

# Slides for Chapter 3: The basic closed-economy general-equilibrium model as an MCP

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1. Introduction to applied general-equilibrium modeling
  - (1) Multiple interacting agents
  - (2) Individual behavior based on optimization
  - (3) Most agent interactions are mediated by markets and prices

- (4) Equilibrium occurs when endogenous variables (e.g., prices) adjust such that
- (a) agents, subject to the constraints they face, cannot do better by altering their behavior
  - (b) markets (generally, not always) clear so, for example, supply equals demand in each market.

## 2. Steps in Applied General-Equilibrium Modeling

- (1) Specify dimensions of the model.
- Numbers of goods and factors
  - Numbers of consumers
  - Numbers of countries
  - Numbers of active markets

- (2) Chose functional forms for production, transformation, and utility functions; specification of side constraints.
  - Includes choice of outputs and inputs for each activity
  - Includes specification of initially slack activities
- (3) Construct micro-consistent data set.
  - Data satisfies zero profits for all activities, or if profits are positive, assignment of revenues
  - Data satisfies market clearing for all markets
- (4) Calibration – parameters are chosen such that functional forms and data are consistent.
  - By “consistent”: data represent a solution to the model
- (5) Replication – run model to see if it reproduces the input data.
- (6) Counter-factual experiments.

Example M3-4a: 2-good, 2-factor closed economy with fixed factor endowments, one representative consumer.

Simply economy, two sectors (X and Y), two factors (L and K), and one representative consumer (utility function  $W$ ).

L and K are in inelastic (fixed) supply, but can move freely between sectors.

$p_x$ ,  $p_y$ ,  $p_l$ , and  $p_k$  are the prices of X, Y, L and K, respectively.

CONS is consumer's income and  $p_w$  will be used later to denote the price of one unit of  $W$ .

$$(1) X = X(L_x, K_x)$$

$$(2) Y = Y(L_y, K_y)$$

$$(3) L^* = L_x + L_y$$

$$(4) K^* = K_x + K_y$$

$$(5) W = W(X, Y)$$

$$(6) \text{CONS} = p_l L^* + p_k K^* = p_x X + p_y Y$$

How do we find equilibrium: prices, and factor allocations?

Equilibrium could be solved for by a constrained optimization problem: Max (5) subject to the constraints (1), (2), (3), (4), (6).

The usefulness of this approach breaks down quickly as the model becomes more complicated.

Alternative approach: convert the problem to a system of equations, and solve that system.

Solve the cost minimization problems for producers and consumers: individual optimizing behavior is embedded in the model.

These give the minimum cost of producing a good.

Unit cost functions for X and Y

$$c_x = c_x(p_l, p_k),$$

$$c_y = c_y(p_l, p_k)$$

Unit cost (expenditure) function for W

$$e = e(p_x, p_y)$$

The next crucial step is provided by theory: Shepard's lemma

$$\frac{\partial c_x}{\partial p_l} = c_{x_{pl}} = \text{X demand for labor per unit of output}$$

$$\frac{\partial c_x}{\partial p_k} = c_{x_{pk}} = \text{X demand for capital per unit of output}$$

$$\frac{\partial e}{\partial p_x} = e_{px} = \text{Consumer's demand for X per unit of utility}$$

Now we are in a position to specify general equilibrium as the solution to a square system of 9 weak inequalities in 9 unknowns.

General-equilibrium formulated as a square system:

- 
- |                                  |                         |                             |
|----------------------------------|-------------------------|-----------------------------|
| (1) Non-positive profits for X   | $cx(p_l, p_k) \geq p_x$ |                             |
| (2) Non-positive profits for Y   | $cy(p_l, p_k) \geq p_y$ | zero-profit<br>inequalities |
| (3) Non-positive "profits" for W | $e(p_x, p_y) \geq p_w$  |                             |



- 
- (4) Supply  $\geq$  Demand for X       $X \geq e_{px}(p_x, p_y)W$
- (5) Supply  $\geq$  Demand for Y       $Y \geq e_{py}(p_x, p_y)W$
- (6) Supply  $\geq$  Demand for W       $W \geq I / p_w$       market-clearing  
inequalities
- (7) Supply  $\geq$  Demand for L       $L^* \geq cx_{pl}X + cy_{pl}Y$
- (8) Supply  $\geq$  Demand for K       $K^* \geq cx_{pk}X + cy_{pk}Y$
- 
- (9) Income balance       $CONS = p_l L^* + p_k K^*$       income  
balance

These weak inequalities can be solved for the unknowns  
 $X, Y, W, p_x, p_y, p_w, p_l, p_k,$  and  $I$ .

These inequalities are of three types, and this is generally true:

- Zero-profit conditions, inequalities (1)-(3) in the above example.
- Market clearing conditions, inequalities (4)-(8) in the above example
- Income balance, equation (9) in the above example.

Formulating equilibrium as a complementarity problem requires that each inequality is associated with a particular variable.

If a zero profit conditions holds as a strict inequality in equilibrium, profits for that activity are negative, that *activity* will not used.

The complementary variable to a *zero-profit condition* is a *quantity*, the activity level.

If a market-clearing condition holds as a strict inequality, supply exceeds demand for that good or factor in equilibrium so its price must be zero.

Thus the complementary variable to a *market clearing inequality* is the *price* of that good or factor.

The complementary variable to an income balance equation is just the income of that agent.

InequalityComplementary Variable

(1) Non-positive profits for X

$$cx(p_l, p_k) \geq p_x$$

X

(2) Non-positive profits for Y

$$cy(p_l, p_k) \geq p_y$$

Y

(3) Non-positive "profits" for W

$$e(p_x, p_y) \geq p_w$$

W

(4) Supply  $\geq$  Demand for X

$$X \geq e_{px}(p_x, p_y)W$$

 $p_x,$ (5) Supply  $\geq$  Demand for Y

$$Y \geq e_{py}(p_x, p_y)W$$

 $p_y$ (6) Supply  $\geq$  Demand for W

$$W \geq I / p_w$$

 $p_w$ (7) Supply  $\geq$  Demand for L

$$L^* \geq cx_{pl}X + cy_{pl}Y$$

 $p_l$ (8) Supply  $\geq$  Demand for K

$$K^* \geq cx_{pk}X + cy_{pk}Y$$

 $p_k$

(9) Income balance

$$\text{CONS} = p_l L^* + p_k K^* \quad \text{CONS}$$

### 3.2 Micro consistent data

A data set is micro consistent when it satisfies the conditions for economic equilibrium (it could be generated as the solution to some model).

Data must satisfy *zero profits, market clearing, and income balance.*

The above problems can be thought of as consisting of  
 three production activities, X, Y, and W,  
 four markets, X, Y, L, and K  
 income balance

Represent the initial data for this economy by a rectangular matrix.  
This matrix is related to the concept of a “SAM” – social accounting matrix

There are two types of *columns* in the rectangular matrix, corresponding to *production activities(sectors)* and *consumers*.

In the model outlined above, there are three production sectors (X, Y and W) and a single consumer (CONS).

*Rows* correspond to *markets*. Complementary variables are prices, so we have listed the price variables on the left to designate rows.

A positive entry signifies a receipt (sale) in a particular market. A negative entry signifies an expenditure (purchase) in a particular market.

Reading down a production column, we then observe a complete list of the transactions associated with that activity.

