Lecture 1: Review of Production Theory

James R. Markusen, University of Colorado, Boulder

1. Production: Functions, Set, Frontier: Transformation Function

3. Production Frontier (2 good case)

(A) Position
- Factor endowments
- Real factor productivities (technology)
- Scale economies

(B) Slope
- Relative factor productivities
- Relative endowments
- Scale Economies

(C) Curvature
- Factor intensity effects
- Scale economies

4. Competitive Equilibrium

5. The gains from trade theorem

Production function:

mapping from inputs to outputs (inputs can include produced intermediates)

Production set:

set of feasible production points. depends on technology and supplies of factor inputs.

Production frontier:

boundary set of the production set: maximum output of one good for a given outputs of other goods.

Transformation function:

the production frontier expressed as a continuous function.

Two-good, one-factor case

economic size of an economy

slope of the frontier (comparative advantage)

curvature of the frontier

$$Y = \beta L_y \qquad X = \alpha L_x^{\gamma} \qquad L^* = L_y + L_x$$

Consider first CRS: $\gamma = 1$

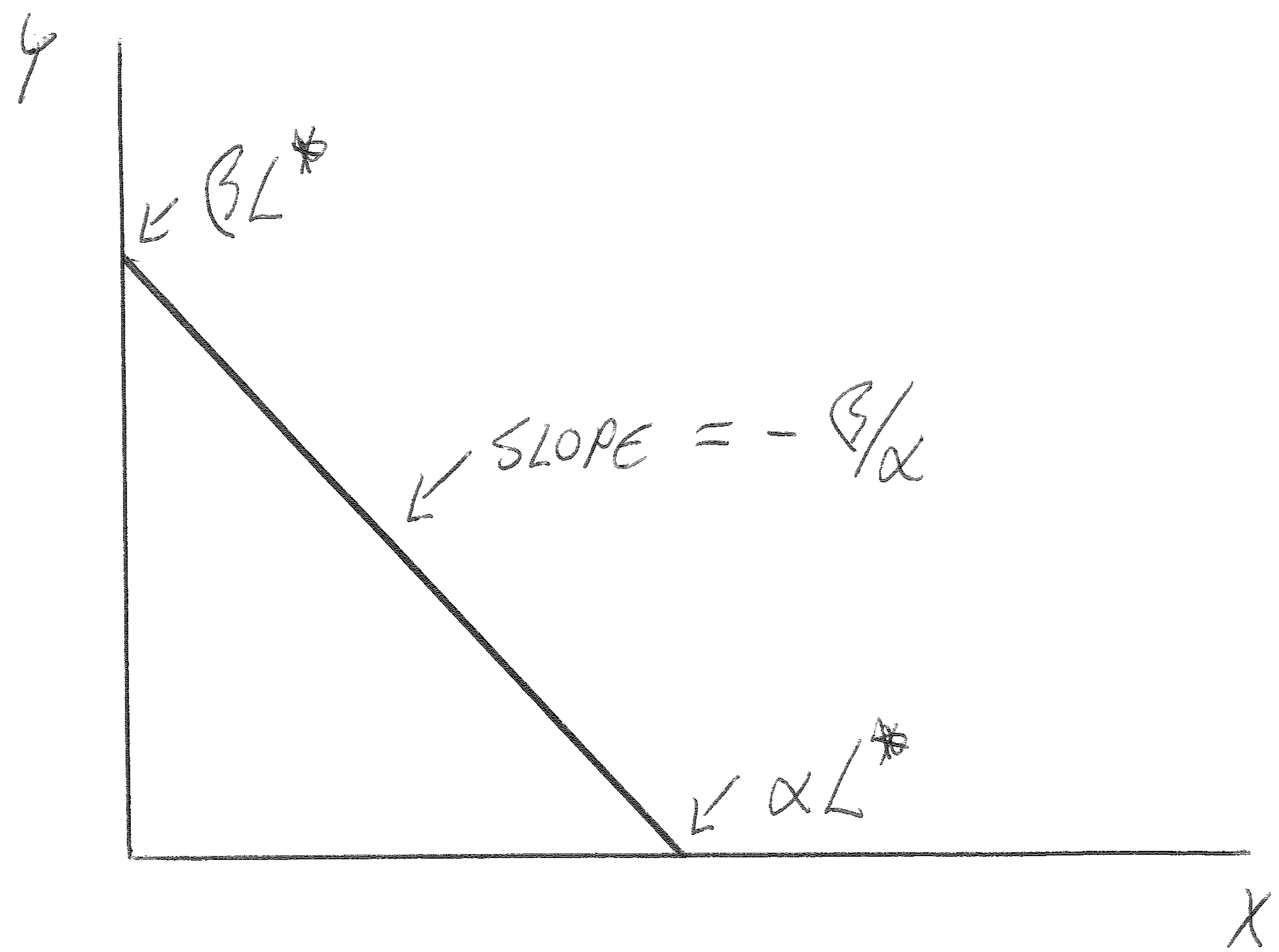
Economic size of the economy, depends on the total endowment and on factor productivity. Endpoints of the frontier are:

$$Y^* = \beta L^* \qquad X^* = \alpha L^*$$

2. Slope of the production frontier depends of <u>relative</u> factor productivities

$$dY = \beta dL_y = -\beta dL_x \qquad dX = \alpha dL_x \qquad \frac{dY}{dX} = -\frac{\beta}{\alpha}$$

3. Curvature of the production frontier: In the case of constant returns and one factor, the production frontier is linear.



