Lecture 7: The Specific-Factors Model (Ronald Jones), Generalizations of Factor-Proportions Models

1. Production functions, convexity of the production set.

2. The direction of trade - modification of the Heckscher-Ohlin theorem.

3. Factor endowments and outputs- modification of the Rybczynski theorem.

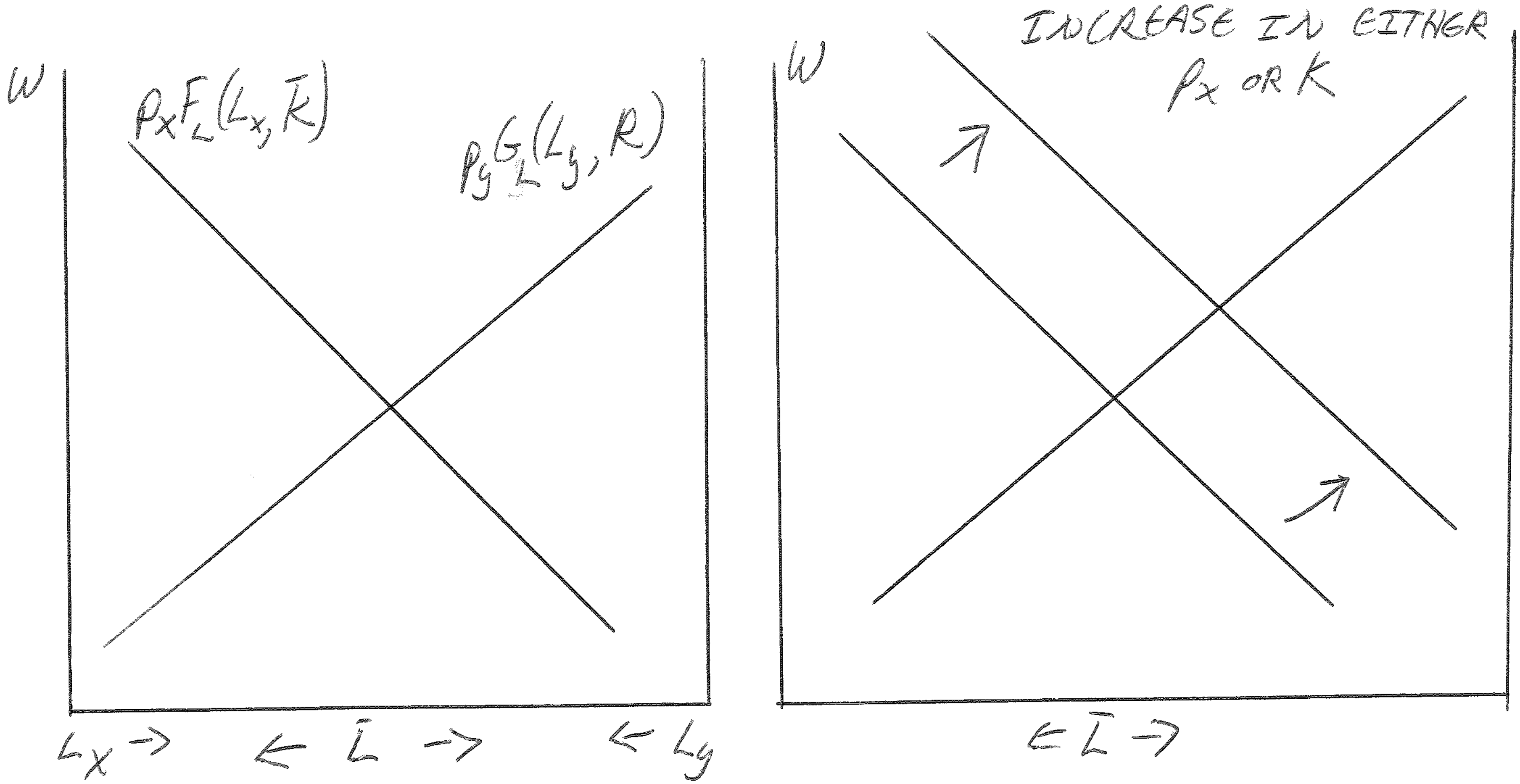
4. Commodity prices and factor prices - modification of the FPE and Stolper-Samuelson theorems.

