $X$  and  $P$  are strictly positive.

*Case 2:* the good, or a particular way to produce or obtain a good (e.g., via imports) is too expensive relative to some alternative: production or trade activity is not used in equilibrium:  $X = 0$ .

$A = 7;$

`SOLVE EQUIL USING MCP;`

*Case 3:* The final possibility is that a good or factor of production may be so plentiful that it commands a zero price in equilibrium

$A = -7;$

`SOLVE EQUIL USING MCP;`

(13) Reading the output (file name G1.LST)

GAMS stores four values for a variable. *LOWER* and *UPPER* are bounds on the variables. Declaring a *NONNEGATIVE VARIABLE* sets the lower bound at 0 (.) and upper bound at +inf.

The *LEVEL* is the solution value of the variables.

*MARGINAL* indicates the degree to which the equation corresponding to the variable is out of equality.

For *P* (price), the equation is *DEMAND* and the value of the marginal is supply minus demand.

For *X* (quantity), the equation is *SUPPLY* and the value of the marginal is the excess of marginal cost over price.



Variables that have *positive values* in the solution should have *zero marginals*.

Variables that have *zero values* in the solution should have *positive marginals*.

Here is the benchmark case.

|            | LOWER | LEVEL | UPPER | MARGINAL |
|------------|-------|-------|-------|----------|
| ---- VAR P | .     | 4.000 | +INF  |          |
| ---- VAR X | .     | 2.000 | +INF  |          |

Case 2: This is the zero-output case. The price equation holds, but the quantity equation is slack. The marginal of 1.0 indicates that, at the solution, marginal cost exceed price by 1.0.

|            | LOWER | LEVEL | UPPER | MARGINAL |
|------------|-------|-------|-------|----------|
| ---- VAR P | .     | 6.000 | +INF  | .        |
| ---- VAR X | .     | .     | +INF  | 1.000    |

Case 3: This is the free-good case. Now the price equation is slack, and the marginal of 1.0 indicates that, at the solution, supply exceeds demand by 1.0.

|            | LOWER | LEVEL | UPPER | MARGINAL |
|------------|-------|-------|-------|----------|
| ---- VAR P | .     | .     | +INF  | 1.000    |
| ---- VAR X | .     | 7.000 | +INF  | .        |

- (14) Example of the use of MPEC: set an endogenous tax rate that maximizes tax revenue.

Declare two (unbounded) variables and equations.

#### VARIABLES

```
T          tax rate on marginal cost
TREV       tax revenue = MC*T*X;
```

#### EQUATIONS

```
SUPPLY2    new supply function incorporating endogenous tax
OBJ        objective function is tax revenue;
```

```
OBJ..      TREV =E= (A + B*X)*T*X;
```

```
SUPPLY2..  (A + B*X)*(1+T) =G= P;
```

```
OPTION MPEC = NLPEC;
```

```
MODEL TREVENUE /OBJ, SUPPLY2.X, DEMAND.P/;
```

```
SOLVE TREVENUE USING MPEC MAXIMIZING TREV;
```

|               | LOWER | LEVEL | UPPER | MARGINAL |
|---------------|-------|-------|-------|----------|
| ---- VAR P    | .     | 5.000 | +INF  | EPS      |
| ---- VAR X    | .     | 1.000 | +INF  | .        |
| ---- VAR T    | -INF  | 0.667 | +INF  | .        |
| ---- VAR TREV | -INF  | 2.000 | +INF  | .        |

## 2.2 Maximization of utility subject to a linear budget constraint

Illustrates the use of the GAMS NLP and MCP solvers

NLP non-linear programming

MCP mixed complementarity problem

Cobb-Douglas utility function with linear budget constraint

$$U = \left( \frac{X_1}{\alpha} \right)^\alpha \left( \frac{X_2}{1 - \alpha} \right)^{1 - \alpha} \quad M = p_1 X_1 + p_2 X_2$$

Result: C-D exponents are expenditure shares:  $\alpha = \frac{p_1 X_1}{M}$

“Primal” formulation as an optimization problem

$$\max U = X_1^\alpha X_2^{1-\alpha} + \lambda(M - p_1 X_1 - p_2 X_2)$$

$$\frac{\partial U}{\partial X_1} = \alpha X_1^{\alpha-1} X_2^{1-\alpha} - \lambda p_1 \leq 0 \quad \perp \quad X_1 \geq 0$$

$$\frac{\partial U}{\partial X_2} = (1-\alpha) X_1^\alpha X_2^{-\alpha} - \lambda p_2 \leq 0 \quad \perp \quad X_2 \geq 0$$

$$\frac{\partial U}{\partial \lambda} = M - p_1 X_1 - p_2 X_2 \leq 0 \quad \perp \quad \lambda \geq 0$$