	Production Sectors			Consumers	
Markets	X	Y	W	CONS	Row sum
PX	100		-100		0
PY		100	-100		0
PW			200	-200	0
PL	-25	-75		100	0
PK	-75	-25		100	0
Column sum	0	0	0	0	

A rectangular SAM is balanced or “micro-consistent” when row and column sums are zeros.

Positive numbers represent the value of commodity flows into the economy (sales or factor supplies),

Negative numbers represent the value of commodity flows out of the economy (factor demands or final demands).

A *row sum is zero* if the total amount of commodity flowing into the economy equals the total amount flowing out of the economy.

Row sum = 0 = *market clearance*, and one such condition applies for each commodity in the model.



Columns in this matrix correspond to production sectors or consumers.

A production sector *column sum is zero* if the value of outputs equals the cost of inputs.

A consumer column is balanced if the sum of primary factor sales equals the value of final demands.

Zero column sums thus = *zero profits or “product exhaustion”* in an alternative terminology.

The numbers of the matrix are *values, prices times quantities*. The modeler decides how to interpret these as prices or quantities.

A good practice is to choose units so that as many things initially are equal to one as possible.

Prices can be chosen as one, and “representative quantities” for activities can be chosen such that activity levels are also equal to one (e.g., activity X at level one produces 100 units of good X).

In the case of taxes, both consumer and producer prices cannot equal one of course, a point we will return to in a later section.

### 3.3 Calibration and replication

Calibration is choosing functional forms and their parameters such that the initial micro-consistent data is a solution to the model.

We use Cobb-Douglas functions for the three activities. The share parameters for the functions are given in the data matrix above.

Goods in the utility function get equal shares of 0.5:

$$e(p_x, p_y) = p_x^{0.5} p_y^{0.5}$$

X is capital intensive: a capital share of 0.75, a labor share of 0.25.

$$c_x(p_l, p_k) = p_l^{0.25} p_k^{0.75}$$

Y is labor intensive with the opposite ordering of shares.

$$c_y(p_l, p_k) = p_l^{0.75} p_k^{0.25}$$

Example of Shepard's lemma: demand for labor in Y.

$$L_y = \frac{\partial c_y(p_l, p_k)}{\partial p_l} Y = 0.75 p_l^{0.75-1} p_k^{0.25} Y = 0.75 p_l^{0.75} p_k^{0.25} Y / p_l$$

If  $p_x$  is the producer price of X, then this can be written as:

$$L_y = \frac{\partial c_y(p_l, p_k)}{\partial p_l} Y = 0.75 p_x / p_l * Y$$

## 3.4 Model 3-4a

- (1) The opening line \$TITLE is not necessary, but used to the model in the listing (output) file.
- (2) \$ONTEXT.....\$OFFTEXT is a way of designating a block of comments, to be ignored by GAMS.

In this case, we put our data matrix inside this block, meaning it is not actually used in the computation.

- (3) A text line can also be preceded by a \*. GAMS ignores any line beginning with a \*. \* must be in the first column.
- (4) We declare the parameter names, then assign them values (note where semi-colons do and do not go).

- (5) Next we declare positive variables and then equation names. We write out the equation names in the syntax shown [equation name], then the equation itself ending with a semi-colon.

Note the use of the reference quantities such as “100”, “75 ” etc. in the equations. Then activity levels will be  $X = Y = W = 1$  in the initial solution.

- (6) GAMS was written to use greater-than-or-equal-to syntax (=G=).
- (7) Then the model is specified, and we chose a numeraire (recall from theory that only relative prices are determined). Here we choose utility as the numeraire, so that factor prices are then real values in terms of utility. The notation is PW.FX, “FX” for “fixed”.
- (8) Before the solve statement, we are going to help the solver by giving starting values for the variables. The syntax is, for example X.L, where the “L” stands for “level”.

- (9) Default values are zero, and in non-linear problems it is very helpful and indeed sometimes necessary to help the solver with some initial guesses. We constructed this problem knowing the answer, so I give those values as .L values.
- (10) Finally, the solve statement.
- (11) After the first solve statement, we do two counterfactual experiments. In the first, we remove the tax and double the labor endowment of the economy.

“LENDOW” and “KENDOW” are multipliers on the initial factor endowment.

This file is saved in the relevant directory as M4-3a.GMS, although you can use any name you want, need not correspond to the model name.

\$TITLE: Model M3-4a: TWOxTWOxONE Economy - Basics

\$ONTEXT

*This model is a closed economy version of the classic Heckscher-Ohlin model: two goods and two factors, one consumer Utility is treated as a produced good: quantity W, price PW*

Markets	Production Sectors			Consumers
	X	Y	W	CONS
PX	100		-100	
PY		100	-100	
PW			200	-200
PL	-25	-75		100
PK	-75	-25		100

\$OFFTEXT

**PARAMETERS**

LENDOW      labor endowment multiplier  
 KENDOW      capital endowment multiplier;

LENDOW = 1;  
 KENDOW = 1;



**NONNEGATIVE VARIABLES**

X activity level for X production  
Y activity level for Y production  
W activity level for the "production" of welfare from X Y

PX price of good X  
PY price of good Y  
PW price of a unit of welfare (real consumer-price index)  
PL price of labor  
PK price of capital

CONS income of the representative consumer;

**EQUATIONS**

PRF\_X zero profit for sector X  
PRF\_Y zero profit for sector Y  
PRF\_W zero profit for sector W (Hicksian welfare index)

MKT\_X supply-demand balance for commodity X  
MKT\_Y supply-demand balance for commodity Y  
MKT\_L supply-demand balance for primary factor L  
MKT\_K supply-demand balance for primary factor L  
MKT\_W supply-demand balance for aggregate demand

I\_CONS *income definition for CONS;*

\* *Zero profit inequalities*

PRF\_X..  $100*(PL^{**0.25} * PK^{**0.75}) =G= 100*PX;$

PRF\_Y..  $100*(PL^{**0.75} * PK^{**0.25}) =G= 100*PY;$

PRF\_W..  $200*(PX^{**0.50} * PY^{**0.50}) =G= 200*PW;$

\* *Market clearance inequalities*

MKT\_X..  $100*X =G= 100 * W * PW / PX;$

MKT\_Y..  $100*Y =G= 100 * W * PW / PY;$

MKT\_W..  $200*W =E= CONS / PW;$

MKT\_L..  $100*LENDOW =G= 25 * X * PX / PL +$   
 $75 * Y * PY / PL;$

MKT\_K..  $100*KENDOW =G= 75 * X * PX / PK +$   
 $25 * Y * PY / PK;$

\* *Income balance equations (don't forget tax revenue)*

I\_CONS..       CONS =E= 100\*LENDOW\*PL + 100\*KENDOW\*PK;

**MODEL** TWOxTWO /PRF\_X.X, PRF\_Y.Y, PRF\_W.W,  
                  MKT\_X.PX, MKT\_Y.PY, MKT\_L.PL, MKT\_K.PK,  
                  MKT\_W.PW, I\_CONS.CONS /;

\* *Chose a numeraire: real consumer price index*

PW.FX = 1;

\* *Set initial values of variables:*

X.L=1; Y.L=1; W.L=1;  
PX.L=1; PY.L=1; PK.L=1; PL.L=1;  
CONS.L=200;

**SOLVE** TWOxTWO USING MCP;

\* *counterfactual 1: double the endowment of labor*

LENDOW = 2;