Consider second IRS: $\gamma > 1$

(1) Economic size of the economy, depends on the total endowment and on factor productivity. Endpoints of the frontier are:

$$Y^* = \beta L^* \qquad X^* = \alpha L^{*\gamma}$$

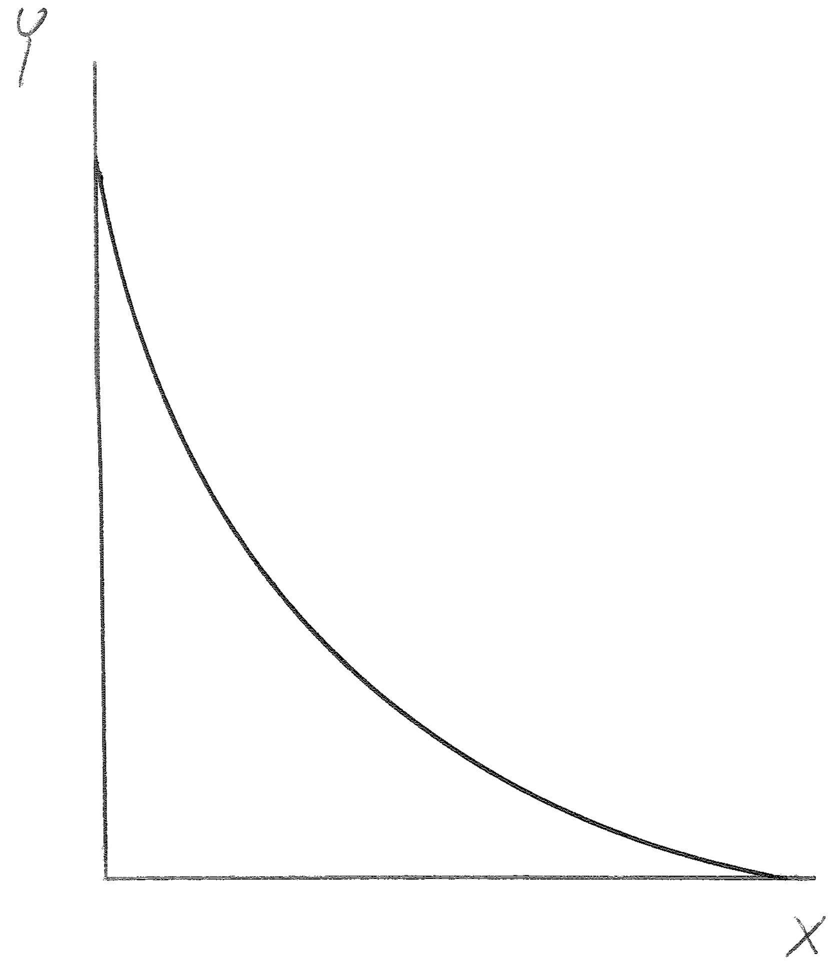
maximum output of Y is homogeneous of degree 1 in L*, maximum output of X is homogeneous of degree γ in L*

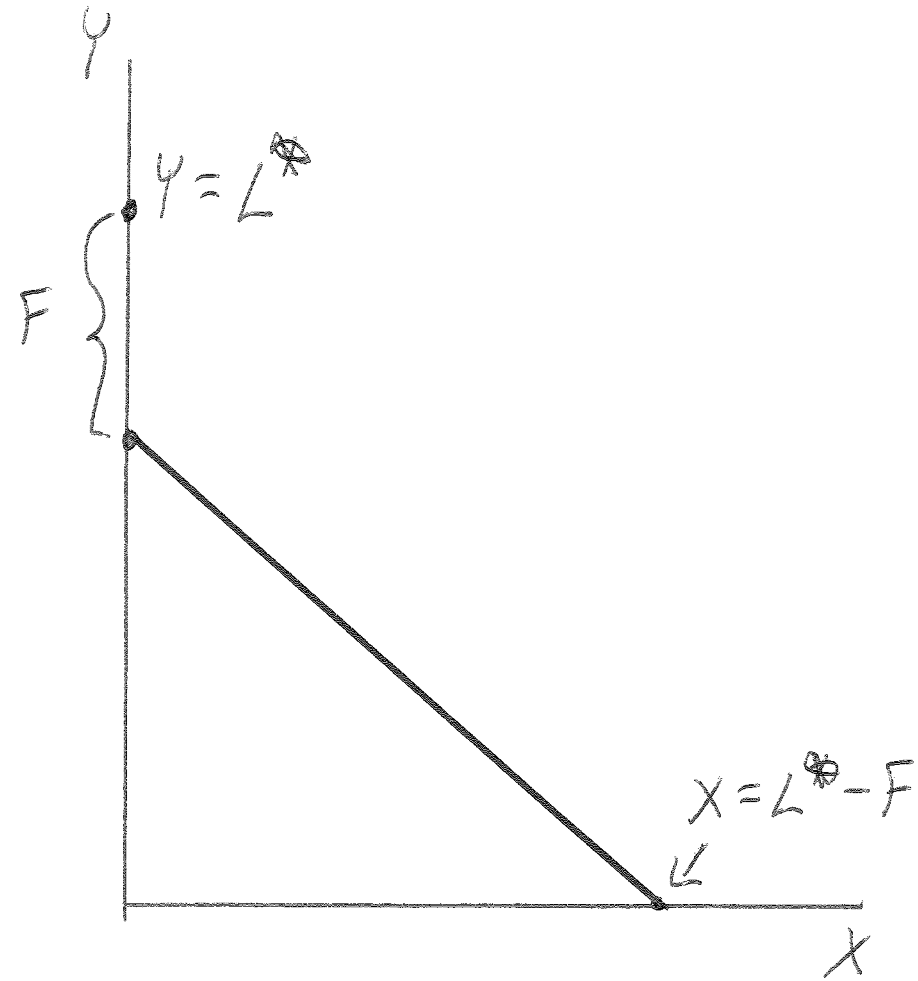
(2) Slope of the production frontier depends of <u>relative</u> factor productivities but also on the size of the economy and the output of X

