Lecture 11
Monopolistic - Competition

1. Dixit-Stiglitz preferences - “love of variety”
   The basic Krugman model - variable markups
   Fixed markups (“large-group” monopolistic competition)

2. Differentiated intermediate inputs and the “division of labor”
   Ethier’s model
   Extensions - traded and non-traded intermediate inputs

3. Differentiated goods and the volume of trade - zero trade costs
   The Helpman-Krugman FPE analysis
   Intra-industry and inter-industry trade volumes
   Extensions - non-homothetic demand

4. Transport costs and home-market effects
   Differences in country size, real wages and Linder effects
   Two-sector models
   Factor-price and agglomeration effects

5. Lancaster's "location" model - " ideal variety"

Krugman's simple model - trade is modeled as the growth in one economy.

C_i - consumption of good i per capita
X_i - production of good i: $X = LC$ (L = number of workers/cons)

Dixit- Stiglitz preferences

$$U = \sum_{i=1}^{n} C_i^{\gamma} \qquad 0 < \gamma < 1 \qquad \text{where n is } \textit{endogenous} \qquad (1)$$

Assertions which you should derive for yourself

(1) The elasticity of substitution between "varieties" is $\sigma = \dfrac{1}{1-\gamma}$

$$\text{if } U = f(C_1, \ldots. C_n) \qquad \sigma \equiv \frac{f_1/f_2}{C_2/C_1} \frac{d(C_2/C_1)}{d(f_1/f_2)}$$

(2) Firm i's perceived elasticity of demand, holding the prices of other goods constant is given by:

$$\epsilon = \sigma - s_i(\sigma - 1) \qquad s_i \equiv \frac{p_i C_i}{\sum_j p_j C_j}$$

so s_i is firm i's market share.

(3) If the elasticity of substitution is greater than one ($0 < \gamma < 1$), then the firm's market share s *falls* as the firm increases its *price* (or *increases* as the firm *increases* its output), other prices constant).

An increase in market share s is a fall in the elasticity of demand

$$\frac{\partial \epsilon_i}{\partial C_i} < 0$$

There is a single factor of production labor in fixed supply L*. Let the labor requirements for good i be given by

$$L_i = \alpha + \beta X_i ; \qquad \sum_i L_i = L^* \tag{2}$$

$$X_i = C_i L^* \qquad \text{supply = demand for goods} \tag{3}$$

$$L^* = \sum_i (\alpha + \beta X_i) \qquad \text{supply = demand for labor} \tag{4}$$

$$\Pi_i = p_i(X_i)X_i - (\alpha + \beta X_i)w_i \qquad \text{profits of firm i} \tag{5}$$

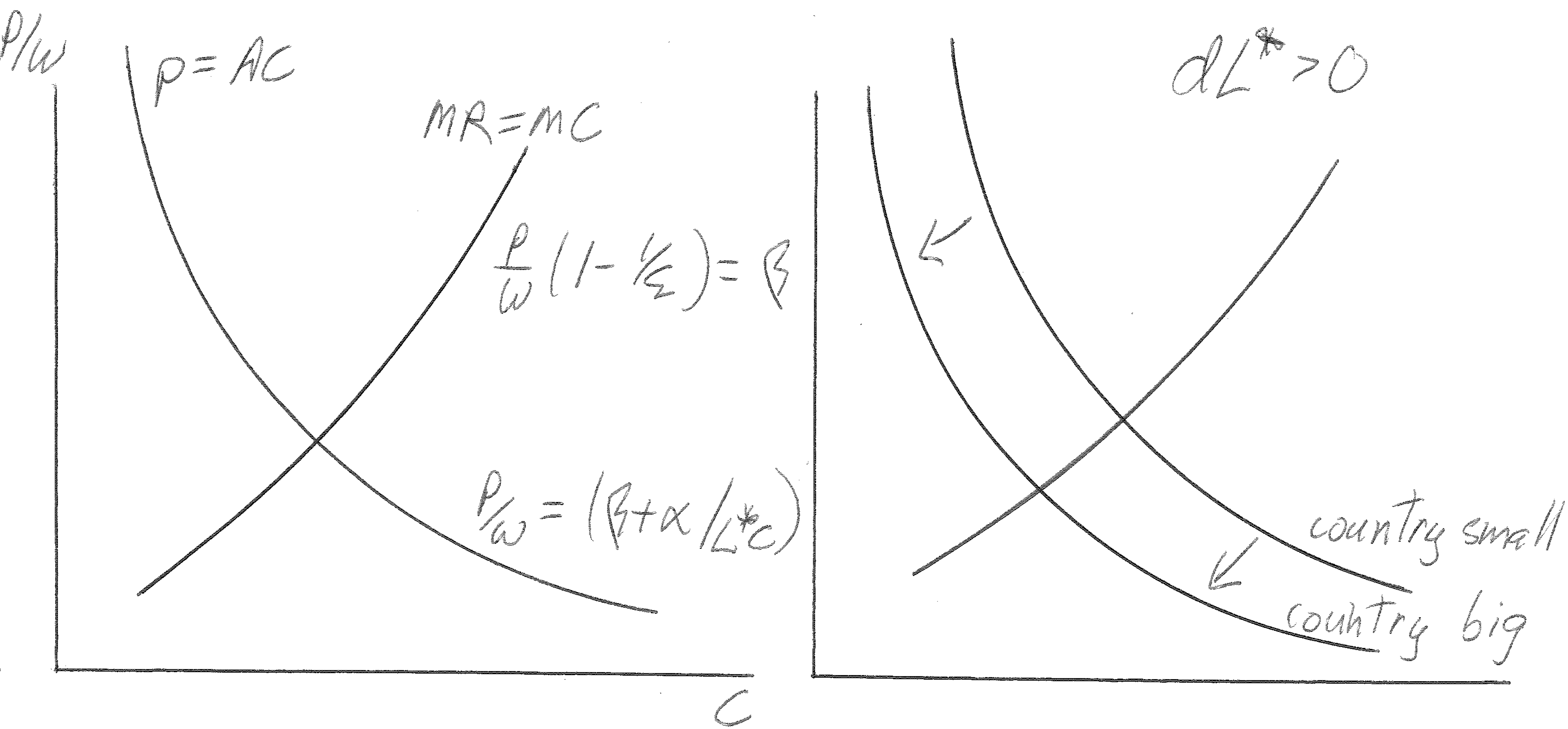
$p_i = p \quad \forall i \qquad X_i = X \quad \forall i$ symmetry

$p(1 - 1/\epsilon) = \beta w$ MR = MC (6)

$pX - (\alpha + \beta X)w = 0$ p = AC (zero profits) (7)

Using (3), these last two equations constitute two equations in two unknowns: p/w (the inverse of the real wage) and C (ε is a function of C).

$$\frac{p}{w}(1 - 1/\epsilon) = \beta \qquad \frac{p}{w} = \beta + \frac{\alpha}{L^*C} \tag{8}$$



Think of free trade as just analogous to growth in one economy, $dL^* > 0$.

(1) the real wage increases, $d(p/w) < 0$.
(2) consumption of a representative variety per capita decreases, $dC < 0$.
(3) But from (7), output per variety increases, $dX > 0$.
(4) Using (3) and (4), n rise, $dn > 0$.

"Large-Group" Monopolistic Competition:

Subsequent work generally assumed, that an individual firm's market share is small enough that the 's' in the elasticity formula can be ignored.
In that case the markup reduces to just $1/\sigma$, which is a constant.

