Why?

Understand the determinants of what goods and services a country produces efficiently and which inefficiently.

Understand how the processes of a market economy produces outcomes (allocations), and whether or not they are efficient outcomes.

Understand general-equilibrium relationships, such as the relationship between barriers to trade, and the domestic distribution of income.

1. The production possibility frontier

   Position:
   (1) factor endowments
   (2) real factor productivities

   Slope:
   (1) relative factor productivities
   (2) factor endowments

   Curvature:
   (1) constant returns to scale
   (2) factor intensity effects
   (3) increasing returns to scale

2. Equilibrium for a single competitive producer

   Value of marginal product = factor price

3. The Edgeworth Box

   Contract curve, efficient allocations
   Contract curve maps into PPF
   Competitive equilibrium implies production efficiency (production point is on the PPF)

4. Competitive equilibrium

   Establish that a competitive equilibrium is a tangency between the PPF and equilibrium prices.

(A) Total factor productivity

$$X = \alpha F(V_1, V_2) \qquad \alpha \text{ is a "neutral" productivity parameter} \qquad (2.1)$$

(B) Returns to scale

(i) $X = V^{\gamma}, \ \gamma = 1$ Constant returns to scale

(ii) $X = V^{\gamma}, \ \gamma > 1$ Increasing returns I

(iii) $X = \max[V - FC, 0]$ Increasing returns II (2.2)

Figure 2.1

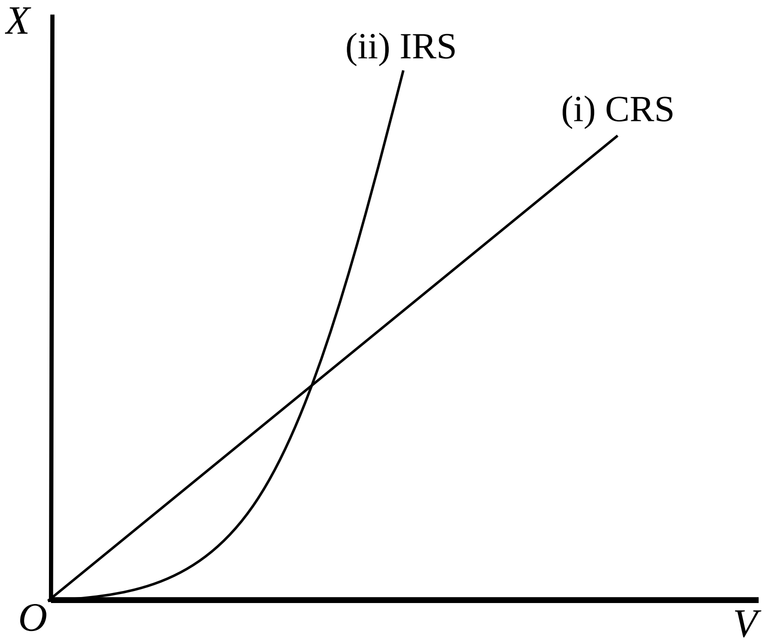
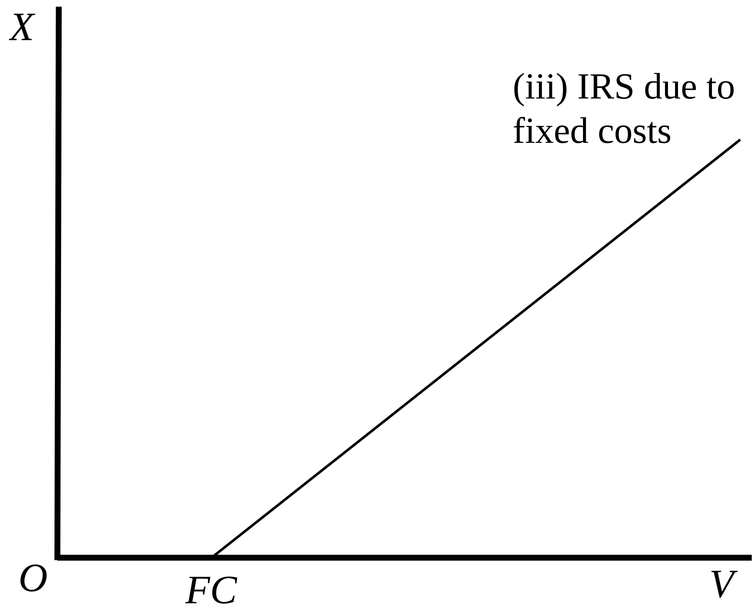


Figure 2.2



(C) Factor substitution and diminishing marginal product.