**SOLVE** TWOxTWO USING MCP;

\* *counterfactual 2: double both endowments from the benchmark*

```
LENDOW = 1;
KENDOW = 2;
SOLVE TWOxTWO USING MCP;
```

*\* counterfactual 3: double the endowment of both factors*

```
LENDOW = 2;
KENDOW = 2;
SOLVE TWOxTWO USING MCP;
```

\$ontext

*We emphasize that the above formulation uses a simplifying trick: the marginal costs of X, Y, and W can be replaced by the producer prices PX, PY, and PW. This should "always" work because when marginal cost and price are not equal in equilibrium, the quantity is zero: marginal cost times quantity = producer price times quantity ( $MC_X \cdot X = PX \cdot X$  regardless of  $X > 0$  or  $X = 0$ ). But below is the model done "properly" with Shepard's lemma*

\$offtext

## **EQUATIONS**

```
MKT_X2
MKT_Y2
MKT_L2
MKT_K2
```

I\_CONS2;

MKT\_X2..           100\*X =G= 100 \* W \* (PX\*\*0.5 \* PY\*\*0.5) / PX;

MKT\_Y2..           100\*Y =G= 100 \* W \* (PX\*\*0.5 \* PY\*\*0.5) / PY;

MKT\_L2..           100\*LENDOW =G= 25 \* X \* PL\*\*0.25 \* PK\*\*0.75 / PL +  
   75 \* Y \* PL\*\*0.75 \* PK\*\*0.25 / PL;

MKT\_K2..           100\*KENDOW =G= 75 \* X \* PL\*\*0.25 \* PK\*\*0.75 / PK +  
   25 \* Y \* PL\*\*0.75 \* PK\*\*0.25 / PK;

\*           *Income balance equations (don't forget tax revenue)*

I\_CONS2..           CONS =E= 100\*LENDOW\*PL + 100\*KENDOW\*PK;

**MODEL** TWOxTWOa /PRF\_X.X, PRF\_Y.Y, PRF\_W.W,  
                           MKT\_X2.PX, MKT\_Y2.PY, MKT\_L2.PL,MKT\_K2.PK,  
                           MKT\_W.PW,I\_CONS2.CONST /;

LENDOW = 1; KENDOW = 1;

**SOLVE** TWOxTWOa USING MCP;

LENDOW = 2;

**SOLVE** TWOxTWOa USING MCP;

\$ontext

*Exercise: declare a parameter alpha, which is a productivity shift parameter*

*producing X. Higher alpha, more output per input.*

*Code this up. Hint: alpha will appear more than in the program.*

*Change alpha and interpret results.*

\$offtext

The output or listing file will automatically be written to the same directory, with name M3-4a.LST.

In the listing file, we will see the following information, along with a statement that the solution is normal and optimal.

The list of 1.000 indicates we successfully reproduced our benchmark data as an equilibrium, since we chose units such that this would be the case.

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	1.000	+INF	.
---- VAR Y	.	1.000	+INF	.
---- VAR W	.	1.000	+INF	.
---- VAR PX	.	1.000	+INF	.
---- VAR PY	.	1.000	+INF	.
---- VAR PW	1.000	1.000	1.000	EPS
---- VAR PL	.	1.000	+INF	.
---- VAR PK	.	1.000	+INF	.
---- VAR CONS	.	200.000	+INF	.

In the first counterfactual, we remove the tax, and double the labor endowment of the economy.

\*\*\*\*\*counterfactual: double labor endowment

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	1.189	+INF	.
---- VAR Y	.	1.682	+INF	.
---- VAR W	.	1.414	+INF	.
---- VAR PX	.	1.189	+INF	.
---- VAR PY	.	0.841	+INF	.
---- VAR PW	1.000	1.000	1.000	6.617E-11
---- VAR PL	.	0.707	+INF	.
---- VAR PK	.	1.414	+INF	.
---- VAR CONS	.	282.843	+INF	.



\$TITLE: EXERCISE 2 Markusen: ETH Zurich GAMS course

\$ONTEXT

$U = (X1^{**ALPHA}) * (X2^{**(1-ALPHA)})$

$X1 = BETA1 * L1$

$X2 = BETA2 * L2$

$LBAR = L1 + L2$

Exercise 2A: model equilibrium as an optimization (NLP)

Exercise 2B: model equilibrium a complementarity problem (MCP)

Let labor be numeraire with price one: then income = LBAR

Hint1: you will need added variables P1, P2

Hint2: get the Marshallian demand functions for X1 and X2

Hint3: convert production functions to unit cost functions  
and COST1 = P1, COST2 = P2

Hint4: hints 2 and 3 should give you a simple 4x4 model

\$OFFTEXT

## PARAMETERS

ALPHA      share of X1 in utility  
BETA1      labor productivity in X1  
BETA2      labor productivity of X2  
LBAR      labor endowment;

ALPHA = 0.5;

BETA1 = 1;

BETA2 = 1;

LBAR = 10;

## Model 3-4b Adds taxes to model 3-4a

There are several important modifications that taxes introduce:

(1) There is no longer “a price” for a good or factor: the price received by a supplier and that paid for by the buyer are different.

(2) The tax base is important:

net basis:	$p = (1+t)mc$	$p$ is consumer price
gross basis:	$p(1-t) = mc$	$mc$ is producer price

(2) Revenue collected must go somewhere for GE adding up to be satisfied.

To consider the effect of purely distortionary taxes, it is typically assumed that the government lump-sum redistributes taxes back to consumers.

Assume that there are commodity taxes on X and Y, on a net basis

$$p_x = (1 + t_x)mc_x \quad p_y = (1 + t_y)mc_y$$

Then the representative consumer must get a redistribution equal to

$$t_x mc_x X + t_y mc_y Y$$

We can write out the expression for marginal cost or, from (1), note that this is equal to consumer price divided by one plus t.

$$t_x (p_x / (1 + t_x)) X + t_y (p_y / (1 + t_y)) Y$$

\$TITLE: Model M3-4b: Closed TWOxTWO Economy - adds taxes

\* adds taxes to model M3-1

\$ONTEXT

Markets	Production Sectors			Consumers
	X	Y	W	
PX	100		-100	
PY		100	-100	
PW			200	-200
PL	-25	-75		100
PK	-75	-25		100

PX and PY will denote the consumer prices of X and Y since  $PX(1+TX) = MC$  (marginal cost), the producer price of X (MC) is  $PX/(1+TX)$  and similarly for Y

\$OFFTEXT

## PARAMETERS

TX            ad-valorem tax rate for X sector inputs  
 TY            ad-valorem tax rate for Y sector inputs  
 LENDOW      labor endowment multiplier  
 KENDOW      capital endowment multiplier;

```
TX = 0; TY = 0;  
LENDOW = 1;  
KENDOW = 1;
```

## POSITIVE VARIABLES

```
X      activity level for X production  
Y      activity level for Y production  
W      activity level for the "production" of welfare from X Y  
  
PX     price of good X  
PY     price of good Y  
PW     price of a unit of welfare (real consumer-price index)  
PL     price of labor  
PK     price of capital  
  
CONS   income of the representative consumer;
```

## EQUATIONS

```
PRF_X  zero profit for sector X  
PRF_Y  zero profit for sector Y  
PRF_W  zero profit for sector W (Hicksian welfare index)  
  
MKT_X  supply-demand balance for commodity X
```