Can solve for “Marshallian” or “uncompensated” demand functions

$$X_i = D_i(p_1, p_2, M)$$

“Dual” formulation as a minimization problem

$$\min p_1 X_1 + p_2 X_2 + \mu (U - X_1^\alpha X_2^{1-\alpha})$$

$$\frac{\partial E}{\partial X_1} = p_1 - \mu \alpha X_1^{\alpha-1} X_2^{1-\alpha} \leq 0 \quad \perp \quad X_1 \geq 0$$

$$\frac{\partial E}{\partial X_2} = p_2 - \mu (1 - \alpha) X_1^\alpha X_2^{-\alpha} \leq 0 \quad \perp \quad X_2 \geq 0$$

$$\frac{\partial E}{\partial \mu} = U - X_1^\alpha X_2^{1-\alpha} \leq 0 \quad \perp \quad \mu \geq 0$$

Can solve for “Hicksian” or “compensated” demand functions

$$X_i = H_i(p_1, p_2, U)$$

\$TITLE: M2-2.GMS: consumer choice, modeled as an NLP and a MCP

\* maximize utility subject to a linear budget constraint

\* two goods, Cobb-Douglas preferences

### \$ONTEXT

*This program introduces economic students to GAMS and GAMS solvers. The problem itself is known and loved by all econ students from undergraduate intermediate micro economics on up:*

*Maximizing utility with two goods and a linear budget constraint.*

*Four versions are considered*

*OPTIMIZE: direct constrained optimization using the NLP (non-linear programming) solver*

*COMPLEM: uses the first-order conditions (FOC) to create a square system of  $n$  inequalities in  $n$  unknowns, solved using the MCP (mixed complementarity problem) solver*

*COMPLEM2: instead of the utility function and FOC, uses the expenditure function and Marshallian demand functions, solved as an MCP*

*COMPLEM3: instead of the utility function and FOC, uses the expenditure function and Hicksian demand functions, solved as an MCP*

### \$OFFTEXT

### PARAMETERS

M                   Income  
P1, P2             prices of goods X1 and X2  
S1, S2             utility shares of X1 and X2;



M = 100;  
P1 = 1;  
P2 = 1;  
S1 = 0.5;  
S2 = 0.5;

### NONNEGATIVE VARIABLES

X1, X2      Commodity demands  
LAMBDA      Lagrangean multiplier (marginal utility of income);

### VARIABLES

U            Welfare;

### EQUATIONS

UTILITY      Utility  
INCOME      Income-expenditure constraint  
FOC1, FOC2    First-order conditions for X1 and X2;

UTILITY..      U =E= 2\*(X1\*\*S1)\*(X2\*\*S2);

INCOME..      M =G= P1\*X1 + P2\*X2;

FOC1..            LAMBDA\*P1 =G= 2\*S1\*X1\*\*(S1-1)\*(X2\*\*S2);

FOC2..            LAMBDA\*P2 =G= 2\*S2\*X2\*\*(S2-1)\*(X1\*\*S1);

*\* set starting values*

U.L = 100;

X1.L = 50;

X2.L = 50;

LAMBDA.L = 1;

*\* modeled as a non-linear programming problem*

**MODEL** OPTIMIZE /UTILITY, INCOME/;

**SOLVE** OPTIMIZE USING NLP MAXIMIZING U;

*\* modeled as a complementarity problem*

**MODEL** COMPLEM /UTILITY.U, INCOME.LAMBDA, FOC1.X1, FOC2.X2/;

**SOLVE** COMPLEM USING MCP;

*\* counterfactuals*

P1 = 2;

**SOLVE** OPTIMIZE USING NLP MAXIMZING U;

**SOLVE** COMPLEM USING MCP;

P1 = 1;  
M = 200;

