Lecture 6: Heckscher-Ohlin Theory

1. Basic assumptions

2. The production set

   Define factor intensities
   The Edgeworth Box
   Strict convexity of the production set
   Slope

3. The Heckscher-Ohlin Theorem

4. The Factor-Price-Equalization Theorem

5. The Stolper-Samuelson Theorem

6. The Rybczynski Theorem

7. Policy Implications

Begin with a model with two goods (X and Y), two factors (L and K), and two countries (h and f)

Factor Intensities

Definition of factor intensities: If at a given wage rental ratio (w/r),

$(K/L)_y > (K/L)_x$

Y is said to be capital intensive and X is labor intensive.

The Edgeworth Box

Factor intensities
Efficient allocations and the contract curve
Proof of strict convexity of the production set (concave transformation curve)
Relationship between (X/Y) produced and (p_x/p_y).

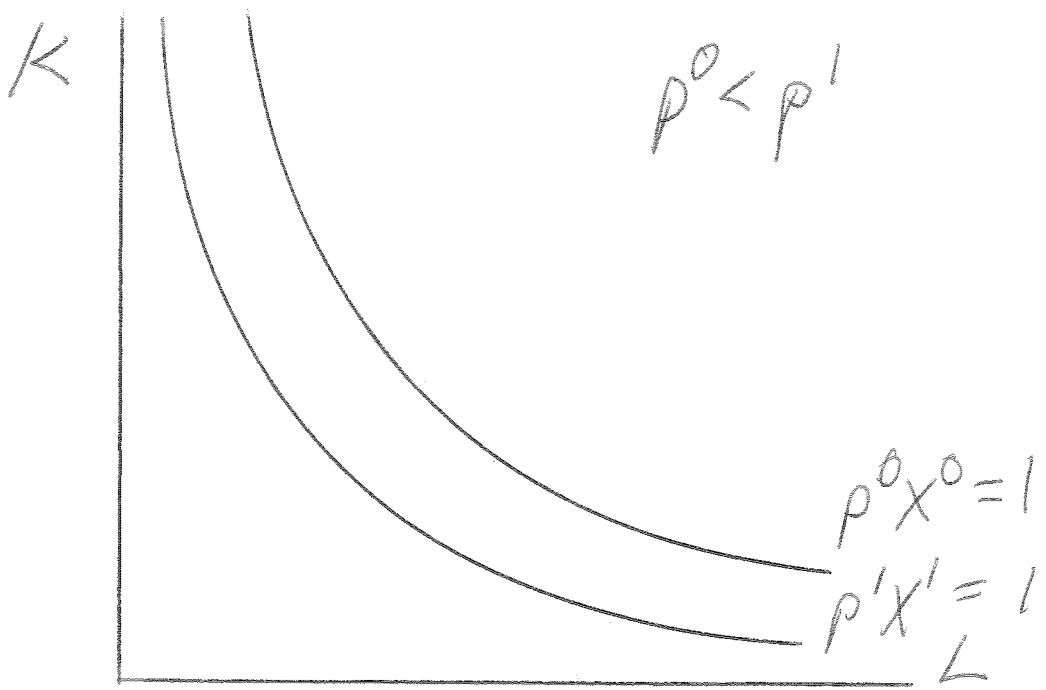
Factor Abundance

If $(K/L)_h > (K/L)_f$, where K's and L's refer to total endowments,

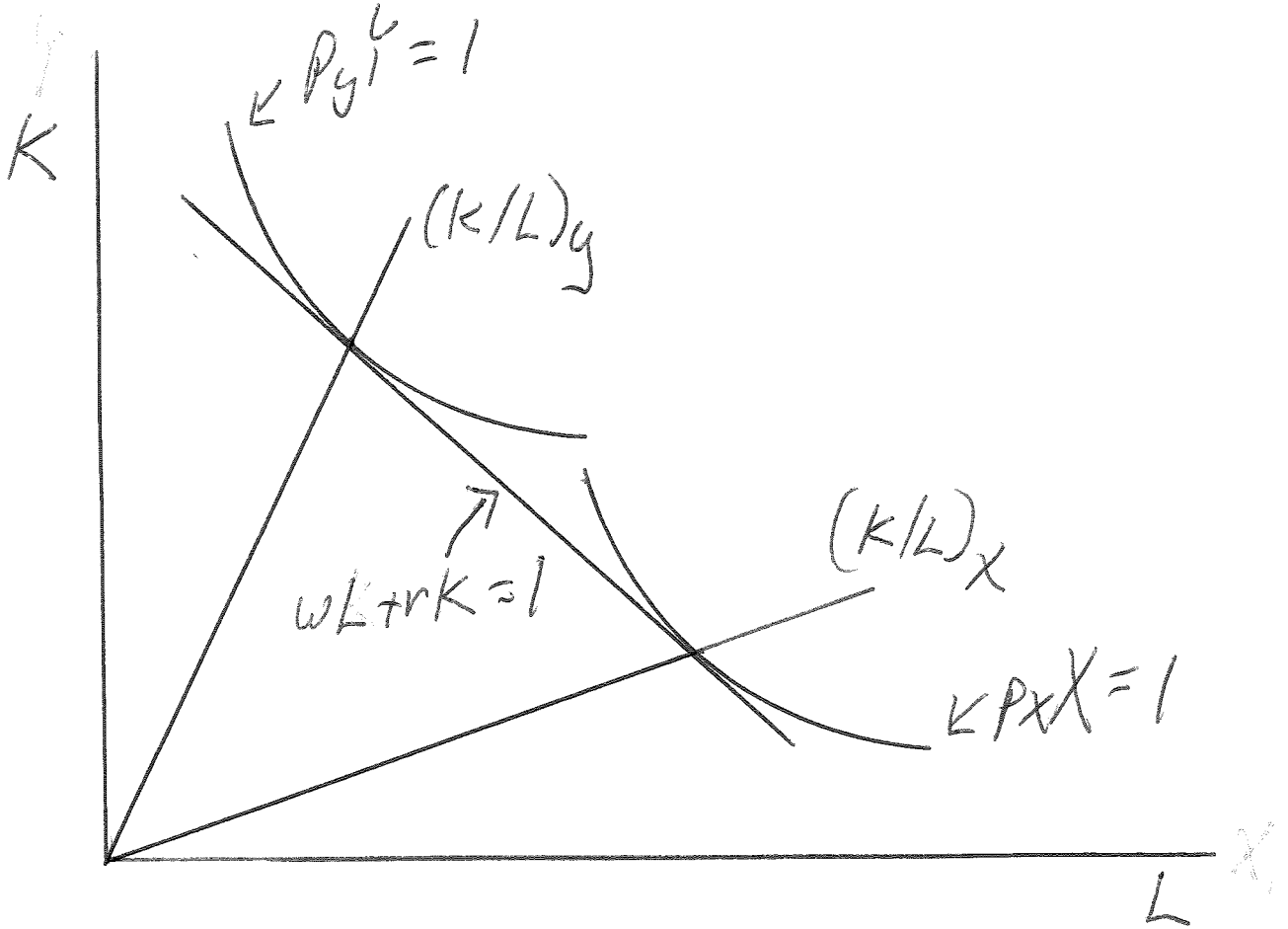
country h is said to be capital abundant, f is labor abundant