(i) $X = V_1 + V_2$ Straight line (perfect substitutes)

(ii) $X = \min[V_1, V_2]$ Right angle (perfect complements)

(iii) $X = V_1^{\beta} V_2^{1-\beta} \quad 0 < \beta < 1$ Smooth Curve (Cobb-Douglas)

(2.3)

(D) Marginal rate of substitution

$$dX = 0 = \beta V_1^{\beta - 1} V_2^{1-\beta} dV_1 + (1 - \beta) V_1^{\beta} V_2^{-\beta} dV_2 \tag{2.4}$$

$$MRS = \frac{dV_2}{dV_1} = \frac{\beta}{1-\beta}\frac{V_2}{V_1} \tag{2.5}$$

Figure 2.3

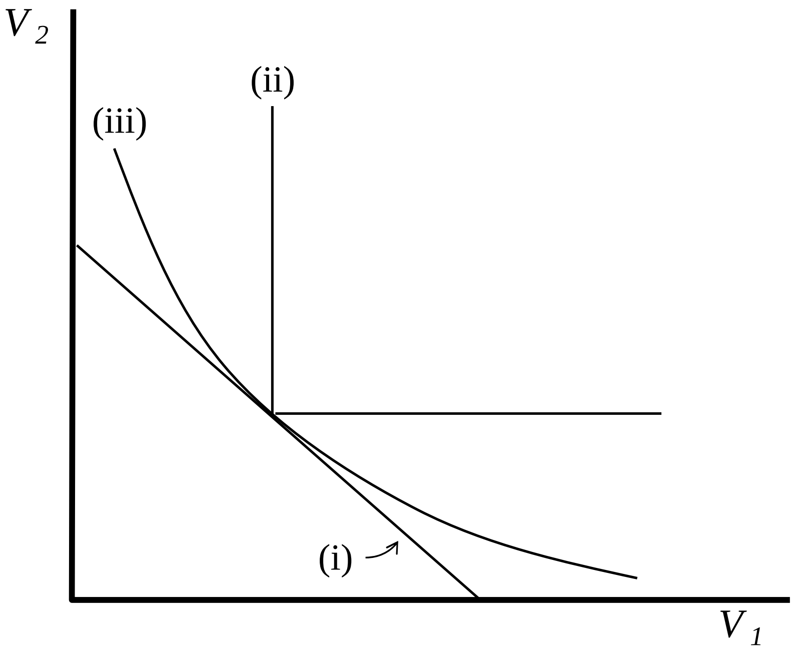
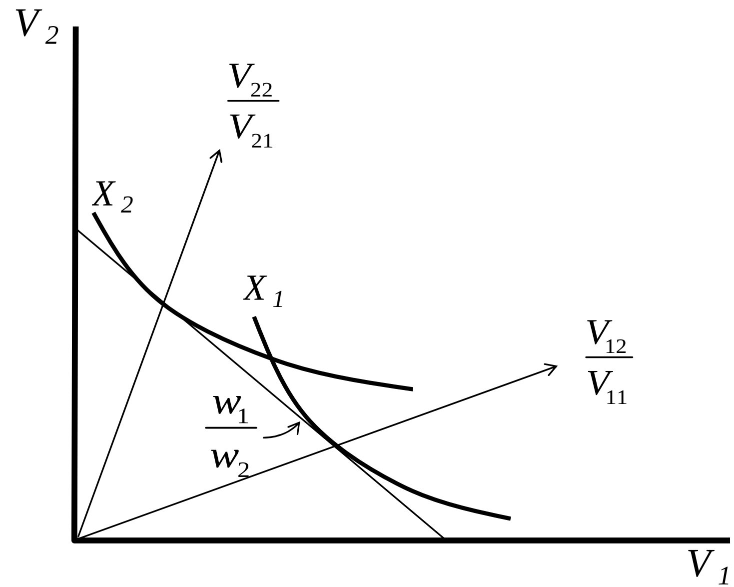