In this case, (6) and (7) can be written as:

$$p(1 - 1/\sigma) = \beta w \qquad \text{MR = MC} \tag{9}$$

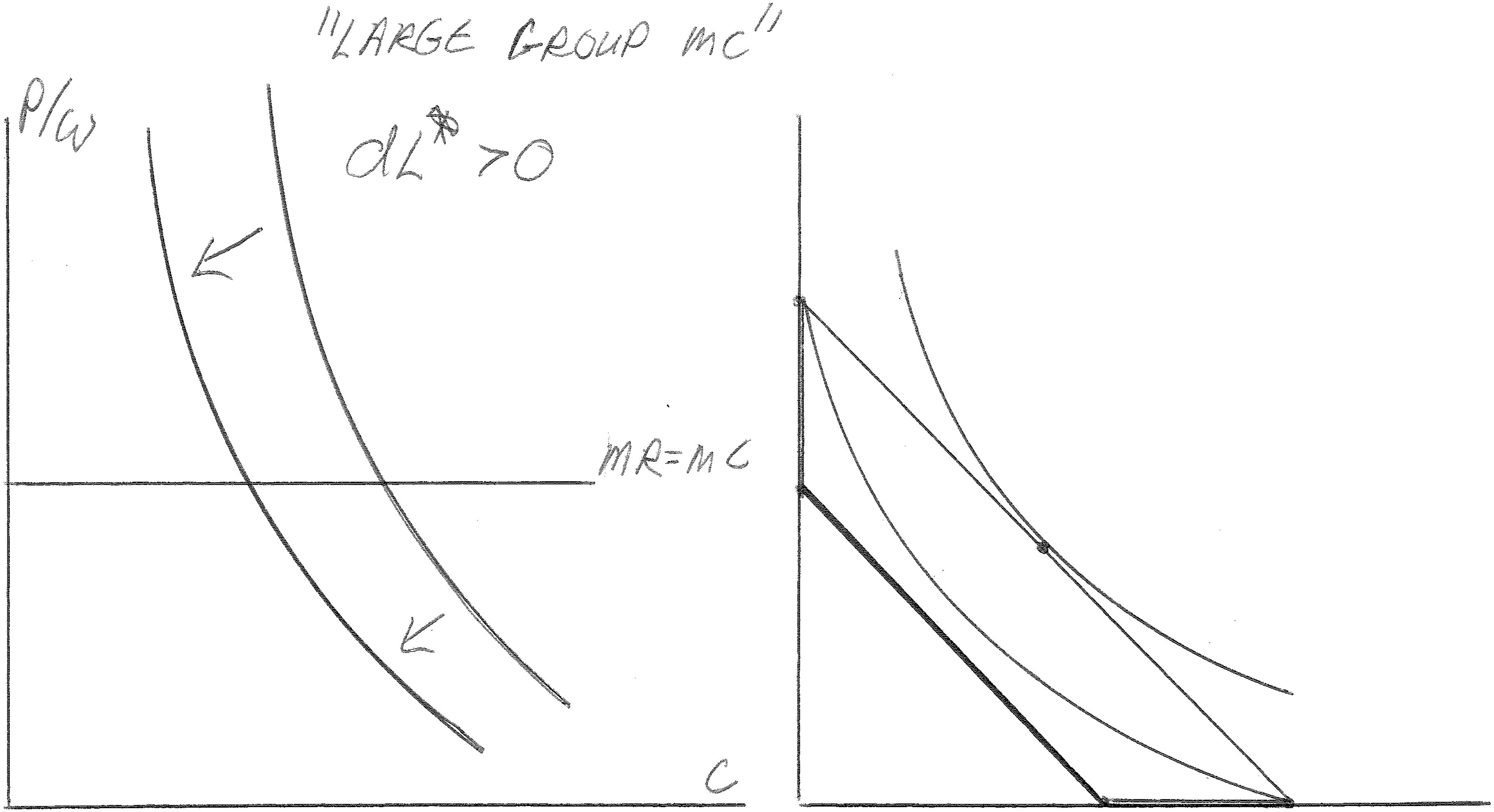
$$p = (\frac{\alpha}{X} + \beta)w \qquad \text{p = AC (zero profits)} \tag{10}$$

Divide one equation by the other to eliminate p and w, and solve for X.

$$X = \frac{\alpha(\sigma - 1)}{\beta} \tag{11}$$

Output of X is now a *constant*.

A growing economy will expand only through the addition of new varieties. From (4), it must be the case that the number of varieties is proportional to L*.



There is no change in firm scale, no pro-competitive effect, and no increase in production efficiency (moving down the AC curve). *Yet there is a welfare gain.*

$$L^* = n(\beta X + \alpha) = n\left[\beta \frac{\alpha(\sigma - 1)}{\beta} + \alpha\right] = n\alpha\sigma \tag{12}$$

Recall that U is defined as per capita.

$$U = nC^\gamma = \frac{L^*}{\alpha\sigma}\left[\frac{X}{L^*}\right]^\gamma = \frac{L^{*1-\gamma}}{\alpha\sigma}\left[\frac{\alpha(\sigma - 1)}{\beta}\right]^\gamma \tag{13}$$

which is increasing in L*, the size of the economy.

If we put two identical economies together in trade, one can produce good 1 and the other good 2, each country trading half its output for half the output of the other country's good.

Ethier AER 1982

Focuses on differentiated intermediate inputs rather than final goods.

W - output of wheat
m - "factor bundles" used in manufacturing production
M - output of manufacturers
x - output of a single manufacturing firm
q - price of a single firm's output
p - price (index) of manufacturing output

(1) $W = T(m)$ transformation between wheat and factor bundles used in manufacturing

(2) $$M = n^{\alpha}\left[\sum_{n} \frac{x_i^{\beta}}{n}\right]^{1/\beta} = \left[\sum_{n} x_i^{\beta}\right]^{1/\beta} \quad \text{if } \alpha = 1/\beta > 1$$

(3) $m = n(ax + b)$ factor bundles (a is marginal cost, b is fixed cost)

(4) $-T'(m) > 0$ cost of a factor bundle using W as numeraire

(5) $c = -T'(m)(ax + b)$ firm's cost function

(6) $MR = q(1 - 1/\sigma) = q(1 - (1 - \beta)) = q\beta$ marginal revenue

(7) $q\beta = -T'(m)a$ MR = MC

(8) $q = -T'(m)(a + b/x)$ q = average cost (free entry)

(9) $x = \dfrac{b\beta}{a(1 - \beta)}$ constant output per firm

using $m = n(ax + b)$

(10) $n = (1 - \beta)\dfrac{m}{b}$

exploiting symmetry in the production function (2)

(11) $$M = n^{\alpha}x = \left[\frac{1-\beta}{b}\right]^{\alpha-1}\frac{\beta}{a}m^{\alpha}$$

Find "supply curve"

(12) $$p_s M = qnx, \qquad p_s M = p_s n^{\alpha}x = qnx$$

(13) $$p_s = n^{1-\alpha}q = n^{1-\alpha}\left[\frac{-T'(m)a}{\beta}\right]$$

(14) $$p_s = -\left[\frac{(1-\beta)m}{b}\right]^{1-\alpha}\frac{-T'(m)a}{\beta}$$

May be positively or negatively sloped. This is similar to the external economies model. The force for concavity (positive slope) is T', the factor-intensity effect and the force for convexity (negative slope) is the first term.

Now consider demand. Assume Cobb-Douglas preferences with share γ spent on M.

