Lecture 10

Oligopoly Models with Homogenous Goods

1. Trade between identical countries in a Cournot duopoly
2. The "linear" model: linear demand and constant marginal cost
3. Cournot duopoly in the linear model
4. A free-entry version of the Cournot linear model
5. Positive trade costs: the "reciprocal dumping" model with segmented markets
6. The "home-market effect": per capital welfare difference in the presence of trade costs and country size differences
7. Notes on segmented versus integrated markets

We consider a two-country, two-good model. Y is a competitive industry producing with constant returns.

X is an imperfectly competitive industry, initially with constant returns (implicitly there must be some barrier to entry).

Y will be used as numeraire, to p measures the price of X in terms of Y.

NB., the choice of numeraire unfortunately matters in these problems. The most theoretically elegant is to use a price index based on the firm owners's preferences as the numeraire. Unfortunately, this generally does not permit a closed-form solution for the markup.

Revenue for a Cournot firm located in country i and selling in country j is given by

$$R_{ij} = p_j(X_{jc})X_{ij}. \qquad \text{where } X_{jc} \text{ is total supply to market j: } X_{jc} = X_{ij} + X_{jj}$$

The Marshallian price elasticity of demand is denoted η.

Cournot conjectures imply that $\partial X_{jc} / \partial X_{ij} = 1$;

that is, a one-unit increase in own supply is a one-unit increase in market supply. Marginal revenue is then:

$$\frac{\partial R_{ij}}{\partial X_{ij}} = p_j + X_{ij} \frac{\partial p_j}{\partial X_{jc}} \frac{\partial X_{jc}}{\partial X_{ij}}$$

$$= p_j + p_j \frac{X_{ij}}{X_{jc}} \left[\frac{X_{jc}}{p_j} \frac{\partial p_j}{\partial X_{jc}} \right] \frac{\partial X_{jc}}{\partial X_{ij}} = p_j \left[1 - \frac{X_{ij}}{X_{jc}} \frac{1}{\eta_j} \right]$$

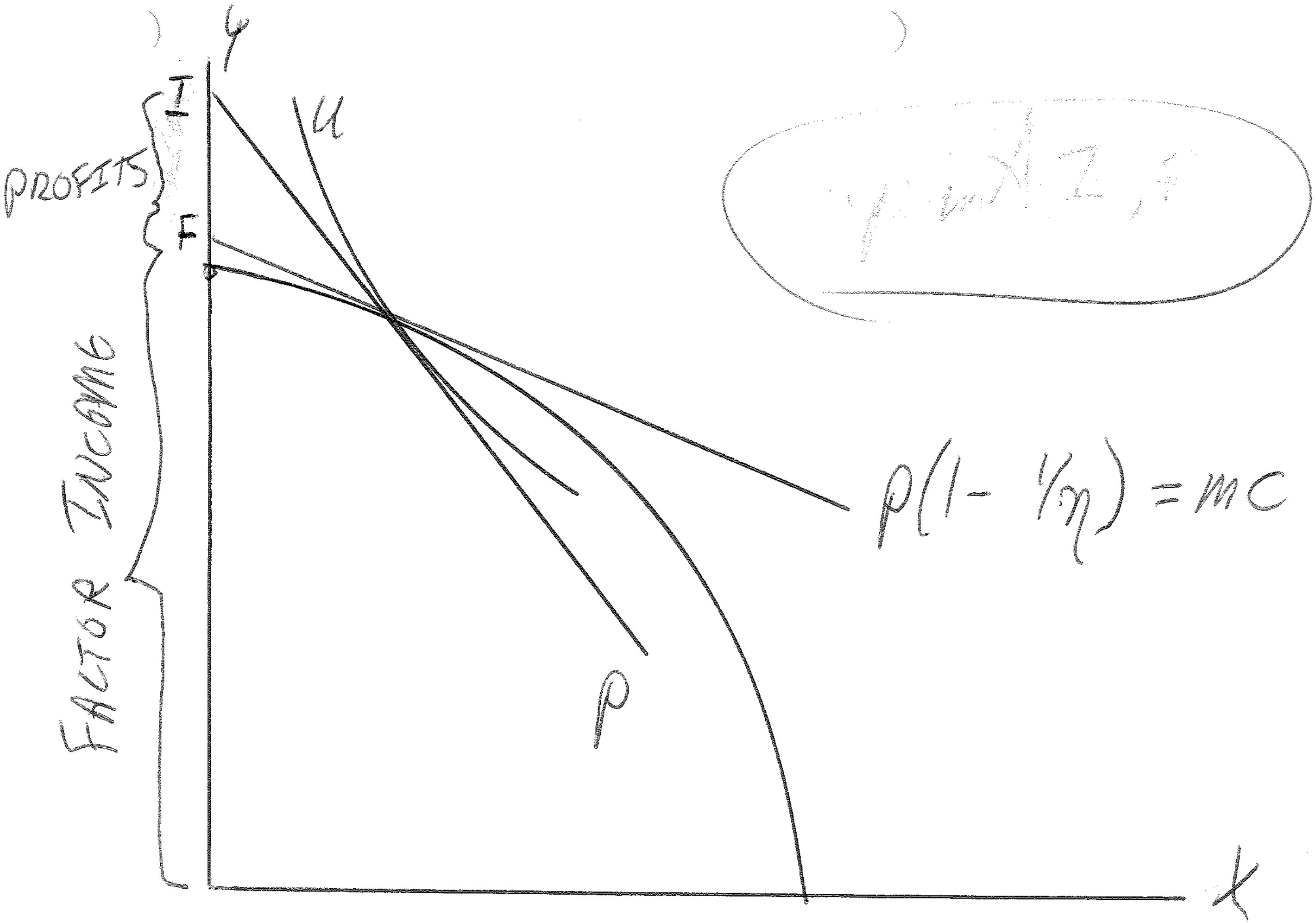
$$\text{since} \quad \frac{\partial X_{jc}}{\partial X_{ij}} = 1 \quad (\text{Cournot conjecture})$$

