Chapter 6

External Economies and Monopolistic Competition

Now we continue our study of increasing returns to scale, turning to models based on external economies of scale and monopolistic-competition models. I have grouped these two together in one chapter since there are close technical similarities between them. A formal demonstration of this equivalence is found in Markusen (CJE vol. 23, 1990, 495-508.).

These models require a few tricks to code in MPS/GE, because MPS/GE requires all activities to have constant returns to scale. Because of this, I want to present MCP versions of the models first, in order to show clearly what equations and inequalities are being solved. Then I present the MPS/GE versions and show the tricks. In very simple models such as the ones presented here, it is generally easiest to stick with the MCP solver and not use the higher-level language. But it more complicated higher-dimension models, it is best to learn the tricks and use MPS/GE.

Models are as follows:

M61-MCP.GMS Closed economy external economies of scale, MCP version

M61-MPS.GMS Closed economy external economies of scale, MPS/GE version

M62-MCP.GMS Closed economy large-group monopolistic competition, MCP version

M62-MPS.GMS Closed economy large-group monopolistic competition, MPS/GE verison

M63-MPC.GMS Two-country model with large-group monopolistic competition, MCP version

M63-MPC.GMS Two-country model with large-group monopolistic competition, MPS/GE version

Model 61-MCP Closed economy external economies of scale, MCP version

The theoretical foundations of this model go back to the 1960s with work by Kemp and Melvin. The idea is that the production function for an individual firm is given by

(1)
$$X_i = (X^{\beta})F(V_i)$$
 $0 < \beta < 1$

where X_i is firm i's output, X is the sum of all firms' output (industry output), F is a function with constant returns to scale, and V_i is the firm's vector of inputs.

The parameter β reflects industry level scale economies, with $\beta = 0$ being the special case of constant returns to scale.

By virtue of the fact that F() has constant returns to scale and is identical for all firms, total industry output can then be written as:

(2)
$$X = (X^{\beta})F(V)$$
 or $X = F(V)^{1/(1-\beta)}$

so X industry output is homogeneous of degree $1/(1 - \beta) > 1$ in primary factor inputs.

To make the point in the simplest fashion, assume that there is just a single factor L with a price w. p denotes the price of X. Each individual firm views total industry output as constant, and thus each firm views itself as producing with constant returns to scale. The firm's optimization condition equates the value of the (private) marginal product of labor to the wage rate.

(3)
$$pX^{\beta} \frac{dX_i}{dL_i} = w \qquad p = X^{-\beta} \left[w \frac{dL_i}{dX_i} \right] \equiv X^{-\beta} c(w)$$

where c(w) is the cost of one "factor bundle", the cost of labor needed to produce F(L) = 1. That is, c(w) is the unit cost function associated with the constant-returns function F(L). This then becomes the pricing equation or zero profit condition for the X sector in the MCP.

The other thing that we need for the market-clearing condition for labor in the MCP

is the total demand for labor in the X sector. This is given by the amount of labor needed for F(L) = 1 times F(L). The unit requirement is given as usual by Shephard's lemma, the derivative of the unit cost function. So using (1), the demand for labor in the X sector is given by

(4)
$$c'(w)F(L) = c'(w)X^{1-\beta}$$

The demand for labor is concave in output, the dual of increasing returns in production. Here is the data matrix for the model, the same one that we have used before:

	Produc	ction Sec	Consumers		
Markets	X	Y	W	CONS	
PX	100		-100		
PY		100	-100		
PU			200	-200	
PW	-40	-60		100	
PZ	-60	-40		100	
	-			•	

We will assume that the production functions are Cobb-Douglas. Units will be chosen such that X = 1 initially, which then greatly simplifies the calibration of the data to the model. The letter "B" is used in place of beta. For some historical reasons now forgotten, PU now denotes the price of utility (welfare), PW is the price of unskilled labor, and the other factor is now called skilled labor, with a price of PZ.

The following block of code is inserted in order to suppress a lot of output in the listing file and you can certainly ignore it for now. But it is valuable in a program with many solve statements.

```
OPTION LIMROW=0;
OPTION LIMCOL=0;
$OFFSYMLIST OFFSYMXREF OFFUELLIST OFFUELXREF
```

Here is the model.

```
$TITLE: Model M61-MCP: External Economies of Scale, uses MCP $ONTEXT
```

The model is based on the benchmark social accounts for model M1-1:

	Prod	duction S	ectors	Consumers		
Markets	X	Y	W		CONS	
PX	100		-100			
PY		100	-100			
PU			200		-200	
PW	-40	-60			100	
PZ	-60	-40			100	

\$OFFTEXT

PARAMETERS

В

ENDOWS

ENDOWL

MODELSTAT;

```
ENDOWS = 100;
ENDOWL = 100;
B = 0.2;
POSITIVE VARIABLES
X
Y
W
PX
PΥ
PU
PZ
PW
CONS;
EQUATIONS
DX
         Demand for X
         Demand for Y
DY
DW
      Demand for W
         MR = MC in X
PRICEX
         Zero profit condition for Y (PY = MC)
PRICEY
         Zero profit condition for W
PRICEW
```

SKLAB Supply-demand balance for skilled labor UNLAB Supply-demand balance for unskilled labor INCOME National income;

PRICEX.. (PW**0.40)*(PZ**0.60)/(X**B) = G = PX;

PRICEY.. (PW**0.60)*(PZ**0.40) = G = PY;

PRICEW.. (PX**0.50)*(PY**0.50) =G= PU;

DX.. X*100 = E = CONS/(2*PX);

DY.. Y*100 = E = CONS/(2*PY);

DW.. 200*W = E = CONS/PU;

SKLAB.. ENDOWS =E= 0.40*(PW**0.60)*(PZ**(0.40-1))*Y*100+ 0.60*(PW**0.40)*(PZ**(0.60-1))*(X**(1-B))*100;

