

Chapter 5

Monopoly, Oligopoly and Increasing Returns

Now we turn to models involving imperfect competition and increasing returns to scale. These topics have attracted considerable attention in both theoretical and empirical analyses of international trade and public economics over the last two decades. Imperfect competition models, at least general-equilibrium models used in international trade and in public economics often fall into one of two cases. First, there are oligopoly models with small numbers of firms strategically interacting with one another. These models often assume that an industry produces a homogeneous good. Second, there are monopolistic-competition models with large numbers of firms producing differentiated goods. Individual firms are small relative to the market, which results in greatly simplified marginal revenue functions or market rules.

Similarly, there are several classes of scale economies. An older literature in international trade assumes that scale economies are external to individual firms but internal to industries. This has some appeal empirically, but has some significant analytical advantages in general-equilibrium models in that competitive pricing rules can be used. A newer literature assumes that scale economies are internal to individual firms so that imperfect competition is an inevitable part of the equilibrium analysis.

In this chapter we will consider monopoly/oligopoly models where markups are endogenous and the industry output is a homogeneous good. In chapter 6, we will consider external economies and monopolistic-competition models, which have more in common with one another than many researchers appreciate.

Because these models have features rather different from the more standard competitive general-equilibrium models, I will present both MCP and MPS/GE versions of each model so that the reader can see exactly what is solved for in the MPS/GE versions. The models in this chapter are as follows:

M51-MCP.GMS	Closed economy model with monopoly in the X sector, MCP version
M51-MPS.GMS	Closed economy model with monopoly in the X sector, MPS/GE version
M52-MCP.GMS	Closed economy model with monopoly and increasing returns in the X sector, MCP version.
M52-MPS.GMS	Closed economy model with monopoly and increasing returns in the X sector, MPS/GE version.

M53-MCP.GMS	Closed economy model with monopoly, increasing returns and free entry/exit in the X sector, MCP version
M53-MPS.GMS	Closed economy model with monopoly, increasing returns and free entry/exit in the X sector, MPS/GE version
M54-MCP.GMS	Two country trade model with monopoly, increasing returns and free entry/exit in the X sector, MCP version
M54-MPS.GMS	Two country trade model with monopoly, increasing returns and free entry/exit in the X sector, MPS/GE version

Model 51-MCP Closed economy model with monopoly in the X sector, MCP version

This is a standard two-good, two-factor, closed-economy general-equilibrium model that is very similar to those used in earlier chapters. Indeed, we start with a data matrix that is very similar to those used in earlier chapters. Activities are X, Y and W (welfare or utility). Factors of production in this and in the next chapter are called unskilled and skilled labor. Unskilled labor is typically called L with a price of PW or just W, and skilled labor is called S with a price of PZ or just Z. PU is the price of a unit of utility (the value of the unit expenditure function).

The monopoly or oligopoly markup will be denoted MK or MARKUP. There are two agents, the representative consumer who receives all the factor incomes and tax revenue (if any) and pays subsidies (if any). Then there may be an agent called ENTRE who receives markup revenue and pays fixed costs (if any).

Here is the data matrix for our first monopoly model.

Markets	Production Sectors			Consumers	
	X	Y	W	CONS	ENTRE
PX	100		-100		
PY		100	-100		
PU			200	-180	-20
PW	-32	-60		92	
PZ	-48	-40		88	
MK	-20				20

We assume that 20% of the value of the X output accrues to the agent ENTRE as a monopoly profit. For reasons connected with theory, we specify the markup as a deduction from the consumer price, not as an addition to marginal cost. Denoting marginal cost by mc , the markup is given by the formula

$$p_x(1 - mk) = mc \quad 100(1 - mk) = 80 \Rightarrow mk = 0.20$$

Written in this form, the markup implied by the data matrix is 20%.

It is desirable that the markup have some basis in theory. Therefore, I will take a short digression into economic theory to derive a marginal revenue function and see how a monopoly markup relates to the underlying elasticity of substitution in preferences and demand.

Suppose demand for good X is just written in inverse form $p(X)$ so the monopolist's revenue is $R = p(X)X$. Marginal revenue is then given by:

$$(1) \quad \frac{\partial R}{\partial X} = p + X \frac{\partial p}{\partial X} = p + p \left[\frac{X}{p} \frac{\partial p}{\partial X} \right] = p \left[1 - \frac{1}{\eta} \right] \equiv MR$$

$$MR = p(1 - mk) \quad mk = \frac{1}{\eta} \quad \eta \equiv - \left[\frac{p}{X} \frac{\partial X}{\partial p} \right]$$

where η is the Marshallian elasticity of demand, defined as positive. The monopoly markup is just the inverse of this elasticity. This looks good, but what is η in general equilibrium?

Suppose now that there are two goods, X_1 and X_2 , and consumer income is given by M . I will assume a symmetric CES utility function and assure the reader that the formula I derive is applicable to a more general case with different weights on the goods. The utility function is given as follows, where σ is the elasticity of substitution between goods. For a monopoly equilibrium to exist, we must have $\sigma > 1$, which in turn imposes the restriction that $1 > \alpha > 0$.

$$(2) \quad U = \left[X_1^\alpha + X_2^\alpha \right]^{\frac{1}{\alpha}}$$

Maximizing utility subject to the usual linear budget constraint, yields demand functions:

$$(3) \quad X_i = p_i^{-\sigma} \left[\sum p_j^{1-\sigma} \right]^{-1} M \quad \sigma = \frac{1}{1 - \alpha}, \quad \alpha = \frac{\sigma - 1}{\sigma}$$

Taking the own-derivative, we have:

$$(4) \quad \frac{\partial X_i}{\partial p_i} = -\sigma p_i^{-\sigma-1} \left[\sum p_j^{1-\sigma} \right]^{-1} M + p_i^{-\sigma} \left[\sum p_j^{1-\sigma} \right]^{-2} (\sigma - 1) p_i^{-\sigma} M$$

Forming the elasticity, we get:

$$(5) \quad \frac{p_i}{X_i} \frac{\partial X_i}{\partial p_i} = -\sigma + (\sigma - 1) p_i^{1-\sigma} \left[\sum p_j^{1-\sigma} \right]^{-1} \quad s_i \equiv \frac{p_i X_i}{M} = p_i^{1-\sigma} \left[\sum p_j^{1-\sigma} \right]^{-1}$$

The second equation gives the share of income spent on good i , s_i , by multiplying through (3) by p_i and dividing both sides by M . We see that this share appears in the second term of the first

equation. Thus the elasticity of demand for X_i can be written as:

$$(6) \quad \eta_i \equiv -\frac{p_i}{X_i} \frac{\partial X_i}{\partial p_i} = \sigma - (\sigma - 1)s_i \quad mk = [\sigma - (\sigma - 1)s_i]^{-1}$$

Note that Cobb-Douglas, $\sigma = 1$, is a special case in which the Marshallian elasticity is also equal to 1.

Now we are able to calibrate our data to an underlying theory. That is, we assume that this data is generated by a standard monopoly pricing model and that preferences are CES. The data show that the share of expenditure on X is 0.50, and the markup is 0.20, so $\eta = 5$. Solving for the implied elasticity of substitution in preferences, we get $\sigma = 9$. We will use this value in the data to follow.

The final issue is also one of calibration. If we let the marginal cost (or producer price) of $X = 1$, then the markup implies that the consumer price is $p_x = 1.25$. Suppose that we wish to chose units so that all other prices are equal to one, a convention that we like to adopt. The cost function for utility (utility price = p_u) which is dual to the utility function given above is given by:

$$(7) \quad p_u = \left[p_x^{1-\sigma} + p_y^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

In our case, the price of good X is 1.25, so we can divide p_x in this cost function to calibrated initially so that both additive terms in the function equal 1. But then the term in square brackets equals 2, so if we want $p_u = 1$ initially we have to compensate with a multiplicative constant on the front of the function. Denote this term as A. The calibrated cost function for utility is then:

$$A * \left((P_x/1.25) ** (1-SIGMA) + P_y ** (1-SIGMA) \right) ** (1 / (1-SIGMA)) = G = P_u;$$

$$A = 0.5 ** (1 / (1-SIGMA))$$

This is satisfied with equality at $p_x = 1.25$, $p_y = 1$, and $p_u = 1$. The parameter A will then also appear in the demand functions for X and Y, which are derived from Shepard's lemma. The demand for X, for example, is the derivative of (7) with respect to p_x , times the level of welfare. Initial X output is X^*80 (i.e., the activity level X is set at one, where $X = 1$ generates 80 units of output) and initial welfare is similarly W^*200 . Thus the demand for X is given by;

$$X^{*80} = E = A * (PX/1.25)^{**(-SIGMA)} * ((PX/1.25)^{*(1-SIGMA)} + PY^{*(1-SIGMA)})^{*(SIGMA/(1-SIGMA))} * W^{*200/1.25};$$

Now you might see why I like MPS/GE (and this is a pretty simple case). The latter will automatically calculate scaling coefficients like A from the data provided in a production block, and then automatically apply Shepard's lemma to get commodity and factor demands.