(1) $X = F(L_x, K) \qquad Y = G(L_y, R) \qquad L_x + L_y = L$

$$(2)\quad \begin{array}{ll} p_x F_l = w & p_y G_l = w \\ p_x F_k = r & p_y G_r = v \end{array} \quad \Rightarrow \quad \frac{p_x}{p_y} = \frac{G_l(L_y, R)}{F_l(L_x, K)}$$

$$(3)\quad \frac{d^2 Y}{dX^2} = \frac{F_l G_{ll} + G_l F_{ll}}{F_l^3} < 0.$$

Introduce the "X" diagram.



The Direction of Trade

1. The specific factors. Suppose that both countries have the same amounts of L and R.

$$(1)\quad p_x F_l(L_x, K) = p_y G_l(L_y, R)$$

$$(2)\quad p_x F_{ll} dL_x + p_x F_{lk} dK = p_y G_{ll} dL_y = -p_y G_{ll} dL_x$$

$$(3)\quad \frac{dL_x}{dK} = -\frac{p_x F_{lk}}{p_x F_{ll} + p_y G_{ll}} = -\frac{dL_y}{dK} > 0$$

$$(4)\quad \frac{dX}{dK} = F_l \frac{dL_x}{dK} + F_k > 0, \qquad \frac{dY}{dK} = G_l \frac{dL_y}{dK} < 0$$

Result

If two countries have the same L and country h has more K than country f and/or country f has more R than country h, then country h exports X and country f exports Y.

2. Now assume that $K_h = K_f$, $R_h = R_f$ and one country has more L. Use the same procedure as above, with dL > 0 for one country. We can show that:

(5)

$$\frac{dL_x}{dL} = \frac{p_y G_{ll}}{p_x F_{ll} + p_y G_{ll}} > 0 \qquad \frac{dL_y}{dL} = \frac{p_x F_{ll}}{p_x F_{ll} + p_y G_{ll}} > 0 \quad \Rightarrow \quad [dX,\, dY] > 0$$

(6) $$\frac{dL_x/L_x}{dL_y/L_y} = \frac{p_y G_{ll} L_y}{p_x F_{ll} L_x} = \frac{G_{ll} L_y / G_l}{F_{ll} L_x / F_l}$$

$$(7)\quad \begin{aligned} \frac{dX}{X} &= \frac{L_x F_l}{X}\frac{dL_x}{L_x} = \frac{p_x F_l L_x}{p_x X}\frac{dL_x}{L_x} = \theta_{lx}\frac{dL_x}{L_x} \\ \frac{dY}{Y} &= \frac{L_y G_l}{Y}\frac{dL_y}{L_y} = \frac{p_y G_l L_y}{p_y Y}\frac{dL_y}{L_y} = \theta_{ly}\frac{dL_y}{L_y} \end{aligned}$$

$$(8)\quad \frac{dX/X}{dY/Y} = \frac{\theta_{lx}}{\theta_{ly}}\left[\frac{G_{ll}L_y/G_l}{F_{ll}L_x/F_l}\right]$$

But using the w equals the value of the marginal product of labor,

$$(9)\quad \frac{L_y}{w}\left[\frac{\partial w}{\partial L_y}\right] = \frac{L_y}{p_y G_l}\left[p_y G_{ll}\right] = \frac{L_y G_{ll}}{G_l} \equiv 1/\eta_y \qquad (\textit{similarly for } X)$$

$$(10) \qquad \frac{dX/X}{dY/Y} = \frac{\theta_{lx}}{\theta_{ly}} \frac{\eta_x}{\eta_y}$$