UNLAB.. ENDOWL =E= 0.60*(PW**(0.60-1))*(PZ**0.40)*Y*100+ 0.40*(PW**(0.40-1))*(PZ**0.60)*(X**(1-B))*100;

```
INCOME.. CONS = E = PZ * ENDOWS + PW * ENDOWL;
MODEL M61 /DX.PX, DY.PY, DW.PU, PRICEX.X, PRICEY.Y,
PRICEW.W,
           SKLAB.PZ, UNLAB.PW, INCOME.CONS/;
OPTION MCP=MILES;
OPTION LIMROW=0;
OPTION LIMCOL=0;
$OFFSYMLIST OFFSYMXREF OFFUELLIST OFFUELXREF
CONS.L = 200;
X.L = 1;
Y.L = 1;
W.L = 1;
PX.L = 1;
PY.L = 1;
PZ.L = 1;
PW.L = 1;
PU.L = 1;
```

```
PY.FX = 1;
SOLVE M61 USING MCP;
MODELSTAT = M61.MODELSTAT - 1.;
DISPLAY MODELSTAT;
        Counterfactual: expand the size of the economy
*
ENDOWS = 200;
ENDOWL = 200;
SOLVE M61 USING MCP;
        Counterfactual: contract the size of the economy
*
ENDOWS = 80;
ENDOWL = 80.
SOLVE M61 USING MCP;
```

The counterfactual-experiments involve changing the size of the economy. The first experiment doubles the size of the economy. This increases the welfare index from 1.000 to 2.181, more than double its initial value representing the scale economies in production.

Model 61-MPS Closed economy external economies of scale, MPS/GE version

The difficulty for translating this model into MPS/GE is that this higher-level language requires constant returns to scale in all activities so that it can generate cost functions and factor demands in a standardized way. There are a couple of tricks that get around this. They may seem awkward and not worth the bother in such a small model, but in big models with many sectors, factors, or countries it is well worth learning the tricks.

How do we model this given that MPS/GE requires constant returns to scale? We will use a trick, in which the X industry produces $X_0 = F(V_0)$ so that the X sector output is now the constant returns function F using factor bundles. Then we will "give" the consumer an additional amount equal to the "true" industry output minus the output from factor bundles.

(5)
$$\Box_1 = X_0^{1/(1-\beta)} - X_0$$

Then the consumer actually receives

(6)
$$X = X_1 + X_0 = F(V_0)^{1/(1-\beta)}$$
.

The value of X_1 is determined by the auxiliary variable XQADJ (X quantity adjustment = X_1) in the program that follows. It is important to note that the value of X listed in the output file of the program is what is called X_0 , not the true output of X.

But now we have an imbalance in that the value of X received by the consumer is more than the value of X produced by the firms. So we will subsidize X production so that the value of payments received by the firm for X_0 is equal to the value of $X = X_1 + X_0$ consumed by the consumer. Let q denote the consumer price and p the producer price of X. Let q be the tax base, so q(1-s) = p. For payments to balance, we need

(7)
$$q(X_0^{1/(1-\beta)}) = pX_0 \quad or \quad (1-s) = X_0^{\beta/(1-\beta)}$$

(8)
$$s = X_0^{\beta/(1-\beta)} - 1$$
 or $s = XQADJ/X_0$

s is determined in the model to follow by the auxiliary variable XPADJ.

The model is calibrated so that all activity levels are one initially, implying that the initial values of XQADJ and XPADJ are zero initially. $\beta = .2$, $\beta/(1-\beta) = .25$

Counterfactual experiments change the size of the economy. Notice the consequences of the scale economies.

\$TITLE: Model M61-MPS: External Economies of Scale, MPS/GE version

\$ONTEXT

The model is based on the benchmark social accounts for model M1-1:

	Prod	duction S	ectors	Consumers		
Markets	X	Y	W	CONS		
PX	100		-100			
PY		100	-100			
PU			200	-200		
PW	-40	-60		100		
PZ	-60	-40		100		

\$OFFTEXT

PARAMETER ENDOW Size index for the economy External economies parameter; В ENDOW = 1;B = 0.2\$ONTEXT \$MODEL:M61 \$SECTORS: ! Activity level for sector X X ! Activity level for sector Y

! Activity level for sector W (welfare index)

\$COMMODITIES:

PX ! Price index for commodity X PY ! Price index for commodity Y

PW ! Price index for primary factor L

PZ ! Price index for primary factor S

PU! Price index for welfare (expenditure

function)

\$CONSUMERS:

CONS ! Income level for consumer CONS

\$AUXILIARY:

XQADJ ! Quantity adjustment (positive when X>1)

XPADJ ! X output subsidy rate (positive when X>1)

\$PROD: X s:1

O:PX Q:100 A:CONS N:XPADJ M:-1

I:PW Q: 40

I:PZ Q: 60

\$PROD: Y s:1

O:PY Q:100

I:PW Q: 60

I:PZ Q: 40

\$PROD: W s:1

O: PU Q:200

I: PX Q:100

I: PY Q:100

\$DEMAND: CONS

D:PU Q:200

E:PW Q: (ENDOW*100)

E:PZ Q: (ENDOW*100)

E:PX Q:100 R:XQADJ

\$CONSTRAINT:XQADJ

XQADJ = E = X * * (1/(1-B)) - X;

\$CONSTRAINT:XPADJ

XPADJ * X = E = XQADJ;

```
$OFFTEXT
$SYSINCLUDE mpsgeset M61
        Adjust bounds so that the auxiliary variables can
        take on negative values:
XQADJ.LO = -INF;
XPADJ.LO = -INF;
        Benchmark replication
*
$INCLUDE M61.GEN
SOLVE M61 USING MCP;
        Counterfactual: expand the size of the economy
*
ENDOW = 2;
$INCLUDE M61.GEN
SOLVE M61 USING MCP;
```

* Counterfactual: contract the size of the economy

```
ENDOW = 0.8;
$INCLUDE M61.GEN
SOLVE M61 USING MCP;
```

Note again in comparing the results of the MCP and MPS/GE versions of the model that in the latter, the reported value of X denotes what we refer to above as X_0 , rather than the "true" value of X industry output given by

$$X = X_0^{1/(1-\beta)}$$

which X is reported by the MCP solver.

Model 62-MCP Closed economy large-group monopolistic competition, MCP version

Now we turn to a favorite model used extensively in the new trade theory, the Dixit-Stiglitz model of monopolistic competition. Y will continue to be a competitive, constant-returns industry while X will consist of an endogenous number of differentiated varieties. Utility of the representative consumer in each country is Cobb-Douglas, and the symmetry of varieties within a group of goods allows us to write utility as follows (0 $< \alpha < 1$).

