Lecture 9: External Economies of Scale

1. Firm and industry production functions

2. Non-tangency and non-convexity

3. Country size as a determinant of trade

4. Possible multiple equilibria and low-level traps

5. International external economies

6. A note on factor prices

X_i firm i's output
X industry output
α externality parameter, $0 \leq \alpha \leq 1$
V_i firm i's vector of inputs
V industry vector of inputs

Individual firm's production function

(1) $X_i = (X^{\alpha})F(V_i)$ where F( ) has constant returns to scale

(2) $\left[\frac{\partial X_i}{\partial V_i}\right]_{\bar{X}} = (X^{\alpha})F_v$ "private" marginal products

Industry production function

(3) $\sum_i X_i = (X^{\alpha})\sum_i F(V_i)$ $X = (X^{\alpha})F(V)$

(4) $X^{1-\alpha} = F(V)$ $X = [F(V)]^{\frac{1}{1-\alpha}}$

(5) $$\frac{dX}{dV} = \frac{1}{1-\alpha}\left[F(V)\right]^{\frac{1}{1-\alpha}-1} F_v = \frac{1}{1-\alpha}\left[F(V)\right]^{\frac{\alpha}{1-\alpha}} F_v$$

(6) $$\left[F(V)\right]^{\frac{\alpha}{1-\alpha}} = X^{\alpha}$$

(7) $$\frac{dX}{dV} = \frac{1}{1-\alpha} X^{\alpha} F_v$$

Side note: competitive equilibrium can exist, since the value of private marginal products just exhaust output.

$$p(X^{\alpha})F_v = w \qquad p(X^{\alpha})F_v v_i = w v_i$$

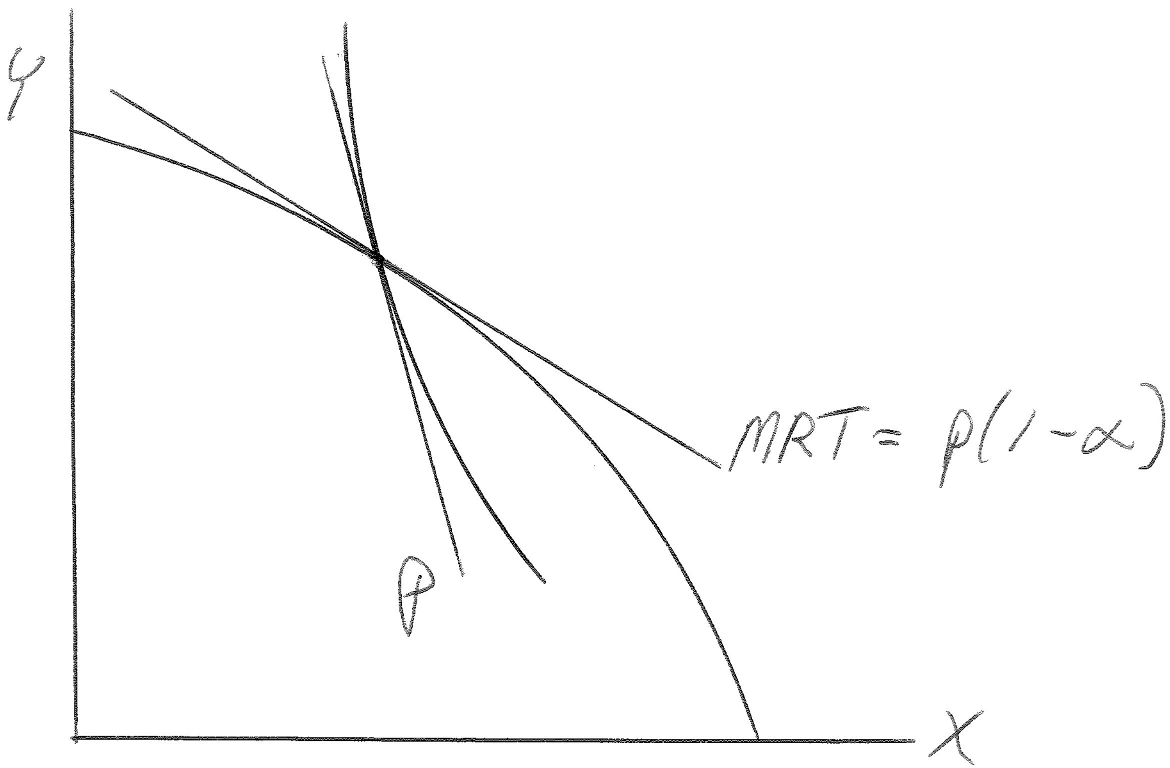
$$p(X^{\alpha})F(V_i) = w v_i = pX_i \quad \text{(zero profits)}$$