Figure 2.4



$$\Pi = pF(V_1, V_2) - w_1 V_1 - w_2 V_2 \tag{2.8}$$

$$\frac{\partial \Pi}{\partial V_1} = pF_1(V_1, V_2) - w_1 = 0 \qquad F_1 \equiv \frac{\partial F(V_1, V_2)}{\partial V_1} = MP_1 \tag{2.9}$$

$$\frac{\partial \Pi}{\partial V_2} = pF_2(V_1, V_2) - w_2 = 0 \qquad F_2 \equiv \frac{\partial F(V_1, V_2)}{\partial V_2} = MP_2 \tag{2.10}$$

$$MRS = \frac{MP_1}{MP_2} = \frac{F_1}{F_2} = \frac{w_1}{w_2} \tag{2.11}$$

$$\frac{V_{22}}{V_{21}} > \frac{V_{12}}{V_{11}} \qquad \text{at} \quad MRS_2 = MRS_1 = \frac{w_1}{w_2}$$

$$\Rightarrow \text{ industry 2 is factor 2 intensive} \tag{2.12}$$

suppose both goods have Cobb-Douglas technologies as in (2.3) above, but with different values of β and specifically $\beta_1 > \beta_2$.

$$\frac{V_{22}}{V_{21}} = \frac{1 - \beta_2}{\beta_2}\frac{w_1}{w_2} > \frac{V_{12}}{V_{11}} = \frac{1 - \beta_1}{\beta_1}\frac{w_1}{w_2} \tag{2.13}$$

(A) Total factor productivity: absolute and relative. One factor of production and constant returns to scale in both of two industries.

The single factor is in fixed supply $\bar{V}$ and is allocated between sectors 1 and 2 in the amounts V_1 and V_2.

$$X_1 = \alpha_1 V_1 \qquad X_2 = \alpha_2 V_2 \qquad V_1 + V_2 = \bar{V} \tag{2.14}$$

$$-\frac{dX_2}{dX_1} = \frac{\alpha_2}{\alpha_1} = MRT \qquad \text{slope of ppf is a constant}$$

Figure 2.5

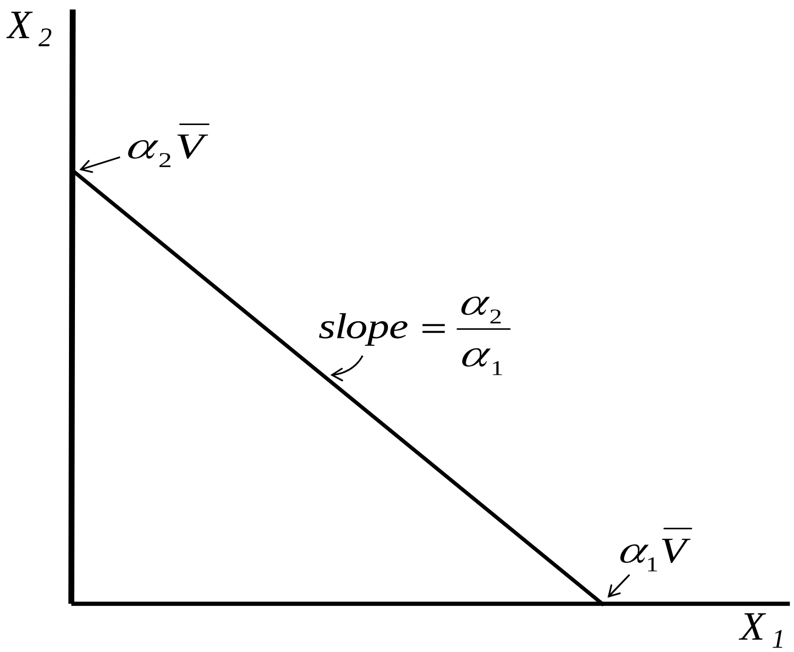
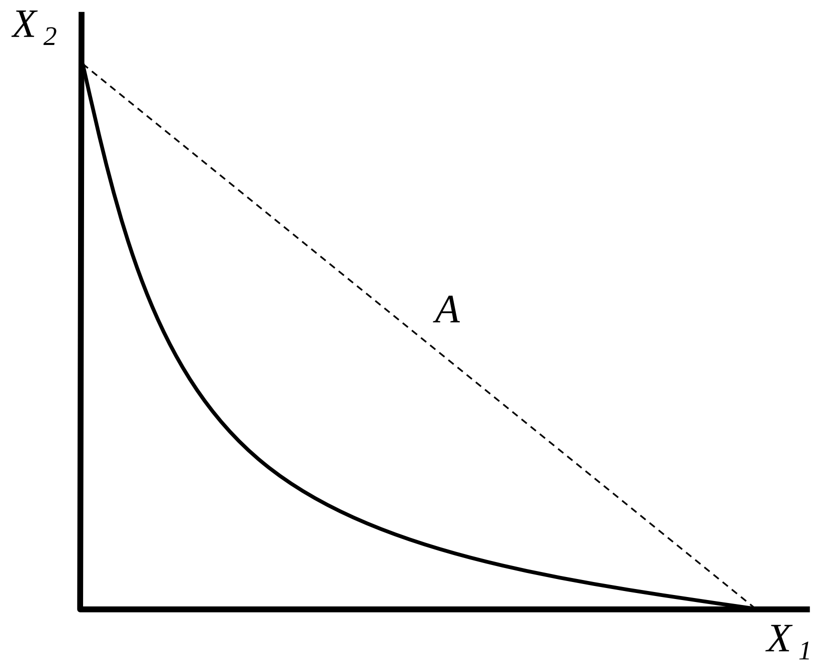