(9)
$$U = X_c^{\beta} Y^{1-\beta}, \qquad X_c \equiv \left[\sum_i^N (X_i)^{\alpha}\right]^{1/\alpha}$$

where the number of varieties N is endogenous. This function permits the use of two-stage budgeting, in which the consumer in country i first allocates total income (M) between Y and X_c . Let X_c be as defined above, and let e denote the minimum cost of buying one unit of X_c at price p for the individual varieties (i.e., e is the unit expenditure function for X_c). Y is numeraire. First-stage budgeting yields:

(10)
$$Y = (1-\beta)M$$
 $X_c = \beta M/e$ $e(p^k) = \min(X_i) \sum_i pX_i$ st $X_c = 1$

Let $M_x = \beta M$ be the expenditure on X in aggregate. We need to solve for the demand for a given X variety, and for the price index e. The consumer's sub-problem maximizing the utility from X goods subject to an expenditure constraint (using λ as a Lagrangean multiplier) and first-order conditions are:

(11)

$$\max X_c = \left[\sum X_i^{\alpha}\right]^{\frac{1}{\alpha}} + \lambda (M_x - \sum p_i X_i) = \frac{1}{\alpha} \left[\sum X_i^{\alpha}\right]^{\frac{1}{\alpha} - 1} \alpha X_i^{\alpha - 1} - \lambda p_i = 0$$

Let σ denote the elasticity of substitution among varieties. Dividing the first-order condition for variety i by the one for variety j,

(12)
$$\left[\frac{X_i}{X_j} \right]^{\alpha - 1} = \frac{p_i}{p_i} \qquad \frac{X_i}{X_j} = \left[\frac{p_i}{p_j} \right]^{\frac{1}{\alpha - 1}} = \left[\frac{p_i}{p_j} \right]^{-\sigma} \qquad \text{since} \quad \sigma = \frac{1}{1 - \alpha}$$

(13)
$$X_{j} = \left[\frac{p_{i}}{p_{j}}\right]^{\sigma} X_{i} \qquad p_{j} X_{j} = p_{j} p_{j}^{-\sigma} p_{i}^{\sigma} X_{i} \qquad \sum p_{j} X_{j} = M_{x} = \left[\sum p_{j}^{1-\sigma}\right] p_{i}^{\sigma} X_{i}$$

Inverting this last equation, we have the demand for an individual variety i:

(14)
$$X_i = p_i^{-\sigma} \left[\sum p_j^{1-\sigma} \right]^{-1} M_x \qquad \sigma = \frac{1}{1-\alpha}, \quad \alpha = \frac{\sigma-1}{\sigma}$$

Now we can use X_i to construct X_c and then solve for e, noting the relationship between α and σ .

(15)
$$X_i^{\alpha} = X_i^{\frac{\sigma-1}{\sigma}} = p_i^{1-\sigma} \left[\sum p_j^{1-\sigma} \right]^{\frac{1-\sigma}{\sigma}} M_x^{\alpha}$$

(16)
$$\sum X_i^{\alpha} = \left[\sum p_i^{1-\sigma}\right] \left[\sum p_j^{1-\sigma}\right]^{\frac{1-\sigma}{\sigma}} M_x^{\alpha} = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{\sigma}} M_x^{\alpha}$$

(17)
$$X_c = \left[\sum X_i^{\alpha}\right]^{\frac{1}{\alpha}} = \left[\sum X_i^{\alpha}\right]^{\frac{\sigma}{\sigma-1}} = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{\sigma-1}} M_x$$

(18)
$$e = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

Having derived e, we can then use equation (13) in (9) to get the demand for an individual variety.

(19)
$$X_i = p_i^{-\sigma} e^{\sigma - 1} M_x \qquad since \qquad e^{\sigma - 1} = \left[\sum p_j^{1 - \sigma} \right]^{-1}$$

The usual assumption in "large-group" monopolistic competition is that there are many firms such that individual firms view e and M as constants. Thus the elasticity of demand for an individual variety is just σ . Equilibrium in the X sector involves two equations in two unknowns. The unknowns are X, output per variety and N, the numbers of varieties or firms. The two equations are the firm's optimization condition, marginal revenue equals marginal cost, and the free-entry or zero profit condition, prices equals average cost. Let c(w,z) denote marginal cost where w and z are the prices of skilled and unskilled labor respectively. Let F(w,z) denote fixed costs per firm. The two equations are given by

(20)
$$p(1-1/\sigma) = c(w,z)$$
 $p = c(w,z) + F(w,z)/X$

(21)
$$\frac{1}{(1-1/\sigma)} = 1 + \frac{F(w,z)}{c(w,z)X}$$

which simplifies to

(22)
$$X = (\sigma - 1) \frac{F(w, z)}{c(w, z)}$$

Another assumption that is typically made in the literature (often implicitly without realizing it) is that F and c have the same functional form, same factor intensities, etc. Under such assumptions, the right-hand side is a constant and does not depend on factor prices. I will use this assumption here. At initial factor prices, c = 1 and c = 1 in the MCP calibration, and so this ratio always equals 20 regardless of equilibrium factor prices. This is reflected in in the zero-profit condition for X below: the equation ZEROP is c = 1.

The starting data matrix is exactly the same as that for model M61. The variables are:

```
POSITIVE VARIABLES
Χ
          Output of an individual X variety
          Output of the Y industry
Υ
          Welfare or utility
W
          Number of varieties produced in equilibrium
Ν
Ε
          Cost of producing one unit of X<sub>c</sub> (unit expenditure
          function for X_c)
          Price of an individual X variety
PX
          Price of Y
PY
          Price of skilled labor
PZ
          Price of unskilled labor
PW
PU
          Price of a unit of utility (the real consumer price
          index)
          Consumer income (M in the notation above)
CONS;
```

The equations of the model are as follows, with complementary variables in parentheses.

EQUATIONS

ZEROP	Zero profits - free entry condition in X	(N)
PRICEY	Zero profit condition for Y	(Y)
PRICEW	Zero profit condition for W	(W)
PRICEX	MR = MC in X	(X)
INDEX	Price index for X sector goods	(E)
DX	Supply-demand balance for X	(PX)
DY	Supply-demand balance for Y	(PY)
DW	Supply-demand balance for utility W(welfare)	(PU)
SKLAB	Supply-demand balance for skilled labor	(PZ)
UNLAB	Supply-demand balance for unskilled labor	(PW)
INCOME	National income;	(CONS)

Here again we see the logic of associating zero profit or pricing equations with quantity variables, and market clearing conditions with price.