$$\eta_j \equiv -\left[\frac{p_j}{X_{jc}} \frac{\partial X_{jc}}{\partial p_j}\right] \qquad (\textit{Marshallian elasticity of demand})$$

Let $s_{ij} = X_{ij}/X_{jc}$ be firm i's market share in market j. Then marginal revenue = marginal cost is given by:

$$MR_{ij} = p\left[1 - \frac{s_{ij}}{\eta_j}\right] = MC_i$$

Consider two identical economies each with a single X producer. Consider first autarky. Since we are using Y as numeraire, the marginal cost of X in terms of Y is just the slope of the production frontier, the marginal rate of transformation.

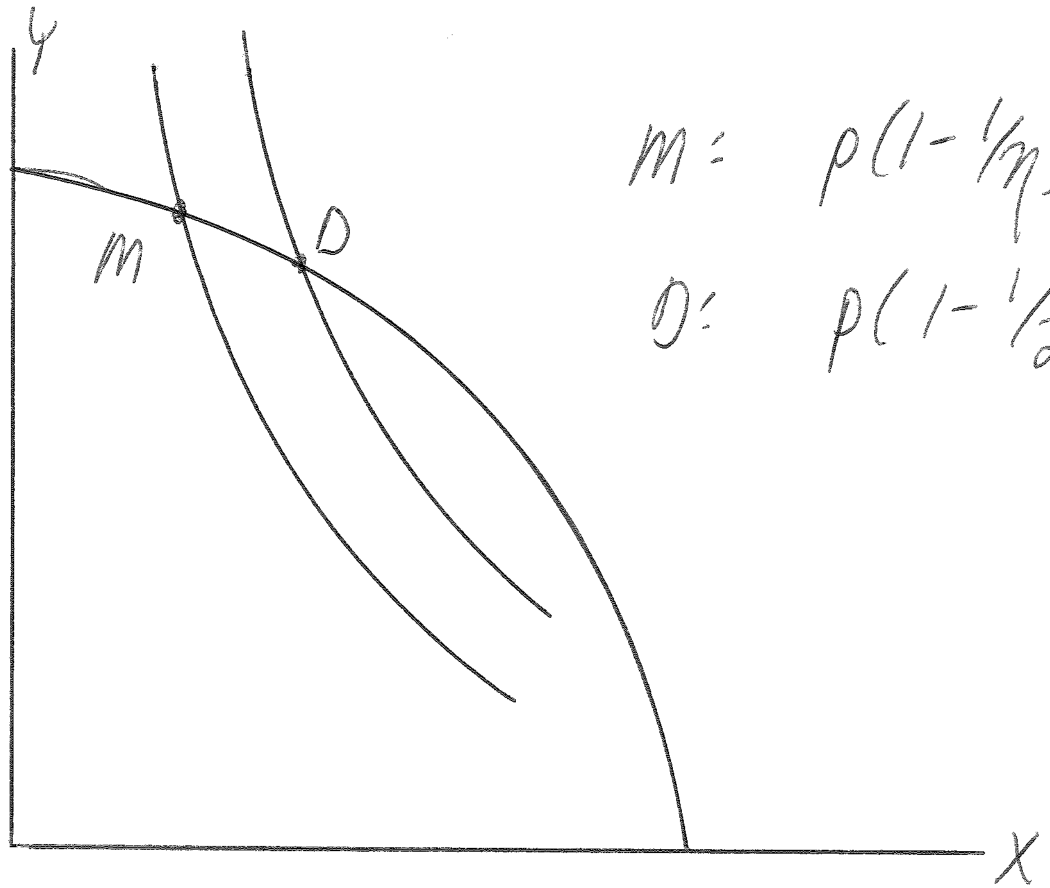


Note that point I can be thought of as the value of total income in terms of good Y. Since factors are paid thir marginal product, point F can then be interpreted as the income of factor owners in terms of Y. The difference between I and F is then profits.