(8)

$$dX = \frac{1}{1-\alpha} X^{\alpha} F_v \, dV_x \qquad p\,dX = \frac{1}{1-\alpha} p X^{\alpha} F_v \, dV_x = \frac{1}{1-\alpha} w \, dV_x$$

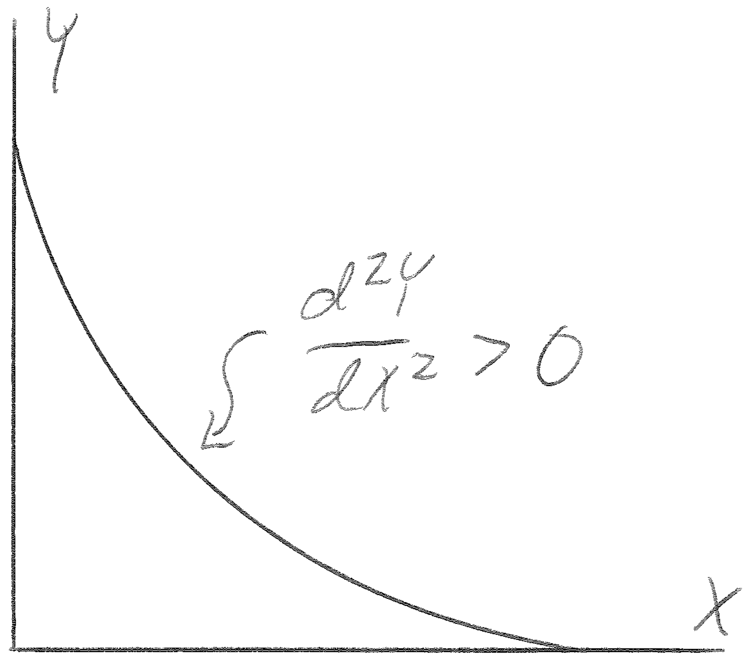
(9) $$dY = G_v dV_y = w\,dV_y = -w\,dV_x$$

(10) $$-\frac{dY}{dX} \equiv MRT = p(1-\alpha) < p$$



Convexity

Case 1: $Y = L_y \qquad X = (X^{\alpha})L_x = L_x^{\frac{1}{1-\alpha}}$



$$dX = \frac{1}{1-\alpha} L_x^{\frac{\alpha}{1-\alpha}} dL_x = \frac{1}{1-\alpha} X^{\alpha} dL_x$$

$$\frac{dY}{dX} = -(1-\alpha)X^{-\alpha} \qquad \frac{d^2Y}{dX^2} = \alpha(1-\alpha)X^{-\alpha-1} > 0$$

Case 2: $Y = L_y \qquad X = (X^{\alpha})L_x^{\beta}K^{1-\beta} = L_x^{\frac{\beta}{1-\alpha}} K^{\frac{1-\beta}{1-\alpha}}$

$$dX = (+)\frac{\beta}{1-\alpha} L_x^{\frac{\beta}{1-\alpha}-1} dL_x = (+)\frac{\beta}{1-\alpha} L_x^{\frac{\beta-1+\alpha}{1-\alpha}} dL_x$$

$$\frac{dY}{dX} = -(+)\frac{1-\alpha}{\beta} L_x^{\frac{1-\alpha-\beta}{1-\alpha}}$$