One thing that is slightly misleading is to call the ZEROP equation "zero profits in X". In the code to follow this is the equation for equilibrium output per firm in the X industry which is derived from using both the zero-profit condition and the pricing equation. This is not really necessary and we could just use the zero-profit condition here. The code of the model follows:

\$TITLE: Model M62-MCP: Large-Group Monopolistic

Competition: uses MCP

\$ONTEXT

PONTEVI		Production Sectors			Consumers		
Markets	XC	X	N	Y	W	CONS	ENTR
PX CX !	100	100 -100			-100	 	
PY				100	-100		
PF			20				-20
PU					200	-200	
PW	-32		-8	-60		100	
PΖ	-48		-12	-40		100	
MK	-20						20
						•	

\$OFFTEXT

```
PARAMETERS
EΡ
FC
ENDOWS
ENDOWL
WELFARE
MODELSTAT;
EP = 5;
FC = 20;
ENDOWS = 100;
ENDOWL = 100;
POSITIVE VARIABLES
Χ
Y
W
Ν
Ε
PΧ
PΥ
```

```
PZ
PW
PU
CONS;
EQUATIONS
       Zero profits free entry condition (associated with N)
ZEROP
PRICEY Zero profit condition for Y (PY = MC)
PRICEW Zero profit condition for W (PU = MC of utility)
PRICEX MR = MC in X (associated with X, output per firm)
INDEX Price index for X sector goods (expenditure function)
DX
       Supply-demand balance for X
       Supply-demand balance for Y
DY
DW
      Supply-demand balance for utility W(welfare)
SKLAB Supply-demand balance for skilled labor
      Supply-demand balance for unskilled labor
UNLAB
INCOME National income;
DX..
          X*80 = E = PX**(-EP)*(E**(EP-1))*CONS/2;
DY.. Y*100 = E = CONS/(2*PY);
```

```
DW.. 200*W = E = (1.25**0.5)*CONS/PU;
ZEROP.. FC*(EP-1) = G = X*80;
PRICEX.. (PW**0.4)*(PZ**0.6) = G = PX*(1-1/EP);
PRICEY.. (PW**0.60)*(PZ**0.40) = G = PY;
PRICEW.. (E^{*}0.5)*(PY^{*}0.5) = G = PU;
INDEX.. E = E = (N*PX**(1-EP))**(1/(1-EP));
INCOME.. CONS =E = PZ*ENDOWS + PW*ENDOWL;
SKLAB.. ENDOWS = E = 0.40*(PW**0.60)*(PZ**(0.40-1))*Y*100
               + 0.6*(PW**0.4)*(PZ**(0.6-1))*N*X*80
               + 0.6*(PW**0.4)*(PZ**(0.6-1))*N*FC;
```

```
UNLAB.. ENDOWL =E= 0.60*(PW**(0.60-1))*(PZ**0.40)*Y*100
               + 0.4*(PW**(0.4-1))*(PZ**0.6)*N*X*80
               + 0.4*(PW**(0.4-1))*(PZ**0.6)*N*FC;
MODEL M62 / PRICEX.X, PRICEY.Y, PRICEW.W, ZEROP.N,
      DX.PX, DY.PY, DW.PU,
           SKLAB.PZ, UNLAB.PW, INDEX.E, INCOME.CONS/;
OPTION MCP=MILES;
OPTION LIMROW=0;
OPTION LIMCOL=0;
$OFFSYMLIST OFFSYMXREF OFFUELLIST OFFUELXREF
E.L = 1.25;
CONS.L = 200;
X.L = 1;
Y.L = 1;
N.L = 1;
W.L = 1;
PX.L = 1.25;
```

```
PY.L = 1;
PZ.L = 1;
PW.L = 1;
PU.L = 1.25**0.5;
PY.FX = 1;
SOLVE M62 USING MCP;
MODELSTAT = M62.MODELSTAT - 1.;
DISPLAY MODELSTAT;
        Counterfactual: expand the size of the economy
ENDOWS = 200;
ENDOWL = 200;
SOLVE M62 USING MCP;
```

The counter-factual experiment doubles the size of the economy. Note that the results are exactly the same as in the external-economies model M61 (although remember that here X is output per firm whereas earlier it referred to total output). These models are in fact operationally identical, as I showed in my 1990 CJE article referenced earlier. In both cases, the X sector's output is homogeneous of degree 1.25 in factor inputs, if by X sector's output here we mean X_c . The X sector expands only through the entry of new firms and X_c is given by

$$X_c = n^{1/\alpha}X$$
 and in the external economies model by $X = F(V)^{\frac{1}{1-\beta}}$

where X in the first equation is a constant. Thus $\alpha = 0.8$ (s = 5) in this model is exactly equivalent to the external economies model with $\beta = 0.2$.

Model 62-MPS Closed economy large-group monopolistic competition, MPS/GE verison

In the case of "large-group monopolistic competition" where markups are assumed to be fixed, we have seen that firms produce at a fixed scale if variable and fixed costs use factors in the same proportion (this point is almost never recognized in the literature). So we could view the industry production function as producing at constant scale, adding new goods instead of more of existing goods. But then we run into trouble modelling preferences, which must have constant returns to scale in MPS/GE. Doubling industry output means more than doubling utility taking into account the value of increased product diversity.

This is the same problem that we ran into with the external economies model M61 in trying to code it into MPS/GE. We will get around this problem by constructing the MPS/GE model using the tricks of the oligopoly model with free entry of chapter 5 and the external economies mode just mentioned. First, there is a "dummy" good called CX, which is produced with constant marginal cost and a markup is assigned to entrepreneurs just as in the oligopoly model. These entrepreneurs "demand" fixed costs just like in the oligopoly model, and the activity level of fixed costs (N) is interpreted as the number of firms active in equilibrium.

In the "large-group" monopolistic-competition case, the markup is given by $1/\sigma$ where σ is the elasticity of substitution among the differentiated goods. In the present example, this elasticity is equal to 5, so the markup is 0.20, and we just treat this as a tax on X production with the revenue assigned to the representative agent ENTRE. This agent demands fixed costs as in our earlier oligopoly model and the activity level for the production of fixed costs is interpreted as the number of firms active in equilibrium (units will be chosen such that N=1 initially; we did not do this in the oligopoly model because the number of firms appears in the markup formula).

But then we have to deal with the consumption side, and this is dealt with in the same fashion as in the external economies case. Let $X = NX_i$ where N is the number of firms (products) and X_i is the output per firm. X_c is defined as in the MCP model.