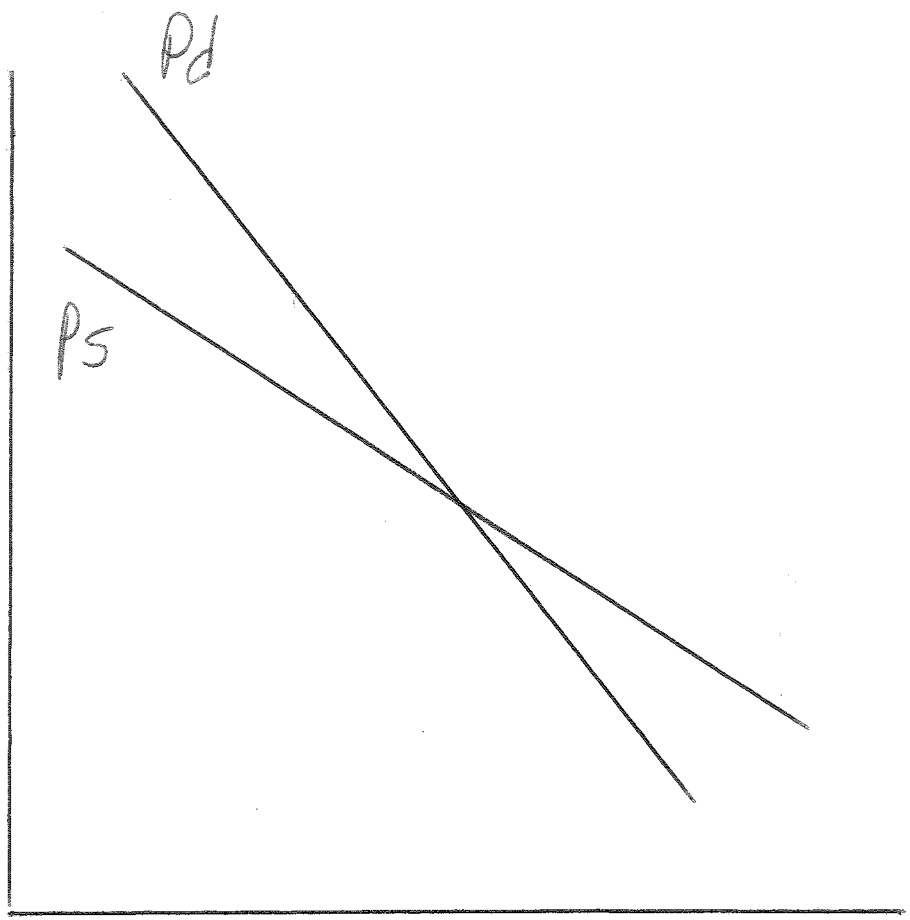
(15) $$p_d M = \gamma(W + p_d M) = \gamma(T(m) + p_d M)$$

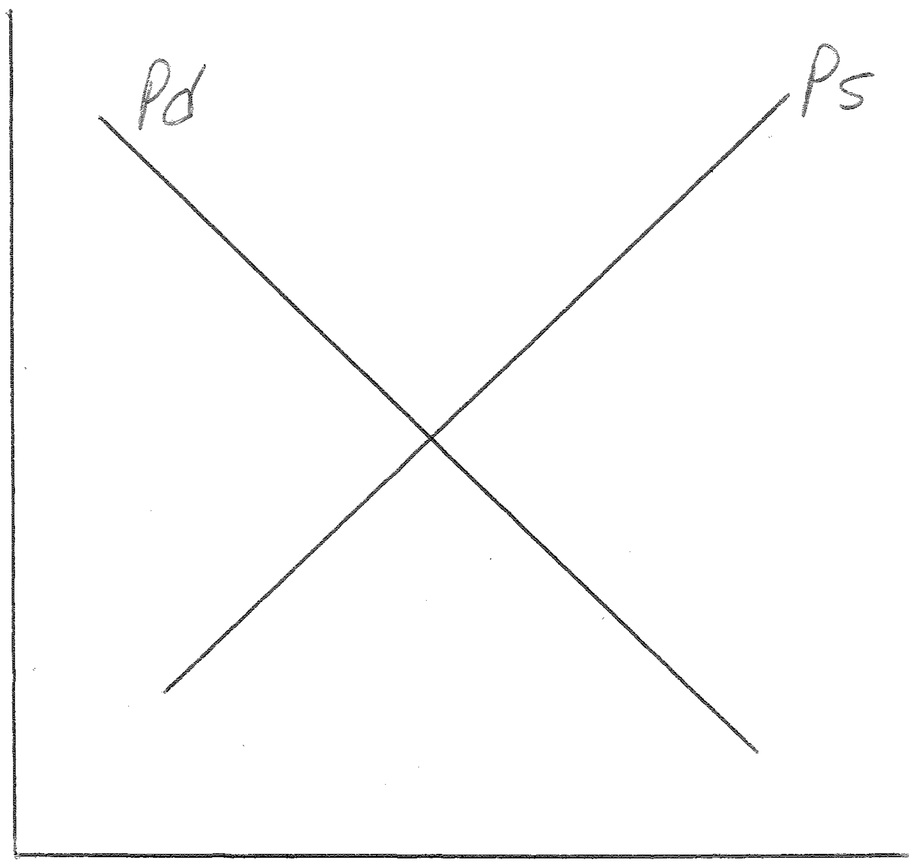
(16) $$p_d = \frac{\gamma}{1-\gamma}\frac{T(m)}{M}$$

(17) $$p_d = \frac{\gamma}{1-\gamma}\frac{a}{\beta}\left[\frac{b}{1-\beta}\right]^{\alpha-1}\frac{T(m)}{m^{\alpha}}$$

Equating p_d and p_s

$$(18) \qquad \frac{T(m)}{mT'(m)} = -\frac{1-\gamma}{\gamma}$$





Open economy, free trade. Output per firm is constant, so

(19) $$m = n(ax + b) \qquad m^* = n^*(ax + b)$$

(20) $$m + m^* = (n + n^*)(ax + b)$$ and using (9)

(21) $$(n + n^*) = \frac{1 - \beta}{b}(m + m^*)$$

(22) $$M = (n + n^*)^\alpha x = \left[\frac{1 - \beta}{b}\right]^{\alpha - 1} \frac{\beta}{\alpha}(m + m^*)^\alpha$$

Using the same procedure as before for the home country, yields

$$(23) \qquad p_s = -\left[\frac{(1-\beta)(m+m^*)}{b}\right]^{1-\alpha} \frac{-T'(m)a}{\beta}$$

$$(24) \qquad p_d = \frac{\gamma}{1-\gamma}\frac{a}{\beta}\left[\frac{b}{1-\beta}\right]^{\alpha-1} \frac{T(m)+S(m^*)}{(m+m^*)^{\alpha}}$$

Let $p_s = p_d$

$$(25) \qquad \gamma(T(m)+S(m^*)) + (1-\gamma)(m+m^*)T'(m) = 0$$

Ethier calls this the "home allocation curve" and, combined with the corresponding curve for the foreign country, solves for free-trade equilibrium.

Consider the special case of two identical economies, so in equilibrium, $m = m^*$ and $T(m) = S(m^*)$.

(26) $$\gamma(2T(m)) + (1-\gamma)(2m)T(m) = 0$$

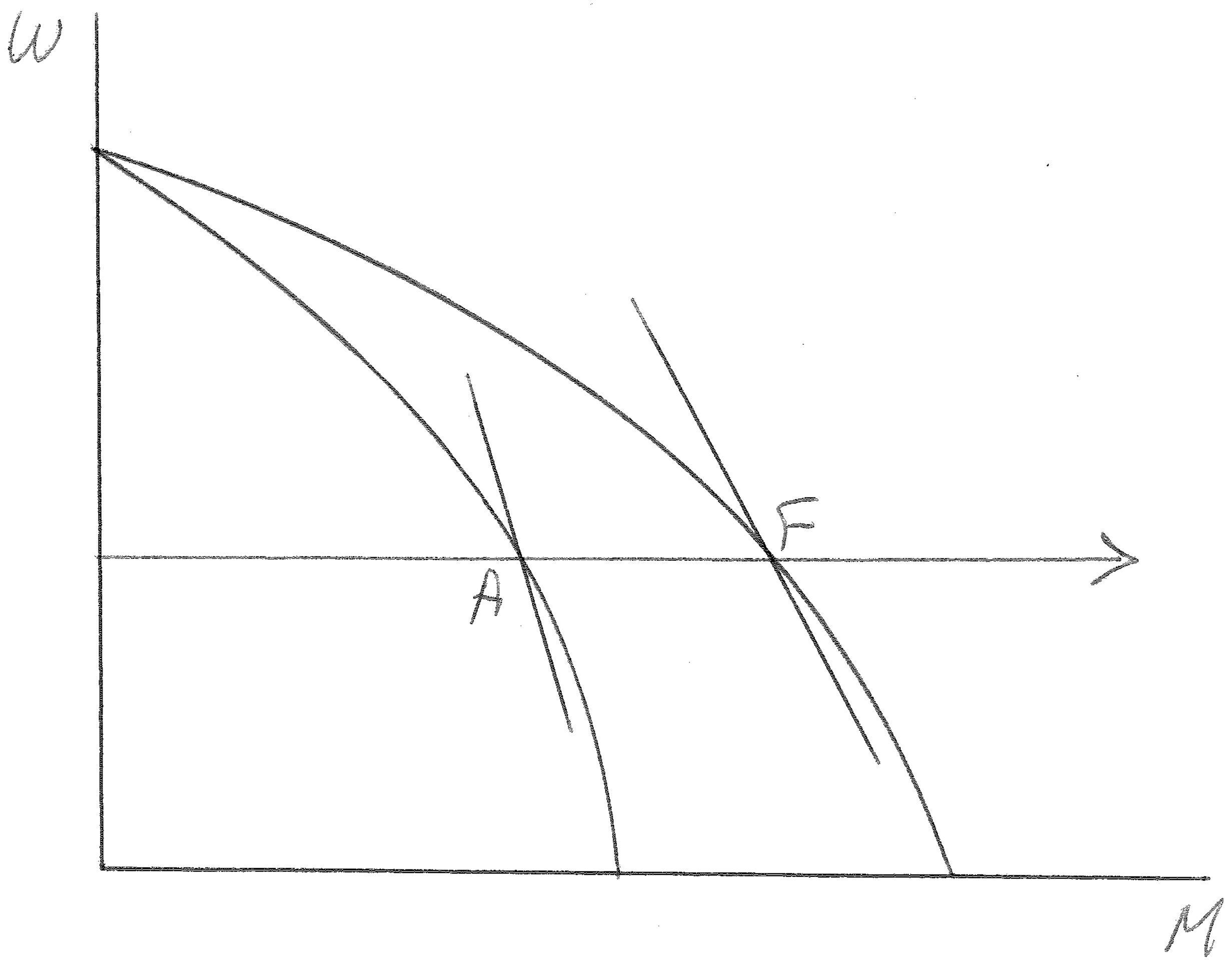
(27) $$\frac{T(m)}{mT'(m)} = -\frac{1-\gamma}{\gamma}$$ but this is the same m that solves equation (18) for autarky