MKT\_Y supply-demand balance for commodity Y  
 MKT\_L supply-demand balance for primary factor L  
 MKT\_K supply-demand balance for primary factor L  
 MKT\_W supply-demand balance for aggregate demand

I\_CONS income definition for CONS;

\* *Zero profit inequalities*

PRF\_X..  $100 * (PL^{**0.25} * PK^{**0.75}) * (1+TX) =G= 100 * PX;$

PRF\_Y..  $100 * (PL^{**0.75} * PK^{**0.25}) * (1+TY) =G= 100 * PY;$

PRF\_W..  $200 * (PX^{**0.50} * PY^{**0.50}) =G= 200 * PW;$

\* *Market clearance inequalities*

MKT\_X..  $100 * X =G= 100 * W * PW / PX;$

MKT\_Y..  $100 * Y =G= 100 * W * PW / PY;$

MKT\_W..  $200 * W =E= CONS / PW;$

MKT\_L..  $100 * LENDOW =G= 25 * X * (PX / (1+TX)) / PL +$   
 $75 * Y * (PY / (1+TY)) / PL;$

```
MKT_K..      100*KENDOW =G= 75 * X * (PX/(1+TX)) / PK +
                25 * Y * (PY/(1+TY)) / PK;
```

\* *Income balance equations (don't forget tax revenue)*

```
I_CONS..     CONS =E= 100*LENDOW*PL + 100*KENDOW*PK +
                TX*100*X*(PX/(1+TX)) +
                TY*100*Y*(PY/(1+TY));
```

```
MODEL TWOxTWO /PRF_X.X, PRF_Y.Y, PRF_W.W,
                MKT_X.PX, MKT_Y.PY, MKT_L.PL, MKT_K.PK,
                MKT_W.PW, I_CONS.CONST /;
```

\* *Chose a numeraire: real consumer price index*

```
PW.FX = 1;
```

\* *Set initial values of variables:*

```
X.L=1; Y.L=1; W.L=1; PX.L=1; PY.L=1; PK.L=1; PL.L=1;
CONS.L=200;
```

```
SOLVE TWOxTWO USING MCP;
```

\* *counterfactual 1: 50% tax on X*

TX = 0.5;

**SOLVE** TWOxTWO USING MCP;

\* *counterfactual 1: 50% tax on X and Y*

TX = 0.5;

TY = 0.5;

**SOLVE** TWOxTWO USING MCP;

\* *counterfactual 2: zero taxes, double the labor endowment*

TX = 0;

TY = 0;

LENDOW = 2;

**SOLVE** TWOxTWO USING MCP;

\* *counterfactual 3: double both endowments from the benchmark*

LENDOW = 2;

KENDOW = 2;

**SOLVE** TWOxTWO USING MCP;



\$ontext

*We emphasize that the above formulation uses a simplifying trick: the marginal costs of X, Y, and W can be replaced by the producer prices  $PX/(1+TX)$ ,  $PY/(1+TY)$ , and  $PW$ . This should "always" work because when marginal cost and price are not equal in equilibrium, the quantity is zero: marginal cost times quantity = producer price times quantity ( $MC\_X * X = PX * X$  regardless of  $X > 0$  or  $X = 0$ ). But below is the model done "properly" with Shepard's lemma*

\$offtext

## EQUATIONS

MKT\_X2

MKT\_Y2

MKT\_L2

MKT\_K2

I\_CONS2;

$$\text{MKT\_X2..} \quad 100 * X = G = 100 * W * (PX^{**0.5} * PY^{**0.5}) / PX;$$

$$\text{MKT\_Y2..} \quad 100 * Y = G = 100 * W * (PX^{**0.5} * PY^{**0.5}) / PY;$$

$$\text{MKT\_L2..} \quad 100 * LENDOW = G = 25 * X * PL^{**0.25} * PK^{**0.75} / PL + \\ 75 * Y * PL^{**0.75} * PK^{**0.25} / PL;$$

```
MKT_K2..      100*KENDOW =G= 75 * X * PL**0.25 * PK**0.75 / PK +
                25 * Y * PL**0.75 * PK**0.25 / PK;
```

\* *Income balance equations (don't forget tax revenue)*

```
I_CONS2..     CONS =E= 100*LENDOW*PL + 100*KENDOW*PK +
                TX*100*X*(PL**0.25*PK**0.75) +
                TY*100*Y*(PL**0.75*PK**0.25);
```

```
MODEL TWOxTWOa /PRF_X.X, PRF_Y.Y, PRF_W.W,
                MKT_X2.PX, MKT_Y2.PY, MKT_L2.PL, MKT_K2.PK,
                MKT_W.PW, I_CONS2.CONST /;
```

```
TX = 0; TY = 0;
```

```
LENDOW = 1; KENDOW = 1;
```

```
SOLVE TWOxTWOa USING MCP;
```

```
TX = 0.5;
```

```
SOLVE TWOxTWOa USING MCP;
```

\$ontext

*Exercise: declare a parameter alpha, which is a productivity shift parameter producing X. Higher alpha, more output per input.*

*Code this up. Hint: alpha will appear more than in the program.*

*Change alpha and interpret results.*

\$offtext

The results for our first counterfactual, in which we place a 50% tax on the inputs to X production.

\*\*\*\*\* counterfactual: tax on X inputs

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	0.845	+INF	.
---- VAR Y	.	1.147	+INF	.
---- VAR W	.	0.985	+INF	.
---- VAR PX	.	1.165	+INF	.
---- VAR PY	.	0.859	+INF	.
---- VAR PW	1.000	1.000	1.000	-1.602E-12
---- VAR PL	.	0.903	+INF	.
---- VAR PK	.	0.739	+INF	.
---- VAR CONS	.	196.946	+INF	.

X production decreases, Y production increases, and welfare falls due to the distortionary nature of the tax, even though the tax revenue is redistributed back to the consumer.

There is also a redistribution of income between factors.

The relative price of capital, the factor used intensively in X falls, and the relative price of labor rises as resources are shifted to Y production.

### 3.5 Initially slack activities

As noted in chapter one, an attractive and powerful feature of GAMS is that it solves complementarity problems in which some production activities can be slack for some values of parameters and active for others.

This allows researchers to consider a much wider set of problems that is allowed using software which can only solve systems of equations.

There are many uses of this in practice:  
alternative energy technologies that are unprofitable initially  
illegal activities that are inefficient relative to legal ones  
trade links that are initially inactive due to high trade costs

Model 3-5 presents an example, motivated by tax evasion activities, or a “green” but expensive technology

A third sector, Z, also produces good X but it is 10% less efficient (10% more costly) than the X activity itself. So initially, Z does not operate.

But when a tax of 25% is imposed on X, this activity goes slack and Z begins to operate.

We could think of Z as a tax evasion or “informal” activity that is less efficient but can successfully avoid the tax.

Another counterfactual assesses the effect of a 25% tax when the Z technology cannot operate, to assess switching costs.

The command Z.FX fixes the level of Z at its current value.

Fixing a variable involves setting both its upper and lower value.

Thus to unfix a variable, both its upper and lower values must be reset.

```
Z.UP = +INF;  
Z.LO = 0;
```

Another thing introduced is a trick to find calibration errors

```
SLACK.ITERLIM = 0;  
SOLVE SLACK USING MCP;
```

```
SLACK.ITERLIM = 2000;  
SOLVE SLACK USING MCP;
```

Please see section 3.8 for an explanation of this command.