Preliminary steps in the theory:

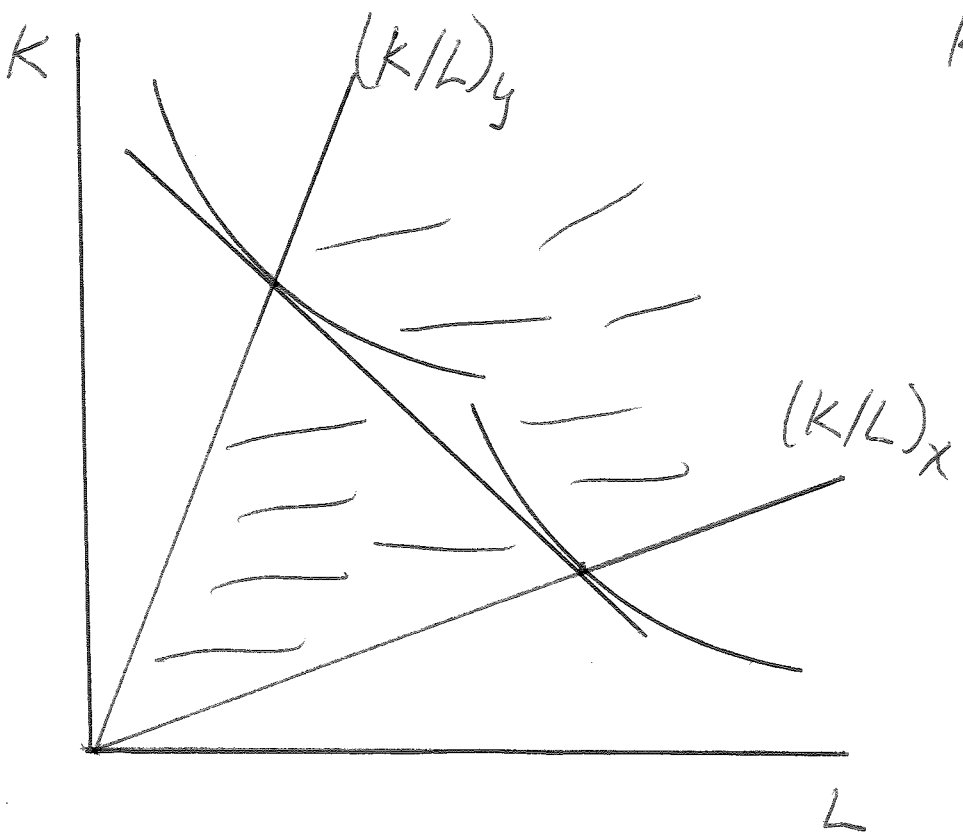
(1) Unit value isoquants give the amounts of capital and labor needed to produce \$1 of X and Y. Thus the *position* of the unit value isoquant depends on relative *prices*. Let p_y be numeraire so that $p_y = 1$.

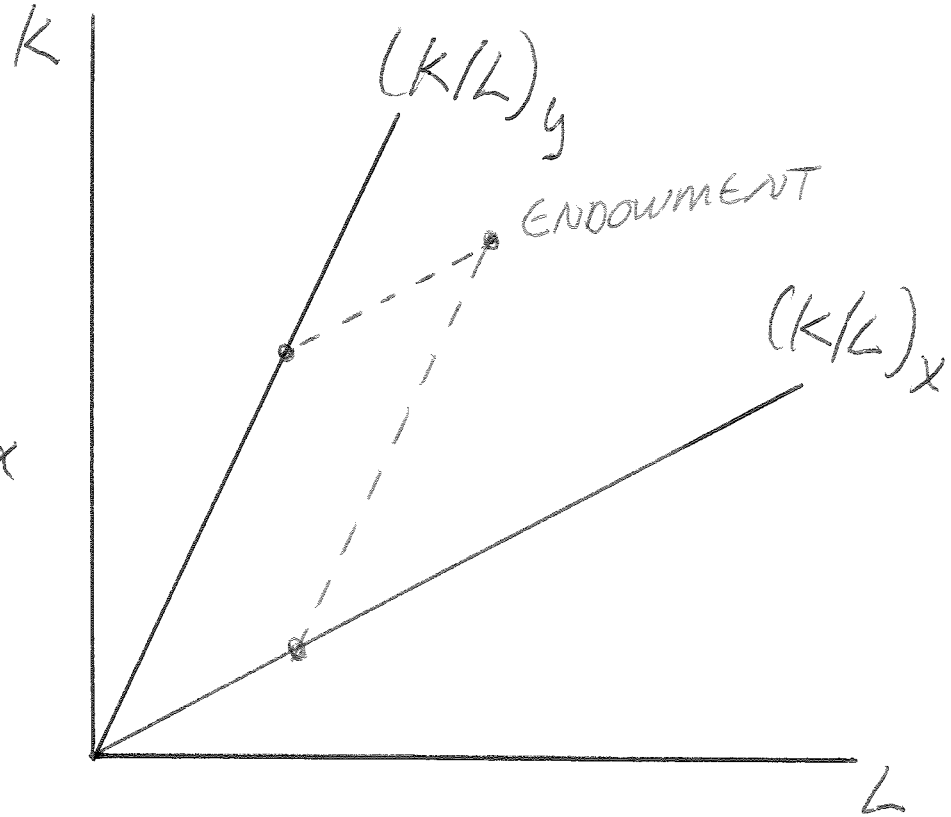


(2) *If* a country produces both goods, the optimal K/L ratios in X and Y depend only on commodity prices (given the technology), not on factor supplies. Refer to these as the <u>diversification K/L ratios</u> at the given price ratio p ($=p_x/p_y$).



(3) The <u>cone of diversification</u> is the set of K/L ratios lying between the diversification K/L ratios.





(4) If the country's K/L endowment ratio lies in the cone of diversification, then the country will be diversified (non-specialized) at the current commodity price ratio.

(5) If the endowment ratio lies on the boundary or outside of the cone, then the country will specialize.

(6) Note that the price ratios at which the country will specialize depend on the endowment ratio. The higher the K/L endowment ratio, the higher the necessary p ratio in order to induce production of X and specialization in X.

The L-abundant country begins to produce X at a lower p_x and specializes in X at a lower p_x relative to the K-abundant country.

(7) Now consider the region of prices where a country produces both goods.

Hold the commodity price ratio constant. Let a_{ij} be the amount of factor i used in the production of commodity j.