(23)
$$X_c = \left[\sum X_i^{\alpha}\right]^{1/\alpha} = \left[NX_i^{\alpha}\right]^{1/\alpha} = N^{1/\alpha}X_i = N^{(1-\alpha)/\alpha}NX_i = N^{(1-\alpha)/\alpha}X$$

Now we can use the trick from the external economies model. The X industry produces $X = (NX_i)$, but consumers receive X_c . So we can "give" the consumer (expand the consumer's endowment by the amount):

$$(24) XQADJ = N^{(1-\alpha)/\alpha}X$$

so that the consumer will receive the correct amount of utility from X and therefore demand the correct amount of X at equilibrium prices.

Finally, we have the same problem as in the external economies case, we must have the value of X received by the consumer equal to the payments received by the producer. Therefore, the consumer has to subsidize the producer of X so that producer revenue equals the payments for X made by the producer.

As in the external economies model, let q be the consumer price, and p be the producer price. We must have

$$q[N^{(1-\alpha)/\alpha}X] = pX = q(1-s)X$$

So we must have

$$s = N^{(1-\alpha)/\alpha} - 1$$

This is given by the endogenous tax rate XPADJ (in the N: field) in the model to follow. Since $\alpha = 0.8$, $\sigma = 5$, the value of s is $s = .25 = (1 - \alpha)/\alpha$.

To avoid having an ad valorem subsidy multiplied on top of an ad valorem tax (the markup in activity XI), we just specify another activity, simply called X. X produces one unit of "final good" PX for each unit of "intermediate good "CX". PX is the good that enters welfare and demand.

The counter-factual experiment in the program that follows doubles the size of the economy. The activity levels for X, XI, and N all double, but welfare more than doubles, reflecting the increased value of product diversity (W = 2.18). For comparison to the MCP version of this model, I also calculate the price index, e, for X_c which is generated after the model solves. This is declared as the parameter INDEX.

\$TITLE: Model M62-MPS: Large-Group Monopolistic Competition, uses MPS/GE

\$ONTEXT

				Production	Co	Consumers			
Markets		ΧI	X	N	Y	W		CONS	ENTR
PX			100			-100			
CX	!	100	-100						
PY					100	-100			
PF				20					-20
PU						200		-200	
PW		-32		- 8	-60			100	
PZ		-48		-12	-40			100	
MK		-20							20

\$OFFTEXT

```
PARAMETERS
  ENDOW Size index for the economy
  INDEX Price index for the X goods
         Elasticity of substitution among X varieties;
  EP
ENDOW = 1;
EP = 5;
$ONTEXT
$MODEL:M62
$SECTORS:
  X
          ! Activity level for sector X
          ! Activity level for sector Y
  Υ
          ! Activity level for sector W (welfare index)
  W
          ! Activity level for fixed costs, no. of firms
 N
          ! Activity level -- marginal cost of X
  ΧI
```

\$COMMODITIES:

PX ! Price index for commodity X (gross of markup)

CX ! Marginal cost index for commodity X (net markup)

PY ! Price index for commodity Y

PW ! Price index for unskilled labor

PZ ! Price index for skilled labor

PF! Unit price of inputs to fixed cost

PU! Price index for welfare (expenditure function)

\$CONSUMERS:

CONS ! Income level for consumer CONS

ENTRE! Entrepreneur (converts markup rev to fixed cost)

\$AUXILIARY:

XQADJ ! Quantity adjustment (positive when X>1)

XPADJ ! X output subsidy rate (positive when X>1)

\$PROD:X s:1

O:PX Q: 80 P:1.25 A:CONS N:XPADJ M:-1

I:CX Q: 80 P:1.25

\$PROD:Y s:1

O:PY Q:100

I:PW Q: 60

I:PZ Q: 40

\$PROD:XI s:1

O:CX Q: 80 A:ENTRE T:0.20

I:PW Q: 32

I:PZ Q: 48

\$PROD:N s:1

O:PF Q:20

I:PZ Q:12

I:PW Q: 8

\$PROD:W s:1.0

O:PU 0:200

I:PX Q: 80 P:1.25

I:PY Q:100

\$DEMAND: CONS

D:PU Q:200

E:PW Q: (100*ENDOW)

E:PZ Q: (100*ENDOW)

E:PX Q:80 R:XQADJ

\$DEMAND: ENTRE

D:PF Q:20

\$CONSTRAINT:XQADJ

XQADJ = E = (N**(1/(EP-1)))*X - X;

\$CONSTRAINT:XPADJ

XPADJ = E = (N**(1/(EP-1))) - 1;

\$OFFTEXT

\$SYSINCLUDE mpsgeset M62

```
Adjust bounds so that the auxiliary variables can
*
        negative values:
*
XOADJ.LO = -INF;
XPADJ.LO = -INF;
        Benchmark replication:
PY.FX = 1;
PX.L = 1.25;
CX.L = 1.25;
$INCLUDE M62.GEN
SOLVE M62 USING MCP;
INDEX = (N.L*CX.L**(1-EP))**(1/(1-EP));
DISPLAY INDEX;
*
        Counterfactual: expand the size of the economy
ENDOW = 2;
```

```
$INCLUDE M62.GEN
SOLVE M62 USING MCP;

INDEX = (N.L*CX.L**(1-EP))**(1/(1-EP));
DISPLAY INDEX;
```

Model 63-MCP Two-country model with large-group monopolistic competition, MCP version

Now I present a two-country version of the monopolistic-competition model. This model has been popular in trade theory and in the so-called new economic geography literature. The countries will be labeled with subscripts i and j. X's will carry double subscripts where the first is the country of production and the second is the country of consumption. Thus X_{ij} is X produced in i and exported to j where it is consumed.

Trade costs are important in the literatures just referred to, so we will want to consider these in our model. It has been popular to use "iceberg" trade costs where some of the good "melts" in transit. The first thing to note is how iceberg trade costs are reflected in prices and demand. For a domestic firm, X_{ij} is the amount produced in country i and shipped to country j. Let t (t > 1) be the ratio of the amount of X exported to the amount that arrives "unmelted". Alternatively 1/t is the proportion of a good that "survives" transit (the proportion "unmelted"). If X_{ij} is shipped, the amount received in country j is X_{ij}/t .