So free trade involves the same output of W in each country, the same number of intermediates in each country.

But each country's M output increases to

(28) $$M = (n + n^*)^{\alpha}x/2 = 2^{\alpha - 1}n^{\alpha}x > n^{\alpha}x$$

and the price of manufacturers must fall.



Extensions: Markusen AER 1989 considers free trade in final goods only versus free trade in intermediates.

Much of the "new growth theory" Romer, Rivera-Batiz and Romer, Grossman and Helpman, etc. start from this Ethier idea.

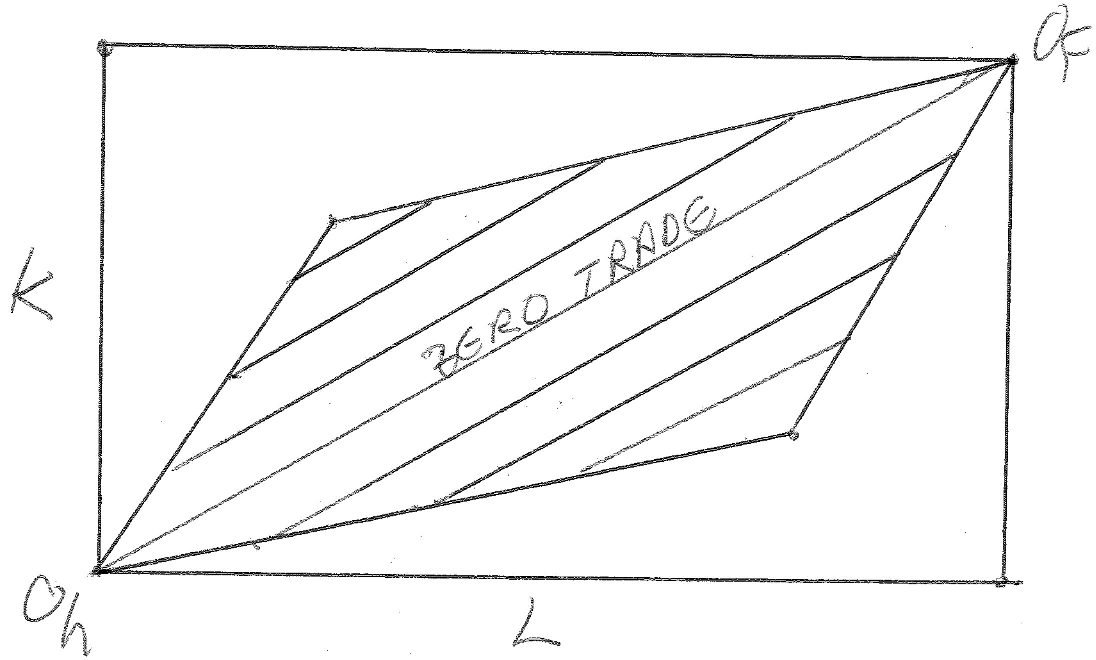
Helpman-Krugman 1985

Assume free trade and use Dixit-Norman FPE set ("integrated equilibrium")

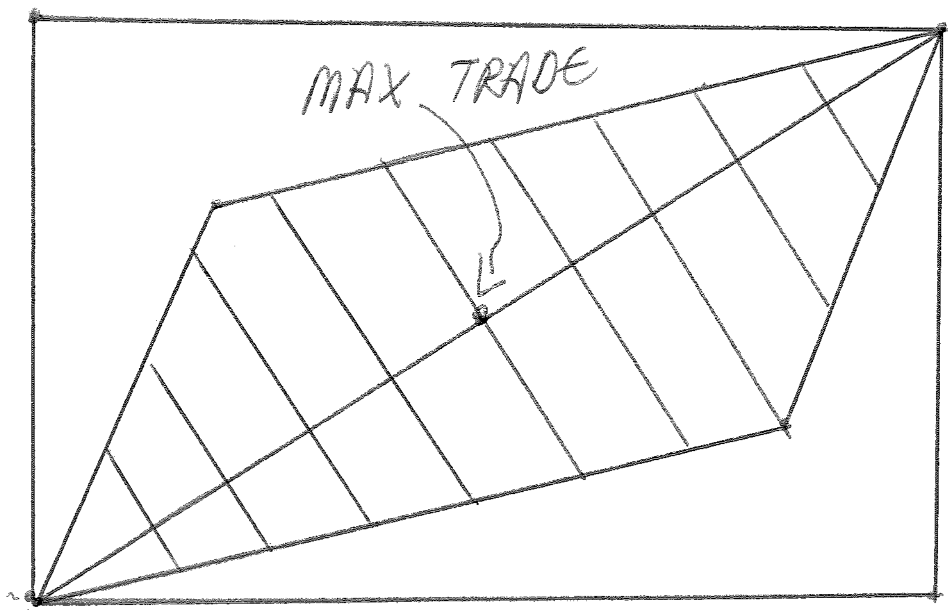
(1) $U = \left[\sum_i X^{\beta}\right]^{\alpha/\beta} Y^{1-\beta}$

Permits both *intra-industry* of X varieties and *inter-industry* trade of X for Y.

Loci of equal volumes of inter industry trade.



Loci of equal volumes of intra industry trade



At A, each country exchanges 1/2 the output of each of its X goods for 1/2 of the other country's goods: VOT in X = 1/2 world output.

Within the FPE set, the total number of goods produced is constant as is the output per good. Normalize the total $n + n^* = 1$.

Countries consume each good in proportion to total income (size).

So at B, F consumers 3/4 of the output of each good, H = 1/4.

At B, F produces 3/4 of the X goods, H produces 1/4.