$$dY = \beta dL_y = -\beta dL_x \qquad dX = \gamma\alpha L_x^{\gamma-1} dL_x \qquad \frac{dY}{dX} = -\frac{\beta}{\alpha}\frac{1}{\gamma}L_x^{1-\gamma}$$

(3) Curvature of the production frontier: In the case of increasing returns and one factor, the production frontier (transformation function) is <u>convex</u>.

The production set is non-convex. e.g., convex combinations of the endpoints of the frontier are not feasible production points and not in the production set.





Alternative representation of increasing returns used in the IO approach to trade.

There are fixed costs required to begin production of X.

$$Y = L_y \qquad X^* = \max[L_x - F,\ 0]$$

Production froniter consists of the the point

(Y = L*, X = 0)

and then the line segment

Y = (L* - F) - X   for all X > 0

The production set is again <u>non-convex</u>. e.g., convex combinations of the endpoints of the frontier are not feasible production points and not in the production set.

Role of Multiple Factors and Factor Intensities

(a) slope of production frontier (comparative advantage)

(b) curvature

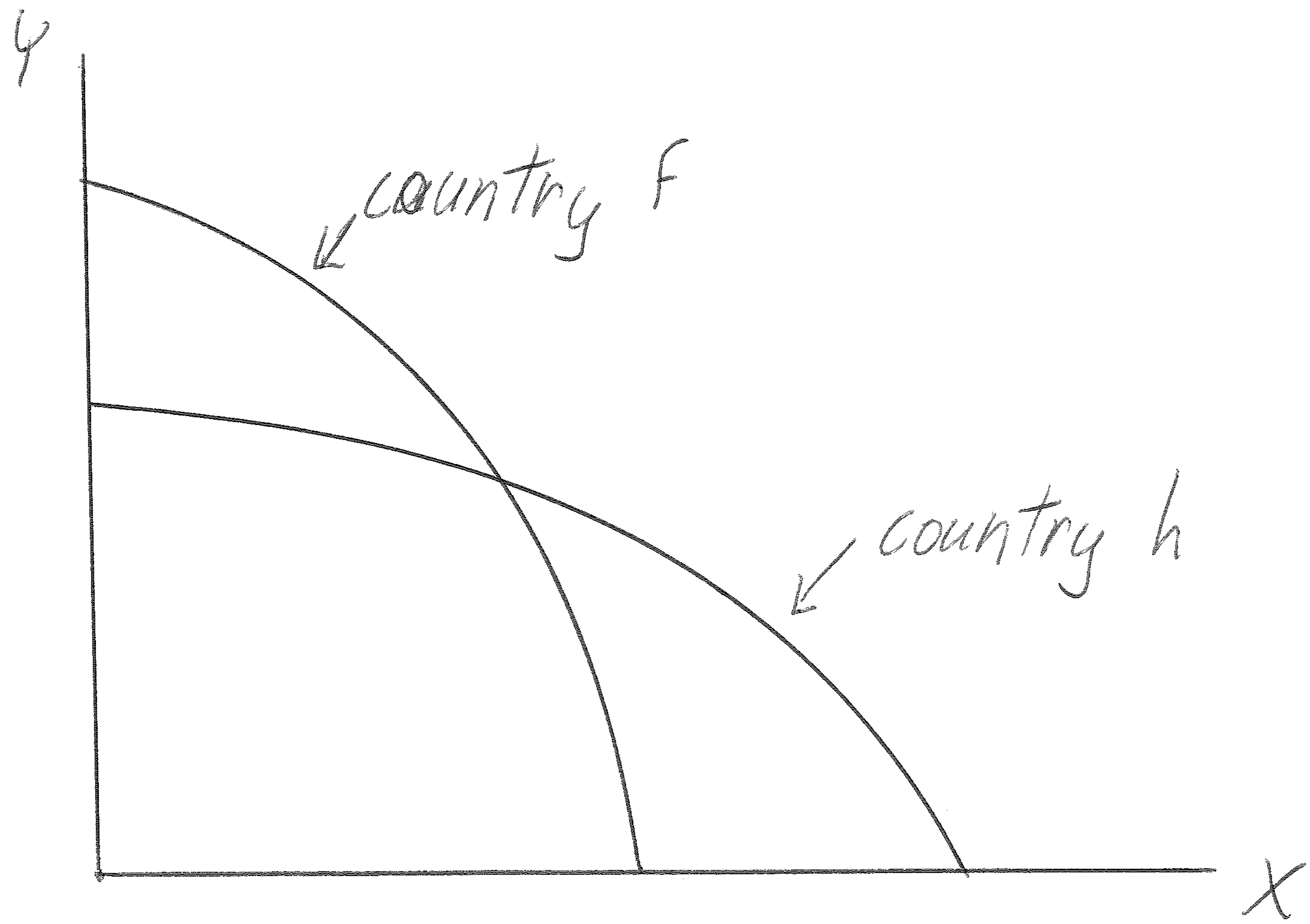
The Specific Factors Model

$$X = F(L_x, K) \qquad Y = G(L_y, R) \qquad L^* = L_x + L_y$$

$$dX = F_l dL_x, \qquad dY = G_l dL_y = -G_l dL_x \qquad \frac{dY}{dX} = -\frac{G_l}{F_l}$$

$$\frac{d^2Y}{dX^2} = \left[F_l G_{ll} + G_l F_{ll}\right] F_l^{-3} \le 0 \qquad (dL_x = F_l^{-1} dX)$$

Consider two countries, h and f. Both have identical endowments of labor. Country h has more K, country f has more R.