\$TITLE M3-5: use of an initially slack activity

\* e.g., modeling tax avoidance or use of "green" but expensive

\* technologies

\$ONTEXT

sector "Z" produces good X, but with a less efficient technology than activity X (sector X). High tax on X leads to switching Z could be interpreted as an "informal" or "illegal" technology such as smuggling. Or, Z is an expensive "green" technology

Markets	Production Sectors			Consumers
	X	Y	W	CONS
PX	100		-100	
PY		100	-100	
PW			200	-200
PL	-40	-60		100
PK	-60	-40		100

\$OFFTEXT

**PARAMETER**

TX Ad-valorem tax rate on X sector inputs

INEF Inefficiency measure in sector Z;

```
TX = 0;  
INEF = 1.1;
```

## NONNEGATIVE VARIABLES

X	Activity level for sector X
Y	Activity level for sector Y
Z	Activity level for sector Z initial inefficient
W	Activity level for sector W
PX	Price index for commodity X
PY	Price index for commodity Y
PL	Price index for primary factor L
PK	Price index for Primary factor K
PW	Price index for welfare (expenditure function)
CONS	Income definition for CONS;

## EQUATIONS

PRF_X	Zero profit for sector X
PRF_Y	Zero profit for sector Y
PRF_Z	Zero profit for sector Z
PRF_W	Zero profit for sector W
MKT_X	Supply-demand balance for commodity X
MKT_Y	Supply-demand balance for commodity Y
MKT_L	Supply-demand balance for primary factor L

MKT\_K     Supply-demand balance for primary factor L

MKT\_W     Supply-demand balance for aggregate demand

I\_CONS    Income definition for CONS;

\*         *Zero profit conditions:*

PRF\_X..   100\*(PL\*\*0.4 \* PK\*\*0.6) \* (1+TX) =G= 100\*PX;

PRF\_Y..   100\*(PL\*\*0.6 \* PK\*\*0.4) =G= 100\*PY;

PRF\_Z..   100\*INEF\*(PL\*\*(0.40) \* PK\*\*(0.60)) =G= 100\*PX;

PRF\_W..   200 \* PX\*\*0.5 \* PY\*\*0.5 =E= 200 \* PW;

\*         *Market clearing conditions:*

MKT\_X..   100\*X + 100\*Z =G= 100\*W\*PW/PX;

MKT\_Y..   100\*Y =G= 100\*W\*PW/PY;

MKT\_W..   200\*W =G= CONS/PW;

MKT\_L..   100 =G= 40\*X\*(PX/(1+TX))/PL + 60\*Y\*PY/PL + 40\*Z\*PX/PL;

MKT\_K..   100 =G= 60\*X\*(PX/(1+TX))/PK + 40\*Y\*PY/PK + 60\*Z\*PX/PK;

\* *Income constraints:*

I\_CONS..      CONS =E= 100\*PL + 100\*PK + TX\*100\*X\*(PX/(1+TX));

**MODEL** SLACK /PRF\_X.X, PRF\_Y.Y, PRF\_Z.Z, PRF\_W.W,  
                  MKT\_X.PX, MKT\_Y.PY, MKT\_L.PL, MKT\_K.PK,  
                  MKT\_W.PW, I\_CONS.CONST /;

\* *Check the benchmark:*

X.L            =1;

Y.L            =1;

W.L            =1;

Z.L            =0;

PL.L           =1;

PX.L           =1;

PY.L           =1;

PK.L           =1;

PW.FX          =1;

CONS.L        =200;

TX             =0;

*\* introducing a calibration check*

```
SLACK.ITERLIM = 0;  
SOLVE SLACK USING MCP;
```

*\* now allow the solver to work*

```
SLACK.ITERLIM = 2000;  
SOLVE SLACK USING MCP;
```

*\* the first counterfactual shows how a tax leads to switching  
\* to the inefficient "informal" technology*

```
TX = 0.25;  
SOLVE SLACK USING MCP;
```

*\* the second counterfactual shows the different in welfare when  
\* the inefficient technology cannot be used*

```
Z.FX = 0;
```

```
TX = 0.25;  
SOLVE SLACK USING MCP;
```

Z.UP = +INF;

Z.LO = 0;

*\* scenario generation: loop over values of the tax*

**SETS** I /I1\*I30/;

**PARAMETERS**

TAXRATE(I)

WELFARE(I)

ZPROD(I)

RESULTS(I, \*);

*\* scenario generation: loop of tax rates*

**LOOP**(I,

TX = 0.01\*ORD(I) - 0.01;

**SOLVE** SLACK USING MCP;

TAXRATE(I) = TX;

WELFARE(I) = W.L;

ZPROD(I) = Z.L;

);

RESULTS(I, "TAXRATE") = TAXRATE(I);

RESULTS(I, "WEFLARE") = WELFARE(I);

RESULTS(I, "ZPROD") = ZPROD(I);

**DISPLAY** TAXRATE, WELFARE, ZPROD, RESULTS;

\$LIBINCLUDE XLDUMP TAXRATE M3-5.XLS SHEET1!B3

\$LIBINCLUDE XLDUMP WELFARE M3-5.XLS SHEET1!B6

\$LIBINCLUDE XLDUMP RESULTS M3-5.XLS SHEET1!B9

## 3.6 Labor/leisure decision

Often general-equilibrium models used in international trade assume that factors of production, especially labor, are in fixed and inelastic supply.

But designing tax, welfare, and education systems, endogenizing labor supply is a crucial part of the story.

Model M3-6 endogenizes labor supply, allowing labor to choose between leisure and labor supply with leisure entering into the workers utility function.

In our formulation, we introduce an additional activity  $T$ , which transforms leisure (price  $PL$ ) into labor supplied (price  $PLS$ ).



The production block for  $W$  specifies that goods and leisure have an elasticity of substitution of 1. This may not be empirically plausible, but we want to stick with Cobb-Douglas for now.

The consumer is endowed with 200 units of labor/leisure, of which 100 units are supplied to the labor market initially.

The use of the labor supply activity is going to imply two separate prices in the presence of the labor tax.

$PL$  is the household (consumer) price of leisure and the take-home wage of labor supplied to the market.  $PLS$  is going to be the producer's cost of labor.

$$PLS = PL(1+TL)$$

Second, note the equations and notation at the end.

```
WELFARE = W.L;
```

```
REALCONS = (PX.L*X.L*100 + PY.L*Y.L*100)
            / (PX.L**0.5*PY.L**0.5*200);
```

```
DISPLAY WELFARE, REALCONS;
```

Parameters (declared earlier) are assigned values following the solution to the model. The first is just the value of welfare, including the value of leisure.

The second, “REALCONS” is short for the real value of goods consumption. The REAL part is due to dividing the nominal value of consumption by the price index for consumption.

As you will see if you run this model, the labor tax leads to a reduction in labor supply.

This of course leads to a fall in commodity consumption but also to a rise in leisure.

Thus in this case, the usual statistics overstate the burden of the tax and would overstate the benefit of removing the tax if labor supply increases (leisure falls).