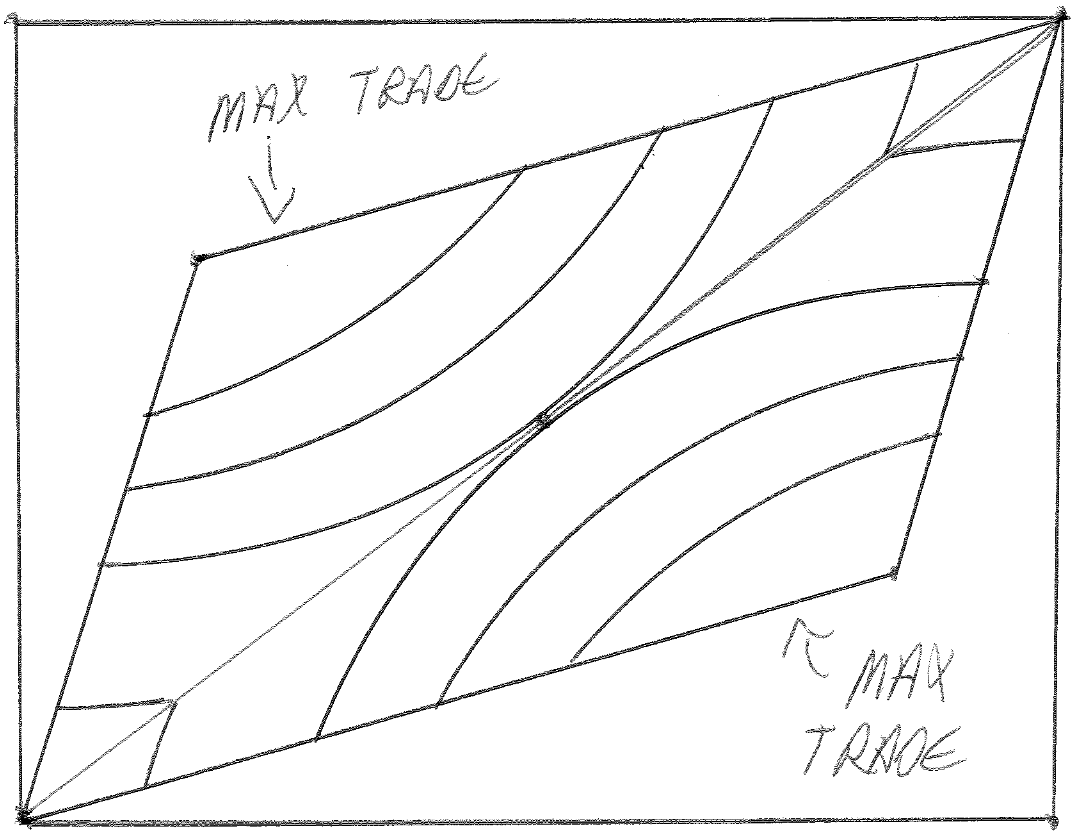
Trade in X at B

$$(1/4)*(3/4) \quad + \quad (3/4)*(1/4) \quad = \quad 6/16 \; = \; 3/8 < 1/2$$

(1/4)	(3/4)	(3/4)	(1/4)
No. of goods produced in H	proportion of each H good exported	no. of goods produced in F	proportion of each F good exported

At B, VOT in X = 0.375 of world X output.

Combined volume of trade in Y and X.



Helps explain the large volume of trade between similar countries.

Extension: Markusen AER 1986 adds non-homothetic demand in a three-region model. Labor-intensive goods are low income elasticity goods, so there is a small volume of N-S trade.

Limitation of the HK analysis: techniques reply totally on costless trade and FPE. Add a tiny trade cost and the whole thing falls apart.

Markusen and Venables JIE 2000

Adds trade costs and multinational firms to the basic HK model.

Good introduction also to the use of price indices that figure so much in the "new economic geography". Country i's utility is given by

$$U_i = X_{ic}^{\beta} Y_{ic}^{1-\beta}, \qquad X_{ic} \equiv \left[N_i (X_{ii})^{\alpha} + N_j (X_{ji})^{\alpha} \right]^{\frac{1}{\alpha}} \tag{4}$$

$$Y_{ic} = (1-\beta) M_i \qquad X_{ic} = \beta M_i / e_i \qquad e_i(p_{ji}) = \min(X_{ji}) \sum_{i,j} p_{ji} X_{ji} \quad st \quad X_{ic} = 1 \tag{5}$$

where M is income. e is referred to as the unit expenditure function or the price index for X. It gives the cost of purchasing one unit of the composite good X_c.

Let's not worry about different sources of X for a minute and just derive the unit expenditure function or price index.

$$\max X_c = \left[\sum X_i^{\alpha}\right]^{\frac{1}{\alpha}} + \lambda(M_x - \sum p_i X_i)$$

$$\frac{1}{\alpha}\left[\sum X_i^{\alpha}\right]^{\frac{1}{\alpha}-1} \alpha X_i^{\alpha-1} - \lambda p_i = 0$$

(6)

Let σ denote the elasticity of substitution among varieties. Dividing the first-order condition for i by the one for good j,

$$\left[\frac{X_i}{X_j}\right]^{\alpha-1} = \frac{p_i}{p_j} \qquad \frac{X_i}{X_j} = \left[\frac{p_i}{p_j}\right]^{\frac{1}{\alpha-1}} = \left[\frac{p_i}{p_j}\right]^{-\sigma} \qquad since \quad \sigma = \frac{1}{1-\alpha} \tag{7}$$

$$X_j = \left[\frac{p_i}{p_j}\right]^{\sigma} X_i \qquad p_j X_j = p_j p_j^{-\sigma} p_i^{\sigma} X_i \qquad \sum p_j X_j = M_x = \left[\sum p_j^{1-\sigma}\right] p_i^{\sigma} X_i \tag{8}$$

Inverting this last equation, we have the demand for variety i:

$$X_i = p_i^{-\sigma}\left[\sum p_j^{1-\sigma}\right]^{-1} M_x \qquad \sigma = \frac{1}{1-\alpha}, \qquad \alpha = \frac{\sigma - 1}{\sigma} \tag{9}$$

Now we can form the expenditure function, noting the relationship between α and σ.

$$X_i^\alpha = X^{\frac{\sigma-1}{\sigma}} = p_i^{1-\sigma}\left[\sum p_j^{1-\sigma}\right]^{\frac{1-\sigma}{\sigma}} M_x^\alpha \tag{10}$$

$$\sum X_i^\alpha = \left[\sum p_i^{1-\sigma}\right]\left[\sum p_j^{1-\sigma}\right]^{\frac{1-\sigma}{\sigma}} M_x^\alpha = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{\sigma}} M_x^\alpha \tag{11}$$

$$X_c = \left[\sum X_i^\alpha\right]^{\frac{1}{\alpha}} = \left[\sum X_i^\alpha\right]^{\frac{\sigma}{\sigma-1}} = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{\sigma-1}} M_x \tag{12}$$

$$e = \left[\sum p_j^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = n^{\frac{1}{1-\sigma}} p \quad if \ \forall p_j = p \tag{13}$$

The price index is homogeneous of degree in the individual prices, but *decreasing* in the range (number of goods) at equal constant prices.

Intuition: as the diversity of goods increases, at constant equal prices, the cost *e* of buying one unit of utility falls.

$$X_i \equiv p_i^{-\sigma} e^{\sigma - 1} M_x \qquad since \qquad e^{\sigma - 1} = \left[\sum p_i^{1-\sigma}\right]^{-1} \tag{14}$$

Now we need to clarify subscripts and superscripts. The first thing to note is how iceberg trade costs are reflected in prices and demand.

For a domestic firm, X_{ij}^d is the amount produced in country i and shipped to country j. Similarly, p_{ij} is the export price per unit in country i.

Let t (t > 1) be the ratio of the amount of X exported to the amount that arrives "unmelted". Alternatively 1/t is the proportion of a good that "survives" transit (the proportion "unmelted").

If X_{ij} is shipped, the amount received in country j is X_{ij}/t.

Second, we make the usual assumption that there is no price discrimination and so the home price of a good for local sales equals its export price. Thus we can use the notation p_i and p_j for the price of all goods *produced* in country i and country j respectively.

The revenues received by the exporter are equal to the costs paid by the importer: $p_i X_{ij}^d$ is the revenue received by the exporter and X_{ij}^d/t are the number of units arriving in the importing country, so the price per unit in the importing country must be $p_i t$ ($p_i X_{ij}^d = (p_i t) X_{ij}^d / t$).

Rather than introduce additional notation, we will therefore use X_{ij}/t and $p_i t$ as the quantity and price in country j of a country i variety exported to country j.