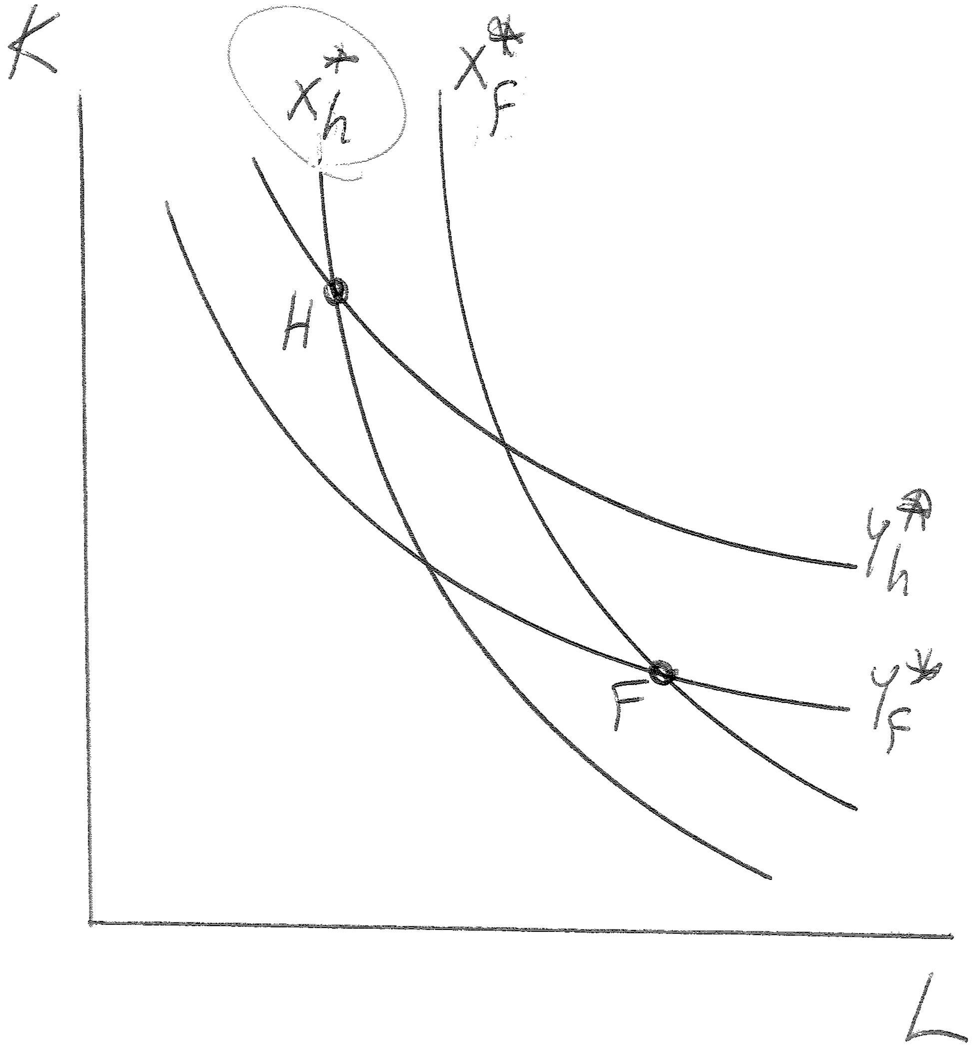
Case 2: The Heckscher-Ohlin Model

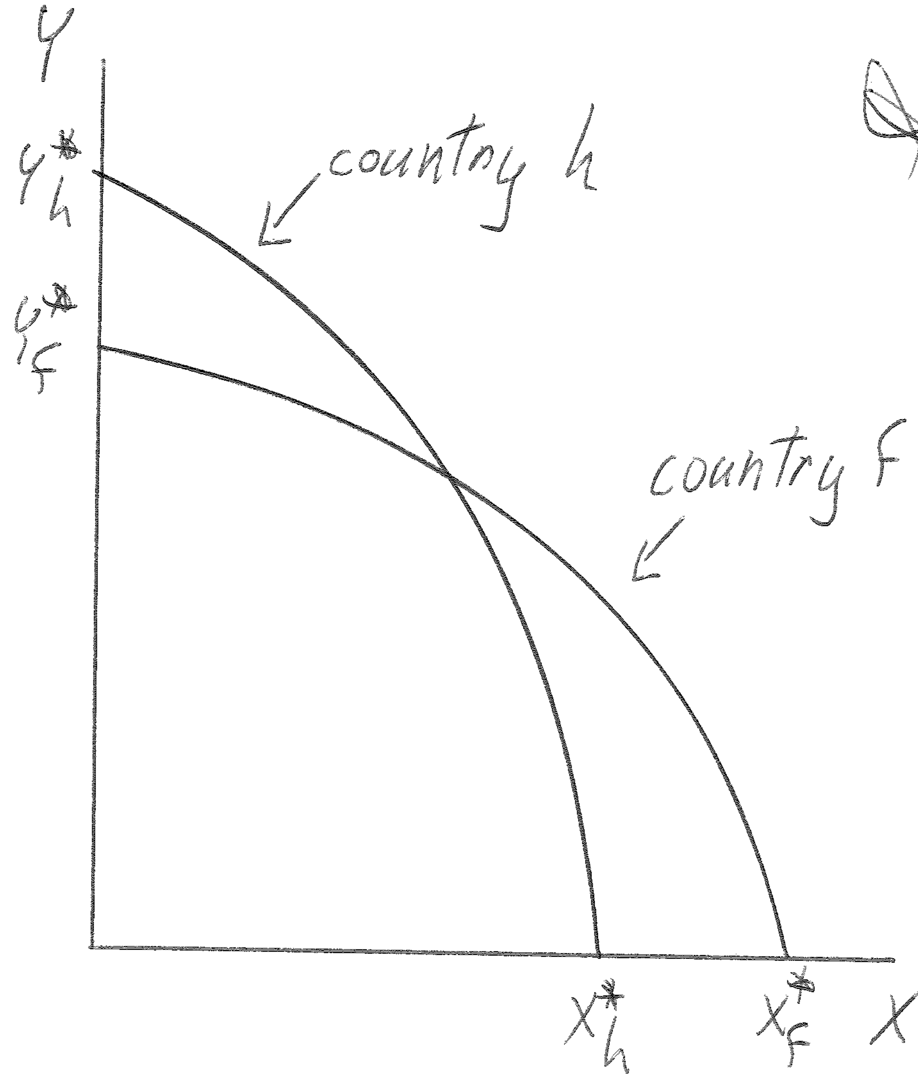
$$X = F(L_x, K_x) \qquad Y = G(L_y, K_y)$$

$$L^* = L_x + L_y \qquad K^* = K_x + K_y$$

If for a given set of factor prices, the optimal K/L ratio in