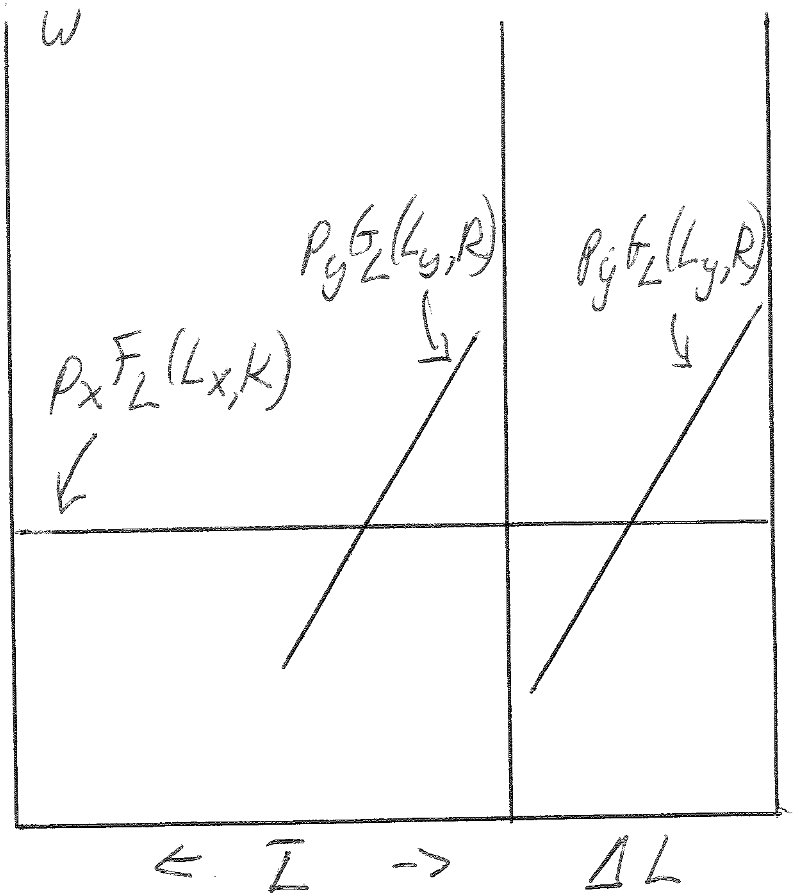
Suppose that we measure labor intensity by the shares. Then this result says that the effects of a labor increase do not just depend on factor intensities, but also on the elasticities of factor demand (inverse elasticities of marginal products).