**SOLVE** OPTIMIZE USING NLP MAXIMZING U;  
**SOLVE** COMPLEM USING MCP;

*\* now use the expenditure function, giving the minimum cost of buying  
\* one unit of utility:  $COSTU = P1**S1 * P2**S2 = PU$   
\* where PU is the "price" of utility: the inverse of lambda  
\* two versions are presented:  
\* one using Marshallian (uncompensated) demand:  $X_i = F_i(P1, P2, M)$   
\* one using Hicksian (compensated) demand:  $X_i = F_i(P1, P2, U)$*

P1 = 1;  
M = 100;

### **NONNEGATIVE VARIABLES**

PU            price of utility;

### **EQUATIONS**

COSTU        expenditure function: cost of producing utility = PU  
DEMANDM1    Marshallian demand for good 1  
DEMANDM2    Marshallian demand for good 2

DEMANDH1 Hicksian demand for good 1  
 DEMANDH2 Hicksian demand for good 2  
 DEMANDU Demand for utility (indirect utility function);

COSTU..  $P1^{**}S1 * P2^{**}S2 =G= PU;$

DEMANDM1..  $X1 =G= S1*M/P1;$

DEMANDM2..  $X2 =G= S2*M/P2;$

DEMANDH1..  $X1 =G= S1*PU*U/P1;$

DEMANDH2..  $X2 =G= S2*PU*U/P2;$

DEMANDU..  $U =E= M/PU;$

PU.L = 1;

**MODEL** COMPLEM2 marshall /COSTU.U, DEMANDM1.X1, DEMANDM2.X2, DEMANDU.PU/;

**MODEL** COMPLEM3 hicks /COSTU.U, DEMANDH1.X1, DEMANDH2.X2, DEMANDU.PU/;

**SOLVE** COMPLEM2 USING MCP;

**SOLVE** COMPLEM3 USING MCP;

\* *counterfactuals*

P1 = 2;

**SOLVE** COMPLEM2 USING MCP;

**SOLVE** COMPLEM3 USING MCP;

P1 = 1;

M = 200;

**SOLVE** COMPLEM2 USING MCP;

**SOLVE** COMPLEM3 USING MCP;

\$TITLE: EXERCISE 1 Markusen: ETH Zurich GAMS course

\$ONTEXT

Maximize profits for a competitive (price taking) firm with decreasing returns, or convex marginal costs:

$$X = L^{**}ALPHA$$

0 < ALPHA < 1 X is output, L is input labor

Exercise 1A: model profit max as an optimization (NLP)

Exercise 1B: model profit max as a complementarity problem

\$OFFTEXT

## PARAMETERS

ALPHA concavity parameter in the cost function

P price of output X

W wage rate of labor L

FC fixed cost (set at zero for now);

ALPHA = 0.5;

P = 2;

W = 1;

FC = 0;

## 2.3 Extension of utility optimization: add a rationing constraint

Illustrate slackness in equilibrium, illustrate “shadow” prices

$RATION \geq X_1$  with Lagrangean multiplier  $\lambda_r$

$$\max U = X_1^\alpha X_2^{1-\alpha} + \lambda(M - p_1 X_1 - p_2 X_2) + \lambda_r (RATION - X_1)$$

$$\frac{\partial U}{\partial X_1} = \alpha X_1^{\alpha-1} X_2^{1-\alpha} - \lambda p_1 - \lambda_r \leq 0$$

$$\frac{\partial U}{\partial X_2} = (1 - \alpha) X_1^\alpha X_2^{-\alpha} - \lambda p_2 \leq 0$$

$$\frac{\partial U}{\partial \lambda} = M - p_1 X_1 - p_2 X_2 \leq 0 \quad \frac{\partial U}{\partial \lambda_r} = RATION - X_1 \leq 0$$

\$TITLE: M2-3.GMS add a rationing constraint to model M2-2  
\* *MAXIMIZE UTILITY SUBJECT TO A LINEAR BUDGET CONSTRAINT*  
\* *PLUS RATIONING CONSTRAINT ON X1*  
\* *two goods, Cobb-Douglas preferences*

### PARAMETERS

M                   Income  
P1, P2             prices of goods X1 and X2  
S1, S2             util shares of X1 and X2  
RATION             rationing constraint on the quantity of X1;

M = 100;  
P1 = 1;  
P2 = 1;  
S1 = 0.5;  
S2 = 0.5;  
RATION = 100.;

### NONNEGATIVE VARIABLES

X1, X2           Commodity demands  
LAMBDAI          Lagrangean multiplier (marginal utility of income)  
LAMBDA R         Lagrangean multiplier on rationing constraint;