Y is great than the optimal K/L ratio in X, then Y is said to be capital intensive and X is labor intensive.

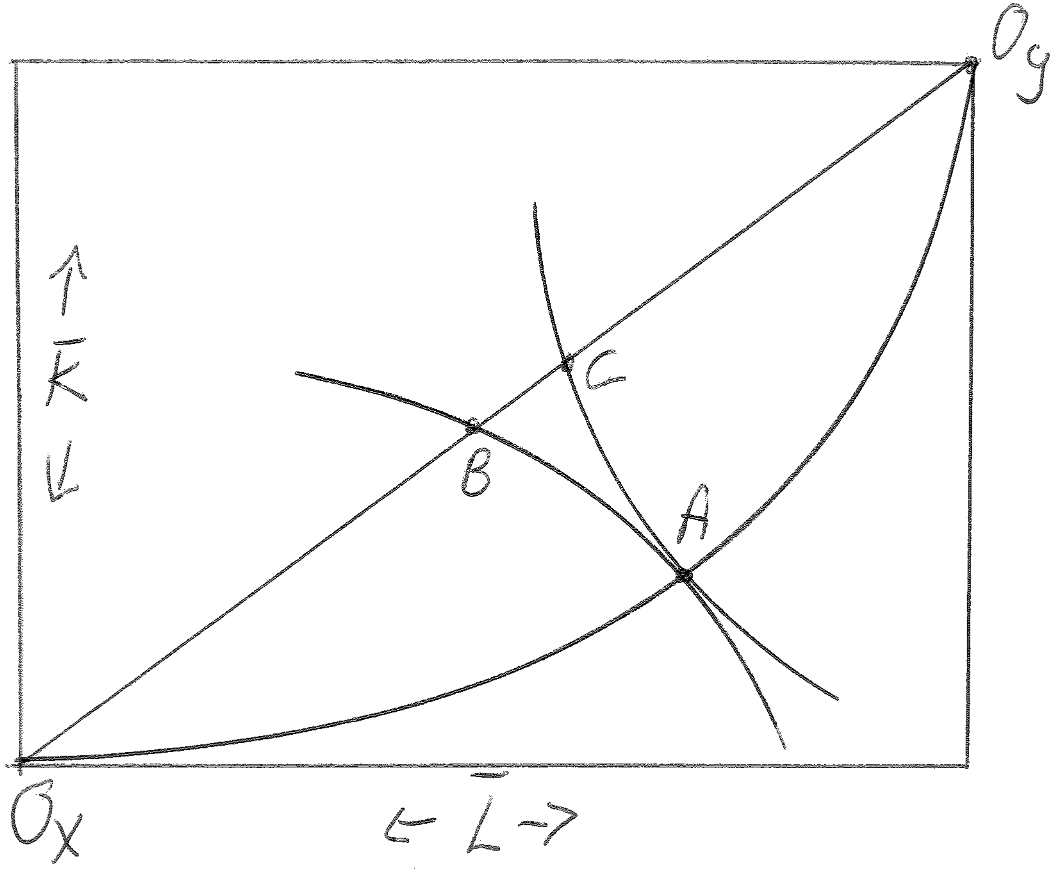
Consider two countries, let h have an absolutely larger K stock and country f have an absolutely larger L stock.