Now put two identical economies together in free trade.

The equilibrium will be symmetric, with equilibrium marginal costs the same for each X firm. With zero trade costs, each firm will have a market share of 1/2 in each market.

Ignoring changes in the elasticity of demand, each firm will produce more in equilibrium. There will be gains from trade, although there will be no net trade. We refer to this as a *pro-competitive gain from trade*.



M: $p(1 - 1/\eta) = MC$

D: $p(1 - 1/2\eta) = MC$

Note that the share of profits in total income decreases with competition, so the gains to factor owners are proportionally larger than suggested in the diagram.

Subtlety: the elasticity of demand is generally not a constant, and its *value* depends on where it is evaluated.
The "Linear" Duopoly Model

Two countries, each with one X firm. X_{ij} denotes sales of firm i in market j

One factor, L.

$Y = L_y$

X requires a fixed cost F and a constant -marginal cost c both in units of labor.

Preferences are quadratic, "quasi-linear".

Utility (Ui) for the representative consumer and the national budget constraint in country i are given by:

$$U_i = \alpha(X_{ii} + X_{ji}) - (\beta/2)(X_{ii} + X_{ji})^2 + (Y_{ii} + Y_{ji}) \tag{1}$$

$$\gamma L_i + \Pi_i = p_i(X_{ii} + X_{ji}) + (Y_{ii} + Y_{ji})$$

Substituting from the budget constraint for good Y, we have the consumer's choice problem, where profits (Π_i) are viewed as exogenous.

$$Max(X)\ U_i = \alpha(X_{ii} + X_{ji}) - (\beta/2)(X_{ii} + X_{ji})^2 + \gamma L_i + \Pi_i - p_i(X_{ii} + X_{ji}) \tag{2}$$

The inverse demand function is given by the first-order condition, and is linear in X.

$$p_i = \alpha - \beta(X_{ii} + X_{ji}) \tag{3}$$

Hold the market structure or regime fixed for the moment. That is, consider the second-stage output decisions first.

Let Π_{ij} denote the profits of firm i on it sales in market j. Profits for firm i on its domestic sales are given by

$$\Pi_{ii} = p_i X_{ii} - c_i X_{ii} = [\alpha - \beta(X_{ii} + X_{ji})]X_{ii} - c_i X_{ii} \qquad (4)$$

Consider first the case of monopoly (autarky), so that $X_{ji} = 0$. In this case, profit maximization gives:

$$X_{ii} = \frac{\alpha - c}{2\beta} = X_{ic} \qquad (6)$$

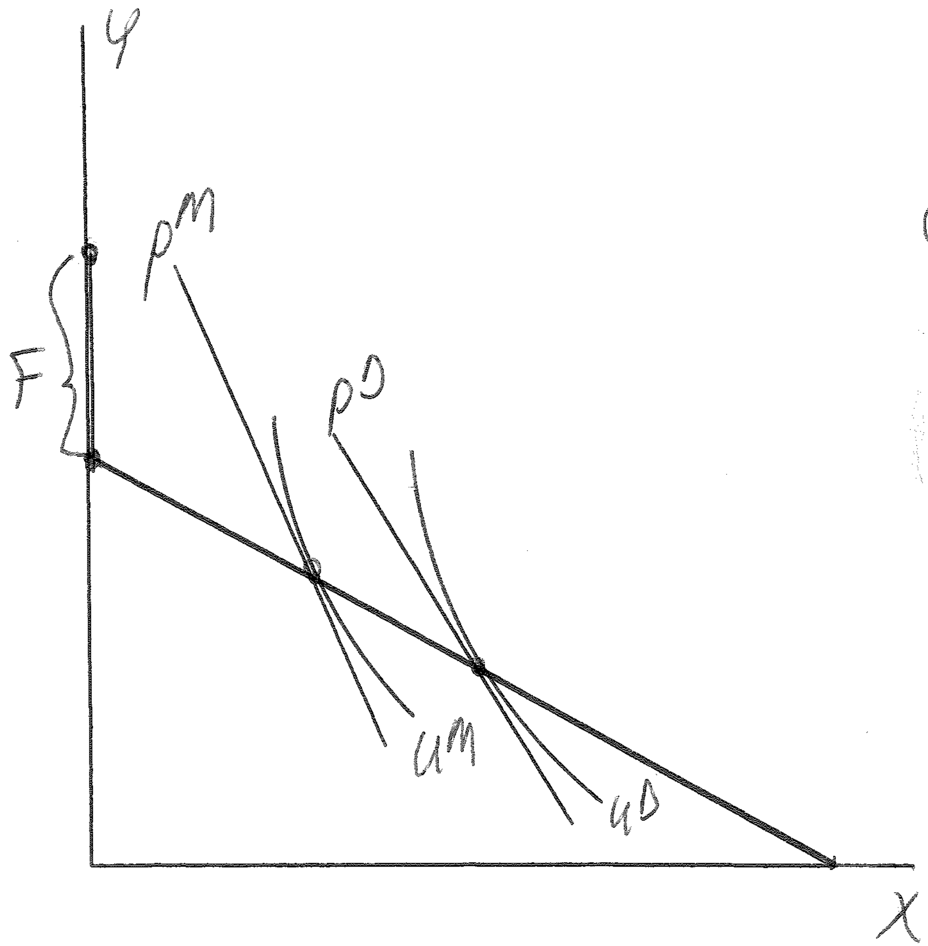
Now assume that two identical economies trade freely. Assume Cournot competition, so each firm makes a best response to the other firm's output, maximizing profits holding the other firm's output fixed.