The rest of the program should be fairly transparent at this point. Two equations are added to calculate the share of expenditure on X (SHX) and the markup (MK). The agent ENTRE receives income from markups and demands utility just like the representative agent. After the model solves, we calculate how total income is divided between the representative factor owner (INCOME) and the monopolist (INCOMEM).

The counter-factual experiment is to impose marginal-cost pricing, which we can do by simply fixing the markup at zero with the statement MARKUP.FX = 0. Not only does welfare increase, but factor owners have a much bigger proportional increase since there is also a redistribution from monopoly profits to the factor owners. Overall welfare rises by 4% following the imposition of marginal-cost pricing, so this is a measure of deadweight loss from monopoly. But consumer (factor owner) welfare rises by 15.6% (0.90 to 1.04) which is much more significant.

\$TITLE Model M51-MCP.GMS: Closed 2x2 Economy, monopoly X producer
 * MCP version

\$ONTEXT

Markets	Production Sectors			Consumers	
	X	Y	W	CONS	ENTRE
PX	100		-100		
PY		100	-100		
PU			200	-180	-20
PW	-32	-60		92	
PZ	-48	-40		88	
MK	-20				20

\$Offtext

PARAMETERS

A Scale parameter for utility function,

SIGMA Elasticity of substitution,

INCOMEM Monopoly profit (in welfare units),

INCOME factor owners' income,

ENDOWS

ENDOWL

MODELSTAT;

SIGMA = 9;

ENDOWS = 88;

ENDOWL = 92;

A = 0.5**(1/(1-SIGMA));

POSITIVE VARIABLES

X

Y

W

PX

PY

PU

PZ

PW

CONS

ENTRE

SHAREX

MARKUP;

EQUATIONS

DX Demand for X
 DY Demand for Y
 DW Demand for W
 PRICEX MR = MC in X
 PRICEY Zero profit condition for Y (PY = MC)
 PRICEW Zero profit condition for W
 SKLAB Supply-demand balance for skilled labor
 UNLAB Supply-demand balance for unskilled labor
 ICONS Consumer (factor owners') income
 IENTRE Entrepreneur's profits
 SHX Share of X in expenditure
 MK Markup equation;

$$\text{PRICEX.. } (PW^{**0.40}) * (PZ^{**0.60}) =G= PX * (1 - \text{MARKUP});$$

$$\text{PRICEY.. } (PW^{**0.60}) * (PZ^{**0.40}) =G= PY;$$

$$\text{PRICEW.. } A * ((PX/1.25)^{(1-\text{SIGMA})} + PY^{(1-\text{SIGMA})})^{1/(1-\text{SIGMA})} \\ =G= PU;$$

$$\text{DX.. } X * 80 =E= A * (PX/1.25)^{-\text{SIGMA}} * \\ ((PX/1.25)^{(1-\text{SIGMA})} + \\ PY^{(1-\text{SIGMA})})^{(\text{SIGMA}/(1-\text{SIGMA}))} \\ * W * 200 / 1.25;$$

$$\text{DY.. } Y * 100 =E= A * PY^{-\text{SIGMA}} * \\ ((PX/1.25)^{(1-\text{SIGMA})} + \\ PY^{(1-\text{SIGMA})})^{(\text{SIGMA}/(1-\text{SIGMA}))} \\ * W * 200;$$

$$\text{DW.. } W * 200 =E= (\text{CONS} + \text{ENTRE}) / PU;$$

$$\text{SKLAB.. } \text{ENDOWS} =E= 0.40 * (PW^{**0.60}) * (PZ^{**0.40-1}) * Y * 100 \\ + 0.60 * (PW^{**0.40}) * (PZ^{**0.60-1}) * X * 80;$$

$$\text{UNLAB.. } \text{ENDOWL} =E= 0.60 * (PW^{**0.60-1}) * (PZ^{**0.40}) * Y * 100 \\ + 0.40 * (PW^{**0.40-1}) * (PZ^{**0.60}) * X * 80;$$

$$\text{ICONS.. } \text{CONS} =E= PZ * \text{ENDOWS} + PW * \text{ENDOWL};$$

$$\text{IENTRE.. } \text{ENTRE} =E= \text{MARKUP} * PX * X * 80;$$

$$\text{SHX.. } \text{SHAREX} =E= 80 * PX * X / (80 * PX * X + 100 * PY * Y) ;$$

$$\text{MK.. } \text{MARKUP} =E= 1 / (\text{SIGMA} - (\text{SIGMA}-1) * \text{SHAREX});$$


```
MODEL M51 /DX.PX, DY.PY, DW.PU, PRICEX.X, PRICEY.Y, PRICEW.W,
          SKLAB.PZ, UNLAB.PW, ICONS.CON, IENTRE.ENTRE
          SHX.SHAREX, MK.MARKUP/;
```

```
OPTION MCP=MILES;
OPTION LIMROW=0;
OPTION LIMCOL=0;
$OFFSYMLIST OFFSYMREF OFFFUELLIST OFFFUELXREF
```

```
CONS.L = 200;
X.L = 1;
Y.L = 1;
W.L = 1;
PX.L = 1.25;
PY.L = 1;
PZ.L = 1;
PW.L = 1;
PU.L = 1;
CONS.L = 180;
ENTRE.L = 20;
SHAREX.L = 0.5;
MARKUP.L = 0.20;
```

```
PY.FX = 1;
```

```
*M51.ITERLIM = 0;
SOLVE M51 USING MCP;
MODELSTAT = M51.MODELSTAT - 1.;
```

```
INCOMEM = W.L*(ENTRE.L/(ENTRE.L + CONS.L));
INCOMECE = W.L*(CONS.L/(ENTRE.L + CONS.L));
```

```
DISPLAY INCOMEM, INCOMECE;
```

```
*      Evaluate the potential gains from first-best (marginal
*      cost) pricing:
```

```
MARKUP.FX = 0;
```

```
SOLVE M51 USING MCP;
```

```
INCOMEM = W.L*(ENTRE.L/(ENTRE.L + CONS.L));
INCOMECE = W.L*(CONS.L/(ENTRE.L + CONS.L));
```

```
DISPLAY INCOMEM, INCOMECE;
```

Model 51-MPS Closed economy model with monopoly in the X sector, MPS/GE version

This model is identical to the previous model, except that this one is coded in MPS/GE. There is not a lot that needs to be said here. The basic simplifying trick is to use the “N” field in a production block to denote an endogenous “tax rate”, which is in this case the markup. The revenue from this tax is assigned to the agent ENTRE as monopoly profits. The endogenous tax rate is an auxiliary variable which is then set in a constraint equation. The production block is given by:

```
$PROD:X  s:1
          O:PX    Q: 80    A:ENTRE  N:MARKUP
          I:PW    Q: 32
          I:PZ    Q: 48
```

Note that if we use the formula derived above, we want the tax set on the output so that we indeed have $PX(1 - \text{MARKUP}) = MC$. If the tax was set on the two input fields, we would have $PX = (1 + \text{MARKUP})MC$. The same markup does not yield the same relationship between price and marginal cost in the two cases since the tax base is different in the two situations. So be careful to specify the production block carefully.

Second, remember to include the price field in the production block for welfare in order for MPS/GE to correctly calibrate the function. This is given by:

```
$PROD:W  s:SIGMA
          O:PU    Q:200
          I:PX    Q:80  P:1.25
          I:PY    Q:100
```

It should be clear that MPS/GE is going to do a lot of work for you, but you must give it the correct information to get a correct calibration.

The endogenous markup is set using two auxiliary variables and two constraint equations.

```
$CONSTRAINT:SHAREX
          SHAREX =E= 80*PX*X / (80*PX*X + 100*PY*Y) ;

$CONSTRAINT:MARKUP
          MARKUP =E= 1/(SIGMA - (SIGMA-1)*SHAREX) ;
```

The rest of the coding should be clear at this point.

\$TITLE Model M51-MPS.GMS: Closed 2x2 Economy, monopoly X producer
 * MPS/GE version

\$ONTEXT

Markets	Production Sectors			Consumers	
	X	Y	W	CONS	ENTRE
PX	100		-100		
PY		100	-100		
PU			200	-180	-20
PW	-32	-60		92	
PZ	-48	-40		88	
MK	-20				20

\$offtext

SCALAR SIGMA Elasticity of substitution,
 INCOMEM Monopoly profit (in welfare units),
 INCOME C factor owners' income;

SIGMA = 9;

\$ONTEXT

\$MODEL:M51

\$SECTORS:

X ! Activity level for sector X
 Y ! Activity level for sector Y
 W ! Activity level for sector W (Hicksian welfare index)

\$COMMODITIES:

PX ! Price index for commodity X
 PY ! Price index for commodity Y
 PW ! Price index for primary factor L (net of tax)
 PZ ! Price index for primary factor K
 PU ! Price index for welfare (expenditure function)

\$CONSUMERS:

CONS ! Representative agent.
 ENTRE ! Entrepreneur (monopolist)

\$AUXILIARY:

SHAREX ! Value share of good X
 MARKUP ! X sector markup on marginal cost

```

$PROD:X  s:1
          O:PX    Q: 80    A:ENTRE  N:MARKUP
          I:PW    Q: 32
          I:PZ    Q: 48

$PROD:Y  s:1
          O:PY    Q:100
          I:PW    Q:60
          I:PZ    Q:40

$PROD:W  s:SIGMA
          O:PU    Q:200
          I:PX    Q:80    P:1.25
          I:PY    Q:100

$DEMAND:CONS
          D:PU    Q:180
          E:PW    Q:92
          E:PZ    Q:88

$DEMAND:ENTRE
          D:PU    Q:20

$CONSTRAINT:SHAREX
          SHAREX =E= 80*PX*X / (80*PX*X + 100*PY*Y) ;

$CONSTRAINT:MARKUP
          MARKUP =E= 1/(SIGMA - (SIGMA-1)*SHAREX);

$OFFTEXT
$SYSINCLUDE mpsgeset M51

*      Benchmark replication:

PX.L    = 1.25;
SHAREX.L = 0.5;
MARKUP.L = 0.20;

PU.FX = 1;

$INCLUDE M51.GEN
SOLVE M51 USING MCP;

INCOMEM = W.L*(ENTRE.L/(ENTRE.L + CONS.L));
INCOME C = W.L*(CONS.L/(ENTRE.L + CONS.L));

DISPLAY INCOMEM, INCOME C;

```

* Evaluate the potential gains from first-best (marginal
* cost) pricing:

MARKUP.FX = 0;

\$INCLUDE M51.GEN

SOLVE M51 USING MCP;

INCOMEM = W.L*(ENTRE.L/(ENTRE.L + CONS.L));

INCOMECE = W.L*(CONS.L/(ENTRE.L + CONS.L));

DISPLAY INCOMEM, INCOMECE;

Model 52-MCP Closed economy model with monopoly and increasing returns in the X sector, MCP version.

One feature of model M51 is that it is not clear what is supporting monopoly power. The most plausible interpretation is that it is supported by some restrictive practice including a government licensing scheme. In the absence of restrictive practices, monopoly power is generally thought to be connected with increasing returns to scale in production. In model M52, we assume increasing returns to scale in X. More specifically, we assume the existence of fixed costs of production, and then constant marginal costs. This produces an average cost curve that is a rectangular hyperbole, with average cost approaching (asymptotic to) marginal cost for high levels of output.

While this is a convenient and trackable technology, it does cause one difficulty, especially for MPS/GE. The code and indeed economic theory has difficulty with negative income for an agent in equilibrium. How is this to be squared with equilibrium and Walras' law? The simplest way around this is to just use one representative agent who both owns the factor endowment and receives markup revenues and pays fixed costs. Then back out monopoly profits once the model is solved. That is what I do here. The data matrix is as follows.

Markets	Production Sectors			W	Consumers CONS
	X	N	Y		
PX	100			-100	
PY			100	-100	
PF		20			-20
PU				200	-200
PW	-32	-8	-60		100
PZ	-48	-12	-40		100
MK	-20				20

We are going to calibrate the model to zero profits initially. So markup revenues of 20 are entirely spent on fixed costs. Fixed costs are produced from unskilled and skilled labor. Since there is only one firm in this model, we do not specify a production or cost function for fixed cost, and it is treated as fixed and as a negative endowment for the representative consumer. The fixed costs appear in three equations of the model: demand for unskilled and skilled labor, and in the consumer's income constraint. Markup revenues are also added to consumers' income.

```
SKLAB..      100*ENDOW =E= 0.40*(PW**0.60)*(PZ**(0.40-1))*Y*100
              + 0.60*(PW**0.40)*(PZ**(0.60-1))*X*80 +12*FCOST;

UNLAB..     100*ENDOW =E= 0.60*(PW**(0.60-1))*(PZ**0.40)*Y*100
              + 0.40*(PW**(0.40-1))*(PZ**0.60)*X*80 + 8*FCOST;

ICONS..     CONS =E= PZ*100*ENDOW + PW*100*ENDOW + MARKUP*PX*X*80
```

$$-PZ*12*FCOST - PW*8*FCOST;$$

After solving the model, we recover monopoly profits. I specify this in welfare units, multiplying welfare (syntax W.L since we have already solved the model - recall that this is the way to report the value of a variable in an MCP or MPS/GE program). I multiply this by the ratio of the monopolist's profits, markup revenues minus fixed costs, to the total value of output. So the parameter INCOMEM gives the share of welfare that is monopoly profits. INCOMECEC gives the income of consumers (factor owners).

$$INCOMEM = W.L * ((MARKUP.L * PX.L * X.L * 80 - PW.L * 8 * FCOST - PZ.L * 12 * FCOST) / (PX.L * X.L * 80 + PY.L * Y.L * 100));$$

$$INCOMECEC = W.L - INCOMEM;$$

DISPLAY INCOMEM, INCOMECEC;

The first counter-factual imposes marginal-cost pricing by setting the markup equal to zero (MARKUP.FX = 0). Welfare increases by 4% as in our previous example, but factor owners have a much bigger increase, assuming that they do not have to pay the deficit (an unrealistic assumption).

The second and third counterfactuals allow monopoly pricing and change the size of the economy. The correct syntax for re-freeing up the markup is to set upper and lower for the variable.

$$\begin{aligned} \text{MARKUP.LO} &= -\text{INF}; \\ \text{MARKUP.UP} &= \text{INF}; \end{aligned}$$

Making the economy larger generates positive monopoly profits and making the economy smaller generates losses, even at the optimal markup. Thus we have a "natural monopoly" in this model. Note the influence of increasing returns in the experiment where we make the economy bigger. Real income and welfare increase more than in proportion to the size of the endowment. Welfare rises from 1.0 to 2.113; however, all of the increase goes to the monopolist and the welfare "per capita" of factor owners is virtually unchanged at 1.998 after doubling the endowment.

\$TITLE Model M52-MCP.GMS: Closed Economy Monopoly with IRS
 * uses MCP

\$ONTEXT

Markets	Production Sectors				Consumers CONS
	X	N	Y	W	
PX	100			-100	
PY			100	-100	
PF		20			-20
PU				200	-200
PW	-32	-8	-60		100
PZ	-48	-12	-40		100
MK	-20				20

\$OFFTEXT

PARAMETERS

A Scale parameter for utility function,
 SIGMA Elasticity of substitution,
 INCOMEM Monopoly profit (in welfare units),
 INCOMEFC Factor owners' income,
 ENDOW Endowment scale multiplier,
 FCOST Fixed costs scale multiplier,
 MODELSTAT;

SIGMA = 9;
 ENDOW = 1;
 FCOST = 1;

$A = 0.5 ** (1 / (1 - SIGMA)) ;$

POSITIVE VARIABLES

X
 Y
 W
 PX
 PY
 PU
 PZ
 PW
 CONS
 ENTRE
 SHAREX
 MARKUP;

EQUATIONS

DX Demand for X
 DY Demand for Y
 DW Demand for W
 PRICEX MR = MC in X
 PRICEY Zero profit condition for Y (PY = MC)
 PRICEW Zero profit condition for W
 SKLAB Supply-demand balance for skilled labor
 UNLAB Supply-demand balance for unskilled labor
 ICONS Consumer (factor owners') income
 IENTRE Entrepreneur's profits
 SHX Share of X in expenditure
 MK Markup equation;

PRICEX.. $(PW^{**0.40}) * (PZ^{**0.60}) =G= PX * (1 - MARKUP);$

PRICEY.. $(PW^{**0.60}) * (PZ^{**0.40}) =G= PY;$

PRICEW.. $A * ((PX/1.25)^{** (1-SIGMA)} + PY^{** (1-SIGMA)})^{** (1/(1-SIGMA))}$
 $=G= PU;$

DX.. $X*80 =E= A * (PX/1.25)^{** (-SIGMA)} * ((PX/1.25)^{** (1-SIGMA)} + PY^{** (1-SIGMA)})^{** (SIGMA/(1-SIGMA))} * W*200/1.25;$

DY.. $Y*100 =E= A * PY^{** (-SIGMA)} * ((PX/1.25)^{** (1-SIGMA)} + PY^{** (1-SIGMA)})^{** (SIGMA/(1-SIGMA))} * W*200;$

DW.. $W*200 =E= CONS/PU;$

SKLAB.. $100*ENDOW =E= 0.40 * (PW^{**0.60}) * (PZ^{** (0.40-1)}) * Y*100 + 0.60 * (PW^{**0.40}) * (PZ^{** (0.60-1)}) * X*80 + 12*FCOST;$

UNLAB.. $100*ENDOW =E= 0.60 * (PW^{** (0.60-1)}) * (PZ^{**0.40}) * Y*100 + 0.40 * (PW^{** (0.40-1)}) * (PZ^{**0.60}) * X*80 + 8*FCOST;$