Second, we make the usual assumption that there is no price discrimination and so the home price of a good for local sales equals its export price. Thus we can use the notation p_i and p_i for the price of all goods *produced* in country i and country j

respectively. The revenues received by the exporter are equal to the costs paid by the importer: $p_i X_{ij}$ is the revenue received by the exporter and X_{ij}/t are the number of units arriving in the importing country, so the price per unit in the importing country must be $p_i t$ ($p_i X_{ij} = (p_i t) X_{ij}/t$). Rather than introduce additional notation, we will therefore use X_{ij}/t and $p_i t$ as the quantity and price in country j of a country i variety exported to country j.

The price index for country i is then given by:

$$e_{i} = \left[N_{i}p_{i}^{1-\sigma} + N_{j}(p_{j}t)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$
(15)

The various X varieties sold in country i are given by:

$$X_{ii} = p_i^{-\sigma} e_i^{\sigma-1} M_{ix} X_{ji} / t = (p_j t)^{-\sigma} e_i^{\sigma-1} M_{ix} (16)$$

where the second equation can also be written as:

$$X_{ji} = p_j^{-\sigma} t^{1-\sigma} e_i^{\sigma-1} M_{ix}$$
 (17)

There are few other complications beyond those discussed in the closed economy model M62-MCP. The model is calibrated to trade free with the countries identical initially. Since X is differentiated, it must therefore be the case that half of each firm's output is sold in its home country and half is exported. There are some numerical coefficients in the problem used to adjust units so that most prices are one initially; this is not necessary but makes the output file more comparable to the MPS/GE version that follows. This arises because I set the number of firms in each country to 1 initially, so that the total number is 2. But if the price of an individual variety is 1, then the price index in each country cannot be equal to 1. I thus use a units adjustment in the equations for WELFAREI, WELFAREJ, PUI and PUJ in order that these all take the value of 1 initially.

Several counterfactual experiments are considered at the end. First, we double the size of the economies and see that there welfare more than doubles. In the second experiment, we return the economy to its original size but set the trade cost at TC = 1.15. This reduces welfare by 3% in each country as consumption of home versus imported goods becomes more unbalanced. The third experiment has country i twice as large as country j, but no trade costs. Welfare is the same for both countries (per capita) in free

trade.

The fourth experiment has country i three times the size of j, but with trade costs TC = 1.15. Here we see some results found in the newer theory literature. There is a "home market effect" whereby country i is better off and has more firms on a per capita basis than country j. At the end of this experiment, we calculate real factor prices in each country by dividing the nominal prices by the real consumer prices indices, the cost of buying one unit of utility. We see that in spite of the countries having identical relative endowments, country i has a higher real price for skilled labor and country j has a higher real price for unskilled labor. Thus this equilibrium is not going to be stable under factor mobility, and skilled labor should flow toward country i increasing further its specialization in X.

The final experiment retains the trade costs, gives both countries the same amount of unskilled labor, but country i has 60% of the skilled labor and country j 40%. If you look at the very end of the listing file, you see that country i has higher real prices for both factors, an outcome noted by Markusen and Venables (JIE 2000).

\$TITLE: Model M63: Two Country Large-Group Monopolistic Competition

* uses MCP

\$ontext

	YI	YJ	IMX	ZMJ	NMI	UMU	WI	ТW	CONI	CONJ	EHTI	ENTJ
DVT	100						100					
PYI	100						-100					
PYJ		100						-100				
PXI			100				-50	-50				
PXJ				100			-50	-50				
FCI					20						-20	
FCJ						20						-20
ZI	-40		-48		-12				100			
ZJ		-40		-48		-12				100		
WI	-60		-32		- 8				100			
ТV		-60		-32		-8				100		
UTILI							200		-200			
UTILJ								200		-200		
MKI			-10	-10							10	10
MKJ			-10	-10							10	10

\$offtext

```
PARAMETERS
EΡ
ΕY
TC
FC
ENDOWIS
ENDOWIL
ENDOWJS
ENDOWJL
MODELSTAT;
EP = 5;
EY = 3;
TC = 1.;
FC = 20;
ENDOWIS = 100;
ENDOWIL = 100;
ENDOWJS = 100;
ENDOWJL = 100;
```

POSITIVE VARIABLES

WFI

WFJ

XII

XIJ

XJJ

XJI

ΥI

ΥJ

NI

NJ

ΡI

РJ

ΡY

PUI

PUJ

EI

ΕJ

ZI

 \mathtt{WI}

ZJ

WJ

```
MΙ
```

MJ;