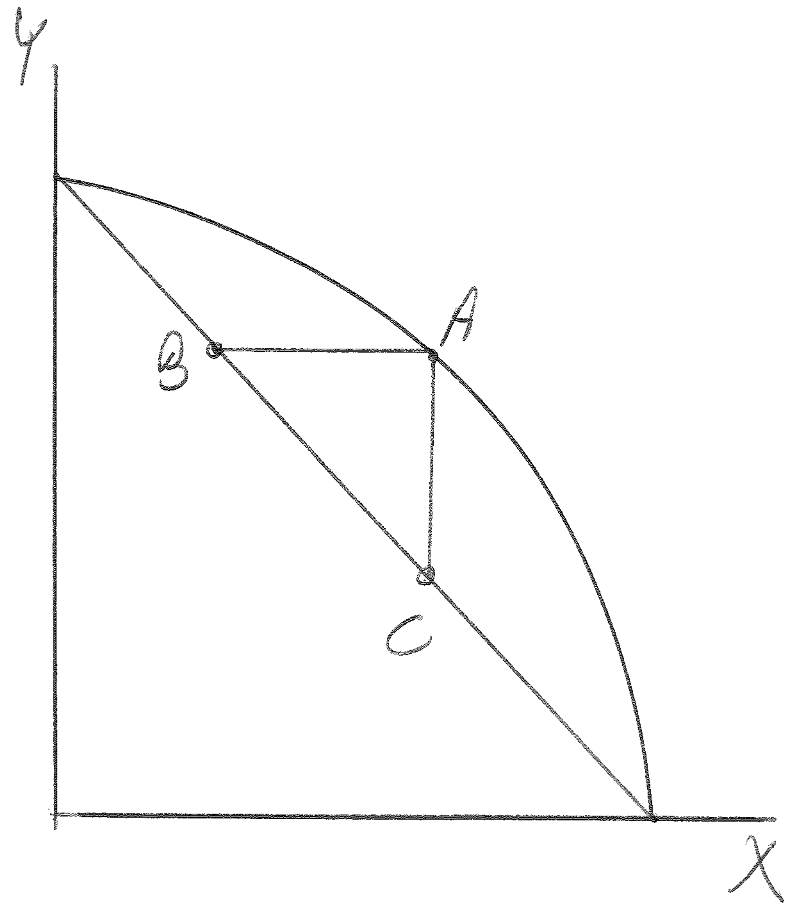




H: endowment of country h
F: endowment of country F

The factor market side of the Heckscher-Ohlin model is represented by the Edgeworth box. Assume that X is *labor intensive*, Y *capital intensive*.





The contract curve of the Edgeworth box maps into the production frontier in output space.

Proposition: If all industries have constant returns to scale, the *production set is convex*: Any convex combination of two feasible production vectors is also feasible (in the production set).

$$X_1 = F_1(V_{11}, \ldots, V_{m1})$$

.... or just $X = F(V)$: X - vector of outputs

$$X_n = F_n(V_{1n}, \ldots, V_{mn})$$

V - matrix of factor use

Suppose that (X^0, V^0) and that (X^1, V^1) are two alternative *feasible* output vectors and corresponding matrices of factor use.

Then $V^2 \equiv \lambda V^0 + (1-\lambda)V^1$ is feasible (satisfies adding up)

Suppose all production functions are concave (CRS), then by definition:

$$\lambda X^0 + (1-\lambda)X^1 = \lambda F(V^0) + (1-\lambda)F(V^1) \leq F(\lambda V^0 + (1-\lambda)V^1) = F(V^2)$$

$\lambda X^0 + (1-\lambda)X^1$ is feasible and is $\leq$ to the feasible output vector $X^2 = F(V^2)$

Competitive Equilibrium

Trade theory generally begins with competitive models in which all agents (firms, households) are price takers. These models are relatively simple to solve, have optimality properties, and allow for a simple representation of equilibrium.