Figure 2.6



(B) Increasing returns to scale I:

$$X_1 = V_1^{\gamma} \quad X_2 = V_2^{\gamma} \qquad \gamma > 1, \qquad V_1 + V_2 = \bar{V} \tag{2.15}$$

$$-\frac{dX_2}{dX_1} = \frac{V_2^{\gamma - 1}}{V_1^{\gamma - 1}}, \qquad V_i^{\gamma - 1} = X_i^{\frac{\gamma - 1}{\gamma}}, \tag{2.16}$$

$$-\frac{dX_2}{dX_1} = \left[\frac{X_2}{X_1}\right]^{\frac{\gamma - 1}{\gamma}} = MRT$$ production frontier in convex

("bowed in")

(C) Increasing returns to scale II (IRS due to fixed costs

$$X_1 = \max[V_1 - FC, 0] \qquad X_2 = \max[V_2 - FC, 0] \qquad V_1 + V_2 = \bar{V} \tag{2.17}$$

The production frontier is "kinked": segment with both goods produced is linear.

But in a crucial way it has the same property as the "smooth" case.

Any point on a line connecting the two endpoints is NOT a feasible production point.

There is an efficiency cost to producing both goods.

Figure 2.7

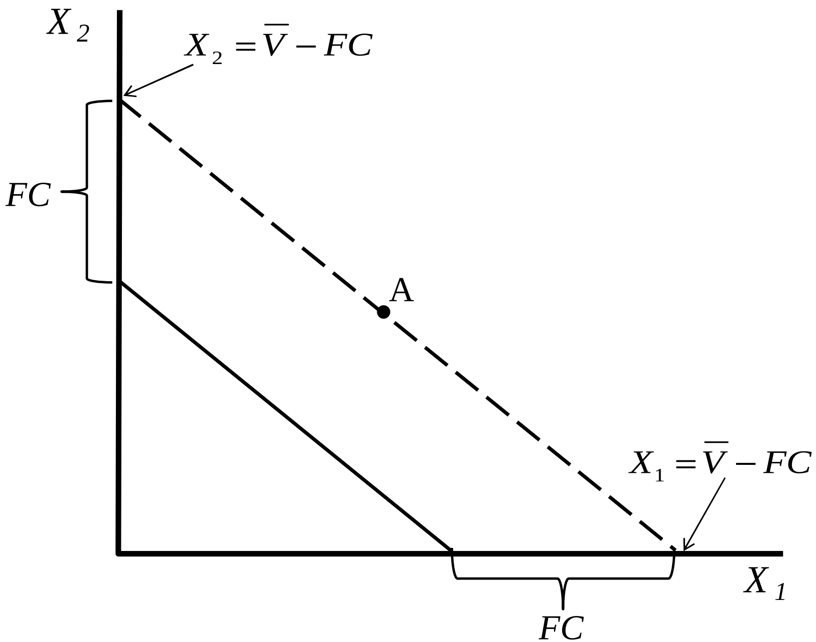
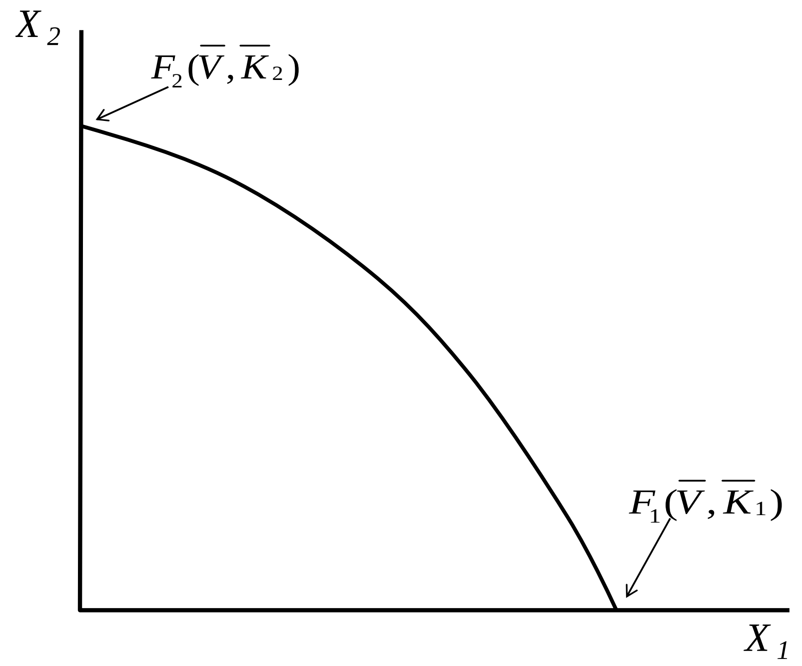