### VARIABLES

U                 Welfare;



**EQUATIONS**

UTILITY           Utility  
 INCOME           Income-expenditure constraint  
 RATION1          Rationing constraint on good X1  
 FOC1, FOC2       First-order conditions for X1 and X2;

UTILITY..         $U = E = 2 * (X1 ** S1) * (X2 ** S2);$

INCOME..         $M = G = P1 * X1 + P2 * X2;$

RATION1..         $RATION = G = X1;$

FOC1..            $LAMBDAI * P1 + LAMBDA R = G = 2 * S1 * X1 ** (S1 - 1) * (X2 ** S2);$

FOC2..            $LAMBDAI * P2 = G = 2 * S2 * X2 ** (S2 - 1) * (X1 ** S1);$

*\* modeled as a non-linear programming problem*

*\* set starting values*

U.L = 100;

X1.L = 50;

X2.L = 50;

LAMBDAI.L = 1;

LAMBDA R.L = 0;

**MODEL** OPTIMIZE /UTILITY, INCOME, RATION1/;

**SOLVE** OPTIMIZE USING NLP MAXIMIZING U;

*\* modeled as a complementarity problem*

**MODEL** COMPLEM /UTILITY.U, INCOME.LAMBDAL, RATION1.LAMBDAR,  
FOC1.X1, FOC2.X2/;

**SOLVE** COMPLEM USING MCP;

*\* try binding rationing constraint at  $X1 \leq RATION = 25$ ;*

RATION = 25;

**SOLVE** OPTIMIZE USING NLP MAXIMIZING U;

**SOLVE** COMPLEM USING MCP;

*\* show that shadow price of rationing constraint increases with income  
\* could lead to a black market in rationing coupons, "scalping" tickets*

M = 200;

**SOLVE** OPTIMIZE USING NLP MAXIMIZING U;

**SOLVE** COMPLEM USING MCP;

*\* illustrate the mpec solver*

*\* suppose we want to enforce the rationing constraint via licenses for  $X1$   
\* consumers are given an allocation of licenses which is RATION*



RATION1.PLIC/;

**OPTION** MPEC = nlpec;

**SOLVE** MPEC USING MPEC MAXIMIZING U;

**SOLVE** COMPLEM2 USING MCP;

M = 200;

**SOLVE** MPEC USING MPEC MAXIMIZING U;

**SOLVE** COMPLEM2 USING MCP;

*\* now use the expenditure function, giving the minimum cost of buying  
 \* one unit of utility:  $COSTU = P1**S1 * P2**S2 = PU$   
 \* where PU is the "price" of utility: the inverse of lambda  
 \* two versions are presented:  
 \* one using Marshallian (uncompensated) demand:  $Xi = F(P1, P2, M)$   
 \* one using Hicksian (compensated) demand:  $Xi = F(P1, P2, U)$*

RATION = 100;

M = 100;

### **NONNEGATIVE VARIABLES**

PU price of utility

M1 income inclusive of the value of rationing allocation;

**EQUATIONS**

COSTU           expenditure function: cost of producing utility = PU  
 DEMANDM1       Marshallian demand for good 1  
 DEMANDM2       Marshallian demand for good 2  
 DEMANDH1       Hicksian demand for good 1  
 DEMANDH2       Hicksian demand for good 2  
 DEMANDU        Demand for utility (indirect utility function)  
 RATION1b       Rationing constraint (same as before)  
 INCOMEb        Income balance equation;

$$\text{COSTU}.. \quad (PLIC+P1)**S1 * P2**S2 =G= PU;$$

$$\text{DEMANDM1}.. \quad X1 =G= S1*M1/(P1+PLIC);$$

$$\text{DEMANDM2}.. \quad X2 =G= S2*M1/P2;$$

$$\text{DEMANDH1}.. \quad X1 =G= S1*PU*U/(P1+PLIC);$$

$$\text{DEMANDH2}.. \quad X2 =G= S2*PU*U/P2;$$

$$\text{DEMANDU}.. \quad U =E= M1/PU;$$

$$\text{RATION1b}.. \quad \text{RATION} =G= X1;$$

$$\text{INCOMEb}.. \quad M1 =E= M + PLIC*\text{RATION};$$

PU.L = 1;