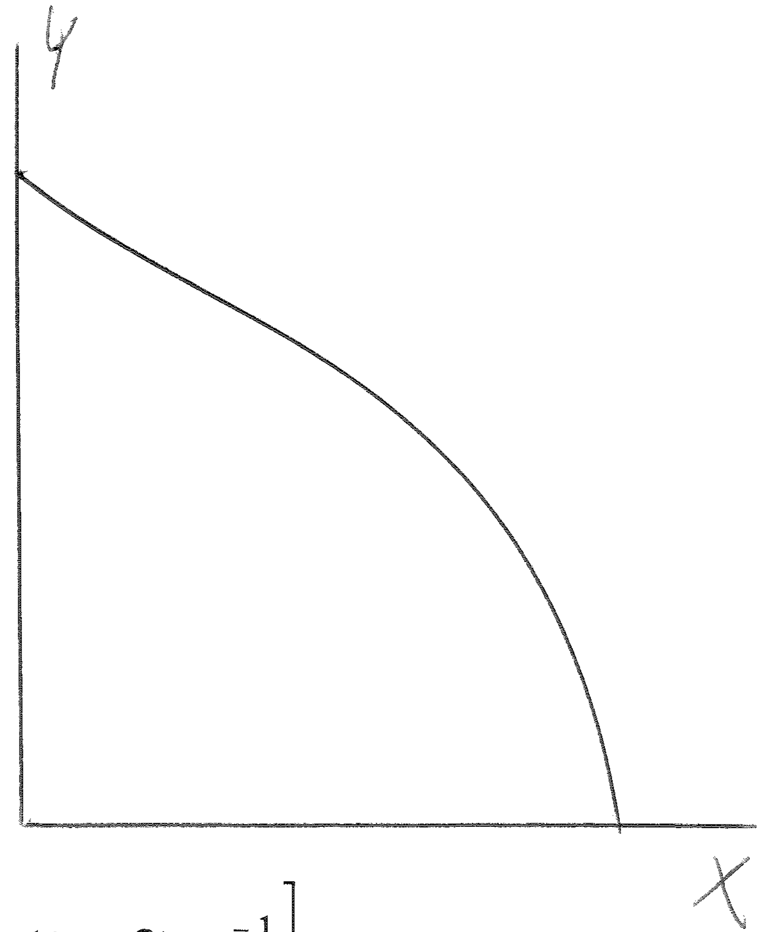
$$\frac{d\left[\frac{dY}{dX}\right]}{dL_x} > 0 \quad iff \quad \beta > (1-\alpha)$$

convex if strong scale effects (α large), weak factor-intensity effects (β large)

Case 3: $Y = L_y^{\beta} K^{1-\beta} \qquad X = (X^{\alpha})L_x = L_x^{\frac{1}{1-\alpha}}$

$$dY = \beta L_y^{\beta-1} K^{1-\beta} dL_y = -\beta L_y^{\beta-1} K^{1-\beta} dL_x$$

$$dX = \frac{1}{1-\alpha} L_x^{\frac{\alpha}{1-\alpha}} dL_x = \frac{1}{1-\alpha} X^{\alpha} dL_x$$



$$\frac{dY}{dX} = -\frac{\beta(1-\alpha)K^{1-\beta}}{L_x^{\alpha/(1-\alpha)}(L - L_x)^{1-\beta}}$$

$$\frac{d\left[\frac{dY}{dX}\right]}{dL_x} = (+)\, L_x^{\alpha/(\alpha-1)} L_y^{\beta-1}\left[\frac{\alpha}{1-\alpha} L_x^{-1} - (1-\beta) L_y^{-1}\right]$$