Suppose throughout that endowments are fixed at V*

Suppose that X^0 , V^0 chosen at commodity and factor prices p^0, w^0
Suppose that X^1 , V^1 chosen at commodity and factor prices p^1, w^1

(1) Approach I: calculus of optimization: value of marg prod = factor price

2x2 case: Four first order conditions: two industries, two factors

$$p_1^0 F_{11}(V_{11}^0, V_{21}^0) = w_1^0 \qquad p_1^0 F_{12}(V_{11}^0, V_{21}^0) = w_2^0$$

$$p_2^0 F_{12}(V_{12}^0, V_{22}^0) = w_1^0 \qquad p_2^0 F_{22}(V_{12}^0, V_{22}^0) = w_2^0$$

Implication 1: efficiency in factor-market allocation: production on the PPF

$$MRS_1 = \frac{F_{11}}{F_{21}} = MRS_2 = \frac{F_{12}}{F_{22}} = \frac{w_1^0}{w_2^0}$$

The allocation must be on the contract curve in the Edgeworth box.

Implication 2: efficient choice of outputs on the PPF.

$$dX_1 = \sum_j F_{j1} dV_{ji} = \sum_j \left[\frac{w_j}{p_1}\right] dV_{j1} = \frac{1}{p_1}\sum_j (w_j dV_{ji})$$

$$dX_2 = \sum_j F_{j2} dV_{ji} = \sum_j \left[\frac{w_j}{p_2}\right] dV_{j2} = \frac{1}{p_2}\sum_j (w_j dV_{j2})$$

But the summations over the factors on the right-hand side are just minus one another: an increase in factor j to industry i must mean an equal decrease in supply from the other industry. $dV_{j1} = -dV_{j2}$

$$MRT = -\frac{dX_2}{dX_1} = \frac{p_1}{p_2}$$

(2) Approach II: profit maximization and "revealed preference"

For each industry i, profit maximization implies that

$$p_i^0 X_i^0 - \sum_j w_j^0 v_{ij}^0 \geq p_i^0 X_i^1 - \sum_j w_j^0 v_{ij}^1$$

Sum over all i industries

$$\sum_i p_i^0 X_i^0 - \sum_i \sum_j w_j^0 v_{ij}^0 \geq \sum_i p_i^0 X_i^1 - \sum_i \sum_j w_j^0 v_{ij}^1$$

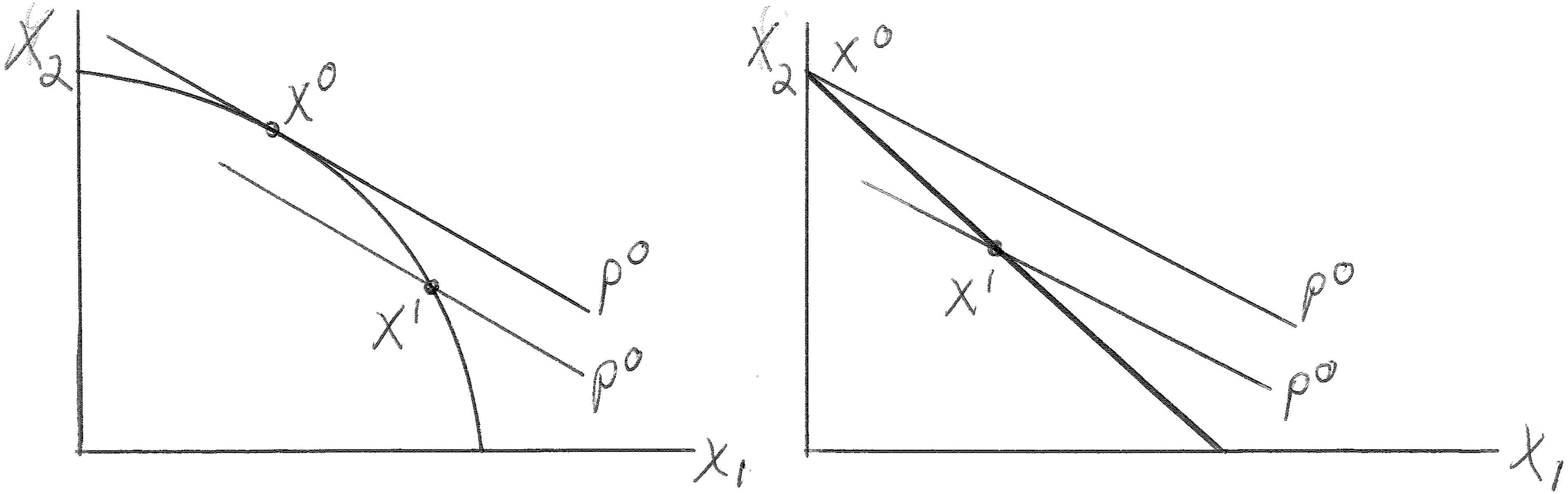
But for each factor j

$$\sum_i w_{ij}^0 V_{ij}^0 = \sum_i w_j^0 V_{ij}^1 = w_j^0 V_j^*$$

Therefore, the outputs X^0 chosen at prices p^0 maximize the value of production at those prices.

$$\sum_i p_i^0 X_i^0 \geq \sum_i p_i^0 X_i^1$$

Geometrically, all feasible production points must lie on or below the price hyperplane p^0. The price plane is "supporting" to the production set.