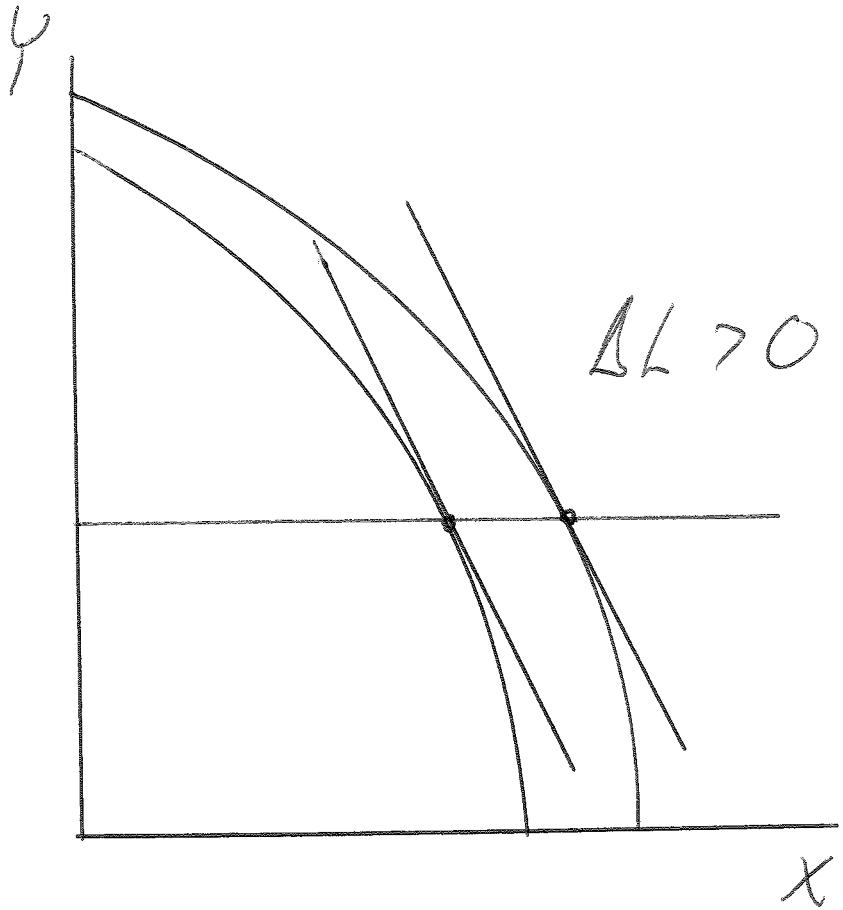
These in turn are related to the elasticities of substitution in production between labor and the specific factors. Jones shows that:

$$(11) \qquad \frac{dX/X}{dY/Y} = \frac{\theta_{lx}}{\theta_{ly}} \frac{(1 - \theta_{ly})}{(1 - \theta_{lx})} \frac{\sigma_x}{\sigma_y}$$ where σ = elasticity of substitution

<u>Result</u> If two countries have the same endowment of both of the specific factors but one country has more labor, then the pattern of trade depends both on factor shares and on elasticities of substitution in production.

<u>Example</u>: Suppose that σ_x = inf: $X = L_x + K$. Then all additional labor must be allocated to the X sector, $dX > 0$, $dY = 0$.





(Non) Factor-Price Equalization

Obvious from "X" diagram. Two commodity prices cannot determine three factor prices. Knowledge about factor-endowments are needed as well.

Variation on Stolper-Samuelson

Suppose that $dp_x > 0$, $dp_y = 0$.

(1) Labor moves to the X sector, K/L_x falls, R/L_y rises.

(2) With constant returns, the marginal product of a factor depends only on the *ratio* in which factors are used.

The effect of the price change is given by

$$
(3) \quad
\begin{aligned}
d(w/p_x) &= d(F_l) < 0 \qquad & d(w/p_y) &= d(G_l) > 0 \\
d(r/p_x) &= d(F_k) > 0 \qquad & d(v/p_y) &= d(G_r) < 0
\end{aligned}
$$

This is a version of the Stolper-Samuelson relationship:

$$(4) \quad \frac{dr}{r} > \frac{dp_x}{p_x} > \frac{dw}{w} > 0 > \frac{dv}{v} \qquad \frac{dv}{v} > \frac{dp_y}{p_y} > \frac{dw}{w} > 0 > \frac{dr}{r}$$

Variations on Rybczynski

First, an increase in one of the specific factors, $dK > 0$.

We note from the "X" diagram that w rises: with prices constant, F_l and G_l both rise, but this means that K/L_x rises, and R/L_y rises.

But this means that $dK/K > dL_x/L_x$ and therefore that $dK/K > dX/X > 0$, and $dY/Y < 0$.

(5) $$\frac{dK}{K} > \frac{dX}{X} > 0 > \frac{dY}{Y}, \qquad \frac{dR}{R} > \frac{dY}{Y} > 0 > \frac{dX}{X}$$

For the mobile factor, we have already seen that:

(6) $$\frac{dL}{L} > \left[\frac{dX}{X}, \frac{dY}{Y}\right] > 0$$

Conclusions from the Specific-Factors Model

1. Illustrates the difficulty of any simple generalizations of Heckscher-Ohlin (e.g., changing the endowment of the mobile factor).