Assume that X is labor intensive, so that (at common factor prices)

$$(1) \quad \frac{a_{lx}}{a_{kx}} > \frac{a_{ly}}{a_{ky}} \Rightarrow a_{lx}a_{ky} > a_{ly}a_{kx} \qquad a_{lx}a_{ky} - a_{ly}a_{kx} \equiv D > 0$$

Factor market clearing requires:

$$(2)\quad \begin{bmatrix} a_{lx} & a_{ly} \\ a_{kx} & a_{ky} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} L \\ K \end{bmatrix}$$

Invert this mapping.

$$(3)\quad \begin{bmatrix} a_{ky}/D & -a_{ly}/D \\ -a_{kx}/D & a_{lx}/D \end{bmatrix} \begin{bmatrix} L \\ K \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$$

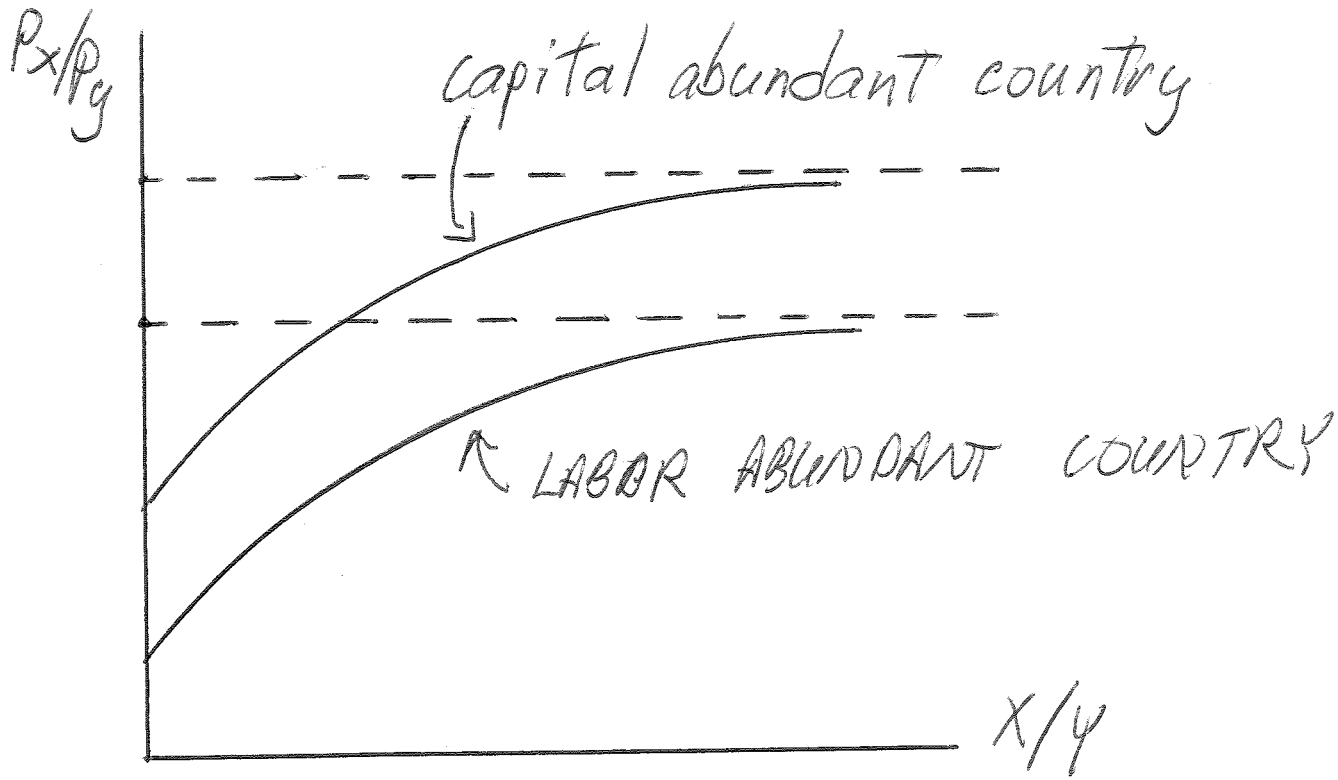
Form the ratio

$$(4)\quad \frac{X}{Y} = \frac{a_{ky} - a_{ly}\dfrac{K}{L}}{-a_{kx} + a_{lx}\dfrac{K}{L}}$$

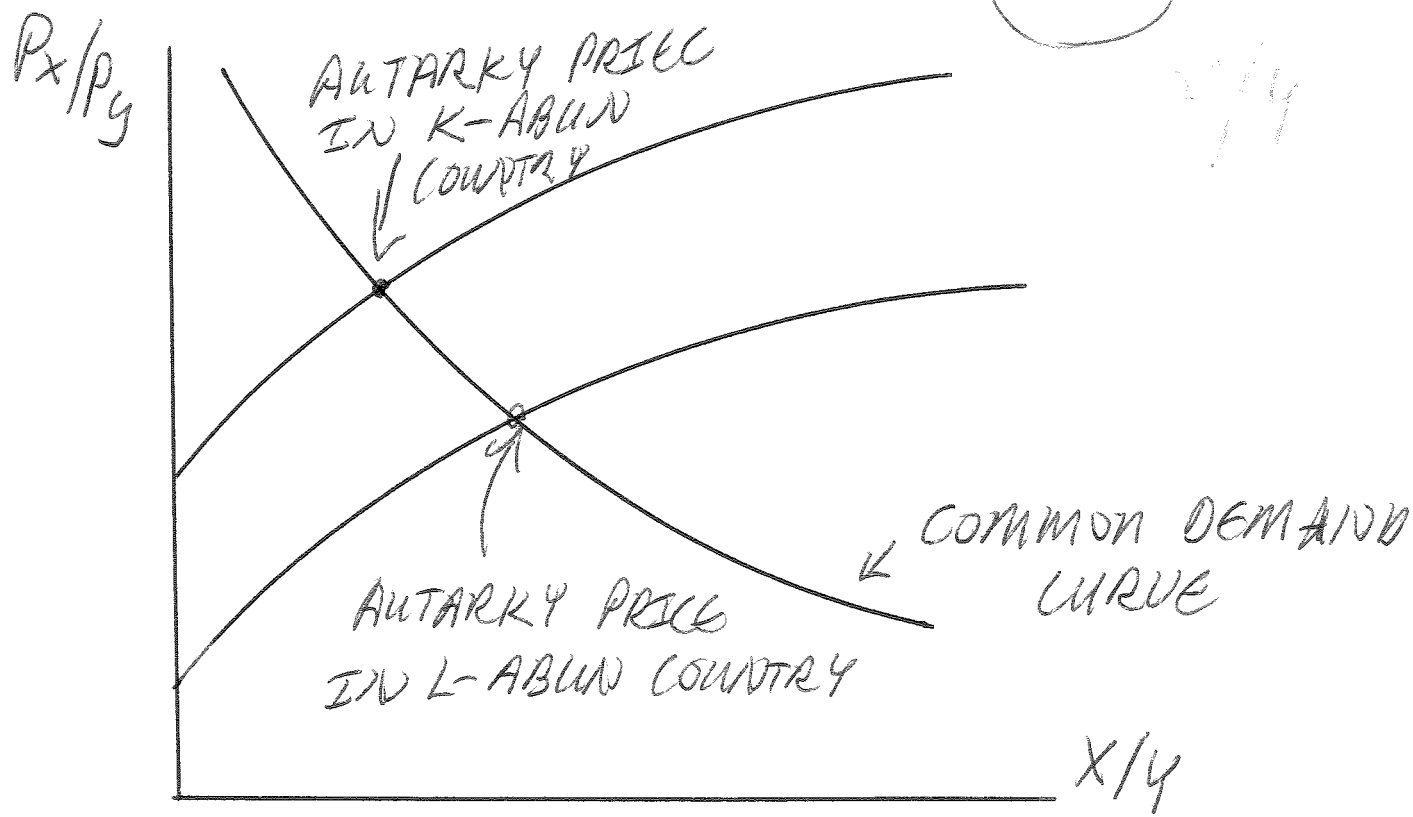
At a given commodity price ratio for which the economy is diversified, the ratio of X/Y production is decreasing in the K/L ratio.