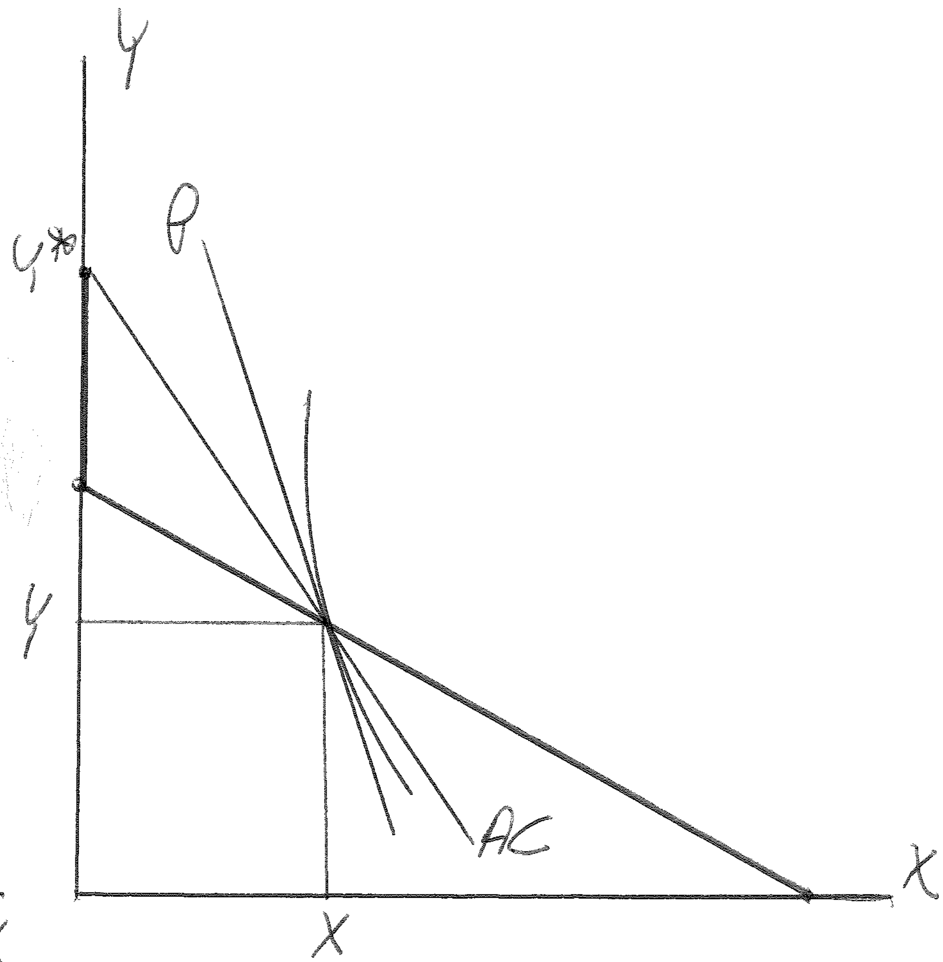
$$\frac{d\Pi_{ii}}{dX_{ii}} = \alpha - 2\beta X_{ii} - \beta X_{ji} - c = 0 \qquad (5)$$

Assume first that firm i faces a rival producing in the domestic market; that is, firm j is a type-h firm. Then, exploiting cost symmetry, we can solve (5) for the Cournot output of the firm i (equal to the country j firm's output from its plant in country i) by setting $X_{ii} = X_{ji}$.

$$X_{ii} = \frac{\alpha - c}{3\beta} \qquad X_{ic} = 2\frac{(\alpha - c)}{3\beta} > \frac{\alpha - c}{2\beta} \qquad (6)$$

Here is a graphical depiction of autarky equilibrium and free trade. Note now that factor income is the average cost of producing X, not the marginal cost.





Since $Y = L_y$, the wage rate also equals one. Average cost is

$$L_x/X = (L - L_y)/X = (Y^* - Y)/X$$

which is the slope of the chord connecting Y^* and the production point.