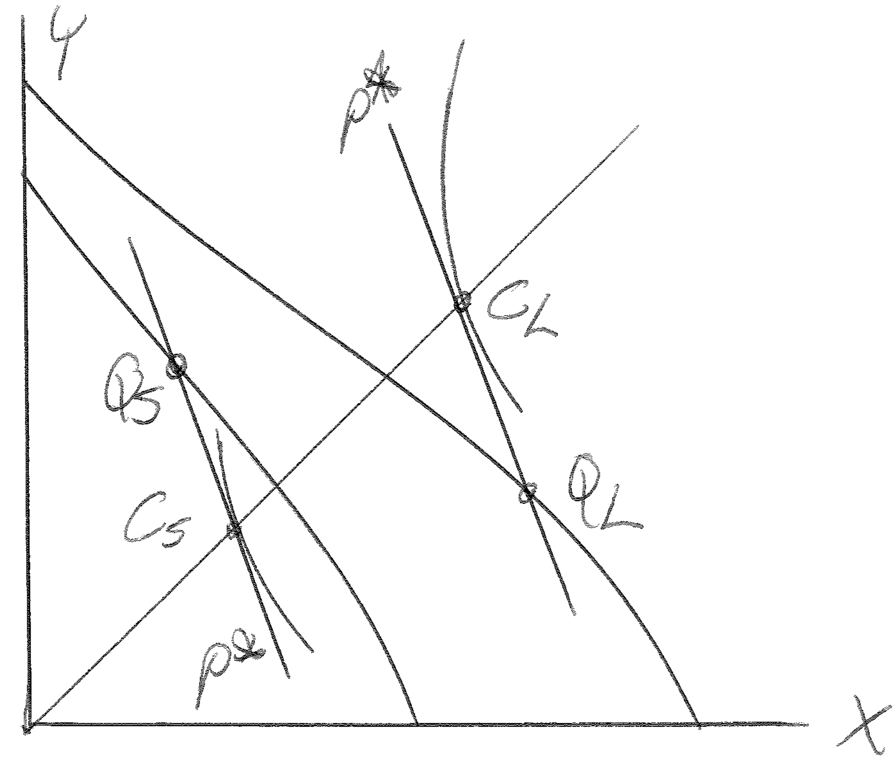
(+) near $L_x = 0$ (convex) and (-) near $L_y = 0$ (concave)

Size as a basis for trade (size as a determinant of comparative advantage)

Suppose two countries are identical, except that one has a proportionately larger endowment of all factors. Y has CRS and X has IRS.

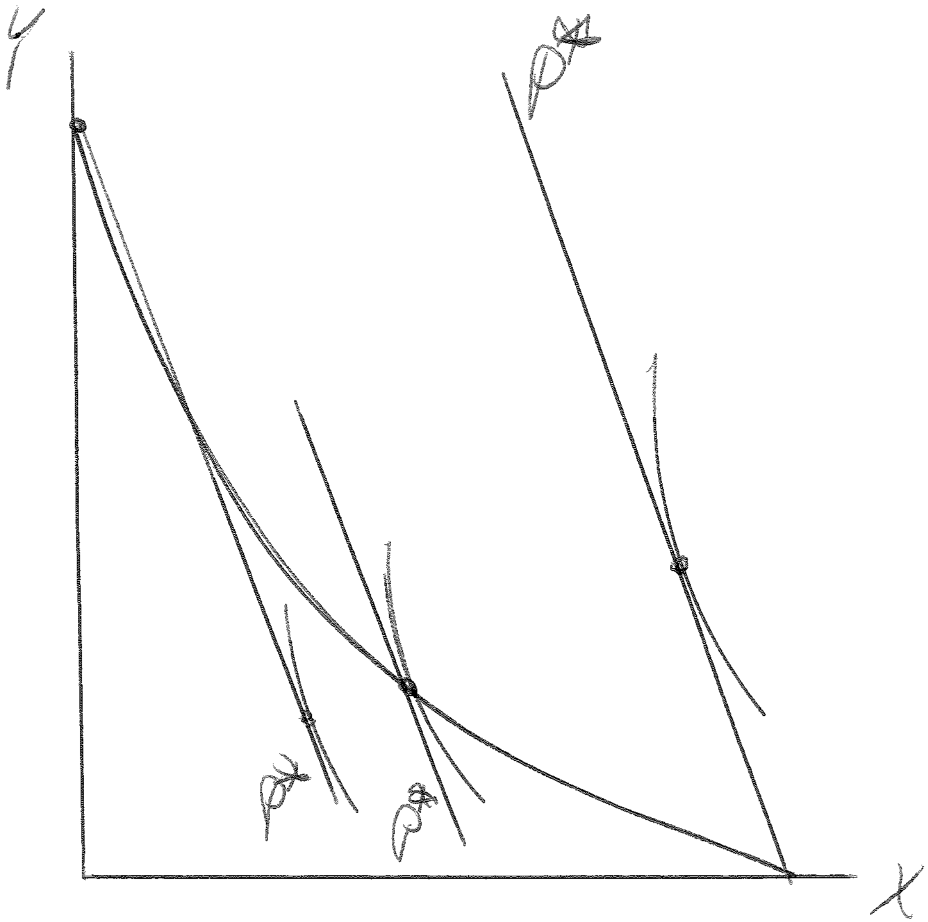
At the same ratio of Y/X production, the larger country will have a lower supply price for X than the small country.