Referring back to the first diagram, an increase in p_x / p_y shifts the unit value isoquants so that the K/L ratios increase in both industries if both goods are produced. From (4), this implies that X/Y increases with p_x / p_y .

Result: At a common supply price ratio, the labor abundant country produces more X relative to Y and the capital abundant country produces more Y relative to X.

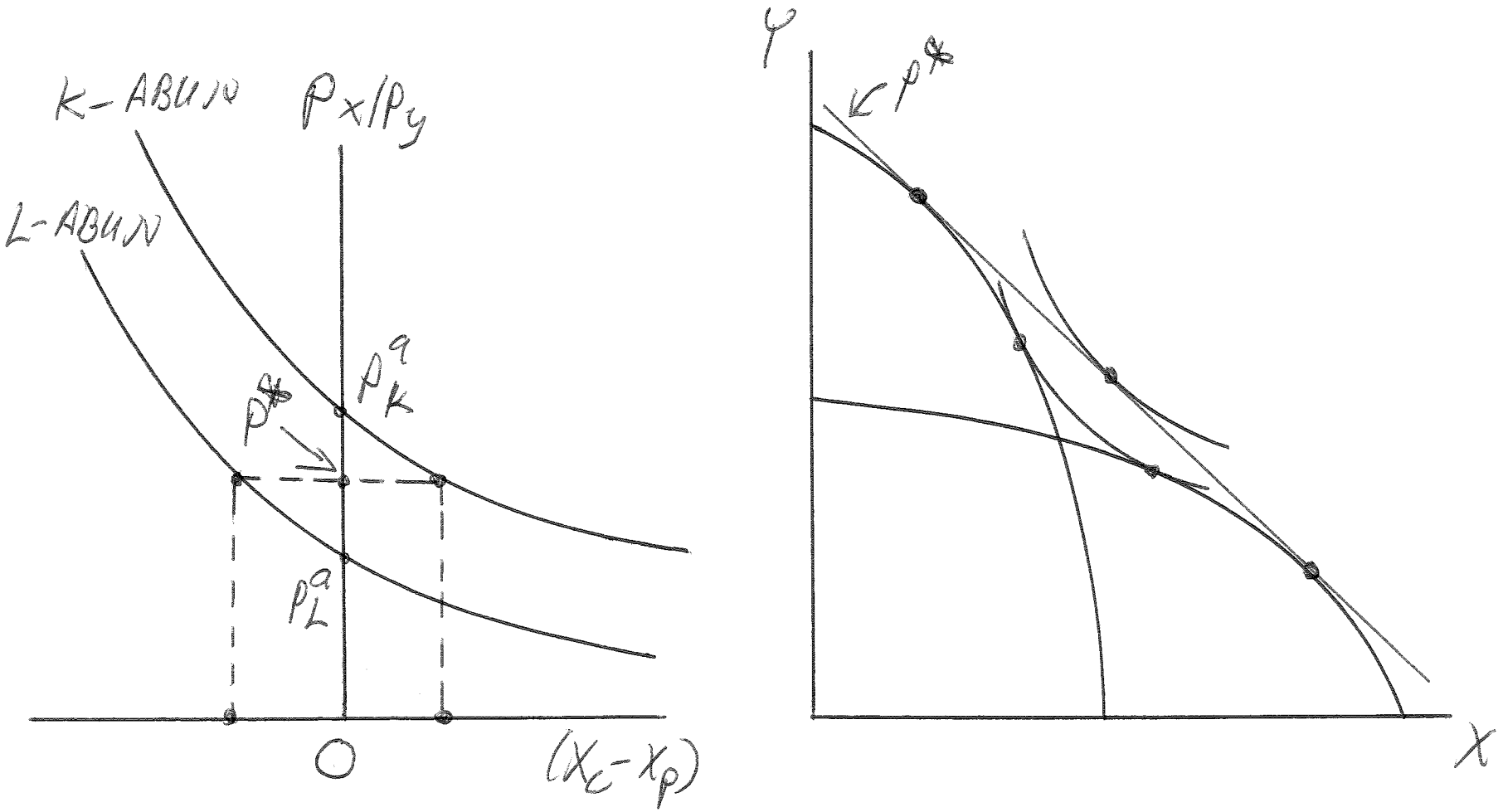


Assume that both countries have identical and homothetic (homogeneous) preferences. Then the relative demand X/Y is identical across countries and depends only on relative prices. Demand curves in (X/Y), (p_x/p_y) space.



Result: In autarky, each country has a relatively low price for the good using intensively its abundant factor.

Construct excess demand or offer curves.



Result, Direction of Trade (HO Theorem): Each country exports the good using intensively its abundant factor.

Factor-Price Equalization Theorem

Suppose that

(1) Commodity prices are equalized between countries by free trade.

(2) Both countries produce both goods in equilibrium (non-specialization).

Then, the price of each factor is equalized across countries.

Traditional Approach to Factor-Price Equalization

Let p_x and p_y be the equilibrium product prices in free trade. Draw the "unit value isoquants for X and Y, that is, the combinations of K and L that produce $1 of X and Y.

Draw the unit isocost line such that there are zero profits in each industry. Draw the implied K/L ratios for the two industries.

If the country's factor endowment lies in the "cone" spanned by these capital-labor ratios, then diversification (non-specialization) is possible. If this is true for both countries, then FPE holds in a free-trade equilibrium.