ICONS.. $CONS =E= PZ*100*ENDOW + PW*100*ENDOW + MARKUP*PX*X*80 - PZ*12*FCOST - PW*8*FCOST;$

SHX.. $SHAREX =E= 80*PX*X / (80*PX*X + 100*PY*Y) ;$

MK.. $MARKUP =E= 1 / (SIGMA - (SIGMA-1)*SHAREX);$

```
MODEL M52 /DX.PX, DY.PY, DW.PU, PRICEX.X, PRICEY.Y, PRICEW.W,
          SKLAB.PZ, UNLAB.PW, ICONS.CON,
          SHX.SHAREX, MK.MARKUP/;
```

```
OPTION MCP=MILES;
OPTION LIMROW=0;
OPTION LIMCOL=0;
$OFFSYMLIST OFFSYMREF OFFUELLIST OFFFUELXREF
```

```
CONS.L = 200;
X.L = 1;
Y.L = 1;
W.L = 1;
PX.L = 1.25;
PY.L = 1;
PZ.L = 1;
PW.L = 1;
PU.L = 1;
CONS.L = 180;
ENTRE.L = 20;
SHAREX.L = 0.5;
MARKUP.L = 0.20;
```

```
PY.FX = 1;
```

```
*M52.ITERLIM = 0;
SOLVE M52 USING MCP;
MODELSTAT = M52.MODELSTAT - 1.;
```

```
INCOMEM = W.L*((MARKUP.L*PX.L*X.L*80 - PW.L*8*FCOST - PZ.L*12*FCOST)/
              (PX.L*X.L*80 + PY.L*Y.L*100));
```

```
INCOMECE = W.L - INCOMEM;
```

```
DISPLAY INCOMEM, INCOMECE;
```

```
* counterfactual: marginal-cost pricing
```

```
MARKUP.FX = 0;
```

```
SOLVE M52 USING MCP;
```

```
INCOMEM = W.L*((MARKUP.L*PX.L*X.L*80 - PW.L*8*FCOST - PZ.L*12*FCOST)/
              (PX.L*X.L*80 + PY.L*Y.L*100));
```

```
INCOMECE = W.L - INCOMEM;
```

```
DISPLAY INCOMEM, INCOMECE;
```

* counterfactual: double the size of the economy

MARKUP.LO = -INF;

MARKUP.UP = INF;

ENDOW = 2;

SOLVE M52 USING MCP;

INCOMEM = W.L* ((MARKUP.L*PX.L*X.L*80 - PW.L*8*FCOST - PZ.L*12*FCOST) /
 (PX.L*X.L*80 + PY.L*Y.L*100));

INCOMECEC = W.L - INCOMEM;

DISPLAY INCOMEM, INCOMECEC;

* counterfactual: cut the size of the economy by 25%

ENDOW = 0.75;

SOLVE M52 USING MCP;

INCOMEM = W.L* ((MARKUP.L*PX.L*X.L*80 - PW.L*8*FCOST - PZ.L*12*FCOST) /
 (PX.L*X.L*80 + PY.L*Y.L*100));

INCOMECEC = W.L - INCOMEM;

DISPLAY INCOMEM, INCOMECEC;

Model 52-MPS Closed economy model with monopoly and increasing returns in the X sector, MPS/GE version.

This model is quite similar to model M51, so there is not a lot to add concerning the MPS/GE version of model M52. The one thing worth noting is how the fixed costs are modeled here. These are specified as negative endowments in the consumer demand block. This is a good time to give a reminder that when a Q, P, or E field specifies an arithmetic operation the expression must be in parentheses. So an expression such as “Q: 100*ENDOW” is not allowed.

```
$DEMAND : CONS
      D : PU      Q : 200
      E : PW      Q : (100*ENDOW)
      E : PZ      Q : (100*ENDOW)
      E : PW      Q : (-8*FCOST)
      E : PZ      Q : (-12*FCOST)
```

This formulation allows fixed costs and the overall endowment to be adjusted independently. The negative endowments are not a problem unless a consumer is left with negative income, a problem discussed earlier.

\$TITLE Model M52-MPS: Monopoly with IRTS - calibrated to zero profits

\$ONTEXT

Markets	Production Sectors				Consumers
	X	N	Y	W	CONS
PX	100			-100	
PY			100	-100	
PF		20			-20
PU				200	-200
PW	-32	-8	-60		100
PZ	-48	-12	-40		100
MK	-20				20

\$OFFTEXT

PARAMETERS

SIGMA Elasticity of substitution in demand,
 FCOST Ratio of fixed costs to benchmark,
 ENDOW Level of factor endowment,
 INCOMEM Income of the monopolist,
 INCOME C Income of the factor owners;

SIGMA = 9;
 FCOST = 1;
 ENDOW = 1;

\$ONTEXT

\$MODEL:M52

\$SECTORS:

X ! Activity level -- monopolist sector X
 Y ! Activity level -- competitive sector Y
 W ! Welfare index for the consumer

\$COMMODITIES:

PU ! Welfare price index for the consumer
 PX ! Price index for X (gross of markup)
 PY ! Price index for Y (gross of markup)
 PW ! Price index for labor
 PZ ! Price index for capital

\$CONSUMERS:

CONS ! All consumers

\$AUXILIARY:

SHAREX ! Value share of X in total consumption
 MARKUP ! Markup based on Marshallian demand

\$PROD:X s:1

O:PX Q: 80 A:CONS N:MARKUP
 I:PW Q: 32
 I:PZ Q: 48

\$PROD:Y s:1

O:PY Q:100
 I:PW Q: 60
 I:PZ Q: 40

\$PROD:W s:sigma

O:PU Q:200
 I:PX Q: 80 P:1.25
 I:PY Q:100

\$DEMAND:CONS

D:PU Q:200
 E:PW Q:(100*ENDOW)
 E:PZ Q:(100*ENDOW)
 E:PW Q:(-8*FCOST)
 E:PZ Q:(-12*FCOST)

\$CONSTRAINT:SHAREX

SHAREX*(80*PX*X + 100*PY*Y) =G= 80*PX*X;

\$CONSTRAINT:MARKUP

MARKUP =E= 1/(SIGMA - (SIGMA-1)*SHAREX);

\$OFFTEXT

\$SYSINCLUDE mpsgeset M52

* Benchmark replication:

PX.L = 1.25;
 SHAREX.L = 0.5;
 MARKUP.L = 0.20;

\$INCLUDE M52.GEN

SOLVE M52 USING MCP;

INCOMEM = W.L*((MARKUP.L*PX.L*X.L*80 - PW.L*8*FCOST - PZ.L*12*FCOST)/
 (PX.L*X.L*80 + PY.L*Y.L*100));

INCOME C = W.L - INCOMEM;

DISPLAY INCOMEM, INCOMECE;

* counterfactual: marginal-cost pricing

MARKUP.FX = 0;

\$INCLUDE M52.GEN
SOLVE M52 USING MCP;

INCOMEM = W.L * ((MARKUP.L * PX.L * X.L * 80 - PW.L * 8 * FCOST - PZ.L * 12 * FCOST) /
(PX.L * X.L * 80 + PY.L * Y.L * 100));
INCOMECE = W.L - INCOMEM;

DISPLAY INCOMEM, INCOMECE;

* counterfactual: double the size of the economy

MARKUP.LO = -INF;
MARKUP.UP = INF;

ENDOW = 2;

\$INCLUDE M52.GEN
SOLVE M52 USING MCP;

INCOMEM = W.L * ((MARKUP.L * PX.L * X.L * 80 - PW.L * 8 * FCOST - PZ.L * 12 * FCOST) /
(PX.L * X.L * 80 + PY.L * Y.L * 100));
INCOMECE = W.L - INCOMEM;

DISPLAY INCOMEM, INCOMECE;

* counterfactual: cut the size of the economy by 25%

ENDOW = 0.75;

\$INCLUDE M52.GEN
SOLVE M52 USING MCP;

INCOMEM = W.L * ((MARKUP.L * PX.L * X.L * 80 - PW.L * 8 * FCOST - PZ.L * 12 * FCOST) /
(PX.L * X.L * 80 + PY.L * Y.L * 100));
INCOMECE = W.L - INCOMEM;

DISPLAY INCOMEM, INCOMECE;

Model 53-MCP Closed economy model with monopoly, increasing returns and free entry/exit in the X sector, MCP version

Cases of pure monopoly or indeed cases with any fixed number of firms are likely rare in economics, and models allowing entry and exit as conditions change are clearly desirable. There is however a significant technical difficulty with allowing the number of firms to change but restricting the number to integer values. The equilibrium number of firms N is found when the N firms make non-negative profits but N+1 firms do make losses. The zero-profit condition cannot be specified as an equality when there are positive numbers of firms. Then the model become an integer complementarity problem and we have no good algorithms for solving this type of problem. The problem could be handled by brute force where there is only one type of firm by iterating over the (integer) number of firms, but even this breaks down when there are several possible firm types (e.g., firms from different countries, multinational versus domestic firms, etc.).

The classic and accepted way around this difficulty is to allow the number of firms to be a continuous variable. This is of course how monopolistic-competition models have always been formulated. While this trick is initially hard to swallow conceptually, it has the powerful analytical advantage of allowing us to specify the entry condition as an inequality, holding with equality if there are a positive number of firms active in equilibrium. This is the technique we adopt here. The data matrix for the problem is:

Markets	Production Sectors			W	Consumers	
	X	N	Y		CONS	ENTRE
PX	100			-100		
PY			100	-100		
PF		20				-20
PU				200	-200	
PW	-32	-8	-60		100	
PZ	-48	-12	-40		100	
MK	-20					20

Firm owners receive markup revenues and “demand” fixed costs. N will be an activity and in equilibrium the activity level will be interpreted as the number of firms producing X. The model is calibrated to zero profits initially, so the markup revenues (20) will cover fixed costs (20). Firm “owners” are represented by the single agent ENTRE.