EQUATIONS

WELFAREI

WELFAREJ

DXII

DXJI

DXJJ

DXIJ

DY

ZEROPI

ZEROPJ

PRICEI

PRICEJ

PRICYI

PRICYJ

PRICEUI

PRICEUJ

DWJ

INDEXI

INDEXJ

```
EXPT
EXPJ
SKTABT
UNTABT
SKLABJ
UNLABJ;
WELFAREI.. 200*WFI =E=
((2**(1/(1-EP))*1.25)**0.5)*MI/(1.025*PUI);
WELFAREJ.. 200*WFJ =E=
((2**(1/(1-EP))*1.25)**0.5)*MJ/(1.025*PUJ);
           XII*40 = E = PI**(-EP)*(EI**(EP-1))*MI/2;
DXII..
DXJI..
            XJI*40/TC = E = (PJ*TC)**(-EP)*(EI**(EP-1))*MI/2;
DXJJ..
            XJJ*40 = E = PJ**(-EP)*(EJ**(EP-1))*MJ/2;
            XIJ*40/TC = E = (PI*TC)**(-EP)*(EJ**(EP-1))*MJ/2;
DXIJ..
DY.. YI*100 + YJ*100 = E = MI/(2*PY) + MJ/(2*PY);
```

```
ZEROPI.. FC*(EP-1) = G= XII*40 + XIJ*40;
ZEROPJ.. FC*(EP-1) = G = XJJ*40 + XJI*40;
PRICEI.. (WI**0.4)*(ZI**0.6) = G = PI*(1-1/EP);
PRICEJ.. (WJ**0.4)*(ZJ**0.6) = G = PJ*(1-1/EP);
PRICYI.. (WI**0.60)*(ZI**0.40) = G = PY;
PRICYJ... (WJ**0.60)*(ZJ**0.40) = G= PY;
PRICEUI.. (EI**0.5)*(PY**0.5)/1.025 =G= PUI;
PRICEUJ.. (EJ**0.5)*(PY**0.5)/1.025 = G = PUJ;
INDEXI.. EI =E= (NI*PI**(1-EP) +
         NJ*(PJ*TC)**(1-EP))**(1/(1-EP));
INDEXJ.. EJ =E= (NI*(PI*TC)**(1-EP) +
             NJ*PJ**(1-EP))**(1/(1-EP));
```

```
EXPI.. MI = E = ZI * ENDOWIS + WI * ENDOWIL;
EXPJ.. MJ = E = ZJ*ENDOWJS + WJ*ENDOWJL;
SKLABI.. ENDOWIS = E=
        0.40*(WI**0.60)*(ZI**(0.40-1))*YI*100
      + 0.6*(WI**0.4)*(ZI**(0.6-1))*NI*((XII+XIJ)*40 + FC);
UNLABI.. ENDOWIL =E=
        0.60*(WI**(0.60-1))*(ZI**0.40)*YI*100
       + 0.4*(WI**(0.4-1))*(ZI**0.6)*NI*((XII+XIJ)*40 + FC);
SKLABJ.. ENDOWJS = E =
         0.40*(WJ**0.60)*(ZJ**(0.40-1))*YJ*100
       + 0.6*(WJ**0.4)*(ZJ**(0.6-1))*NJ*((XJJ+XJI)*40 + FC);
UNLABJ.. ENDOWJL =E=
```

```
0.60*(WJ**(0.60-1))*(ZJ**0.40)*YJ*100
       + 0.4*(WJ**(0.4-1))*(ZJ**0.6)*NJ*((XJJ+XJI)*40 + FC);
MODEL M63 / WELFAREI.WFI, WELFAREJ.WFJ,
              PRICYI.YI, PRICYJ.YJ, DXII.XII, DXJI.XJI,
              DXJJ.XJJ, DXIJ.XIJ, DY.PY,
              ZEROPI.NI, ZEROPJ.NJ,
              PRICEI.PI, PRICEJ.PJ,
              PRICEUI.PUI, PRICEUJ.PUJ,
              SKLABI.ZI, SKLABJ.ZJ, UNLABI.WI,
              UNLABJ.WJ, INDEXI.EI, INDEXJ.EJ, EXPI.MI,
              EXPJ.MJ/;
OPTION MCP=MILES;
OPTION LIMROW=0;
OPTION LIMCOL=0;
$OFFSYMLIST OFFSYMXREF OFFUELLIST OFFUELXREF
WFI.L = 1;
WFJ.L = 1;
PUI.L = 1.25**0.5;
```

```
PUJ.L = 1.25**0.5;
EI.L = 1;
EJ.L = 1;
MI.L = 200;
MJ.L = 200;
XII.L = 1;
XIJ.L = 1;
XJJ.L = 1;
XJI.L = 1;
YI.L = 1;
YJ.L = 1;
NI.L = 1;
NJ.L = 1;
PI.L = 1.25;
PJ.L = 1.25;
PY.L = 1;
ZI.L = 1;
WI.L = 1;
ZJ.L = 1;
WJ.L = 1;
```

```
PY.FX = 1;
TC = 1.;
SOLVE M63 USING MCP;
MODELSTAT = M63.MODELSTAT - 1.;
ENDOWIS = 200;
ENDOWIL = 200;
ENDOWJS = 200;
ENDOWJL = 200;
SOLVE M63 USING MCP;
TC = 1.15;
ENDOWIS = 100;
ENDOWIL = 100;
ENDOWJS = 100;
ENDOWJL = 100;
```

```
SOLVE M63 USING MCP;
TC = 1.0;
ENDOWIS = 150;
ENDOWIL = 150;
ENDOWJS = 50;
ENDOWJL = 50;
SOLVE M63 USING MCP;
TC = 1.15;
ENDOWIS = 150;
ENDOWIL = 150;
ENDOWJS = 50;
ENDOWJL = 50;
SOLVE M63 USING MCP;
```

```
REALWI = WI.L/PUI.L;
REALWJ = WJ.L/PUJ.L;
REALZI = ZI.L/PUI.L;
REALZJ = ZJ.L/PUJ.L;
DISPLAY REALWI, REALWJ, REALZI, REALZJ;
ENDOWIS = 120;
ENDOWIL = 100;
ENDOWJS = 80;
ENDOWJL = 100;
SOLVE M63 USING MCP;
REALWI = WI.L/PUI.L;
REALWJ = WJ.L/PUJ.L;
REALZI = ZI.L/PUI.L;
REALZJ = ZJ.L/PUJ.L;
DISPLAY REALWI, REALWJ, REALZI, REALZJ;
```

Model 63-MPS Two-country model with large-group monopolistic competition, MPS/GE version

This is the MPS/GE version of the same two-country monopolistic-competition model. At this point, I don't think it requires much additional explanation. Note how iceberg trade costs conveniently fit into the production blocks for export sales.

```
$PROD:XIJ
O:PXIJ Q:(40./TC) A:CONSJ N:SUBI M:-1.0
I:PXI Q:(40.)
```

If 40 units are sent out, then 40/TC arrive unmelted.

It is also important to remember this in the XQADJ constraints. X_{ij} is the amount exported from i under iceberg trade costs, but only X_{ij} /TC is received in country j.

```
$CONSTRAINT:XQADJIJ

XQADJIJ =E= (NI**(.25))*XIJ/TC - XIJ/TC;
```

\$TITLE: Model M63: Two Country Large-Group Monopolistic Competition

* uses MPS/GE

\$ontext

	YI	YJ	XI	ХJ	NI	NJ	WI	UM	CONI	CONJ	EHTI	ENTJ
DVT	100						1.00					
PYI	100						-100					
PYJ		100						-100				
PXI			100				-50	-50				
PXJ				100			-50	-50				
FCI					20						-20	
FCJ						20						-20
ZI	-40		-48		-12				100			
ZJ		-40		-48		-12				100		
WI	-60		-32		- 8				100			
WJ		-60		-32		-8				100		
UTILI							200		-200			
UTILJ								200		-200		
MKI			-10	-10							10	10
MKJ			-10	-10							10	10

\$offtext

PARAMETERS

ENDOWIL

```
ENDOWJL
 ENDOWIS
 ENDOWJS
 TC
 SUBSIDY
 SIGMA
 REALWI
 REALWJ
 REALZI
 REALZJ;
ENDOWIL = 1.;
ENDOWJL = 1.;
ENDOWIS = 1.;
ENDOWJS = 1.;
TC = 1;
SUBSIDY = 0;
```

```
SIGMA = 5;
```