The price index for country i is then given by:

$$e_i = \left[N_i p_i^{1-\sigma} + N_j (p_j t)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{15}$$

Assuming that the relevant firm types are active in equilibrium, the demand functions for the various X varieties sold in country i are given by:

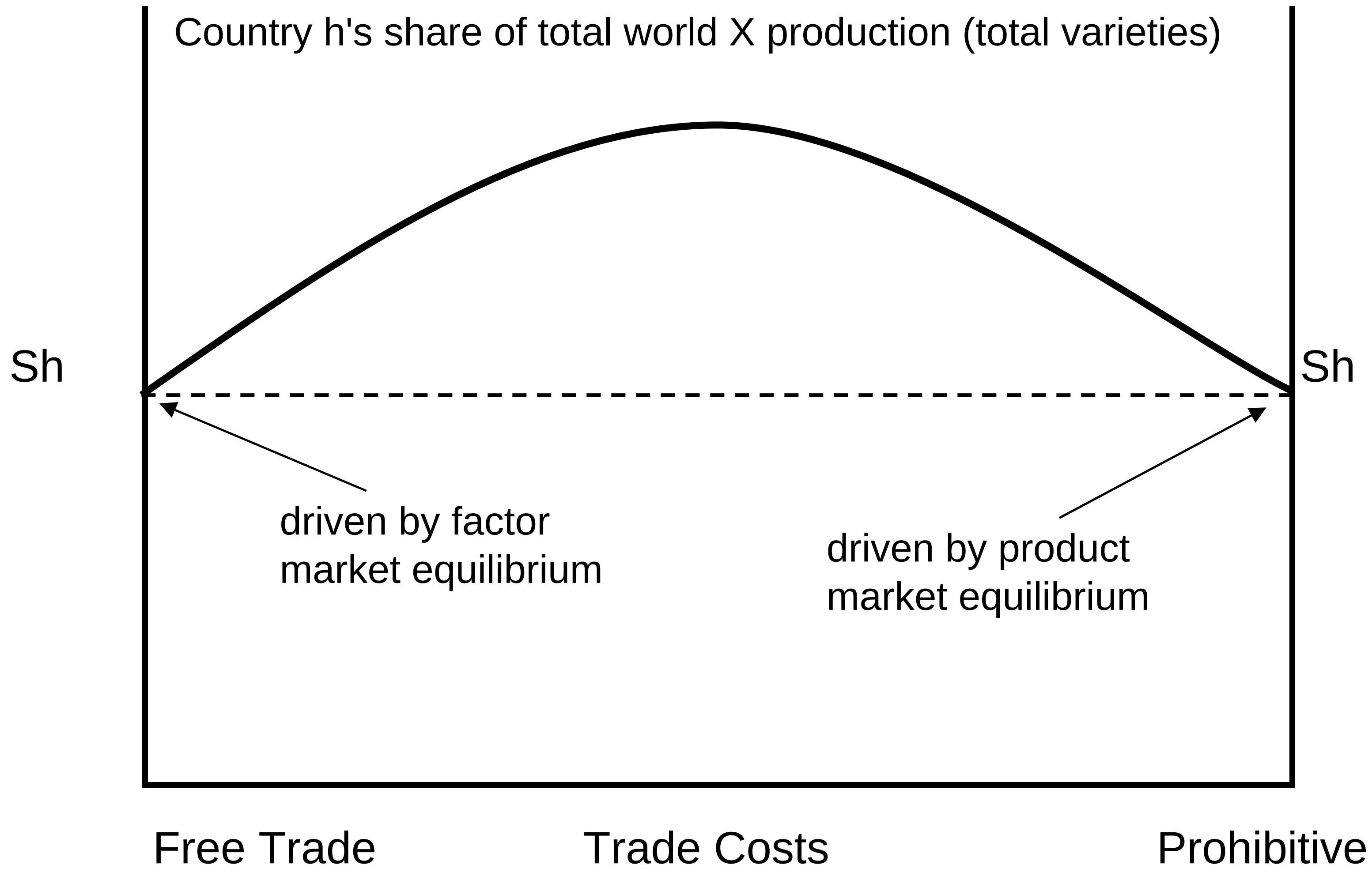
$$X_{ii} = p_i^{-\sigma} e_i^{\sigma - 1} M_{ix} \qquad X_{ji}/t = (p_j t)^{-\sigma} e_i^{\sigma - 1} M_{ix} \tag{16}$$

where the second equation can also be written as:

$$X_{ji} = p_j^{-\sigma} t^{1-\sigma} e_i^{\sigma - 1} M_{ix} \tag{17}$$

Let's just focus on the case where the countries have identical relative endowments of capital and labor, but country i is larger than country j.

Country h large: its share of total world income Sh > 1/2



Here is a proof that countries will not simply produced number of differentiated goods in proportion to size. If they did, demand for a large country goods would exceed demand for a small country good.

Assume that $\frac{N_i}{M_i} = \frac{N_j}{M_j} \quad X_{ii} + X_{ji} = X_{jj} + X_{ji} \quad => \quad p_i = p_j$

Since both countries have identical relative endowments, if they produce the same amount of each good and the number of goods is in proportion to size then average costs and prices must be equal.

Then using the price indices above, it must be that if country i is larger,

$$\left[\frac{e_i}{e_j}\right]^{\sigma - 1} = \frac{N_j(1 + t^{1-\sigma}N_i/N_j)}{N_i(1 + t^{1-\sigma}N_j/N_i)} > \frac{N_j}{N_i}$$

Then using (16) and (17)

$$X_{ii} - X_{ji} = p^{-\sigma}(1 - t^{1-\sigma})e_i^{\sigma-1}M_i$$

by assumption

$$\frac{X_{ii} - X_{ji}}{X_{jj} - X_{ij}} = \frac{e_i^{\sigma-1}M_i}{e_j^{\sigma-1}M_j} > \frac{M_i/N_i}{M_j/N_j} = 1 \qquad \text{implies}$$

$$X_{ii} + X_{ij} > X_{jj} + X_{ji}$$

Thus demand for a country i good exceeds the demand for a country j good and, since every good is produced in the same amount, we have a contradiction. It cannot be that countries produce a number of goods in proportion to their sizes.

(1) An equilibrium requires that the large country produce more varieties relative to its size, driving down the demand for its individual varieties.

(2) But this in turn means that the cost of producing an fixed amount of X in country i is higher due to factor intensity effects. Since the output of any good produced in either country is the same, zero profits then requires that country i varieties are more expensive than country j varieties. $p_i > p_j$

(3) Result (1) also implies that country i is relatively specialized in the X sector (since the output of each variety if fixed) and so is a net exporter of X goods.