\$TITLE M3-6.GMS: Introduces a labor supply or labor/leisure activity

\$ONTEXT

*Activity T transforms leisure into labor supply*

*Initially, the consumer has 200 units of leisure and supplies 100 to the market, retaining 100 as leisure*

*X, Y and Leisure are all Cobb-Douglas substitutes*

Markets	Production Sectors				Consumers	
	A	B	W	T	CONS	
PX	100		-100			
PY		100	-100			
PW			300			-300
PLS	-40	-60		100		
PL			-100	-100		200
PK	-60	-40				100

*PL will denote the CONSUMER (HOUSEHOLD) price of labor*

*PLS will denote the PRODUCER price or COST of labor to the firms*

\$OFFTEXT

## PARAMETERS

TL ad-valorem tax rate on labor

WELFARE true welfare - including the value of leisure  
REALCONS observed market value of consumption of X and Y;

TL = 0;

### POSITIVE VARIABLES

X Activity level for sector X  
Y Activity level for sector Y  
T Labor supply (transforms leisure to labor)  
W Activity level for sector W  
PX Price index for commodity X  
PY Price index for commodity Y  
PL Price index for primary factor L (household price)  
PLS Price index for labor supply (producer cost)  
PK Price index for primary factor K  
PW Price index for welfare (expenditure function)  
CONS Income definition for CONS;

### EQUATIONS

PRF\_X Zero profit for sector X  
PRF\_Y Zero profit for sector Y  
PRF\_T Zero profit for sector T  
PRF\_W Zero profit for sector W  
  
MKT\_X Supply-demand balance for commodity X  
MKT\_Y Supply-demand balance for commodity Y

MKT\_L     Supply-demand balance for primary factor L  
MKT\_LS    Supply-demand balance for Leisure  
MKT\_K     Supply-demand balance for primary factor K  
MKT\_W     Supply-demand balance for aggregate demand

I\_CONS    Income definition for CONS;

\*           *Zero profit conditions:*

PRF\_X..   100\*(PLS\*\*0.4 \* PK\*\*0.6) =G= 100\*PX;

PRF\_Y..   100\*(PLS\*\*0.6 \* PK\*\*0.4) =G= 100\*PY;

PRF\_T..   100\*(PL \*(1+TL)) =G= 100 \* PLS;

PRF\_W..   300\*(PY\*\*(1/3) \* PX\*\*(1/3)) \* PL\*\*(1/3)  
          =G= 300 \* PW;

\*           *Market clearing conditions:*

MKT\_X..   100\*X =G= 300\*W\*(1/3)\*PW /PX;

MKT\_Y..   100\*Y =G= 300\*W\*(1/3)\*PW /PY;

MKT\_W..   300\*W =G= CONS/PW;

MKT\_L.. 200 =G= 100\*T + 300\*W\*(1/3)\*PW/PL;

MKT\_LS.. 100\*T =G= 40\*X\*PX/PLS + 60\*Y\*PY/PLS;

MKT\_K.. 100 =G= 60\*X\*PX/PK + 40\*Y\*PY/PK;

\* *Income constraints:*

I\_CONS.. CONS =E= 200\*PL + 100\*PK + TL\*100\*T\*PL;

**MODEL** LABELS /PRF\_X.X, PRF\_Y.Y, PRF\_T.T, PRF\_W.W,  
MKT\_X.PX, MKT\_Y.PY, MKT\_L.PL,  
MKT\_LS.PLS, MKT\_K.PK, MKT\_W.PW, I\_CONS.CONS /;

X.L =1;

Y.L =1;

W.L =1;

T.L =1;

PL.L =1;

PLS.L =1;

PX.L =1;

PY.L =1;

PK.L =1;

PW.FX =1;

```
CONS.L = 300;
```

```
TL = 0;
```

```
LABLEIS.ITERLIM = 0;
```

```
SOLVE LABLEIS USING MCP;
```

```
LABLEIS.ITERLIM = 1000;
```

```
SOLVE LABLEIS USING MCP;
```

```
WELFARE = W.L;
```

```
REALCONS = (PX.L*X.L*100 + PY.L*Y.L*100)/  
            (PX.L**0.5*PY.L**0.5*200);
```

```
DISPLAY WELFARE, REALCONS;
```

```
TL = 0.5;
```

```
SOLVE LABLEIS USING MCP;
```

```
WELFARE = W.L;
```

```
REALCONS = (PX.L*X.L*100 + PY.L*Y.L*100)/  
            (PX.L**0.5*PY.L**0.5*200);
```

```
DISPLAY WELFARE, REALCONS;
```



### 3.7 Two households with different preferences and endowments

Questions of interest to trade and public finance economists involve issues of distribution rather than or in addition to issues of aggregate welfare.

Households may differ in their preferences and more importantly in their sources of income (or their factor endowments).

Adding multiple household types is a straightforward extension of our earlier models. M3-7 assumes two households.

Household A is relatively well endowed with labor, and also has a preference for good Y, which is the labor-intensive good.

Household B is relatively well endowed with capital and has a relative preference for the capital intensive good.

Our counterfactual experiment is to place a tax on the factor inputs to X, assigning half the revenue to each consumer. This lowers the welfare of household B.

However, household A is better off.

This welfare gain for A is a combination of

- (a) a lowering of the relative consumer price of Y, the good favored by A
- (b) an increase in the real return to labor, due to the shift toward the labor-intensive sector (Stolper-Samuelson)
- (c) neutral redistribution of tax revenue.

\$TITLE M3-7.GMS: two households with different preferences  
 \* and different endowments

\$ONTEXT

*Two household: differ in preferences and in endowments*

*Household A: well endowed with labor,  
 preference for labor-int good Y*

*Household B: well endowed with capital,  
 preference for capital-int good X*

*Allows for tax to be redistributed unevenly between households*

Markets	Production Sectors				Consumers	
	X	Y	WA	WB	A	B
PX	100		-40	-60		
PY		100	-60	-40		
PWA			100		-100	
PWB				100		-100
PL	-25	-75			90	10
PK	-75	-25			10	90

*The tax redistribution or sharing rule can also be interpreted as the relative number of households in each group, with all households getting an equal share of tax receipts*

\$OFFTEXT

**PARAMETERS**

TX Ad-valorem tax rate for X sector inputs  
SHA Share of tax revenue given back to household A  
SHB share of tax revenue given back to household B;

TX = 0;  
SHA = 0.5;  
SHB = 0.5;

**NONNEGATIVE VARIABLES**

X Activity level for sector X  
Y Activity level for sector Y  
WA Activity level for sector WA  
WB Activity level for sector WB  
PX Price index for commodity X  
PY Price index for commodity Y  
PK Price index for primary factor K  
PL Price index for primary factor L  
PWA Price index for welfare A(expenditure function)  
PWB Price index for welfare B(expenditure function)

CONSA Income definition for CONSA  
CONSB Income definition for CONSB;

**EQUATIONS**

PRF\_X Zero profit for sector X  
 PRF\_Y Zero profit for sector Y  
 PRF\_WA Zero profit for sector WA  
 PRF\_WB Zero profit for sector WB

MKT\_X Supply-demand balance for commodity X  
 MKT\_Y Supply-demand balance for commodity Y  
 MKT\_L Supply-demand balance for primary factor L  
 MKT\_K Supply-demand balance for primary factor K  
 MKT\_WA Supply-demand balance for consumer A  
 MKT\_WB Supply-demand balance for consumer B

I\_CONSA Income definition for CONSA  
 I\_CONSB Income definition for CONSB;