Solve for monopoly profits for the firm in country i

$$\Pi_{ii} = (p_i - c)X_{ii} = [\alpha - \beta X_{ii} - c]\frac{\alpha - c}{2\beta} \qquad (7)$$

$$\Pi_{ii} = \left[\alpha - c - \frac{(\alpha - c)}{2}\right]\frac{\alpha - c}{2\beta} = \frac{\alpha - c}{2}\,\frac{\alpha - c}{2\beta} \qquad (8)$$

$$\Pi_i^m = \beta\left[\frac{\alpha - c}{2\beta}\right]^2 \qquad (9)$$

Now solve for duopoly (free trade) profits. Substitute (6) back into the profit equation (4).

$$\Pi_{ii} = (p_i - c)X_{ii} = [\alpha - \beta(X_{ii} + X_{ji}) - c]\frac{\alpha - c}{3\beta} \tag{7}$$

$$\Pi_{ii} = \left[\alpha - c - \frac{2(\alpha - c)}{3}\right]\frac{\alpha - c}{3\beta} = \frac{\alpha - c}{3}\frac{\alpha - c}{3\beta} \tag{8}$$

But these profits are earning in market j as well, so the firm's total profits are:

$$\Pi_{ii} + \Pi_{ij} = \Pi_i^d = 2\beta\left[\frac{\alpha - c}{3\beta}\right]^2 < \Pi_i^m \quad \text{(d-duopoly, m- monopoly)} \tag{9}$$

There is an overall welfare gain from trade and, since profits shrink, factor owner welfare rises proportionately more than total real income.

The linear model with free entry.

Now it is useful to think in terms of per capita utility.

Since I am assuming no trade costs, let's just think about one market and think of trade between two countries as doubling the size of the market. This allows me to simplify the notation (I now need a subscript indexing firms).

Assume that there are L individuals and so per capita consumption is

$$\sum_i X_i/L$$

where i indexes the number of firms. n, the number of firms is *endogenous*.

Demand and profits for the ith firm are then given by

$$p_i = \alpha - \beta\left[\sum_i X_i/L\right] \tag{10}$$

$$\Pi_i = p_i X_i - cX_i - F = \left[\alpha - \beta\left[\sum_j X_i/L\right]\right]X_i - cX_i - F \tag{11}$$

Marginal revenue minus marginal cost for firm i is given by:

$$MR - MC = \alpha - 2(\beta/L)X_i - (\beta/L)\sum_{j \neq i} X_j - c = 0 \tag{12}$$

Now impose symmetry. X will denote the output of a representative firm, and n the number of firms. All firms that are active in equilibrium will produce the same amount.

$$MR - MC = \alpha - (\beta/L)(n + 1)X - c = 0 \tag{13}$$

The second equation we need for equilibrium is the free-entry condition that will be associated with the number of firms.

This literature generally follows a long, if not entirely satisfactory tradition of allowing the number of firms to be a continuous variable.

Then the zero profit condition is that the profits of the representative firm are exactly zero.

$$\alpha X - (\beta/L)nX^2 - cX - F = 0 \tag{14}$$

Multiple (13) through by X. We then have two equations in two unknowns, n and X.

$$\alpha X - (\beta/L)(n + 1)X^2 - cX = 0 \tag{15}$$

$$\alpha X - (\beta/L)nX^2 - cX - F = 0 \tag{16}$$

Solving these, we get

$$X = \left[\frac{FL}{\beta}\right]^{1/2} \tag{17}$$

So output per firm increases with the size of the economy (L).