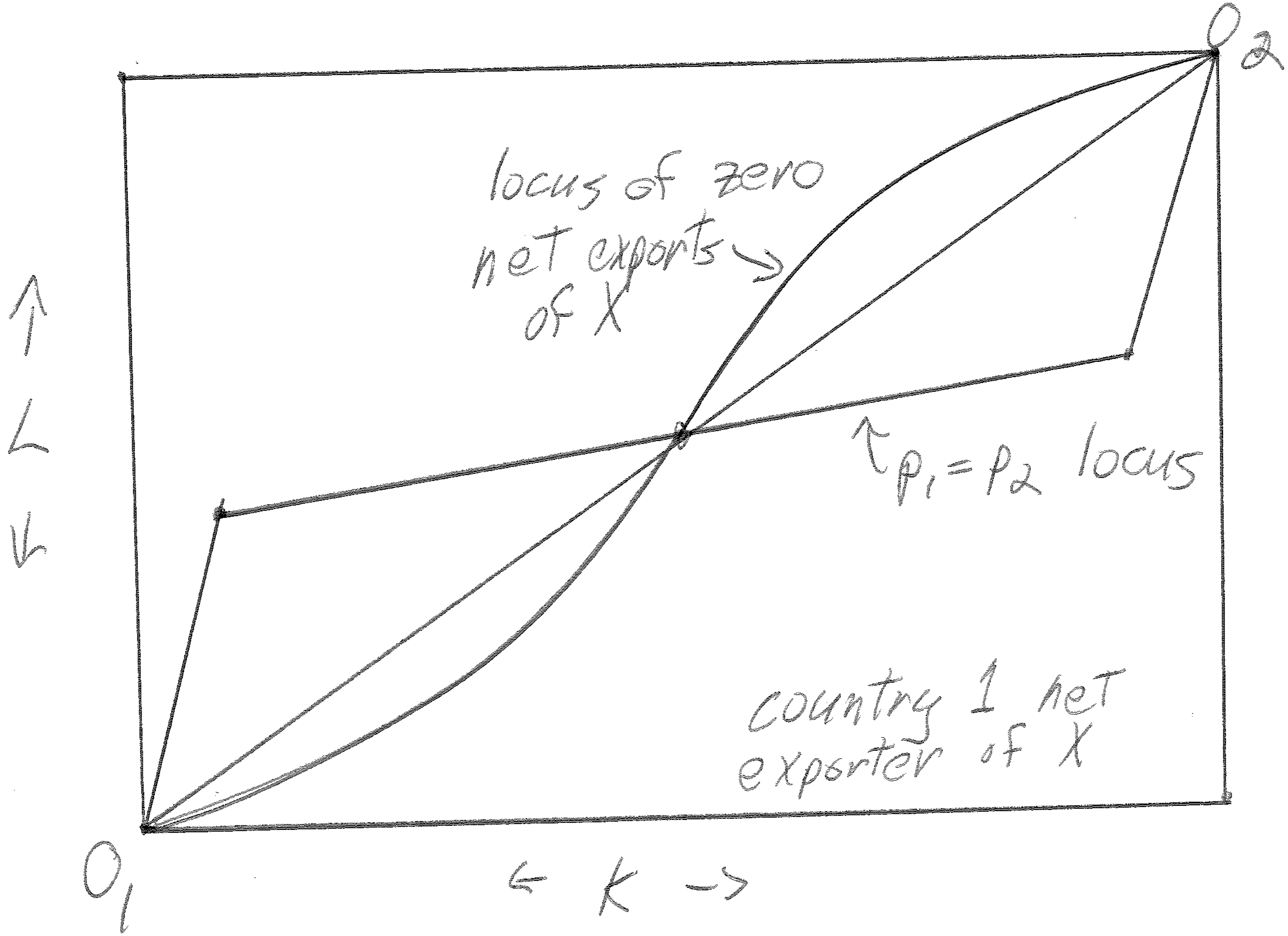
Conclusions:

(1) Country i is relatively specialized in and exports X goods.

(2) The price of X goods is higher in country i

(3) The price of capital, the factor used intensively in X is higher in country i and the price of labor is higher in country j.

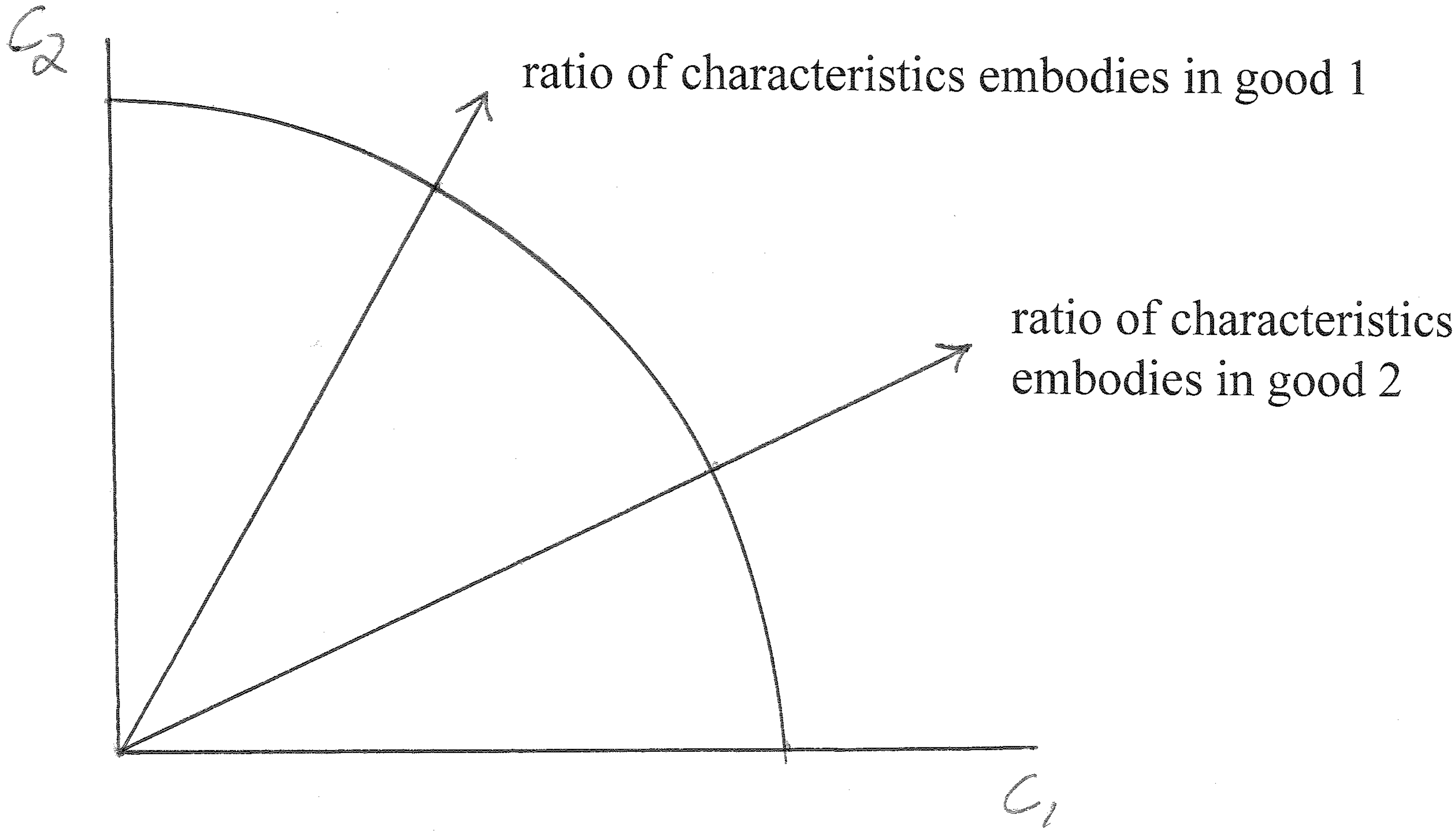
Later, we will return to this model in order to look at the incentives for factors to move and agglomerate.



O_2
locus of zero net exports of X
$p_1 = p_2$ locus
L
country 1 net exporter of X
O_1
K

Lancaster's Approach: "location" models, "ideal variety"