Here I am going to introduce a somewhat different calibration scheme than that used above. Instead of assuming a given number of firms and using the data and markup formula to solve for the implied elasticity of substitution in demand, I am going to assume Cobb-Douglas preferences and solve for the implied number of firms in the benchmark equilibrium. To do this, we need to derive the Cournot markup formula when there is more than one firm.

Revenue for a Cournot firm i is given by $R_i = p(X_c)X_i$. The Marshallian price elasticity

of demand is denoted η . η is just -1 in our formulation with Cobb-Douglas demand in (6) since here $\sigma = 1$. Cournot conjectures imply that $\partial X_c / \partial X_i = 1$; that is, a one-unit increase in own supply is a one-unit increase in market supply. Marginal revenue is then:

$$MR_i \equiv \frac{\partial R_i}{\partial X_i} = p + X_i \frac{\partial p}{\partial X_c} \frac{\partial X_c}{\partial X_i} = p + p \frac{X_i}{X_c} \left[\frac{X_c}{p} \frac{\partial p}{\partial X_c} \right] \frac{\partial X_c}{\partial X_i} = p \left[1 - \frac{X_i}{X_c} \frac{1}{\eta} \right]$$

$$MR_i = p(1 - mk_i) \quad mk_i = \frac{s_i}{\eta} \quad \eta \equiv - \left[\frac{p}{X_c} \frac{\partial X_c}{\partial p} \right] \quad \frac{X_i}{X_c} \equiv s_i$$

The combined assumptions of Cournot pricing and Cobb-Douglas preferences lead to a very simple expression for a firm's markup, which is given by the firm's market share.

$$mk_i = \frac{X_i}{X_c} \equiv s_i \quad \text{since } \eta = 1 \text{ (Cobb-Douglas)}$$

In the data given above, the implied markup is 20%, thus there must be five firms active initially given Cobb-Douglas preferences. The model below is calibrated with $N = 5$ initially. We now have two additional equations and unknowns relative to the previous model. There is the quantity (N) and unit price of fixed costs (PF) which are the complementary variables for the zero-profit equation for fixed costs ($PRICEF$) and the market clearing equation for fixed costs (DF) respectively. These are given by:

$$PRICEF.. \quad (PW**0.40) * (PZ**0.60) =G= PF; \quad (\text{complementary to } N)$$

$$DF.. \quad N*4 =G= ENTRE/PF; \quad (\text{complementary to } PF)$$

We have to remember that total fixed costs are 20 and therefore we must use $N*4 (= 20)$ for the demand for fixed costs for a consistent calibration at $N = 5$. N then appears in the markup equation as given by the algebra above and the markup revenue in turn provides the entrepreneur with income ($ENTRE$).

$$MK.. \quad MARKUP*N =E= 1; \quad (\text{complementary to } MARKUP)$$

$$IENTRE.. \quad ENTRE =E= MARKUP*PX*X*80; \quad (\text{complementary to } ENTRE)$$

Thus the entrepreneur is treated as a consumer who derives income from markup revenues with that income spent on a good called fixed costs. Total fixed costs are interpreted as the number of

firms in equilibrium, which in turn feeds into the markup formula.

Alternatively, I could have chosen units to let $N = 1$, and then the correct quantity in the factor-demand equations and fixed-cost pricing equations would be $N*20$ and the market equation would be $MARKUP*N*5 = E = 1$. This latter case is convenient for the same reason that it is always convenient to have activity levels and prices calibrated to one initially: the new equilibrium levels in counterfactual experiments are proportional changes from the benchmark. Many calibration errors are due to inconsistent use of units and representative quantity and price units in different functions. So it is a useful exercise here to recalibrate the model to $N = 1$. N appears in four equations in the model and as just indicated, something must be changed in each of those four equations.

The counterfactual-experiment is to double the size of the economy. Note that this more than doubles welfare, which rises to 2.072. X production more than doubles but the number of firms much less than doubles, rising from 5.0 to 7.071. Increases in the number of firms leads each individual firm to perceive demand as more elastic and the equilibrium markup falls from 0.20 to 0.14. There is an increase in the output per firm, firms move down their average costs curves and become more technically efficient.

\$TITLE Model M53-MCP: Oligopoly with Free Entry, MCP version

\$ONTEXT

Markets	Production Sectors				Consumers	
	X	N	Y	W	CONS	ENTRE
PX	100			-100		
PY			100	-100		
PF		20				-20
PU				200	-200	
PW	-32	-8	-60		100	
PZ	-48	-12	-40		100	
MK	-20					20

\$OFFTEXT

PARAMETERS

SIGMA Elasticity of substitution,

ENDOW Endowment scale multiplier,

MODELSTAT;

SIGMA = 1;

ENDOW = 1;

POSITIVE VARIABLES

X

Y

W

N

PX

PY

PU

PF

PZ

PW

CONS

ENTRE

MARKUP;

EQUATIONS

DX Demand for X

DY Demand for Y

DW Demand for W

DF Demand for fixed costs

PRICEX MR = MC in X

PRICEY Zero profit condition for Y (PY = MC)

PRICEW Zero profit condition for W
 PRICEF Zero profit condition for fixed costs
 SKLAB Supply-demand balance for skilled labor
 UNLAB Supply-demand balance for unskilled labor
 ICONS Consumer (factor owners') income
 IENTRE Entrepreneur's profits
 MK Markup equation;

PRICEX.. $(PW^{**0.40}) * (PZ^{**0.60}) =G= PX * (1 - MARKUP);$

PRICEY.. $(PW^{**0.60}) * (PZ^{**0.40}) =G= PY;$

PRICEW.. $((PX/1.25)^{**0.5}) * (PY^{**0.5}) =G= PU;$

PRICEF.. $(PW^{**0.40}) * (PZ^{**0.60}) =G= PF;$

DX.. $X*80 =E= 0.5*CONS/PX;$

DY.. $Y*100 =E= 0.5*CONS/PY;$

DW.. $W*200 =E= CONS/PU;$

DF.. $N*4 =G= ENTRE/PF;$

SKLAB.. $100*ENDOW =E= 0.40*(PW^{**0.60}) * (PZ^{** (0.40-1)}) * Y*100$
 $+ 0.60*(PW^{**0.40}) * (PZ^{** (0.60-1)}) * X*80$
 $+ 0.60*(PW^{**0.40}) * (PZ^{** (0.60-1)}) * N*4;$

UNLAB.. $100*ENDOW =E= 0.60*(PW^{** (0.60-1)}) * (PZ^{**0.40}) * Y*100$
 $+ 0.40*(PW^{** (0.40-1)}) * (PZ^{**0.60}) * X*80$
 $+ 0.40*(PW^{** (0.40-1)}) * (PZ^{**0.60}) * N*4;$

ICONS.. $CONS =E= PZ*100*ENDOW + PW*100*ENDOW;$

IENTRE.. $ENTRE =E= MARKUP*PX*X*80;$

MK.. $MARKUP*N =E= 1;$

MODEL M53 /DX.PX, DY.PY, DW.PU, DF.PF, PRICEX.X, PRICEY.Y, PRICEW.W,
 PRICEF.N, SKLAB.PZ, UNLAB.PW, ICONS.CON, IENTRE.ENTRE,
 MK.MARKUP/;

OPTION MCP=MILES;
 OPTION LIMROW=0;
 OPTION LIMCOL=0;

```
$OFFSYMLIST OFFSYMXREF OFFUELLIST OFFFUELXREF
```

```
CONS.L = 200;  
X.L = 1;  
Y.L = 1;  
W.L = 1;  
N.L = 4;  
PX.L = 1.25;  
PY.L = 1;  
PZ.L = 1;  
PW.L = 1;  
PU.L = 1;  
PF.L = 1;  
CONS.L = 180;  
ENTRE.L = 20;  
MARKUP.L = 0.20;
```

```
PY.FX = 1;
```

```
*M53.ITERLIM = 0;  
SOLVE M53 USING MCP;  
MODELSTAT = M53.MODELSTAT - 1.;
```

```
* counterfactual: double the size of the economy
```

```
ENDOW = 2;
```

```
SOLVE M53 USING MCP;
```

Model 53-MPS Closed economy model with monopoly, increasing returns and free entry/exit in the X sector, MPS/GE version

Relative to M52, we now add one activity (N) and one commodity (PF) to the model. As in the MCP version just discussed, we need to be careful about calibration. If we choose $N = 5$ as the calibration, then the representative quantities for the \$PROD:N block are the total quantities from the data matrix divided by 5, and the markup equation just uses N.

```
$PROD:N  s:1
          O:PF    Q: (20/5)
          I:PW    Q: (8/5)
          I:PZ    Q: (12/5)
```

```
$CONSTRAINT:MARKUP
          MARKUP*N =E= 1;
```

If instead, we want an initial value of $N = 1$, then we would need

```
$PROD:N  s:1
          O:PF    Q: 20
          I:PW    Q: 8
          I:PZ    Q: 12
```

```
$CONSTRAINT:MARKUP
          MARKUP*N*5 =E= 1;
```

As I have emphasized several times, MPS/GE automatically generates the appropriate factor demand equations, and so there is no need to have to worry about consistency of units for these equations as we have to do in the MCP version.