Putting (17) into (15), we can solve for n

$$n = \frac{\alpha - c}{2}\left[\frac{L}{\beta F}\right]^{1/2} - 1 \qquad (18)$$

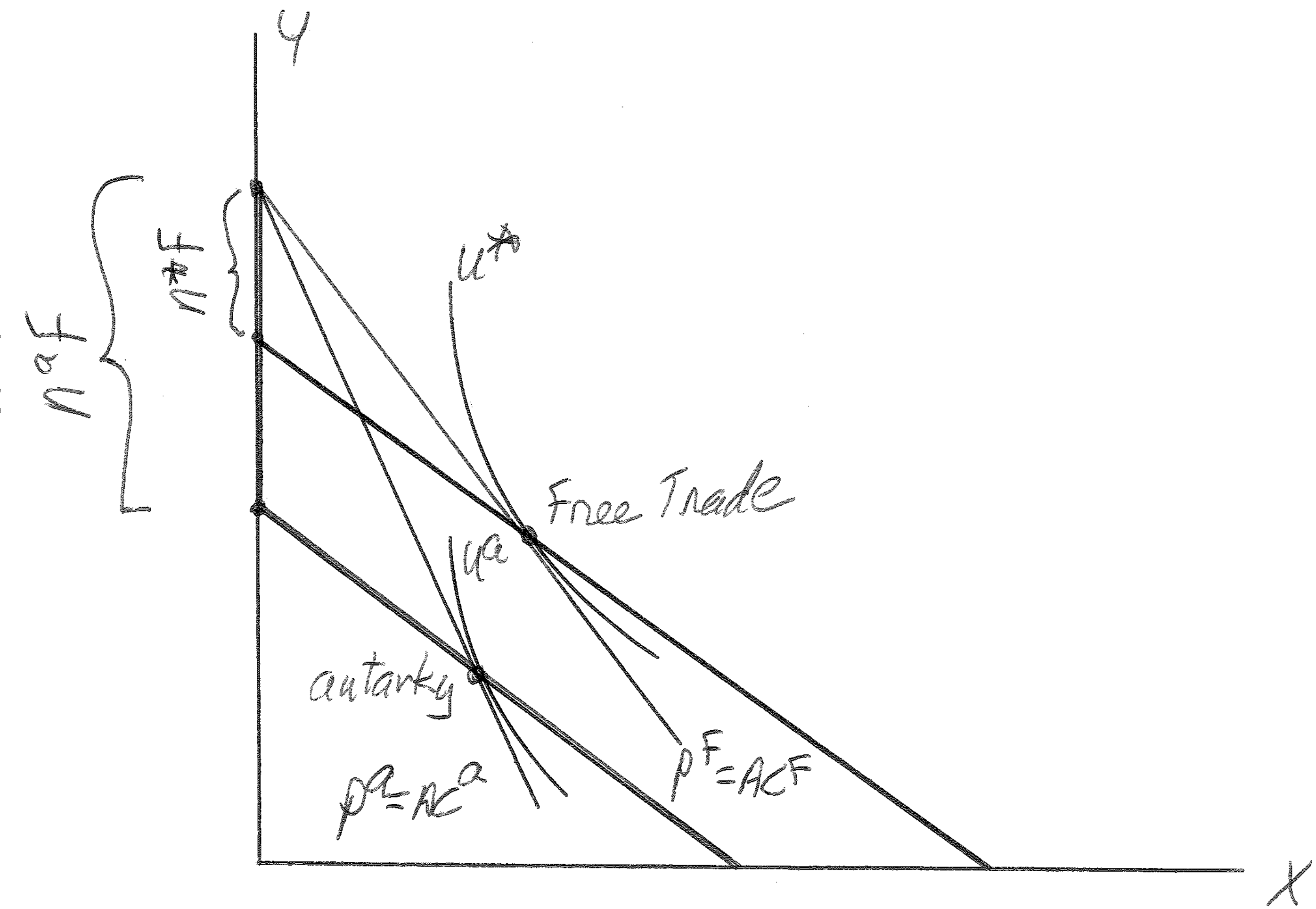
(17) and (18) then give us total output (nX) and output per consumer.

$$nX = \frac{\alpha - c}{2\beta}L - \left[\frac{FL}{\beta}\right]^{1/2} \qquad \frac{nX}{L} = \frac{\alpha - c}{2\beta} - \left[\frac{F}{\beta L}\right]^{1/2} \qquad (19)$$

As suggested above, think of trade as expanding the size of the economy; e.g., opening two identical economies to trade doubles L. The effect of this is to:

(1) increase output per firm
(2) decrease average cost, and hence the price of X
(3) increase total output of X and output per consumer
(4) Since the total number of firms increases slower than the size of the market, the number of firms in each country *falls*.

Effect on one country of opening trade with an identical country.



Positive trade costs, cross-hauling, "reciprocal dumping"

Unit trade cost (in both directions is t)

$$\frac{d\Pi_{ii}}{dX_{ii}} = \alpha - 2\beta X_{ii} - \beta X_{ji} - c = 0 \qquad (20)$$

$$\frac{d\Pi_{ij}}{dX_{ij}} = \alpha - 2\beta X_{ij} - \beta X_{jj} - c - t = 0 \qquad (21)$$

Exploiting cost symmetry, we can solve (5) for the Cournot output of the firm i (equal to the country j firm's output from its plant in country i) by setting $X_{ii} = X_{jj}$ and $X_{ij} = X_{ji}$.

$$X_{ii} = \frac{\alpha - c + t}{3\beta} \qquad X_{ij} = \frac{\alpha - c - 2t}{3\beta} \qquad (22)$$

There is going to be positive trade, "cross-hauling" or "reciprocal dumping" if $t < (\alpha - c)/2$.

Total X consumption of a country is:

$$X_i = X_{ii} + X_{ji} = \frac{2(\alpha - c - t/2)}{3\beta} \tag{23}$$

Refer back to equation 2. We see that "consumer surplus" from X consumption is:

$$CS_i = \alpha X_i - (\beta/2)X_i^2 - pX_i =$$

$$\alpha X_i - (\beta/2)X_i^2 - \alpha X_i + (\beta)X_i^2 = (\beta/2)X_i^2 \tag{24}$$