\* *Zero profit conditions:*

PRF\_X..  $100 * (PL^{**0.25} * PK^{**0.75}) * (1+TX) =G= 100 * PX;$

PRF\_Y..  $100 * (PL^{**0.75} * PK^{**0.25}) =G= 100 * PY;$

PRF\_WA..  $100 * (PX^{**0.4} * PY^{**0.6}) =G= 100 * PWA;$

PRF\_WB..  $100 * (PX^{**0.6} * PY^{**0.4}) =G= 100 * PWB;$

\* *Market clearing conditions:*

$$\text{MKT\_X..} \quad 100 \cdot X = G = 40 \cdot \text{WA} \cdot \text{PWA} / \text{PX} + 60 \cdot \text{WB} \cdot \text{PWB} / \text{PX};$$

$$\text{MKT\_Y..} \quad 100 \cdot Y = G = 60 \cdot \text{WA} \cdot \text{PWA} / \text{PY} + 40 \cdot \text{WB} \cdot \text{PWB} / \text{PY};$$

$$\text{MKT\_WA..} \quad 100 \cdot \text{WA} = G = \text{CONSA} / \text{PWA};$$

$$\text{MKT\_WB..} \quad 100 \cdot \text{WB} = G = \text{CONSB} / \text{PWB};$$

$$\text{MKT\_L..} \quad 90 + 10 = G = 25 \cdot X \cdot (\text{PX} / (1 + \text{TX})) / \text{PL} + 75 \cdot Y \cdot \text{PY} / \text{PL};$$

$$\text{MKT\_K..} \quad 10 + 90 = G = 75 \cdot X \cdot (\text{PX} / (1 + \text{TX})) / \text{PK} + 25 \cdot Y \cdot \text{PY} / \text{PK};$$

\* *Income constraints:*

$$\text{I\_CONSA..} \quad \text{CONSA} = E = 90 \cdot \text{PL} + 10 \cdot \text{PK} + \text{SHA} \cdot \text{TX} \cdot 100 \cdot X \cdot \text{PX} / (1 + \text{TX});$$

$$\text{I\_CONSB..} \quad \text{CONSB} = E = 10 \cdot \text{PL} + 90 \cdot \text{PK} + \text{SHB} \cdot \text{TX} \cdot 100 \cdot X \cdot \text{PX} / (1 + \text{TX});$$

**MODEL** TWOHOUSE /PRF\_X.X, PRF\_Y.Y, PRF\_WA.WA, PRF\_WB.WB,  
MKT\_X.PX, MKT\_Y.PY, MKT\_L.PL,  
MKT\_K.PK, MKT\_WA.PWA, MKT\_WB.PWB,  
I\_CONSA.CONSA, I\_CONSB.CONSB /;

\* *Check the benchmark:*

X.L =1;

Y.L =1;

WA.L =1;

WB.L =1;

PL.FX =1;

PX.L =1;

PY.L =1;

PK.L =1;

PWA.L =1;

PWB.L =1;

CONSA.L =100;

CONSB.L =100;

TX =0;

TWOHOUSE.ITERLIM = 0;

**SOLVE** TWOHOUSE USING MCP;

TWOHOUSE.ITERLIM = 1000;

**SOLVE** TWOHOUSE USING MCP;

TX = 0.50;

**SOLVE** TWOHOUSE USING MCP;

\* *counterfactual: give a larger share of tax to household B*  
\* *illustrates impossibility of making both types better off*

TX = 0.50;  
SHA = 0.25;  
SHB = 0.75;

**SOLVE** TWOHOUSE USING MCP;

\* *suppose that 80% of households are type A, 20% type B*  
\* *and all individual households get an equal share*  
\* *of tax receipts*

TX = 0.50;  
SHA = 0.80;  
SHB = 0.20;

**SOLVE** TWOHOUSE USING MCP;



### 3.8 Cobb-Douglas functions, calibration and calibration errors

Let good  $X$  be produced from inputs of labor  $L$  and capital  $K$ .

Let  $X^0$ ,  $L^0$ ,  $K^0$ ,  $p_X^0$ ,  $p_L^0$ ,  $p_K^0$  denote the initial values of quantities and prices observed in the data. Assume that these are micro-consistent, meaning that they satisfy adding up:

$$p^0 X^0 = p_L^0 L^0 + p_K^0 K^0 \quad (1)$$

The generic Cobb-Douglas production function is:

$$X = A L^\alpha K^{1-\alpha} \quad (2)$$

Calibration involves choose  $\alpha$  and  $A$  such that the production function (1) is consistent with the initial (benchmark) data.

Assertion 1:  $\alpha$  and  $(1 - \alpha)$  will always be the value shares of the inputs in the value of output, that is:

$$\alpha = \frac{p_L L}{p_X X} \quad 1 - \alpha = \frac{p_K K}{p_X X} \quad (3)$$

so the initial data will allow us to calculate  $\alpha$ , We need to find A.

Assertion 2: Let all prices be one initially. Then the correct calibration of the Cobb-Douglas function in this special case is:

$$X = \left( \frac{L}{\alpha} \right)^\alpha \left( \frac{K}{1 - \alpha} \right)^{1 - \alpha} \quad \text{in (2), } A = \alpha^{-\alpha} (1 - \alpha)^{\alpha - 1} \quad (4)$$

Check that this equation is satisfied if  $X = 100$ ,  $L = 75$  and  $K = 25$  with all prices initially equal to one, and  $\alpha = 0.75$ .

Assertion 3: the unit cost function corresponding to this is:

$$c(p_L, p_K) = p_L^\alpha p_K^{1-\alpha} \quad \text{where } c(p_L, p_K) = p_X \text{ in equilibrium} \quad (5)$$

Note that this equation is satisfied when all prices equal one and marginal (unit) cost equals the price of  $X$ , which equals one.

Let  $l_x$  and  $k_x$  be the amounts of  $L$  and  $K$  that are optimal to produce one unit of  $X$ . Shepard's lemma gives the optimal inputs:

$$l_x = \frac{\partial c(p_l, p_k)}{\partial p_l} = \alpha p_l^{\alpha-1} p_k^{1-\alpha} = \alpha p_l^\alpha p_k^{1-\alpha} / p_l \quad (6)$$

$$l_x = \alpha c(p_l, p_k) / p_l = \alpha p_x / p_l \quad \text{with total factor demands} \quad (7)$$

$$L_x = \alpha (p_x / p_l) X \quad K_x = (1 - \alpha) (p_x / p_k) X \quad (8)$$

Check that this gives the correct factor demands for the above numerical example when all prices are one initially.

What if the prices of  $L$  and  $K$  are not equal to one initially? Suppose that the values 75 and 25 for labor and capital and 100 for  $X$  are broken down into price and quantity as:

$$p_L^0 = 1.5, \quad L_X^0 = 50, \quad p_K^0 = 0.5, \quad K_X^0 = 50 \quad p_X^0 = 2, \quad X^0 = 50$$

The simple form of the production and cost functions are not correct when prices do not equal one, but the general form is not too different.

Let  $X^0, L^0, K^0, p_X^0, p_L^0, p_K^0$  denote the initial values of quantities and prices observed in the data.