Figure 2.8



(D) Factor intensities I: specific factors. Constant returns to scale

$$X_1 = F_1(V_1, \bar{K}_1) \qquad X_2 = F_2(V_2, \bar{K}_2) \qquad V_1 + V_2 = \bar{V}$$

$$-\frac{dX_2}{dX_1} = \frac{F_{21}(V_1, \bar{K}_1)}{F_{11}(V_2, \bar{K}_2)} \quad \text{Convex by diminishing MP} \qquad (2.18)$$

("bowed out")

(E) Factor intensities II: Heckscher-Ohlin. Constant returns to scale

$$X_1 = F_1(V_{11}, V_{12}) \qquad X_2 = F_2(V_{21}, V_{22})$$

$$V_{11} + V_{21} = \bar{V}_1 \qquad V_{12} + V_{22} = \bar{V}_2 \qquad (2.19)$$

Figure 2.9

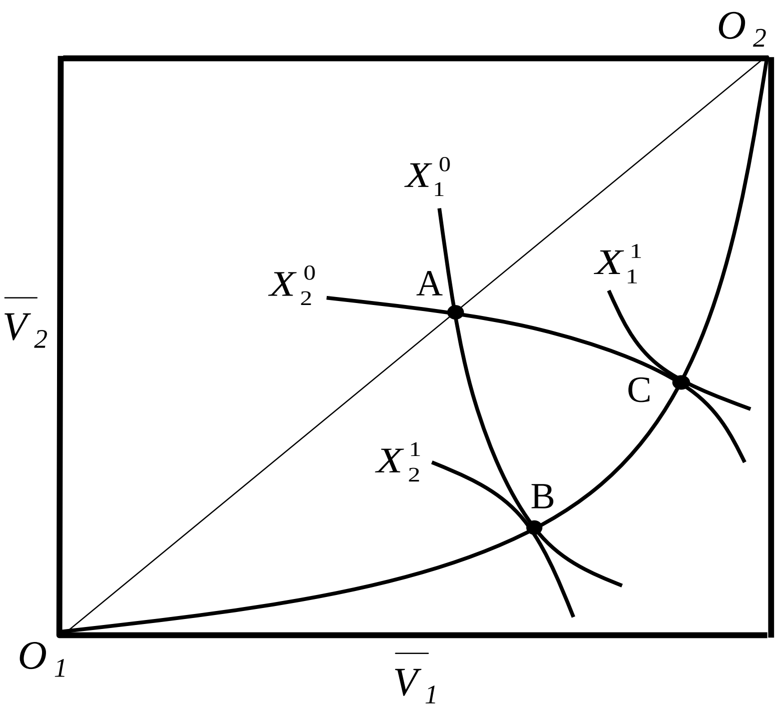
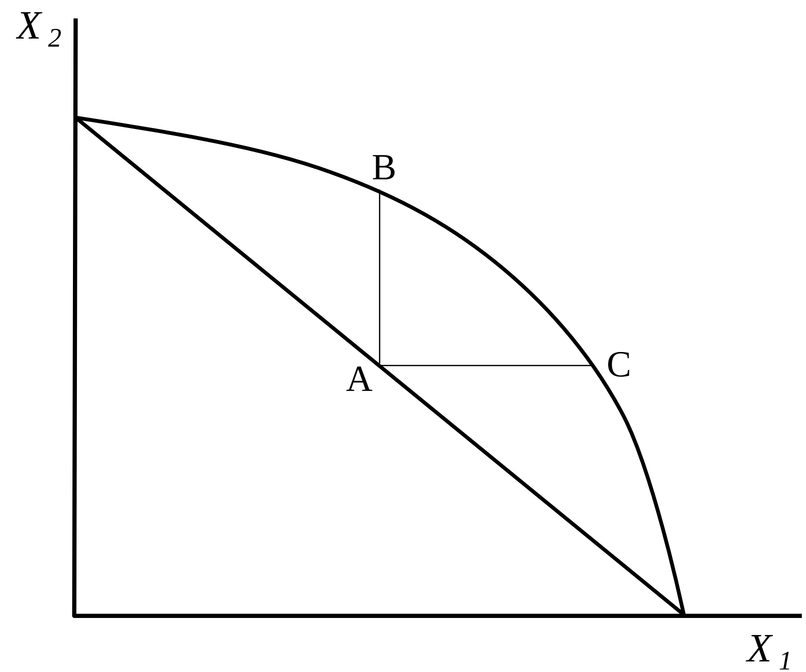