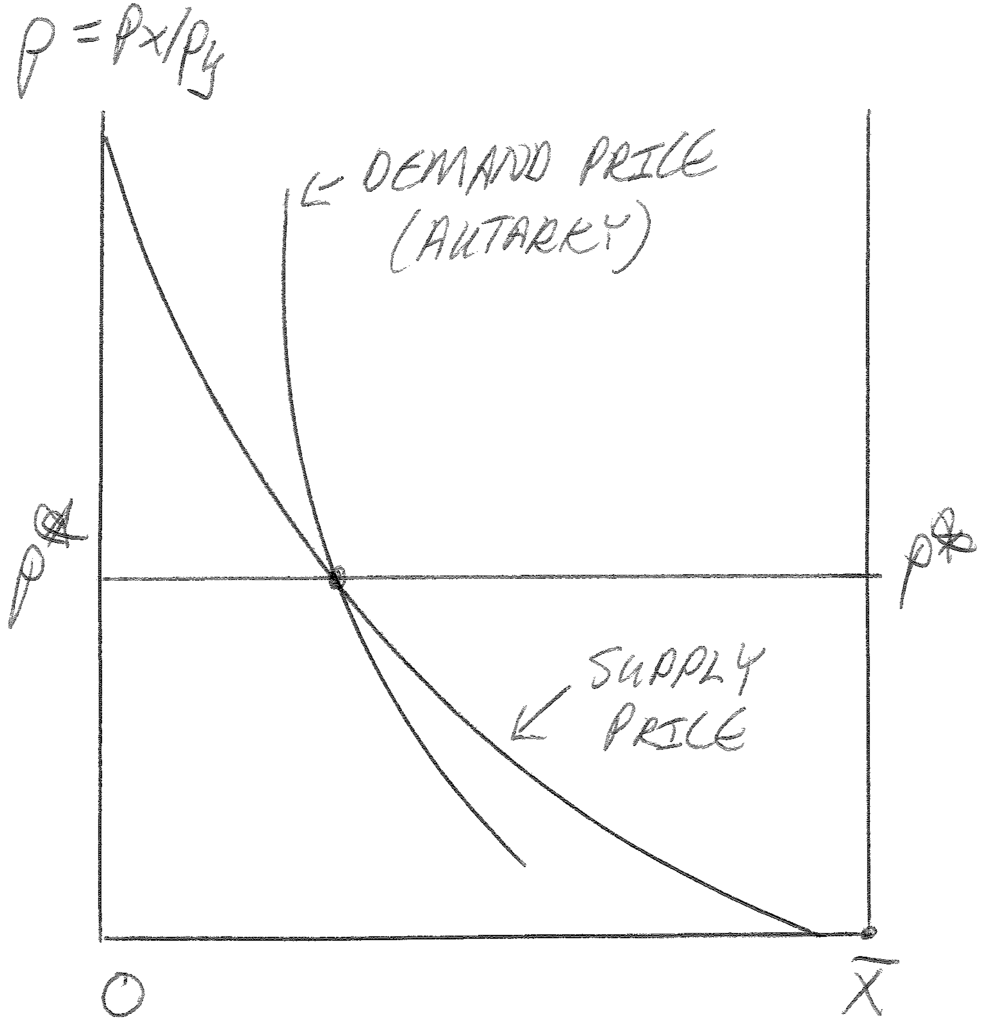
Thus if producer prices are the same in the two countries, the large country must produce a lower ratio of Y/X that the small country. The large country will export X, the IRS good.