2. Factor-intensities can of course be defined tautologically so that countries always export goods using intensively their abundant factors. But direction of trade cannot be predicted from factor shares alone.

3. Trade economists view the generalization of the Stolper-Samuelson theorem as realistic and plausible. Specific factors are very vulnerable to changes and, in a particular industry, labor and capital may lobby together.

4. Most "workers" have some industry-specific skills or industry-specific human capital. Or they may own some of the specific factor in their sector (e.g., farmers owning their land). Thus "workers" in different sectors can have very different view about trade liberalization.

5. Interesting in some cases to think of the Specific-Factors Model as a short-run version of the Heckscher-Ohlin Model. Some implications are intuitive. For example, short-run responses of outputs to price changes are less elastic than long-run changes (when all factors may move between sectors).

Generalizations of Factor-Proportions Trade Models

<u>General Problem of n not equal to m (n goods, m factors)</u>

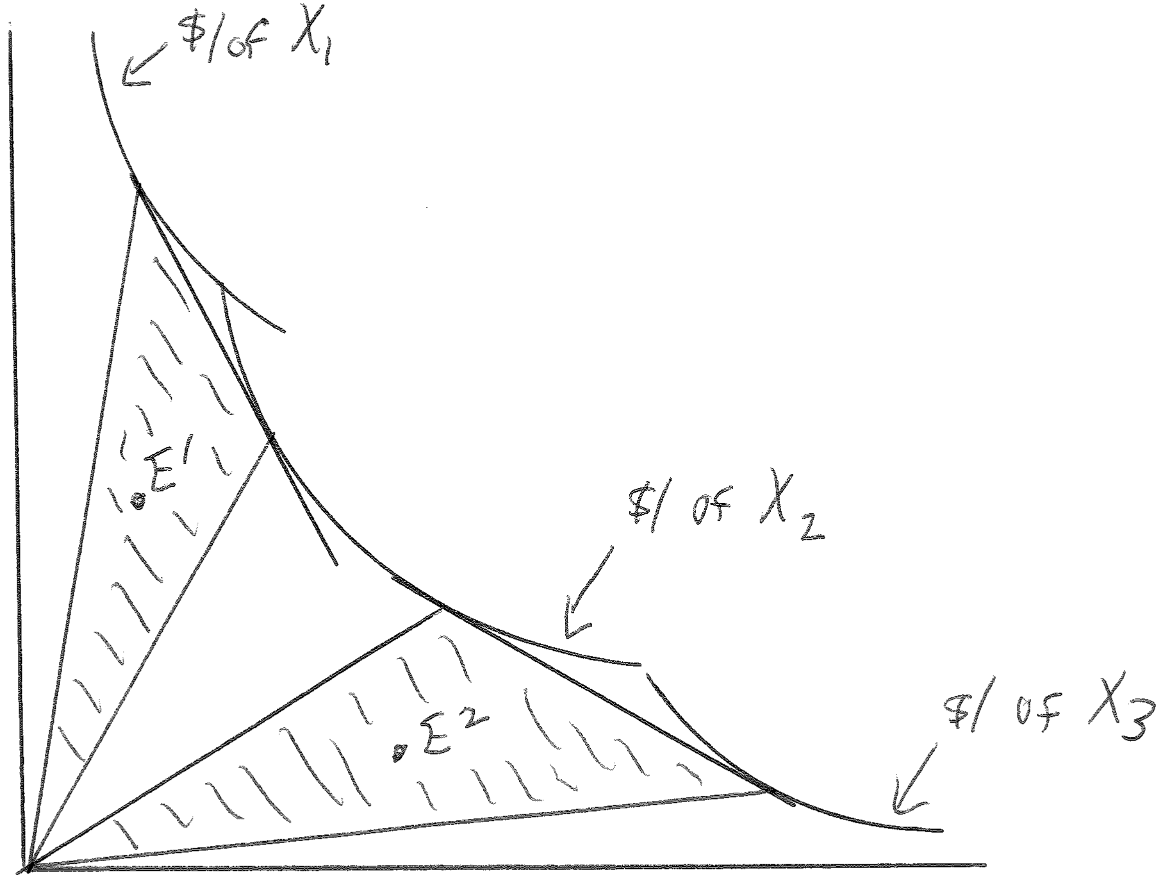
Let a_{ij} = the amount of factor i needed for one unit of good j.