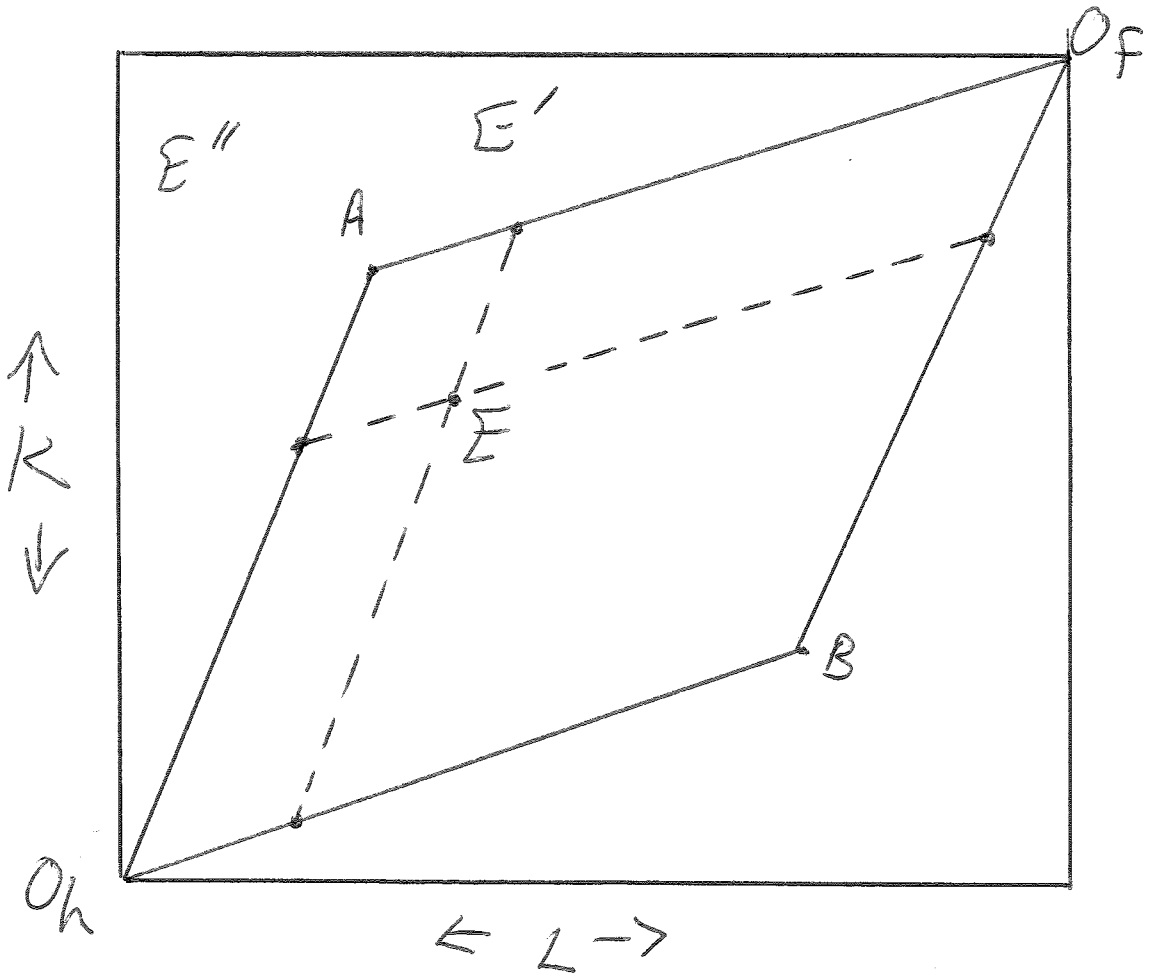
The Dixit-Norman integrated equilibrium approach (helps determine whether or not non-specialization holds).

Suppose that the endowments of the two countries were combined into one big country (with no internal trade costs, etc.).

Solve for the competitive equilibrium. Refer to this as the "Integrated World Equilibrium".

Consider the World Edgeworth Box, with country h graphed from the lower left and country f from the upper right. Let O_hA be the vector of factors necessary to produce the integrated-equilibrium amount of Y and O_hB the vector of factors necessary to produce the integrated-equilibrium amount of X.

Whether or not FPE holds in a trading equilibrium reduces to the question of whether or not the endowment point E lies inside the parallelogram O_hAO_fB.



If E lies in the parallelogram, then E can be written as $\lambda_a A + \lambda_b B$ where $0 < \lambda_i < 1$. That is, each country's endowment can be written as the sum of "positive" vectors that allocate factors to X and Y in the FPE proportions.

Rybczynski Theorem

Holding commodity prices constant, an increase in the endowment of one factor leads to a more than proportional increase in the output of that good and a fall in the output of the other good.

Since the a_{ij} depend only on commodity prices (which fix factor prices)

$$(1)\quad a_{lx}dX + a_{ly}dY = dL = \left[\frac{L_x}{X}\right]dX + \left[\frac{L_y}{Y}\right]dY$$

$$(2)\quad \left[\frac{L_x}{L}\right]\frac{dX}{X} + \left[\frac{L_y}{L}\right]\frac{dY}{Y} = \frac{dL}{L}$$

$$(3)\quad \left[\frac{K_x}{K}\right]\frac{dX}{X} + \left[\frac{K_y}{K}\right]\frac{dY}{Y} = \frac{dK}{K}$$

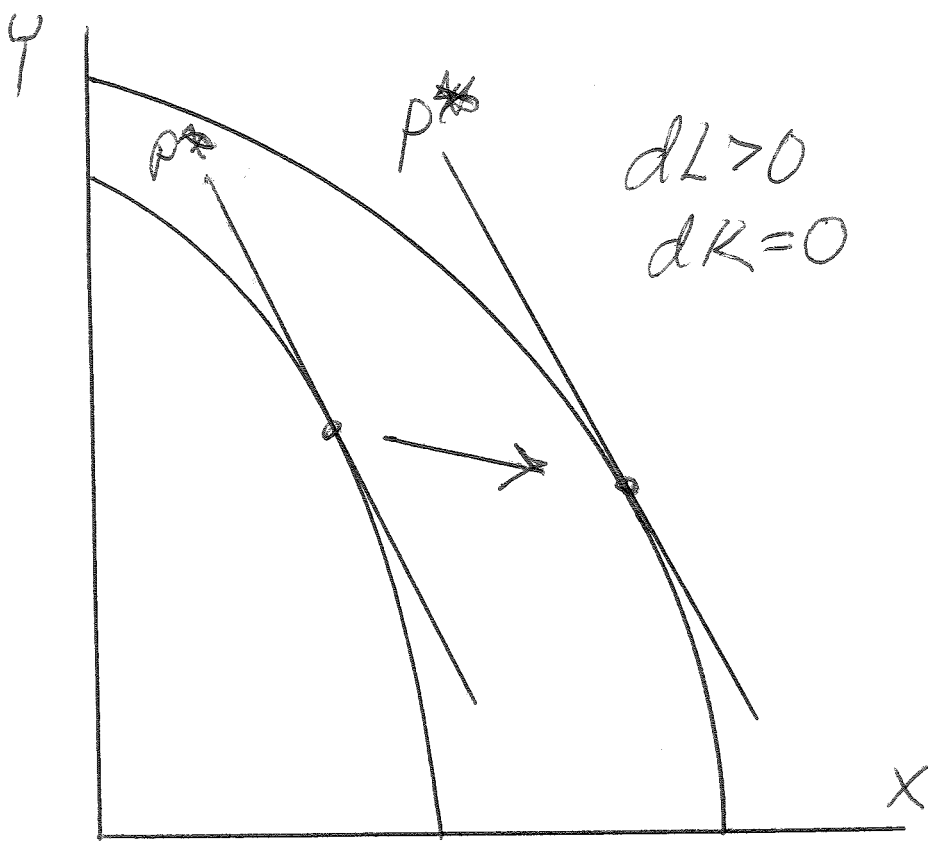
$$(4)\quad \begin{bmatrix} \lambda_{lx} & \lambda_{ly} \\ \lambda_{kx} & \lambda_{ky} \end{bmatrix} \begin{bmatrix} \hat{X} \\ \hat{Y} \end{bmatrix} = \begin{bmatrix} \hat{L} \\ \hat{K} \end{bmatrix}$$

$$(5)\quad D_\lambda = \lambda_{lx}\lambda_{ky} - \lambda_{kx}\lambda_{ly} > 0. \qquad \frac{\lambda_{ky}}{\lambda_{lx}\lambda_{ky} - \lambda_{kx}\lambda_{ly}} > 1$$