**MODEL** COMPLEM3 /COSTU.U, DEMANDM1.X1, DEMANDM2.X2, DEMANDU.PU,  
RATION1b.PLIC, INCOMEb.M1/;

**MODEL** COMPLEM4 /COSTU.U, DEMANDH1.X1, DEMANDH2.X2, DEMANDU.PU,  
RATION1b.PLIC, INCOMEb.M1/;

**SOLVE** COMPLEM3 USING MCP;

**SOLVE** COMPLEM4 USING MCP;

*\* counterfactuals*

RATION = 25;

**SOLVE** COMPLEM3 USING MCP;

**SOLVE** COMPLEM4 USING MCP;

M = 200;

**SOLVE** COMPLEM3 USING MCP;

**SOLVE** COMPLEM4 USING MCP;

*\*\$exit*

*\* scenario generation*

**SETS** I indexes different values of rationing constraint /I1\*I10/  
J indexes income levels /J1\*J10/;

**PARAMETERS**

RLEVEL(I)  
PCINCOME(J)  
LICENSEP(I,J);

U.L = 50;  
X1.L = 25;  
X2.L = 25;  
PLIC.L = 0. ;  
LAMBDAI.L = 1;

*\* the following is to prevent solver failure when evaluating  $X1^{*(S1-1)}$   
\* at  $X1 = 0$  (given  $S1-1 < 0$ )*

X1.LO = 0.01;  
X2.LO = 0.01;

**LOOP**(I,  
**LOOP**(J,

RATION = 110 - 10\***ORD**(I);  
M = 25 + 25\***ORD**(J);

**SOLVE** MPEC USING MPEC MAXIMIZING U;

RLEVEL(I) = RATION;

PCINCOME(J) = M;

LICENSEP(I,J) = PLIC.L;

);

);

**DISPLAY** RLEVEL, PCINCOME, LICENSEP;

\$LIBINCLUDE XLDUMP LICENSEP M2-3.XLS SHEET1!B3



\$TITLE: M2-4.GMS quick introduction to sets and scenarios using M2-2  
\* *MAXIMIZE UTILITY SUBJECT TO A LINEAR BUDGET CONSTRAINT*  
\* *same as UTIL-OPT1.GMS but introduces set notation*

**SET** I Prices and Goods / X1, X2 /;

**ALIAS** (I, II);

### **PARAMETER**

M Income  
RATION ration of X1 (constraint on max consumption of X1)  
P(I) prices  
S(I) util shares;

M = 100;  
P("X1") = 1;  
P("X2") = 1;  
S("X1") = 0.5;  
S("X2") = 0.5;  
RATION = 100;

### **NONNEGATIVE VARIABLES**

X(I) Commodity demands  
LAMBDAI Marginal utility of income (Lagrangean multiplier)  
LAMBDA R Marginal effect of ration constraint;

**VARIABLES**

U           Welfare;

**EQUATIONS**

UTILITY  
INCOME  
RATION1  
FOC(I);

UTILITY..     U =E= 2\***PROD**(I, X(I)\*\*S(I));

INCOME..     M =G= **SUM**(I, P(I)\*X(I));

RATION1..    RATION =G= X( "X1" );

FOC(I)..     LAMBDAI\*P(I) + LAMBDAR\$(**ORD**(I) EQ 1) =G=  
              S(I)\*X(I)\*\*(-1)\*2\***PROD**(II, X(II)\*\*S(II));

U.L = 100;

X.L(I) = 50;

RATION = 100;

\* *first, solve the model as an nlp, max U subject to income*  
\* *rationing constraint in non-binding*

**MODEL** UMAX /UTILITY, INCOME, RATION1/;  
**SOLVE** UMAX USING NLP MAXIMIZING U;

\* *second, solve the model as an mcp, using the two FOC and income*  
LAMBDAI.L = 1;  
LAMBDAI.L = 0;

**MODEL** COMPLEM /UTILITY.U, INCOME.LAMBDAI, RATION1.LAMBDAI, FOC.X/;  
**SOLVE** COMPLEM USING MCP;

\* *scenario generation*

**SETS** J indexes different values of rationing constraint /J1\*J10/;

#### **PARAMETERS**

RLEVEL(J)  
WELFARE(J)  
LAMRATION(J)  
RESULTS(J, \*);

**LOOP**(J,  
RATION = 110 - 10\***ORD**(J);

**SOLVE** COMPLEM USING MCP;

RLEVEL(J) = RATION;

WELFARE(J) = U.L;