This can also be thought of in terms of cost minimization. Rewrite the profit inequality for industry i as:

$$p_i^0 X_i^0 - \sum_j w_j^0 v_{ij}^0 \geq p_i^0 X_i^1 - \sum_j w_j^0 v_{ij}^1$$

$$\left[p_i^0 - \sum_j w_j^0 \frac{v_{ij}^0}{X_i^0} \right] X_i^0 \geq \left[p_i^0 - \sum_j w_j^0 \frac{v_{ij}^1}{X_i^1} \right] X_i^1$$

$$\left[p_i^0 - \sum_j w_j^0 a_{ij}^0 \right] X_i^0 \geq \left[p_i^0 - \sum_j w_j^0 a_{ij}^1 \right] X_i^1$$

Where a_{ij}^k is the *cost minimizing amount* of factor j needed to produce one unit of good i at factor prices k.

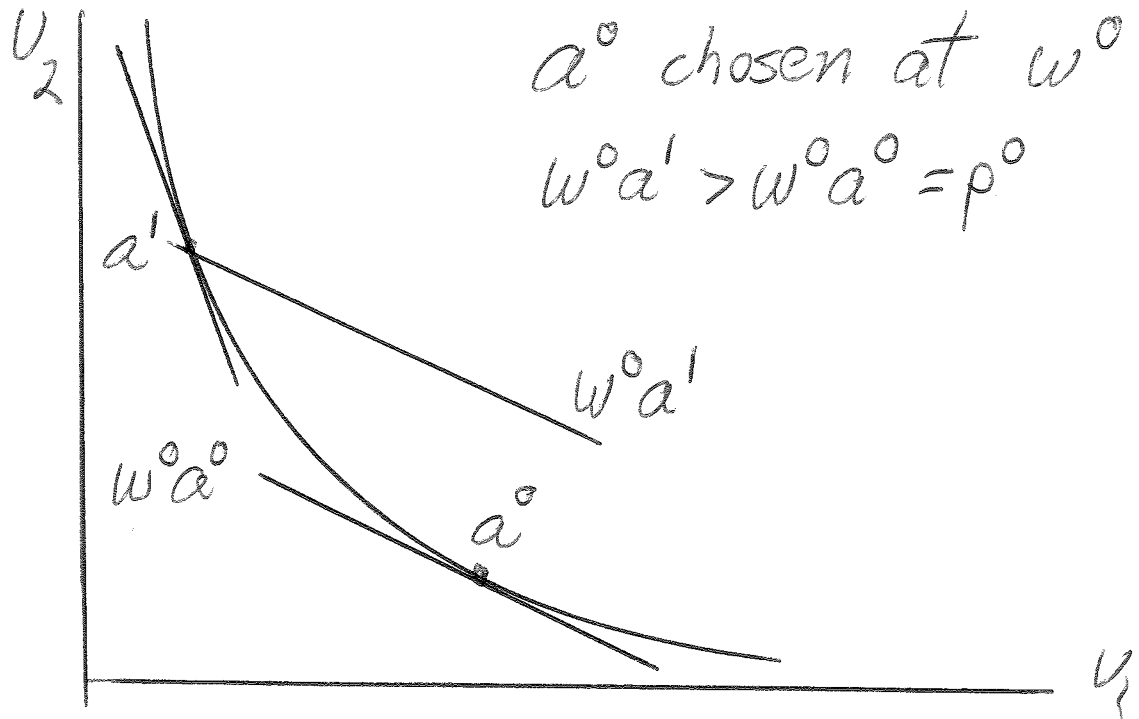
Therefore, $\sum_j w_j^0 a_{ij}^0 \leq \sum_j w_j^0 a_{ij}^1$

Thus cost minimization implies that the left-bracketed term is less than the right-bracketed term in the previous equation.

Furthermore, the left side of that equation is zero by free entry, and so the inequality must hold.

$$0 \geq \left[p_i^0 - \sum_j w_j^0 a_{ij}^1 \right] X_i^1$$

Graphically, this looks as follows:



Summary points

1. The "economic size" of a country is a combination of
   (a) the size of its factor endowment
   (b) real factor productivity; the latter could also just be called "technological sophistication".
   (c) scale economies can multiply size differences in endowments, technology

2. A country's relative ability to produce different goods is determined by
   (a) relative factor productivity across industries
   (b) relative factor endowments combined with factor intensity differences across industries
   (c) with IRS in some industries, country size also matters

3. The curvature of the production frontier (transformation function) is determined by factor intensities and scale economies
   (a) in the special case of one factor, the frontier is linear with constant returns, convex with increasing returns
   (b) in the special case of multiple factors and constant returns, the production frontier is concave (factor intensities must differ across industries)

4. When all agents are price takers, competitive equilibrium is efficient.
   (a) MRS are equated across industries, so production takes place on the contract curve of the Edgeworth box, and therefore on the PPF
   (b) For given goods prices, the efficient point on the PPF is chosen, the point of tangency between the frontier and the commodity price ratio.