$$\left[A_{ij}\right]\left[X\right] = \left[V\right]$$ factor-market clearing: m equations in n unknowns

$$\left[A_{ij}^{\prime}\right]\left[w\right] = \left[p\right]$$ Zero profits: n equations in m unknowns

n > m (more goods than factors)

There are "flats" in the transformation surface (e.g., the two-good, one-factor case). Not all goods produced, in general at most m goods (possibly m + 1) produced.



m > n (more factors than goods).

The goods prices are not sufficient to determine the factor prices (i.e., the FPE theorem does not hold). Factor prices will depend on factor supplies as well as on commodity prices; e.g., the two-good, three-factor Jones (specific-factors) model.

Other Issues

1. Commodity prices are generally not equalized by trade, implying different factor prices, implying different choices of techniques.

2. Countries may be partially specialized in equilibrium, meaning that they produce different sets of goods.

3. Even if commodity prices are equalized by trade and countries produce the same goods, this rarely implies factor-price equalization (e.g., the specific factors model).

Notation

X - vector of production
C - vector of consumption
M - vector of net imports
E - vector of net exports (unfortunately also used for the expenditure function)

p - vector of commodity prices
v - vector of endowments
w - vector of factor prices

Let $c_i(w)$ be the unit cost function for good i. Then by duality theory

(1) $$a(w)_{ij} = \frac{\partial c_i}{\partial w_j}$$

where a_{ij} is the *optimal* amount of factor j used to produce one unit of good i.

The matrix of the a_{ij} is sometimes called the Rybczynski matrix. Suppose that there really was FPE in the world, so that each country produces with the same $[A_{ij}(w)]$ matrix. Use the factor-market adding up condition for one country, and also for the world as a whole.

(2) $$v = [A_{ij}(w)]X \qquad v_u = [A_{ij}(w)]X_u$$

Where the subscript u refers to "universe" (world). Let s be a scalar, equal to the individual countries share of total world income. the second equation of (2) can be written as:

$$(3) \quad sV_u = [A_{ij}(w)]sX_u$$

Subtract (3) from the first equation of (2).

$$(4) \quad (v - sv_u) = [A_{ij}(w)](X - sX_u)$$

If commodity prices are equalized by trade, and if demand is identical and homothetic between countries, then sX_u is simply the countries consumption vector C, and (X - C) = E, the net export vector.

$$(5) \quad (v - sv_u) = [A_{ij}(w)](X - C) = [A_{ij}(w)]E$$

Pre-multiply both sides by the vector on the left, the vector of "excess factor supplies" to form a scalar.

(6) $(v - sv_u)'(v - sv_u) = (v - sv_u)'[A_{ij}(w)]E > 0$

Interpret $[A_{ij}]$ as the matrix of factor intensities. The result in (6) says that the country exports goods using intensively its abundant factors.

Having made many outrageous assumptions, make some more and assume that A is square and invertible.

<u>Result 1</u>

(7) $E = [A^{-1}(w)](v - sv_u)$

Often A^{-1} is referred to as the Rybczynski matrix rather than A. This equation in (7) is often used in empirical work.

Anything weaker?

Mean Value Theorem

Let $y^0 = F(X^0)$, and $y^1 = F(X^1)$, then there exists a $X^* = \lambda X^0 + (1-\lambda)X^1$ such that $(y^1 - y^0) = F'(X^*)(X^1 - X^0)$.