LAMRATION(J) = LAMBDA.R.L;

);

RESULTS(J, "RLEVEL") = RLEVEL(J);

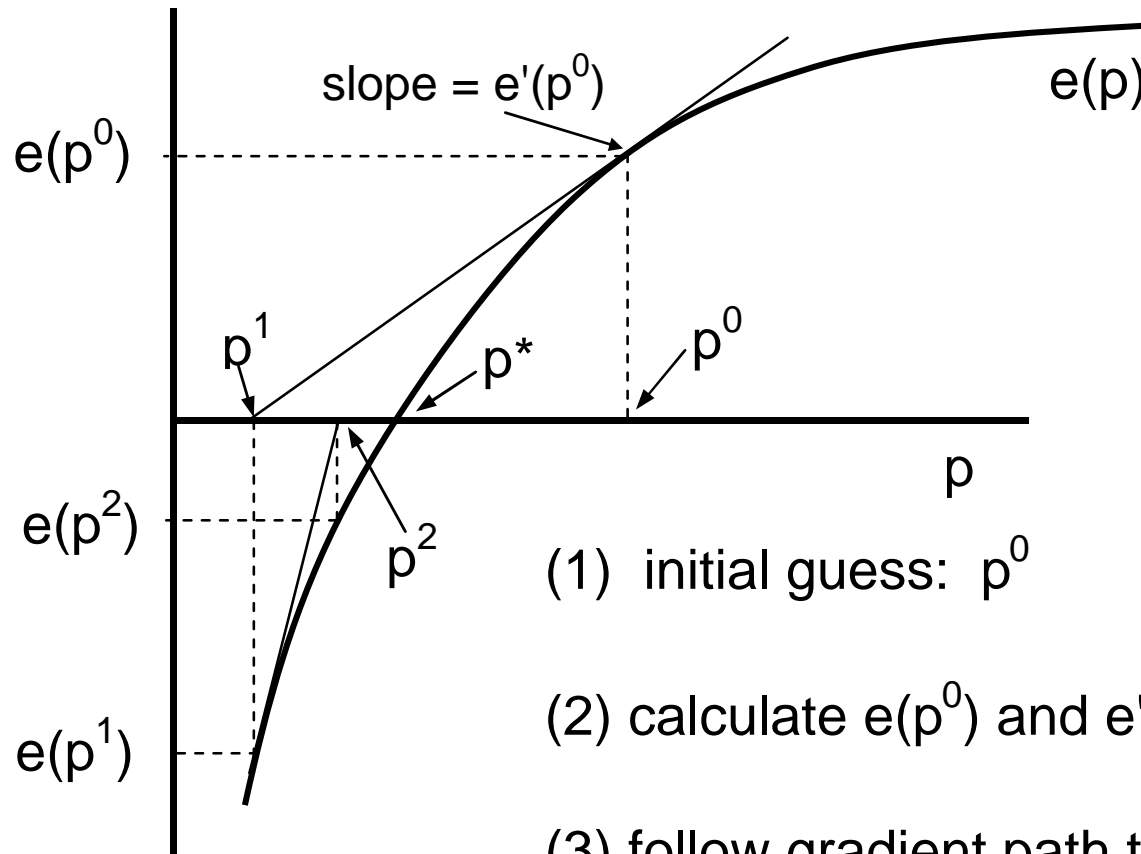
RESULTS(J, "WELFARE") = WELFARE(J);

RESULTS(J, "LAMRATION") = LAMRATION(J);

**DISPLAY** RLEVEL, WELFARE, LAMRATION, RESULTS;

\$LIBINCLUDE XLDUMP RESULTS M2-3.XLS SHEET2!B3

Illustration of gradient method: find the zero of the excess supply function:  $e(p^*) = s(p^*) - d(p^*) = 0$



iteration rule:

$$p^{i+1} = p^i - [e'(p^i)]^{-1} e(p^i)$$

(1) initial guess:  $p^0$

(2) calculate  $e(p^0)$  and  $e'(p^0)$  (exit if  $|e(p^0)| < \varepsilon$ )

(3) follow gradient path to  $e = 0$

(4) new guess  $p^1$  implicitly given by  

$$e(p^0)/(p^0 - p^1) = e'(p^0)$$

$$p^{i+1} = p^i - [e'(p^i)]^{-1} e(p^i) \quad (\text{generalizes to } n \times n)$$