Multiple equilibria for a small open economy (with a non-convex production set).

Suppose that the autarky price ratio just happens to equal the world price ratio. There could be three equilibria at this price ratio. One could be welfare inferior to autarky.

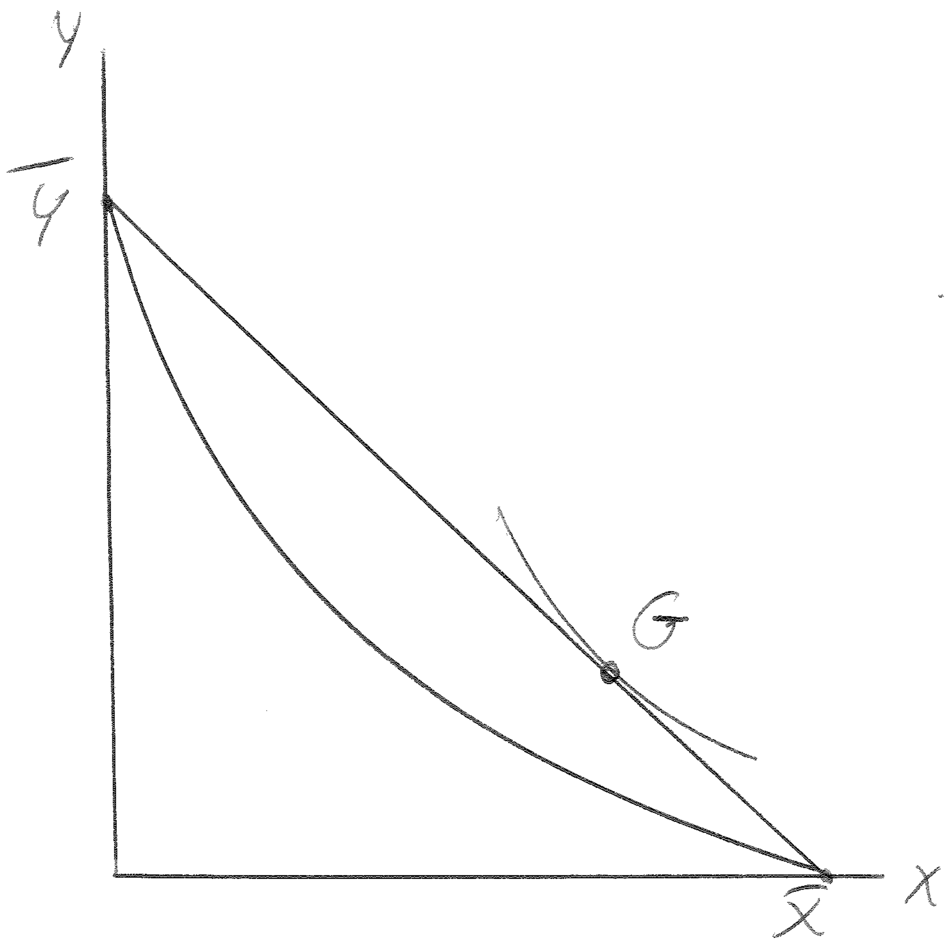


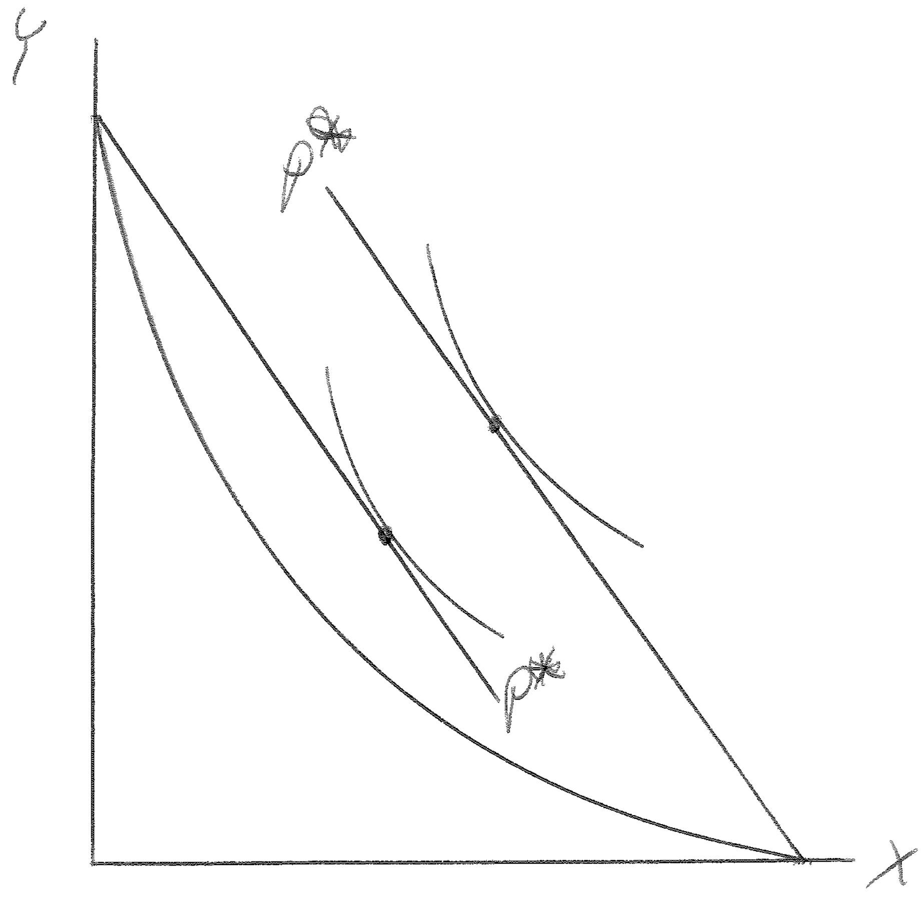


Multiple equilibria for two identical non-convex economies.

What if both countries want to consume at point G when the price ratio is the cord connecting the end points of the ppf?

The equilibrium price ratio must be steeper than the diagonal. The country that gets (accident?) to specialize in X's better off.





"International" external economies. Two countries, s (small) and b(big).

$$X_s = (X_s + X_b)^{\alpha} F(V_s) \qquad X_b = (X_s + X_b)^{\alpha} F(V_b)$$

Size ceases to be a source of comparative advantage. However, to the extent that trade is needed to transmit the externality, there are gains from trade, especially for the large country.

External Economies of Scale - Summary Points

1. Increasing returns to scale provide a source of gains from trade for identical economies or for economies of different sizes.

2. Because of the inevitable distortion between price and marginal cost, positive gains from trade cannot be assured.

3. So on the one hand, IRS offer an important additional, non-comparative-advantage source of gains, but on the other hand gains are not assured.

4. Multiple equilibria may exist, with very different welfare levels. This represents a huge challenge for public policy, and requires "non-marginal analysis".

5. Factor prices are generally not equalized by free trade, and in many cases each country has the relatively high price for the factor used intensively in its export industry. We will return to this point later.

6. International external economies of scale further add to the possible gains from trade. We will also return to this point later in connection with the monopolistic-competition model.