Using an analysis similar to (6) - (9), profits are given by

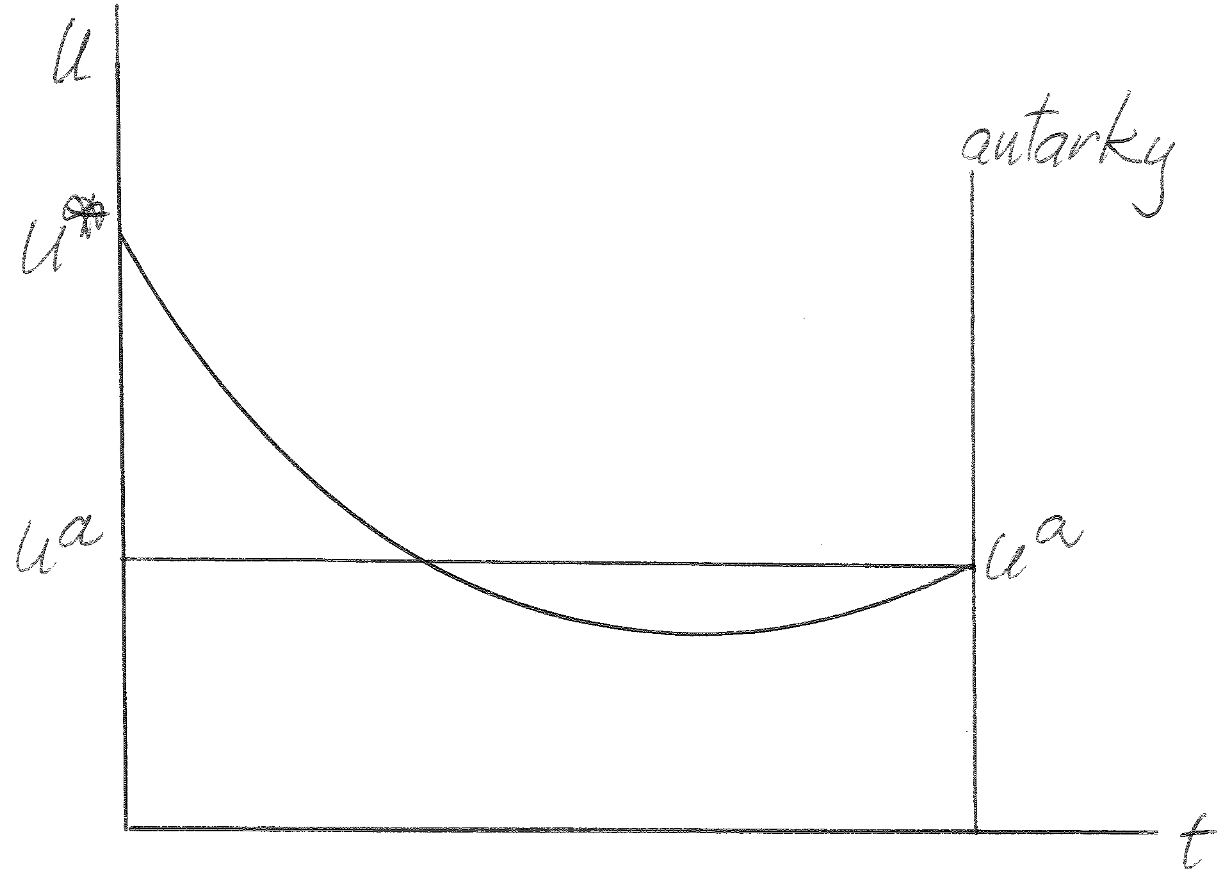
$$\Pi_i = \beta X_{ii}^2 + \beta X_{ij}^2$$

Therefore, using (1) welfare is given by

$$U_i = (\beta/2)(X_{ii} + X_{ji})^2 + \beta X_{ii}^2 + \beta X_{ij}^2$$

Reduction of trade costs is going to have an ambiguous effect possibly. Consumers gain (consumer surplus increases) but profits are going to fall.

This will be left as an exercise: show that welfare as a function of profits looks like this:



Home - Market Effect

This term has unfortunately been used in several different senses, but one is to refer to higher per capita income and/or higher real factor prices in a larger country.

This phenomenon is important for a number of reasons, including agglomeration and the consequences of factor mobility.

Let the number of consumers in the two counties be L_i and L_j. $L_i > L_j$.

Single X firm in each country.

With no entry, the above results are going to imply that consumer surplus (per capita) is going to be the same in the two countries.

$$\frac{X_i}{L_i} = \frac{X_{ii} + X_{ji}}{L_i} = \frac{2(\alpha - c - t/2)}{3\beta} \tag{25}$$

Profits per capita for the firm in the *smaller* country will be lower. Using earlier results:

$$\Pi_i = \beta\left[\frac{\alpha - c + t}{3\beta}\right]^2 L_i + \beta\left[\frac{\alpha - c - 2t}{3\beta}\right]^2 L_j - F \tag{26}$$

However, the profits *per capita* in the smaller country may be *bigger* (sum of last two terms is positive) or *smaller* (sum of the last two terms is negative).

$$\frac{\Pi_i}{L_i} = \beta\left[\frac{\alpha - c + t}{3\beta}\right]^2 + \beta\left[\frac{\alpha - c - 2t}{3\beta}\right]^2 \frac{L_j}{L_i} - \frac{F}{L_i} \tag{27}$$

If fixed costs are very small, the firm in the smaller country makes higher profits per capita and that country is unambiguously the better off per capita.