Invert the mapping.

$$(6)\quad \lambda^{-1} \begin{bmatrix} \hat{L} \\ \hat{K} \end{bmatrix} = \begin{bmatrix} >1 & <0 \\ <0 & >1 \end{bmatrix} \begin{bmatrix} \hat{L} \\ \hat{K} \end{bmatrix} = \begin{bmatrix} \hat{X} \\ \hat{Y} \end{bmatrix}$$

This is the Rybczynski Theorem



Stolper-Samuelson Theorem

Holding factor endowments constant, an increase in the price of one good increases the real income of the factor used intensively in the production of that good, and decreases the real income of the other factor.

Zero profit conditions give

$$(1)\quad \begin{bmatrix} a_{lx} & a_{kx} \\ a_{ly} & a_{ky} \end{bmatrix} \begin{bmatrix} w \\ r \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \end{bmatrix}$$

Differentiate these using the envelop theorem: $\dfrac{dc_x(w,r)}{dw} = \dfrac{\partial c_x(w,r)}{\partial w} = a_{lx}$

$$(2)\quad a_{lx}dw + a_{kx}dr + \left[w da_{lx} + r da_{kx}\right] = dp_x = a_{lx}dw + a_{kx}dr$$

because $\frac{da_{kx}}{da_{lx}}$ is the slope of the unit isoquant, and equals the factor-price ratio in equilibrium. So (1) can be written as

$$(3)\quad \begin{bmatrix} a_{lx} & a_{kx} \\ a_{ly} & a_{ky} \end{bmatrix} \begin{bmatrix} dw \\ dr \end{bmatrix} = \begin{bmatrix} dp_x \\ dp_y \end{bmatrix}$$

Now we can do the same thing transform this mapping into proportion changes that we did with the Rybczynski theorem.

$$(4)\quad \left[\frac{L_x}{X}\right] dw + \left[\frac{K_x}{X}\right] dr = dp_x \quad \Rightarrow \quad \left[\frac{wL_x}{p_x X}\right] \frac{dw}{w} + \left[\frac{rK_x}{p_x X}\right] \frac{dr}{r} = \frac{dp_x}{p_x}$$

$$(5)\quad \begin{bmatrix} \Theta_{lx} & \Theta_{kx} \\ \Theta_{ly} & \Theta_{ky} \end{bmatrix} \begin{bmatrix} \hat{w} \\ \hat{r} \end{bmatrix} = \begin{bmatrix} \hat{p}_x \\ \hat{p}_y \end{bmatrix} \qquad where \qquad \left[\Theta_{lx}\Theta_{ky} - \Theta_{kx}\Theta_{ly}\right] > 0$$

If we invert this mapping, we get:

$$(6)\quad \Theta^{-1} \begin{bmatrix} \hat{p}_x \\ \hat{p}_y \end{bmatrix} = \begin{bmatrix} >1 & <0 \\ <0 & >1 \end{bmatrix} \begin{bmatrix} \hat{p}_x \\ \hat{p}_y \end{bmatrix} = \begin{bmatrix} \hat{w} \\ \hat{r} \end{bmatrix}$$

This is the Stolper-Samuelson Theorem.
Summarize the Rybczynski and SS effects.

$$(7)\quad \hat{w} > \hat{p}_x > 0 > \hat{r} \qquad \hat{r} > \hat{p}_y > 0 > \hat{w}$$

$$\hat{X} > \hat{L} > 0 > \hat{Y} \qquad \hat{Y} > \hat{K} > 0 > \hat{X}$$

Summary: Differences between countries in relative factor abundance combined with differences between goods in relative factor intensities produces a basis for trade. Four theorems are associated with this model

The Heckscher-Ohlin Theorem
The Factor-Price-Equalization Theorem
The Rybczynski Theorem
The Stolper-Samuelson Theorem.

Some Policy Implications

Heckscher-Ohlin Theorem

Cautions policy makers not to make spurious interpretations about the *direction of trade*. e.g., the fact that the US runs a deficit with Japan in manufactured goods is a natural consequence of their differences in factor endowments.

FPE and Rybczynski Theorems

Changes in factor supplies may impact chiefly on the *composition of output*, and not on factor prices and the *distribution of income*.

Stolper-Samuelson Theorem

Changes in protection levels do not just cause aggregate gains or losses to society, they *redistribute income* within society as well.

Table 8.1 Measures of Factor Intensity for US Manufacturing Industries

	2000					2005				
Industry	Value Added	Production	Capital Exp.	Nonproduction		Value Added	Production	Capital Exp.	Nonproduction	Evident
	($ millions)	Labor (000)	per PL	labor per PL		($ millions)	Labor (000)	per PL	labor per PL	Intensity
Petroleum and coal products	$ 45,748	67	$ 74,624	0.51		$ 117,541	65	$ 169,501	0.58	Capital, Skill
Chemical products	$ 235,614	508	$ 41,112	0.75		$ 328,440	433	$ 38,971	0.76	Capital, Skill
Computer & electronic products	$ 291,125	848	$ 33,227	0.94		$ 226,319	465	$ 33,972	1.16	Capital, Skill
Mineral products	$ 55,722	408	$ 14,820	0.28		$ 64,545	360	$ 14,334	0.29	Capital
Transportation equipment	$ 240,989	1,349	$ 12,529	0.36		$ 254,665	1,104	$ 13,842	0.41	Capital, Skill
Food, beverages & tobacco	$ 255,245	1,244	$ 11,714	0.35		$ 316,389	1,177	$ 13,090	0.34	Capital, Skill
Wood & paper products	$ 114,260	914	$ 12,234	0.24		$ 120,651	765	$ 11,268	0.27	Capital
Miscellaneous products	$ 70,621	501	$ 8,219	0.49		$ 92,974	422	$ 11,044	0.61	Skill
Plastic & rubber products	$ 92,333	862	$ 10,086	0.26		$ 96,348	688	$ 10,127	0.29	Capital
Machinery	$ 148,798	920	$ 10,116	0.52		$ 142,488	683	$ 9,947	0.56	Skill
Printing	$ 63,446	597	$ 7,398	0.39		$ 58,930	457	$ 9,510	0.41	Skill
Metal products	$ 215,545	1,839	$ 8,729	0.30		$ 232,106	1,418	$ 8,545	0.33	Skill
Electrical equipment & appliances	$ 62,991	431	$ 9,069	0.37		$ 54,318	294	$ 6,551	0.43	Skill
Textile products	$ 35,225	475	$ 5,130	0.20		$ 32,395	285	$ 4,633	0.23	Labor
Leather products	$ 4,510	55	$ 2,813	0.25		$ 2,865	29	$ 3,527	0.29	Labor
Furniture & related products	$ 42,267	515	$ 4,011	0.25		$ 46,801	414	$ 3,404	0.29	Labor
Apparel	$ 28,210	423	$ 2,302	0.24		$ 16,319	171	$ 2,882	0.31	Labor