\$ONTEXT

\$MODEL:M63

\$SECTORS:

WFI WFJ

ΧI

XII

XIJ

ХJ

XJI

XJJ

YI YJ

NI NJ

\$COMMODITIES: PΥ PUI PUJ WI WJ ZIZJPXII PXJI PXJJ PXIJ PXI PXJ FCI FCJ \$CONSUMERS: CONSI CONSJ ENTI ENTJ \$AUXILIARY: XQADJII XQADJJI XQADJJJ XQADJIJ XPADJI

XPADJJ

\$PROD:YI s:1.0

O:PY Q:100.0

I:WI Q: 60.0

I:ZI Q: 40.0

\$PROD:YJ s:1.0

O:PY Q:100.0

I:WJ Q: 60.0

I:ZJ Q: 40.0

\$PROD:XI s:1.0

O:PXI Q:80. A:ENTI T:.20

I:ZI Q:48

I:WI Q:32

\$PROD:XII

O:PXII Q:40. A:CONSI N:XPADJI M:-1.0

I:PXI Q:40.

\$PROD:XIJ

O:PXIJ Q:(40./TC) A:CONSJ N:XPADJI M:-1.0

I:PXI Q: (40.)

\$PROD:XJ s:1.0

O:PXJ Q:80. A:ENTJ T:.20

I:ZJ Q:48

I:WJ Q:32

\$PROD:XJI

O:PXJI Q:(40./TC) A:CONSI N:XPADJJ M:-1.0

I:PXJ Q:(40.)

\$PROD:XJJ

O:PXJJ O:40. A:CONSJ N:XPADJJ M:-1.0

I:PXJ 0:40.

\$PROD:NI s:1.0

I:ZI Q:12

I:WI Q:8

```
$PROD:NJ s:1.0
O:FCJ
      Q:20
I:ZJ
     0:12
     Q:8
I:WJ
$PROD:WFI s:1.0 a:5.0
     Q:200.
O:PUI
I:PXII Q:40. P:1.25 a:
I:PXJI Q:40 P:1.25 a:
I:PY
     Q:100.
$PROD:WFJ s:1.0 a:5.0
O:PUJ
      Q:200.
I:PXJJ 0:40. P:1.25 a:
I:PXIJ 0:40. P:1.25
                      a:
     0:100.
I:PY
$DEMAND: CONSI
D:PUI
E:WI
    O: (100*ENDOWIL)
    Q: (100*ENDOWIS)
E:ZI
E:PXII
      O:40 R:XOADJII
```

```
E:PXJI Q:40 R:XQADJJI
$DEMAND: CONSJ
D:PUJ
E:WJ O:(100*ENDOWJL)
E:ZJ Q: (100*ENDOWJS)
E:PXIJ Q:40 R:XQADJIJ
E:PXJJ Q:40 R:XQADJJJ
$DEMAND: ENTI
D:FCI
$DEMAND: ENTJ
D:FCJ
$CONSTRAINT:XQADJII
XQADJII = E = (NI * * (.25)) * XII - XII;
$CONSTRAINT:XQADJJI
XQADJJI = E = (NJ**(.25))*XJI/TC - XJI/TC;
```

```
$CONSTRAINT:XQADJJJ
 XOADJJJ = E = (NJ * * (.25)) * XJJ - XJJ;
$CONSTRAINT:XOADJIJ
 XQADJIJ = E = (NI * * (.25)) * XIJ/TC - XIJ/TC;
$CONSTRAINT:XPADJI
 XPADJI = E = NI * * .25 - 1;
$CONSTRAINT: XPADJJ
 XPADJJ = E = NJ**.25 - 1;
$OFFTEXT
$SYSINCLUDE MPSGESET M63
XQADJII.LO = -INF;
XQADJIJ.LO = -INF;
XQADJJI.LO = -INF;
XQADJJJ.LO = -INF;
XPADJI.LO = -INF;
XPADJJ.LO = -INF;
```

```
PY.FX = 1;
PXII.L = 1.25;
PXIJ.L = 1.25;
PXJJ.L = 1.25;
PXJI.L = 1.25;
PXI.L = 1.25;
PXJ.L = 1.25;
XII.L = 1;
XIJ.L = 1;
XJJ.L = 1;
XJI.L = 1;
M63.ITERLIM = 5000;
OPTION LIMROW=0;
OPTION LIMCOL=0;
$OFFSYMLIST OFFSYMXREF OFFUELLIST OFFUELXREF
$INCLUDE M63.GEN
```

SOLVE M63 USING MCP;

```
*counterfactual: double size of world economy
ENDOWIL = 2;
ENDOWJL = 2;
ENDOWIS = 2;
ENDOWJS = 2;
$INCLUDE M63.GEN
SOLVE M63 USING MCP;
TC = 1.15;
ENDOWIL = 1;
ENDOWJL = 1;
ENDOWIS = 1;
ENDOWJS = 1;
$INCLUDE M63.GEN
SOLVE M63 USING MCP;
```

```
TC = 1.0;
ENDOWIL = 1.5;
ENDOWJL = .5;
ENDOWIS = 1.5;
ENDOWJS = .5;
$INCLUDE M63.GEN
SOLVE M63 USING MCP;
TC = 1.15;
ENDOWIL = 1.5;
ENDOWIS = 1.5;
ENDOWJL = 0.5;
ENDOWJS = 0.5;
$INCLUDE M63.GEN
SOLVE M63 USING MCP;
```

```
REALWI = WI.L/PUI.L;
REALWJ = WJ.L/PUJ.L;
REALZI = ZI.L/PUI.L;
REALZJ = ZJ.L/PUJ.L;
DISPLAY REALWI, REALWJ, REALZI, REALZJ;
ENDOWIL = 1.0;
ENDOWIS = 1.2;
ENDOWJL = 1.0;
ENDOWJS = 0.8;
$INCLUDE M63.GEN
SOLVE M63 USING MCP;
REALWI = WI.L/PUI.L;
REALWJ = WJ.L/PUJ.L;
REALZI = ZI.L/PUI.L;
REALZJ = ZJ.L/PUJ.L;
DISPLAY REALWI, REALWJ, REALZI, REALZJ;
```