\$TITLE Model M53-MPS: Oligopoly with free entry, MPS/GE version

\$ONTEXT

Markets	Production Sectors				Consumers	
	X	N	Y	W	CONS	ENTRE
PX	100			-100		
PY			100	-100		
PF		20				-20
PU				200	-200	
PW	-32	-8	-60		100	
PZ	-48	-12	-40		100	
MK	-20					20

\$OFFTEXT

PARAMETERS

SIGMA
ENDOW;

SIGMA = 1;
ENDOW = 1;

\$ONTEXT

\$MODEL:M53

\$SECTORS:

X ! Activity level - sector X output
Y ! Activity level - competitive sector Y
W ! Welfare index for the representative consumer
N ! Activity level - sector X fixed costs = no. of firms

\$COMMODITIES:

PU ! Price index for representative agent utility
PX ! Price of good X (gross of markup)
PY ! Price of good Y
PF ! Unit price of inputs to fixed cost
PW ! Price index for labor
PZ ! Price index for capital

\$CONSUMERS:

CONS ! Representative agent
ENTRE ! Entrepreneur (converts markup revenue to fixed cost)

\$AUXILIARY:

MARKUP ! Optimal markup based on Marshallian demand elasticity

\$PROD:X s:1

O:PX Q: 80 A:ENTRE N:MARKUP
 I:PW Q: 32
 I:PZ Q: 48

\$PROD:Y s:1

O:PY Q:100
 I:PW Q: 60
 I:PZ Q: 40

\$PROD:N s:1

O:PF Q: (20/5)
 I:PW Q: (8/5)
 I:PZ Q: (12/5)

\$PROD:W s:1

O:PU Q:200
 I:PX Q: 80 P:1.25
 I:PY Q:100

\$DEMAND:CONS

D:PU Q:200
 E:PW Q:(ENDOW*100)
 E:PZ Q:(ENDOW*100)

\$DEMAND:ENTRE

D:PF Q: 20

\$CONSTRAINT:MARKUP

MARKUP*N =E= 1;

\$OFFTEXT

\$SYSINCLUDE mpsgeset M53

* Benchmark replication:

N.L = 5;

PX.L = 1.25;

MARKUP.L = 0.20;

\$INCLUDE M53.GEN

SOLVE M53 USING MCP;

* Counterfactual double the size of economy.

ENDOW = 2;

\$INCLUDE M53.GEN

SOLVE M53 USING MCP;

Model 54-MCP Two country trade model with monopoly, increasing returns and free entry/exit in the X sector, MCP version

Model M54 is a two-country version of M53. The initial data specifies that the countries are identical, and trade is costless. There is really not a lot new here, except that one has to be careful about the initial calibration of the markups. Markup revenues in the two markets are 20% of sales and once again we will assume Cobb-Douglas preferences. Thus there are once again 5 firms in total. This in turn implies that there are 2.5 firms in each country initially. I realize that this might be troublesome for some readers, and there are of course alternative calibrations such as the procedure discussed earlier where we choose the number of firms and calibrate the elasticity of substitution. Or in some cases research just use a “conjectural variation” parameter which is a multiplicative constant on the markup to reconcile the actual markup with the one given by the assumed number of firms. The modeler often has no alternative to this last procedure when trying to calibrate a model to real data.

A couple of features deserve discussion. First, this is a model in which I assume that markets are segmented, meaning that firms can price independently in the two markets (the two prices need not differ by exactly the transport/tariff cost). In general, firms will have different market shares and thus different markups in the two countries, and thus it is important to have different pricing conditions for the two supplies of a given firm. Here I use a technique that is very useful in many models, which is to add additional activities. I first have one activity that converts factors into aggregate firm output. Then this output is the input into two separate activities, one which supplies output to the domestic market and one which supplies the foreign (export) market.

For country j , the $PXDJ$ denotes the producer price, or cost of aggregate X output, XJ , so the value of this variable is the marginal cost of production. This aggregate supply is then divided by supply to j , XJJ , and exports to i , XJI ($XJ = XJJ + XJI$). Consumer price of XJ in country j is given by PXJ and that in country i by PXI . The relationship between marginal cost and consumer prices depends on the markups in the two markets and on transport costs on exported output. The relevant pricing equations for firms in j , with complementary variables in parenthesis are:

$$PRXDJ \dots (WJ^{**0.40}) * (ZJ^{**0.60}) =G= PXDJ ; \quad (XJ)$$

$$PRXJJ \dots PXDJ =G= PXJ * (1 - MARKJJ) ; \quad (XJJ)$$

$$PRXJI \dots PXDJ * (1+TC) =G= PXI * (1 - MARKJI) ; \quad (XJI)$$

Given Cobb-Douglas preferences, a firm’s markup is just its market share. I have coded the model such that the X variables just introduced are the total outputs of all firms located in country j . Thus to get the output (sales) of an individual firm from j in a market, we have to divide these quantities by the number of j firms NJ . Thus, for example, the supply of a single j firm to market j is given by

$$(XJJ/NJ)/(XIJ + XJJ) \text{ or } XJJ/(NJ*(XIJ + XJJ));$$

The markup equations are thus given by:

$$\begin{aligned} \text{MKII} \dots \text{MARKII} &= \text{E} = \text{XII} / (\text{NI} * (\text{XII} + \text{XJI})); \\ \text{MKIJ} \dots \text{MARKIJ} &= \text{E} = \text{XIJ} / (\text{NI} * (\text{XIJ} + \text{XJJ})); \\ \text{MKJJ} \dots \text{MARKJJ} &= \text{E} = \text{XJJ} / (\text{NJ} * (\text{XIJ} + \text{XJJ})); \\ \text{MKJI} \dots \text{MARKJI} &= \text{E} = \text{XJI} / (\text{NJ} * (\text{XII} + \text{XJI})); \end{aligned}$$

I coded the MPS/GE version of this model first, which took very little time. Then I tried to code this corresponding MCP. I made a few mistakes, things that MPS/GE does automatically, and it took me quite a while. So I pass on comments about my mistakes. First, trade costs require that more output must be shipped than is received. The relationships between firms' total outputs and supplies to the two markets are given by:

$$\text{DXDI} \dots \text{XII} * 40 + \text{XIJ} * 40 * (1 + \text{TC}) = \text{E} = \text{XI} * 80;$$

$$\text{DXDJ} \dots \text{XJJ} * 40 + \text{XJI} * 40 * (1 + \text{TC}) = \text{E} = \text{XJ} * 80;$$

Remember this when specifying factor demands. Second, remember the choice of units in correctly calibrating the functions. The markup equations given above are calibrated to $\text{NI} = \text{NJ} = 2.5$. Thus in the equations for demand for fixed costs and factor demands the 20 units shown in benchmark data must be written as $\text{N} * 8$ ($=20$). Unit are chosen such that variables such as YI , XII , and XIJ are all equal to one, and so quantity multipliers must be chosen that are consist with this choice of units. The correct market clearing equation for skilled labor in country i is:

$$\begin{aligned} \text{SKLABI} \dots 100 * \text{ENDOWIS} &= \text{E} = 0.40 * (\text{WI} ** 0.60) * (\text{ZI} ** (0.40 - 1)) * \text{YI} * 100 \\ &+ 0.60 * (\text{WI} ** 0.40) * (\text{ZI} ** (0.60 - 1)) * (\text{XII} + \text{XIJ} * (1 + \text{TC})) * 40 \\ &+ 0.60 * (\text{WI} ** 0.40) * (\text{ZI} ** (0.60 - 1)) * \text{NI} * 8; \end{aligned}$$

Study this equation and make sure you understand it: the first line is demand from Y , the second line is demand for factors used in marginal costs, and the third line is demand for factors in fixed costs.

Finally, one experiment is to put a subsidy on production in country i . The pricing equation PRXDI is rather obvious. However, don't forget to charge the consumer for this subsidy. Country i 's consumer's income is given by:

$$\begin{aligned} \text{ICONSI} \dots \text{CONSI} &= \text{E} = \text{ZI} * 100 * \text{ENDOWIS} + \text{WI} * 100 * \text{ENDOWIL} \\ &- (\text{WI} ** 0.40) * (\text{ZI} ** 0.60) * \text{SUBSIDY} * \text{XI} * 80; \end{aligned}$$

This is of course another thing that MPS/GE does automatically for you. Note from the output that the subsidy (reminiscent of strategic trade policy) does not improve the welfare of country i , the potential benefits are dissipated by entry (Horstmann and Markusen, JIE 1986).