Source: Compiled by authors from US Department of Commerce, *Annual Survey of Manufactures*

Table 8.2 Measures of Relative Factor Endowments								
	2000				2005			
Country	Capital Stock per worker	Arable land per worker (HA)	R&D Scientists per 1000 people		Capital Stock per worker	Arable land per worker (HA)	R&D Scientists per 1000 people	Evident Abundance
Singapore	$ 239,044	0.00	8.08		$ 247,608	0.00	10.45	Capital, R&D
Japan	$ 182,196	0.07	9.55		$ 194,375	0.07	10.55	Capital, R&D
USA	$ 153,689	1.19	8.64		$ 181,856	1.13	8.97	Capital, R&D
Australia	$ 149,347	4.91	6.86		$ 169,374	4.68	6.76	Capital, Land
Germany	$ 160,918	0.29	6.38		$ 162,214	0.29	6.71	Capital, R&D
Canada	$ 142,345	2.82	6.69		$ 156,814	2.55	6.55	Capital, Land
Finland	$ 149,338	0.84	13.42		$ 155,699	0.85	15.00	Capital, R&D
Rep. of Korea	$ 102,235	0.08	4.80		$ 123,959	0.07	7.56	Capital, R&D
UK	$ 102,447	0.20	5.43		$ 117,232	0.19	5.86	R&D
Mexico	$ 48,140	0.64	1.12		$ 50,827	0.58	1.11	Labor
Brazil	$ 39,311	0.70	0.77		$ 37,885	0.63	0.77	Labor
South Africa	$ 31,060	0.95	0.96		$ 30,532	0.86	0.99	Labor
China	$ 13,183	0.18	0.95		$ 20,090	0.18	1.44	Labor
India	$ 7,556	0.42	0.29		$ 9,465	0.37	0.31	Labor

Sources: computed by authors with data available from World Bank, *World Development Indicators*; Food and Agricultural Organization, *FAO-Stat Database*; and Penn World Tables version 6.2.

Table 8.3 Proportions of World GDP and World Factor Endowments, 2000							
				Primary	Secondary	Post-secondary	R&D Research
	GDP	Capital Stock	Arable Land	School	School	School	Scientists
USA	27.10%	23.89%	19.42%	2.25%	11.96%	30.22%	29.20%
Canada	2.45%	2.43%	5.07%	0.80%	0.76%	1.65%	2.49%
Germany	6.18%	6.83%	1.31%	2.15%	4.13%	3.69%	5.89%
UK	4.22%	3.20%	0.65%	2.02%	1.54%	2.43%	3.69%
Australia	1.36%	1.51%	5.24%	0.48%	0.74%	1.19%	1.51%
Japan	8.91%	12.97%	0.50%	3.29%	4.92%	7.63%	14.78%
Rep. of Korea	2.16%	2.44%	0.19%	1.29%	3.10%	3.10%	2.49%
Mexico	2.90%	2.00%	2.78%	2.61%	1.98%	1.77%	1.01%
Brazil	3.39%	3.42%	6.39%	2.87%	1.49%	2.84%	1.46%
China	8.26%	10.11%	14.75%	32.62%	33.33%	9.79%	15.80%
India	4.29%	3.11%	18.02%	16.93%	9.84%	9.00%	2.57%
Countries	43	43	43	43	43	43	36

Sources: computed by authors with data available from World Bank, *World Development Indicators*; Food and Agricultural Organization, *FAO-Stat Database*; and Penn World Tables version 6.2. Figures for GDP are measured with PPP exchange rates at constant 2005 $US.

Table 8.4 Results of Statistical Testing in Davis-Weinstein				
			HOV	HOV
	HOV	HOV-HN	non-FPE	non-FPE & gravity
Statistic	H1	H2	H3	H4
Slope	-0.002	-0.05	0.43	0.82
Standard Error	0.005	0.02	0.02	0.03
R^2	0.01	0.31	0.96	0.98
Sign Test	32%	50%	86%	91%
Observations	22	22	22	22

Source: Davis and Weinstein (2001).

Figure 8.1

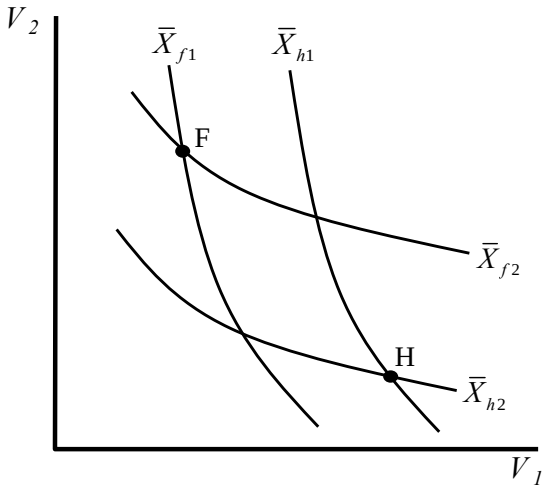


Figure 8.2

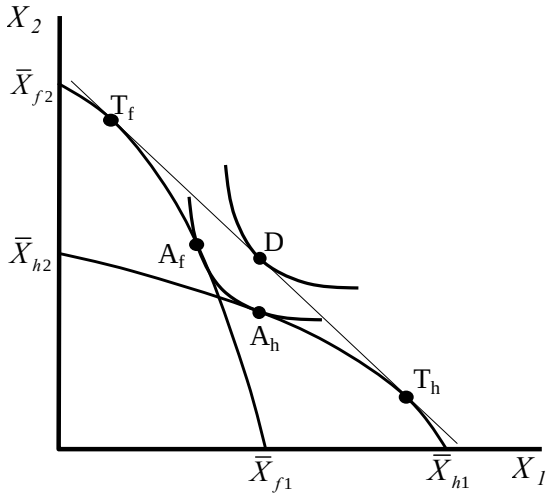
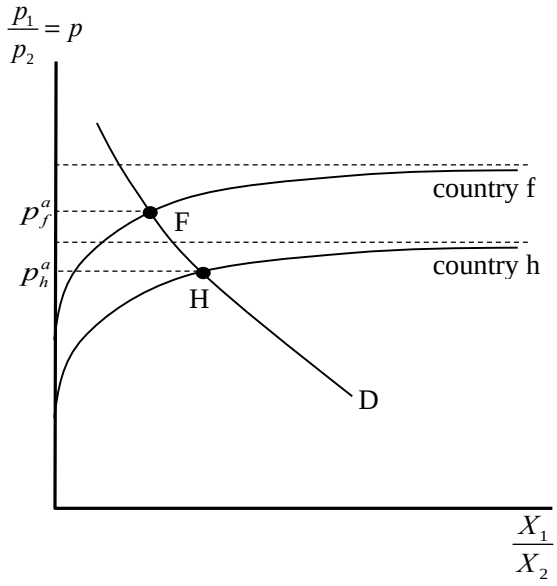