Assertion 4: Then the correctly calibrated production, cost, and factor-demand functions for the Cobb-Douglas are:

$$X = A \left( \frac{L}{\alpha} \right)^\alpha \left( \frac{K}{1-\alpha} \right)^{1-\alpha} \quad A = (p_L^0)^\alpha (p_K^0)^{1-\alpha} / p_X^0 \quad \alpha = \frac{p_L^0 L^0}{p_X^0 X^0} \quad (9)$$

$$c(p_L, p_K) = p_X^0 (p_L/p_L^0)^\alpha (p_K/p_K^0)^{1-\alpha} \quad (= p_L^\alpha p_K^{1-\alpha} / A) \quad (10)$$

$$L_x = \alpha (p_X/p_L) X \quad K_x = (1-\alpha) (p_X/p_K) X \quad (11)$$

Note that (11) is the same as (8), hence they don't depend on any assumption about the initial values of prices.

Check that this is valid for the numbers given above.

Go through this in more detail using (1):

First, of all for assertion 1. The profits for a firm can be written as:

$$\Pi = p_X A L^\alpha K^{1-\alpha} - p_L L - p_K K \quad (12)$$

The first-order condition for profit maximization is:

$$\frac{\partial \Pi}{\partial L} = \alpha p_X A L^{\alpha-1} K^{1-\alpha} - p_L = 0 \quad (13)$$

$$= \alpha p_X [A L^\alpha K^{1-\alpha}] L^{-1} - p_L = \alpha p_X X/L - p_L = 0$$

$$\alpha = \frac{p_L L}{p_X X} \quad \text{Note this is independent of } A, \text{ so valid for (4), (9)}$$

Assertion 5: For initial, micro-consistent values

$X^0, L^0, K^0, p_X^0, p_L^0, p_K^0$ , the correct production function can be written as:

$$X = X^0 \left( \frac{L}{L^0} \right)^\alpha \left( \frac{K}{K^0} \right)^{1-\alpha} \quad \text{where } \alpha = \frac{p_L^0 L^0}{p_X^0 X^0} \quad (14)$$

Check that this is satisfied for the initial values just mentioned.

Some multiplying and dividing will give us:

$$X = \left( \frac{L}{L^0/X^0} \right)^\alpha \left( \frac{K}{K^0/X^0} \right)^{1-\alpha} \quad (15)$$

$$X = \left[ (p_L^0)^\alpha (p_K^0)^{1-\alpha} / p_X^0 \right] \left( \frac{L}{p_L^0 L^0 / (p_X^0 X^0)} \right)^\alpha \left( \frac{K}{p_K^0 K^0 / (p_X^0 X^0)} \right)^{1-\alpha} \quad (16)$$

$$X = A \left( \frac{L}{\alpha} \right)^\alpha \left( \frac{K}{1-\alpha} \right)^{1-\alpha} \quad A = (p_L^0)^\alpha (p_K^0)^{1-\alpha} / p_X^0 \quad (17)$$

which is (9). Now use our earlier result on the value of  $\alpha$  to get the optimal amounts of L and K to produce one unit of X:  $l_x$  and  $k_x$

$$\alpha = \frac{p_L L}{p_X X} \Rightarrow l_x = \alpha p_X / p_L \quad k_x = (1-\alpha) p_X / p_K \quad (18)$$

Setting  $X = 1$  in (17), this equation is:

$$1 = A \left( \frac{l_x}{\alpha} \right)^\alpha \left( \frac{k_x}{1-\alpha} \right)^{1-\alpha} \quad (19)$$

Now substitute (18) into (19).



$$1 = A \left( \frac{\alpha p_X / p_L}{\alpha} \right)^\alpha \left( \frac{(1 - \alpha) p_X / p_K}{1 - \alpha} \right)^{1 - \alpha} \quad (20)$$

$$1 = A p_X p_L^{-\alpha} p_K^{\alpha - 1} = A c(p_L, p_k) p_L^{-\alpha} p_K^{\alpha - 1} \quad \text{since } p_x = c(p_L, p_K) \quad (21)$$

Now invert (21) and substitute in for A from (17) to get the cost function

$$c(p_L, p_k) = p_X^0 (p_L / p_L^0)^\alpha (p_K / p_K^0)^{1 - \alpha} \quad (22)$$

The application of Shepard's lemma to (22) gives factor demands

$$\begin{aligned} \frac{\partial c(p_L, p_k)}{\partial p_L} &= \alpha p_X^0 (p_L / p_L^0)^{\alpha - 1} (p_K / p_K^0)^{1 - \alpha} / p_L^0 \\ &= \alpha c(p_L, p_k) (p_L^0 / p_L) / p_L^0 = \alpha c(p_L, p_k) / p_L \end{aligned} \quad (23)$$

$$l_x = \frac{\partial c(p_L, p_K)}{\partial p_L} = \alpha p_X / p_L \quad (24)$$

So total factor demands are

$$L_x = \alpha (p_X / p_L) X \quad K_x = (1 - \alpha) (p_X / p_K) X \quad (25)$$

Use the example quantities and prices given above to show that this gives the correct figures

## Identifying and correcting calibration errors.

A calibration error occurs when parameters are set at incorrect values such that the model does not reproduce the benchmark data when run.

Run model M3-4a setting `LENDOW = 2` initially (which is the first counterfactual).

The problem is that you will have no idea (in general) where the mistake occurs.

Solution is to (at first) not allow the model to solve, returning the initial imbalances in the model: `syntax <MODELNAME>.ITERLIM = 0;`

```
TWOxTWO.ITERLIM = 0;  
SOLVE TWOxTWOa USING MCP;
```

This will return the solution:

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	2.000	+INF	.
---- VAR Y	.	2.000	+INF	.
---- VAR W	.	2.000	+INF	.
---- VAR PX	.	1.000	+INF	.
---- VAR PY	.	1.000	+INF	.
---- VAR PW	1.000	1.000	1.000	3.3316E-9
---- VAR PL	.	1.000	+INF	.
---- VAR PK	.	1.000	+INF	-100.000
---- VAR CONS	.	400.000	+INF	100.000

This gives you a big hint as to where the error is located:

There is an error in the equationS MKT\_L and in I\_CONS (after finding the error return iterlim to = 1000;)

However, this can create a problem if there is not actually an error in setting parameters, but in setting the starting values.

Try running M3-4a with  $PX.L = 2$ ; Here is what you will get:

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR X	.	2.000	+INF	-100.000
---- VAR Y	.	2.000	+INF	.
---- VAR W	.	2.000	+INF	82.843
---- VAR PX	.	2.000	+INF	100.000
---- VAR PY	.	1.000	+INF	.
---- VAR PW	1.000	1.000	1.000	3.3316E-9
---- VAR PL	.	1.000	+INF	-150.000
---- VAR PK	.	1.000	+INF	-250.000
---- VAR CONS	.	400.000	+INF	200.000

This is a big mess. Always start with trying to find the error in the pricing equation, in this case for X: equation PRF\_X.

But before putting in too much sweat, simply run the model a second time with ITERLIM set at 1000:

if this reproduces the benchmark, then the model is correctly calibrated and there is a starting value error.

For the first two solve statements, use:

```
TWOxTWO.ITERLIM = 0;  
SOLVE TWOxTWOa USING MCP;
```

```
TWOxTWO.ITERLIM = 1000;  
SOLVE TWOxTWOa USING MCP;
```

A true calibration error means non-zero marginals in the first, and the second does not reproduce the benchmark.

A starting value error reproduces the benchmark in the second solve