\$TITLE Model M54-MCP: Two-Country Oligopoly with free entry
 * MCP version

\$ONTEXT

	YI	YJ	XI	XJ	NI	NJ	WI	WJ	CONI	CONJ	EHTI	ENTJ
PYI	100						-100					
PYJ		100						-100				
PXI			100				-50	-50				
PXJ				100			-50	-50				
FCI					20						-20	
FCJ						20						-20
ZI	-40		-48		-12				100			
ZJ		-40		-48		-12				100		
WI	-60		-32		-8				100			
WJ		-60		-32		-8				100		
PUI							200		-200			
PUJ								200		-200		
MKI			-10	-10							10	10
MKJ			-10	-10							10	10

\$OFFTEXT

PARAMETERS

ENDOWIL

ENDOWIS

ENDOWJL

ENDOWJS

TC

SUBSIDY

MODELSTAT

REALWI

REALWJ

REALZI

REALZJ;

ENDOWIL = 1;

ENDOWIS = 1;

ENDOWJL = 1;

ENDOWJS = 1;

TC = 0;

SUBSIDY = 0;

POSITIVE VARIABLES

YI
 YJ
 WFI
 WFJ
 XI
 XII
 XIJ
 XJ
 XJJ
 XJI
 NI
 NJ
 PY
 PUI
 PUJ
 WI
 WJ
 ZI
 ZJ
 PXI
 PXJ
 PXDI
 PXDJ
 PFI
 PFJ
 CONSI
 CONSJ
 ENTI
 ENTJ
 MARKII
 MARKIJ
 MARKJI
 MARKJJ;

EQUATIONS

DXDI X output in country i
 DXI Demand for X in country i
 DXDJ X output in country j
 DXJ Demand for X in country j
 DY Demand for Y
 DWI Demand for welfare in country i
 DWJ Demand for welfare in country j
 DFI Demand for fixed costs in i
 DFJ Demand for fixed costs in j
 PRXDI Marginal cost of X in i

PRXII MR = MC for XII
 PRXIJ MR = MC for XIJ
 PRXDJ Marginal cost of X in j
 PRXJJ MR = MC for Xjj
 PRXJI MR = MC for Xji
 PRYI Zero profits for YI
 PRYJ Zero profits for YJ
 PRWI Zero profits for WFI
 PRWJ Zero profits for WFJ
 PRFI Zero profits for FI
 PRFJ Zero profits for FJ
 SKLABI Market clearing for SI
 SKLABJ Market clearing for SJ
 UNLABI Market clearing for LI
 UNLABJ Market clearing for LJ
 ICONSI Consumer income in i
 ICONSJ Consumer income in j
 IENTREI Entrepreneur's income (markups) in i
 IENTREJ Entrepreneur's income (markups) in j
 MKII Markup ii
 MKIJ Markup ij
 MKJJ Markup jj
 MKJI Markup ji;

PRXDI.. $(WI^{**0.40}) * (ZI^{**0.60}) * (1 - \text{SUBSIDY}) =G= \text{PXDI};$

PRXDJ.. $(WJ^{**0.40}) * (ZJ^{**0.60}) =G= \text{PXDJ};$

PRXII.. $\text{PXDI} =G= \text{PXI} * (1 - \text{MARKII});$

PRXIJ.. $\text{PXDI} * (1 + \text{TC}) =G= \text{PXJ} * (1 - \text{MARKIJ});$

PRXJJ.. $\text{PXDJ} =G= \text{PXJ} * (1 - \text{MARKJJ});$

PRXJI.. $\text{PXDJ} * (1 + \text{TC}) =G= \text{PXI} * (1 - \text{MARKJI});$

PRYI.. $(WI^{**0.60}) * (ZI^{**0.40}) =G= \text{PY};$

PRYJ.. $(WJ^{**0.60}) * (ZJ^{**0.40}) =G= \text{PY};$

PRWI.. $((\text{PXI} / 1.25)^{**0.5}) * (\text{PY}^{**0.5}) =G= \text{PUI};$

PRWJ.. $((\text{PXJ} / 1.25)^{**0.5}) * (\text{PY}^{**0.5}) =G= \text{PUJ};$

PRFI.. $(WI^{**0.40}) * (ZI^{**0.60}) =G= \text{PFI};$

PRFJ.. $(WJ^{**0.40}) * (ZJ^{**0.60}) =G= \text{PFJ};$

DXDI.. $XII*40 + XIJ*40*(1+TC) =E= XI*80;$

DXDJ.. $XJJ*40 + XJI*40*(1+TC) =E= XJ*80;$

DXI.. $(XII*40 + XJI*40) =E= 0.5*CONSI/PXI;$

DXJ.. $(XJJ*40 + XIJ*40) =E= 0.5*CONSJ/PXJ;$

DY.. $(YI + YJ)*100 =E= 0.5*(CONSI + CONSJ)/PY;$

DWI.. $WFI*200 =E= CONSI/PUI;$

DWJ.. $WFJ*200 =E= CONSJ/PUJ;$

DFI.. $NI*8 =G= ENTI/PFI;$

DFJ.. $NJ*8 =G= ENTJ/PFJ;$

SKLABI.. $100*ENDOWIS =E= 0.40*(WI**0.60)*(ZI**(0.40-1))*YI*100$
 $+ 0.60*(WI**0.40)*(ZI**(0.60-1))*(XII+XIJ*(1+TC))*40$
 $+ 0.60*(WI**0.40)*(ZI**(0.60-1))*NI*8;$

SKLABJ.. $100*ENDOWJS =E= 0.40*(WJ**0.60)*(ZJ**(0.40-1))*YJ*100$
 $+ 0.60*(WJ**0.40)*(ZJ**(0.60-1))*(XJJ+XJI*(1+TC))*40$
 $+ 0.60*(WJ**0.40)*(ZJ**(0.60-1))*NJ*8;$

UNLABI.. $100*ENDOWIL =E= 0.60*(WI**(0.60-1))*(ZI**0.40)*YI*100$
 $+ 0.40*(WI**(0.40-1))*(ZI**0.60)*(XII+XIJ*(1+TC))*40$
 $+ 0.40*(WI**(0.40-1))*(ZI**0.60)*NI*8;$

UNLABJ.. $100*ENDOWJL =E= 0.60*(WJ**(0.60-1))*(ZJ**0.40)*YJ*100$
 $+ 0.40*(WJ**(0.40-1))*(ZJ**0.60)*(XJJ+XJI*(1+TC))*40$
 $+ 0.40*(WJ**(0.40-1))*(ZJ**0.60)*NJ*8;$

ICONSI.. $CONSI =E= ZI*100*ENDOWIS + WI*100*ENDOWIL$
 $- (WI**0.40)*(ZI**0.60)*SUBSIDY*XI*80;$

ICONSJ.. $CONSJ =E= ZJ*100*ENDOWJS + WJ*100*ENDOWJL;$

IENTREI.. $ENTI =G= MARKII*PXI*XII*40 + MARKIJ*PXJ*XI*40;$

IENTREJ.. $ENTJ =G= MARKJJ*PXJ*XJJ*40 + MARKJI*PXI*XJI*40;$

```

MKII..   MARKII =E= XII/(NI*(XII + XJI));
MKIJ..   MARKIJ =E= XIJ/(NI*(XIJ + XJJ));
MKJJ..   MARKJJ =E= XJJ/(NJ*(XIJ + XJJ));
MKJI..   MARKJI =E= XJI/(NJ*(XII + XJI));

```

```

MODEL M54 /DXDI.PXDI, DXDJ.PXDJ, DXI.PXI, DXJ.PXJ, DY.PY,
           DWI.PUI, DWJ.PUJ, DFI.PFI, DFJ.PFJ,
           PRXDI.XI, PRXII.XII, PRXIJ.XIJ,
           PRXDJ.XJ, PRXJJ.XJJ, PRXJI.XJI,
           PRYI.YI, PRYJ.YJ, PRWI.WFI, PRWJ.WFJ,
           PRFI.NI, PRFJ.NJ, SKLABI.ZI, SKLABJ.ZJ,
           UNLABI.WI, UNLABJ.WJ, ICONSI.CONSI, ICONSJ.CONSJ,
           IENTREI.ENTI, IENTREJ.ENTJ,
           MKII.MARKII, MKIJ.MARKIJ, MKJJ.MARKJJ, MKJI.MARKJI/;

```

```

OPTION MCP=MILES;
OPTION LIMROW=0;
OPTION LIMCOL=0;
$OFFSYMLIST OFFSYMREF OFFUELLIST OFFFUELXREF

```

```

CONSI.L = 200;
CONSJ.L = 200;
ENTI.L = 20;
ENTJ.L = 20;
XI.L = 1;
XJ.L = 1;
XII.L = 1;
XIJ.L = 1;
XJJ.L = 1;
XJI.L = 1;
YI.L = 1;
YJ.L = 1;
WFI.L = 1;
WFJ.L = 1;
NI.L = 2.5;
NJ.L = 2.5;
PXDI.L = 1;
PXDJ.L = 1;
PXI.L = 1.25;
PXJ.L = 1.25;
PY.L = 1;
ZI.L = 1;
ZJ.L = 1;
WI.L = 1;
WJ.L = 1;
PUI.L = 1;
PUJ.L = 1;