Figure 8.3



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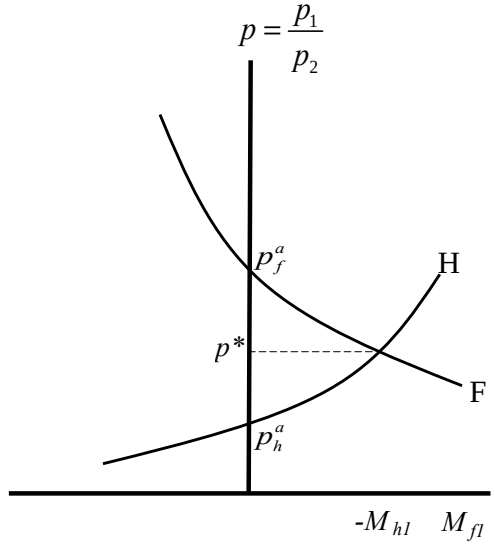

Figure 8.5

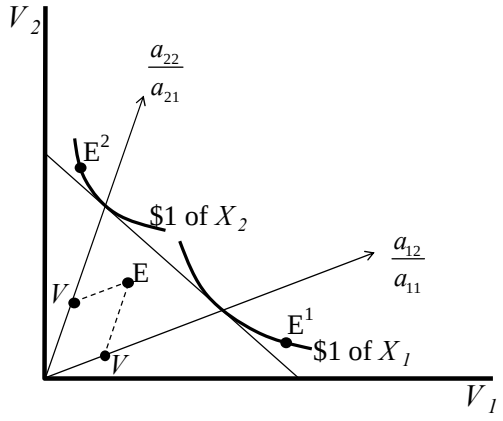

Figure 8.6

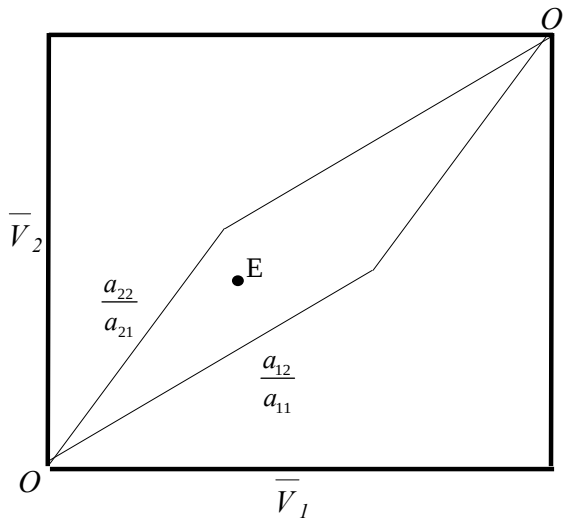

Figure 8.7

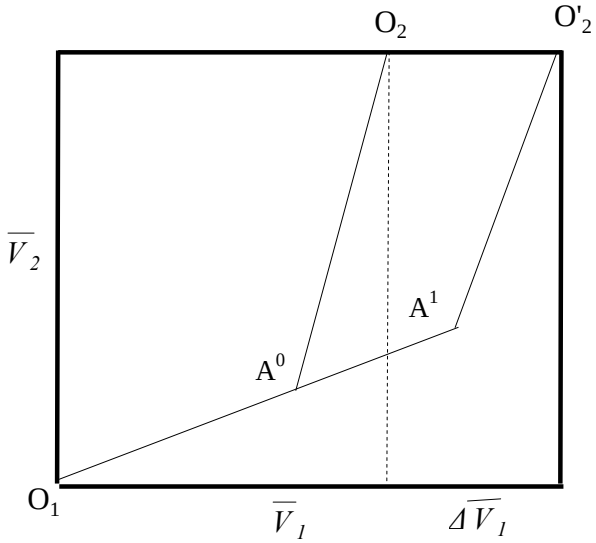

Figure 8.8

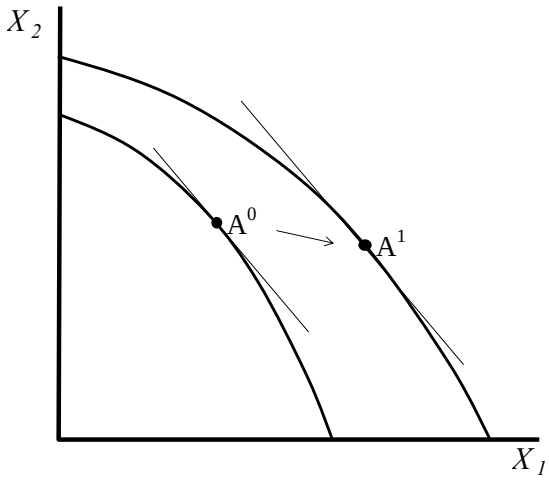

Figure 8.9

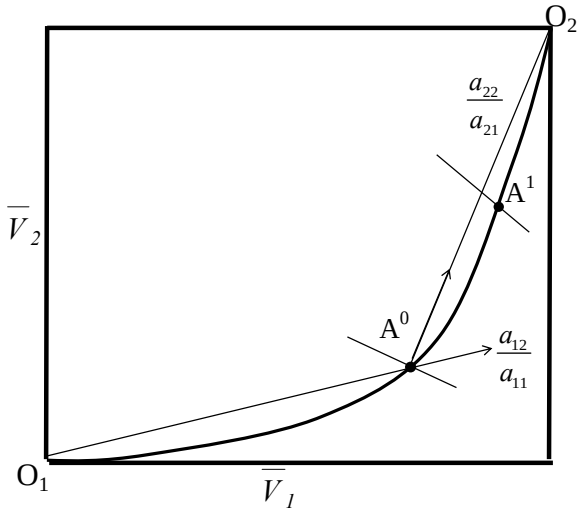

Figure 8.10

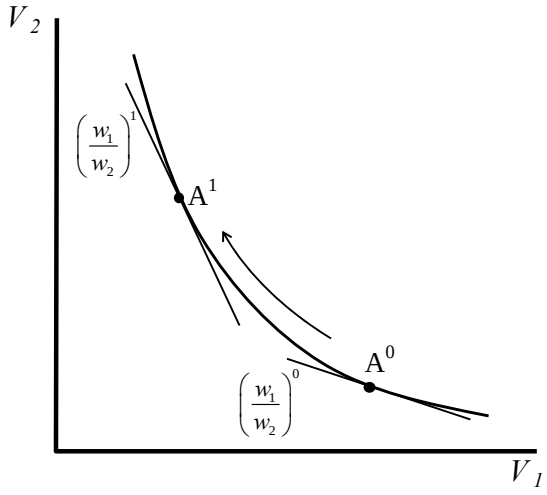

Figure 8.11