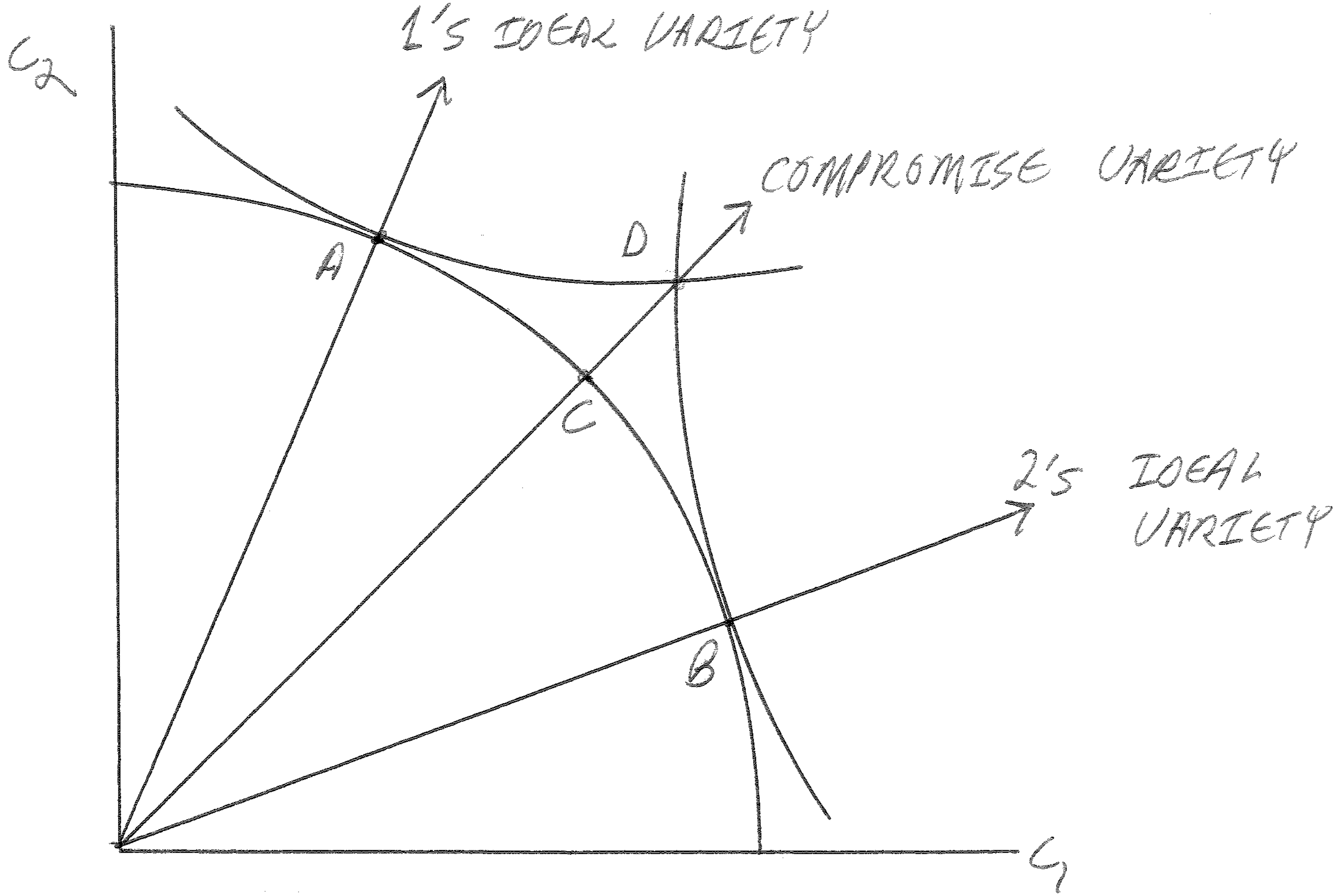
Suppose that preferences are defined over two characteristics (C_1, C_2) of goods rather than the goods themselves. Different goods embody different combinations of (C_1, C_2). Suppose that there are two goods, (X_1, X_2).



Alternative combinations of (C_1, C_2) produced by a *fixed* amount of resources.

A, B are amounts of X_1, X_2 which would cost the same. Assume that

X_1 is individual 1's most preference good ("ideal variety")
X_2 is individual 2's most preference good ("ideal variety")



1 unit of A and 1 unit of B are better than each getting 1 unit of C from the consumer's point of view.

DC/CO is called the "compensation ratio", But 2C *costs less* than 1A + 1B is *increasing returns to scale.*

If 2D costs less to produce than (1A + 1B) then the "compromise variety" is preferred.

Example: L is the only factor of production

$L_a = L_b = L_c = aC + F$ labor needed for one unit of A, B, or C.

$L_d = aD + F$ labor needed for one unit of D. $D > C$

The question is whether or not the labor needed for two units of D is greater than or less than the labor needed for 1A + 1B

$L_{2d} = 2aD + F$ $L_a + L_b = 2aC + 2F$

Compromise variety is preferred if

$$2aD + F < 2aC + 2F \qquad 2a(D - C) - F < 0 \qquad (1)$$

Suppose that this inequality "marginally" holds. Now suppose that we double the size of the economy, so that we need to compare 4D with 2A + 2B.

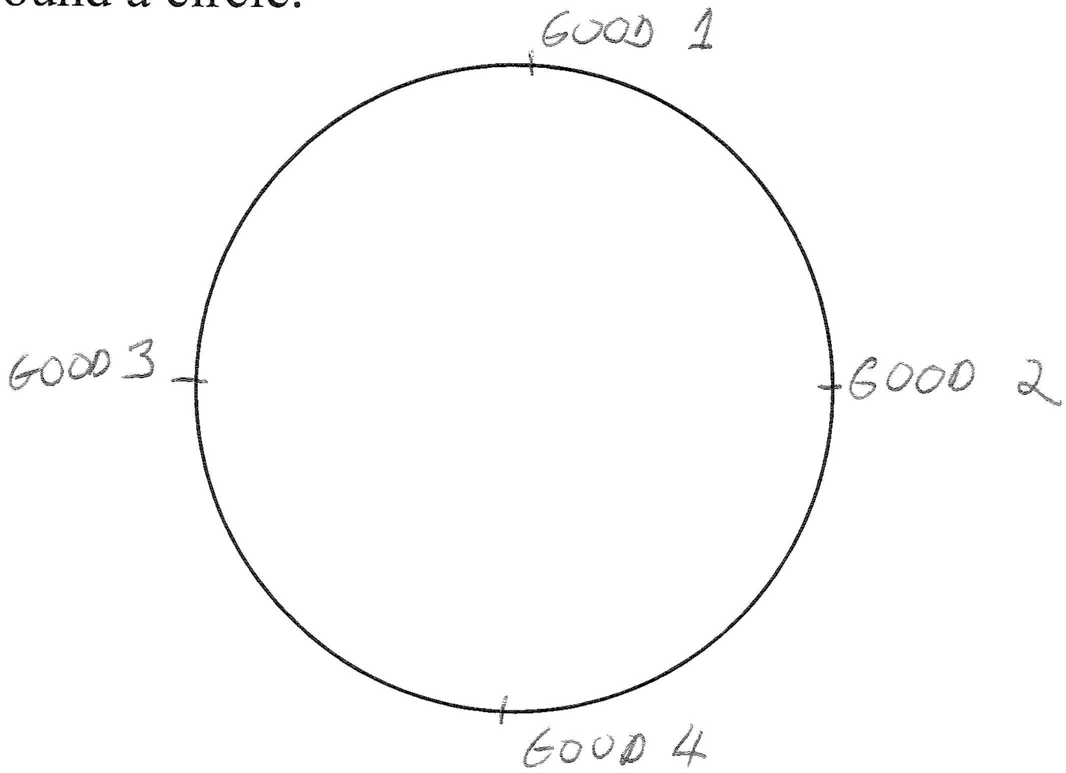
$$L_{4d} = 4aD + F \qquad L_{2a} + L_{2b} = 4aC + 2F$$

Compromise variety is preferred if

$$4aD + F < 4aC + 2F \qquad 4a(D - C) - F < 0 \qquad (2)$$

If (1) "marginally" holds, then (2) will not hold. The larger economy will have a more diverse set of goods. Each consumer type will get their most preferred variety.

A more complex approach used by Helpman. Goods characteristics are indexed around a circle. Consumers' most preferred varieties are distributed uniformly around a circle.



Zero-profit conditions (free entry) determine how many firms can exist in equilibrium. We can then calculate the average distance a consumer is away from his/her "ideal variety".

Again, think of trade as increasing the number of consumers (density of demand).

This will permit more firms in equilibrium.

There is a welfare gain in the sense that each consumer is "on average" closer to his/her ideal variety.

Internal Economies of Scale II: Differentiated Goods - Summary Points

1. Product differentiation, combined with scale economies (which limit differentiation in autarky), is a source of gains from trade.

2. Gains may be captured through a combination of increased diversity and increased scale of in individual goods. "Large group" monopolistic competition only involves the former.

3. Two-good, two-sector models produce equilibria in which ther is a combination of inter-industry and intra-industry trade. Inter-industry trade increases with the difference in the relative endowments of the two countries, intra-industry trade decreases with differences in country size (total income).

4. Positive transport costs make country size a basis for comparative advantage. If countries have identical relative endowments, the large country will be a net exporter of differentiated goods, and have a high price for the factor of production used intensively in the IRS sector.

5. This last result forms the basis for understanding some of the "new economic geography" in which permitting factors to move leads to agglomeration.