```



```
PFI.L = 1;  
PFJ.L = 1;  
MARKII.L = 0.20;  
MARKIJ.L = 0.20;  
MARKJJ.L = 0.20;  
MARKJI.L = 0.20;
```

```
PY.FX = 1;
```

```
*M54.ITERLIM = 0;  
SOLVE M54 USING MCP;  
MODELSTAT = M54.MODELSTAT - 1.;
```

```
* counterfactual: trade costs of 15%
```

```
TC = 0.15;
```

```
SOLVE M54 USING MCP;
```

```
* counterfactual: home production subsidy of 10%, trade costs 0
```

```
TC = 0.;  
SUBSIDY = .10;
```

```
SOLVE M54 USING MCP;
```

```
* counterfactual: country's identical except for size,  
* positive trade costs (home market advantage)
```

```
SUBSIDY = 0;  
TC = 0.15;
```

```
ENDOWIL = 1.5;  
ENDOWJL = 0.5;  
ENDOWIS = 1.5;  
ENDOWJS = 0.5;
```

```
SOLVE M54 USING MCP;
```

```
REALWI = WI.L/PUI.L;  
REALWJ = WJ.L/PUJ.L;  
REALZI = ZI.L/PUI.L;  
REALZJ = ZJ.L/PUJ.L;
```

```
DISPLAY REALWI, REALWJ, REALZI, REALZJ;
```

Model 54-MPS Two country trade model with monopoly, increasing returns and free entry/exit in the X sector, MPS/GE version

As models get bigger, the advantages of MPS/GE become more and more obvious. It is not just the amount of coding saved by using MPS/GE, but the avoidance of errors due to the fact that MPS/GE automatically generates factor demands, income balance, and market clearing conditions. It took me about 40 minutes to code the MPS/GE version of this model, and about three or four hours to get the MCP version correct. Even then, I only knew that there were some errors in the MCP version because results from a couple of the counterfactuals did not match those from the MPS/GE version. I do think that it is very important for new modelers to code MCP versions of simple problems, so that the modeler understands exactly what MPS/GE is doing and exactly what square system is being generated in the background.

There is virtually nothing to add at this point. One thing that I might alert modelers to is possible divide by zero errors that I have had to deal with in my models of multinationals. When there are several types of firms, not all of which are active in equilibrium, there can be a divide-by-zero problem in the markup equations with NI zero if there are no type-NI firms in equilibrium. The simplest solution is just to set a lower bound on such a variable by using an expression like

```
NI.LO = 0.0001;
```

before the solve statement. This is not perfectly satisfactory since if that firm type is not active, some resources will still be consumed in fixed costs, but the approximation error is small. A more sophisticated solution to this difficulty is found in appendix 5 of my book on multinational firms (“Multinational Firms and the Theory of International Trade”, MIT press, 2002).

A related problem is if there is an entrepreneur who is the sole demander of a particular (fixed cost) good and that firm type is not active, then there is no demand for that good. Then the price of that good is zero, which in turn causes the solution algorithm problems insofar as that prices may appear in the denominator of some expression in the solution routine. This problem only occurs in the initialization of the model and is easily solved. In the case of this model, this problem is easily solved by including the following statements before the solve statements to ensure that the initial values of incomes are not zero.

```
FCI.L = MAX(FCI.L, 0.0001);
FCJ.L = MAX(FCJ.L, 0.0001);
```

```
$INCLUDE M54.GEN
SOLVE M54 USING MCP;
```

Note again the difference between the suffixes “.L” which stands for “level”, and sets the current value of a variable but allows the variable full freedom to change, and “.LO” which stands for “fix the lower bound of the variable at...”.

\$TITLE: M54-MPS.GMS. Two country oligopoly model with free entry
 * uses MPS/GE

\$ONTEXT

	YI	YJ	XI	XJ	NI	NJ	WI	WJ	CONI	CONJ	EHTI	ENTJ
PYI	100						-100					
PYJ		100						-100				
PXI			100				-50	-50				
PXJ				100			-50	-50				
FCI					20						-20	
FCJ						20						-20
ZI	-40		-48		-12				100			
ZJ		-40		-48		-12				100		
WI	-60		-32		-8				100			
WJ		-60		-32		-8				100		
PUI							200		-200			
PUJ								200		-200		
MKI			-10	-10							10	10
MKJ			-10	-10							10	10

\$OFFTEXT

PARAMETERS

ENDOWIL

ENDOWJL

ENDOWIS

ENDOWJS

REALWI

REALWJ

REALZI

REALZJ

TC

SUBSIDY;

TC = .0;

SUBSIDY = 0;

ENDOWIL = 1;

ENDOWJL = 1;

ENDOWIS = 1;

ENDOWJS = 1;

\$ONTEXT

\$MODEL:M54

\$SECTORS:

YI YJ
 WFI WFJ
 XI
 XII
 XIJ
 XJ
 XJI
 XJJ
 NI NJ

\$COMMODITIES:

PY
 PUI PUJ
 WI WJ
 ZI ZJ
 PXI PXJ
 PXDI
 PXDJ
 FCI FCJ

\$CONSUMERS:

CONSI CONSJ
 ENTI ENTJ

\$AUXILIARY:

MARKII
 MARKIJ
 MARKJI
 MARKJJ

\$PROD:YI s:1.0
 O:PY Q:100.0
 I:WI Q:60.0
 I:ZI Q:40.0

\$PROD:YJ s:1.0
 O:PY Q:100.0
 I:WJ Q:60.0
 I:ZJ Q:40.0

\$PROD:XI s:1
 O:PXDI Q:80.
 I:WI Q:32 A:CONSI S:SUBSIDY
 I:ZI Q:48 A:CONSI S:SUBSIDY

\$PROD:XII

O:PXI Q:40. A:ENTI N:MARKII
I:PXDI Q:40.

\$PROD:XIJ s:0.0

O:PXJ Q:40. A:ENTI N:MARKIJ
I:PXDI Q:(40.*(1+TC))

\$PROD:XJ s:1

O:PXDJ Q:80.
I:WJ Q:32.
I:ZJ Q:48.

\$PROD:XJI

O:PXI Q:40. A:ENTJ N:MARKJI
I:PXDJ Q:(40.*(1+TC))

\$PROD:XJJ s:0.0

O:PXJ Q:40. A:ENTJ N:MARKJJ
I:PXDJ Q:40.

\$PROD:NI s:1

O:FCI Q:(20/2.5)
I:WI Q:(8/2.5)
I:ZI Q:(12/2.5)

\$PROD:NJ s:1

O:FCJ Q:(20/2.5)
I:WJ Q:(8/2.5)
I:ZJ Q:(12/2.5)

\$PROD:WFI s:1.0

O:PUI Q:200.
I:PXI Q:80. P:1.25
I:PY Q:100.

\$PROD:WFJ s:1.0

O:PUJ Q:200.
I:PXJ Q:80. P:1.25
I:PY Q:100.

\$DEMAND:CONSI

D:PUI Q:200
E:WI Q:(100.*ENDOWIL)
E:ZI Q:(100.*ENDOWIS)

\$DEMAND:CONSJ

D:PUJ Q:200

E:WJ Q:(100.*ENDOWJL)

E:ZJ Q:(100.*ENDOWJS)

\$DEMAND:ENTI

D:FCI Q:20

\$DEMAND:ENTJ

D:FCJ Q:20

\$CONSTRAINT:MARKII

MARKII*NI*(XII + XJI) =G= XII;

\$CONSTRAINT:MARKIJ

MARKIJ*NI*(XIJ + XJJ) =G= XIJ;

\$CONSTRAINT:MARKJI

MARKJI*NJ*(XII + XJI) =G= XJI;

\$CONSTRAINT:MARKJJ

MARKJJ*NJ*(XIJ + XJJ) =G= XJJ;

\$OFFTEXT

\$SYSINCLUDE MPSGESET M54

PXI.L = 1.25;

PXJ.L = 1.25;

MARKII.L = .2;

MARKJI.L = .2;

MARKIJ.L = .2;

MARKJJ.L = .2;

NI.L = 2.5;

NJ.L = 2.5;

PY.FX = 1.0;

NI.LO = 0.0001;

NJ.LO = 0.0001;

*OPTION SOLPRINT=OFF;

OPTION LIMROW=0;

OPTION LIMCOL=0;

\$OFFSYMLIST OFFSYMREF OFFUELLIST OFFUELXREF

```
$INCLUDE M54.GEN  
SOLVE M54 USING MCP;
```

```
* counterfactual: trade costs of 15%
```

```
TC = 0.15;
```

```
$INCLUDE M54.GEN  
SOLVE M54 USING MCP;
```

```
* counterfactual: home production subsidy of 10%, trade costs 0
```

```
TC = 0.;  
SUBSIDY = .10;
```

```
$INCLUDE M54.GEN  
SOLVE M54 USING MCP;
```

```
* counterfactual: country's identical except for size,  
* positive trade costs (home market advantage)
```

```
SUBSIDY = 0;  
TC = 0.15;
```

```
ENDOWIL = 1.5;  
ENDOWJL = 0.5;  
ENDOWIS = 1.5;  
ENDOWJS = 0.5;
```

```
$INCLUDE M54.GEN  
SOLVE M54 USING MCP;
```

```
REALWI = WI.L/PUI.L;  
REALWJ = WJ.L/PUJ.L;  
REALZI = ZI.L/PUI.L;  
REALZJ = ZJ.L/PUJ.L;
```

```
DISPLAY REALWI, REALWJ, REALZI, REALZJ;
```