Since $p = c(w)$, and since the a_{ij} are the derivatives of $c(w)$, this allows us to write:

$$(8)\quad (p^1 - p^0) = [A_{ij}(w*)](w^1 - w^0)$$

Result 2

$$(9)\quad (p^1 - p^0)'(p^1 - p^0) = (p^1 - p^0)'[A_{ij}(w*)](w^1 - w^0) > 0$$

Interpret the one and zero as the prices of two countries in autarky. This result says that in autarky goods are cheap where the factors they use intensively are cheap.

Let R(p,v) and E(p,U) be a country's national product and expenditure functions respectively, where U is utility. Let p^1 and p^0 be each country's autarky prices and let superscript f denote variables at free trade.

$$(10) \qquad R(p^1, v^1) \geq p^{1\prime} X^{1f} \qquad E(p^1, U^{1f}) \leq p^{1\prime} C^{1f}$$

$$(11) \qquad R(p^1, v^1) - E(p^1, U^{1f}) \geq p^{1\prime}(X^{1f} - C^{1f}) = -p^{1\prime} M^{1f}$$

Now note that free-trade utility is at least as large as autarky utility and that in autarky the value of production and expenditure are equal at autarky prices.

$$(12) \qquad E(p^1, U^{1f}) > E(p^1, U^1), \quad R(p^1, v^1) - E(p^1, U^1) = 0 \quad \Rightarrow \quad R(p^1, v^1) - E(p^1, U^{1f}) < 0$$

$$(13) \qquad p^{1\prime} M^{1f} > 0 \qquad p^{0\prime}(-M^{1f}) > 0$$

Result 3

$$(14) \qquad (p^1 - p^0)' M^{1f} > 0$$

Countries import goods that are relatively expensive in autarky.

Now pre-multiply both sides of equation (8) by M.

Result 4

$$(15) \qquad M^{1f'}(p^1 - p^0) = M^{1f'}[A_{ij}(w*)](w^1 - w^0) > 0$$

Countries import goods that use intensively the countries' relatively expensive (in autarky) factors.

In addition to R(p,v) being the *maximum* value of output at prices p and endowment v, it is the *minimum* value of factor payments w'v subject to zero profits c(w) ≥ p.

$$(16)\quad \begin{aligned} R(p^1, v^1) &= w^{1\prime} v^1, \quad R(p^0, v^0) = w^0 v^0 \\ R(p^1, v^0) &\leq w^{1\prime} v^0, \quad R(p^0, v^1) \leq w^{0\prime} v^1 \end{aligned}$$

Now assume that preferences are homothetic, E(p,U) = E(p)U and adopt the price normalization such that E(p) = 1.

We then have the equilibrium condition for the economy that:

$$(17)\qquad E(p, v) = E(p)U = U = R(p, v)$$

Since we know that there are gains from trade, and gains from being allowed to trade at any prices other than autarky prices, we must then have

$$(18)\qquad R(p^0, v^1) \geq R(p^1, v^1) \qquad R(p^0, v^0) \leq R(p^1, v^1)$$

Combining these inequalities, we have

$$(19) \qquad \begin{aligned} w^{1\prime}v^1 &= R(p^1, v^1) \le R(p^0, v^1) \le w^{0\prime}v^1 \\ w^{0\prime}v^0 &= R(p^0, v^0) \le R(p^1, v^0) \le w^{1\prime}v^0 \end{aligned}$$

Result 5

$$(20) \qquad (w^1 - w^0)'(v^1 - v^0) \le 0$$

Factors are cheap where they are abundant.

If A(w*) is inverted, we could also get the result that:

Result 6

$$(21) \qquad (p^1 - p^0)'[A_{ij}(w*)']^{-1}(v^1 - v^0) \le 0$$

Goods are cheap where the factors that they use intensively are cheap.