Intuition: without fixed costs, the firms make the same profits per capita in autarky. Trade gives each firm one competitor. But while trade allows the firm in the small country access to a big market, but the firm in large country gets access to only a small market.

Suppose that the profits of the firm in the small country are "close" to zero. Then the sum of the last two terms in (26) and (27) are negative, profits per capita are *lower* in the small country, and welfare per capital is *lower* in the small country.

Or, if profits are "close" to zero in the large country, then profits will be negative in the small country and that firm exits.

Let j be the small country. The firm in the large country is then a monopolist and supplies to the two markets are then:

$$\frac{X_i}{L_i} = \frac{\alpha - c}{2\beta} > \frac{X_j}{L_j} = \frac{\alpha - c - t}{2\beta} \qquad (28)$$

The small country has a lower consumer surplus per capita, plus it has no profit income, and so it is worse off than the large country per capita.

This does *not* imply that it does not gain from trade.

With free entry, there are no firm profits, so we need only worry about consumer surplus. For consumer surplus, we need only worry about prices in the two countries.

I will not derive the formal proof, but the argument (by contradiction) runs like this.

(1) Suppose that prices are the same in the two countries.

(2) Each firm sells more in its home market than in its export market.

(3) The ratio of what a local firm sells to the amount an importing firm sells is the same across markets.

(4) For each firm, its high margin sales $(p - c + t)$ are in its local market

(5) For a firm located in the large country, its sales are relatively concentrated in its high margin market, whereas for a firm located in the small country, its sales are relatively concentrated in its low margin market.

(6) If prices are the same in the two markets, then if a representative firm in the large country is making zero profits, a representative firm in the small country must be making losses.

(7) All firms will be located in the large country. Equilibrium must involve a lower price, and higher welfare per consumer, in the large country.

Segmented versus integrated markets

This is a topic that I will not be able to get into. It is potentially quite important in that some authors have proposed that market integration reduces the ability of firms to price discriminate.

Segmented markets - means firms may price independently in two markets. There is no arbitrage by independent agents.

Integrated markets - it is somewhat less clear what this means.

(1) there are independent agents that may arbitrage. So the firm may not charge a price differential between markets that is in excess of the transport cost or tariff.

In a symmetric duopoly model, this constraint is generally non-binding. That is, each firm absorbs some of the transport cost on its exports, so the price charged in the export market minus the home price is less than the transport cost.

(2) there is a single factory-gate price, so that the firm must sell all its output to independent wholesalers at the same price for all units, regardless of where a particular unit is going.

This makes a great deal of difference and is not at all equivalent to the first definition.

This latter definition is widely used in the monopolistic-competition model, the next lecture.

Consider a monopolist serving two identical markets, producing in i and incurring a transport cost t to the foreign market. We have already implicitly solved for the firm's optimal supplies

$$X_i = \frac{\alpha - c}{2\beta} \qquad X_j = \frac{\alpha - c - t}{2\beta}$$

Now assume that the firm must sell for one factory price, with exports shipped by independent, competitive arbitrageurs. Then

$$p_j = p_i + t \qquad p_j - t = p_i \qquad \text{implying}$$

$$\alpha - \beta X_j - t = \alpha - \beta X_i \qquad \beta X_i - t = \beta X_j$$

Profits are then given by:

$$\Pi = (p_i - c)X_i + (p_i - c)X_j$$

$$\Pi = (\alpha - \beta X_i - c)X_i + (\alpha - \beta X_i - c)(X_i - t/\beta)$$

$$\frac{\partial \Pi}{\partial X_i} = \alpha - c - 2\beta X_i + \alpha - c - 2\beta X_i + t = 0$$

$$X_i = \frac{\alpha - c + t/2}{2\beta}$$

$$X_j = \frac{\alpha - c + t/2}{2\beta} - \frac{t}{\beta} = \frac{\alpha - c - 3t/2}{2\beta}$$

These results tell us that (a) with integrated markets the firm's total output is the same as with segmented markets, but with integrated markets more is supplied to the home market and less to the foreign market. Interestingly, market integration makes home consumers better off and foreign consumers worse off.

Oligopoly Models with Homogeneous Goods - Summary Points

1. For similar economies, the opening of trade results in pro-competitive gains from trade that improve the welfare of both countries, even with no basis for comparative advantage.

2. This occurs with either a single firm (or equal number of firms) in each country, or with free entry/exit.

3. However, examples can be constructed where one country is worse off in free trade than in autarky. These situations can arise when there are asymmetries in costs or in country size (with no entry/exit).

4. The term "home market effect" has sometimes referred to a comparison of factor prices or per capita utility across countries in the presence of trade costs. In the present models, per capita utility *may* be higher in the small country in the duopoly case, but lower under free entry.

5. The effects of market segmentation is a potentially important topic, but the research is not very far advanced.