Figure 2.10



First-order conditions for profit maximization

$$p_1F_{11}(V_{11},V_{12}) = w_1 \qquad p_1F_{12}(V_{11},V_{12}) = w_2 \tag{2.20}$$

$$p_2F_{21}(V_{21},V_{22}) = w_1 \qquad p_2F_{22}(V_{21},V_{22}) = w_2$$

The first implication of these condition is factor market allocation efficiency. If we divide the first equation in each row of (2.20) by the second, we see that

$$MRS_1 = \frac{F_{11}}{F_{12}} = MRS_2 = \frac{F_{21}}{F_{22}} = \frac{w_1^0}{w_2^0} \tag{2.21}$$

The fact that the MRS in each industry is equal tells us that the competitive outcome must be on the contract curve in Figure 2.9 => on ppf.

Second, we can differentiate the production function for each good, and replace the physical marginal products F_{ij} with w_j/p_i from (2.20).

$$dX_1 = \sum_j F_{1j} dV_{1j} = \sum_j \left[\frac{w_j}{p_1}\right] dV_{1j} = \frac{1}{p_1} \sum_j (w_j dV_{1j}) \qquad (2.22)$$

$$dX_2 = \sum_j F_{2j} dV_{2j} = \sum_j \left[\frac{w_j}{p_2}\right] dV_{2j} = \frac{1}{p_2} \sum_j (w_j dV_{2j})$$

The summations over the factors on the right-hand side are simply one subtracted from the

$$MRT = -\frac{dX_2}{dX_1} = \frac{p_1}{p_2} \text{ tangency of ppf and price ratio} \qquad (2.23)$$

Figure 2.11

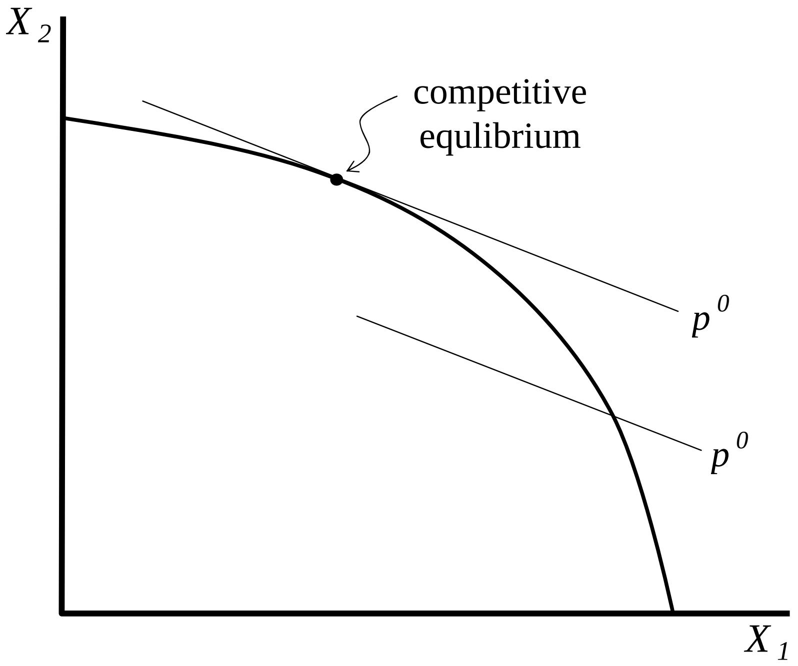
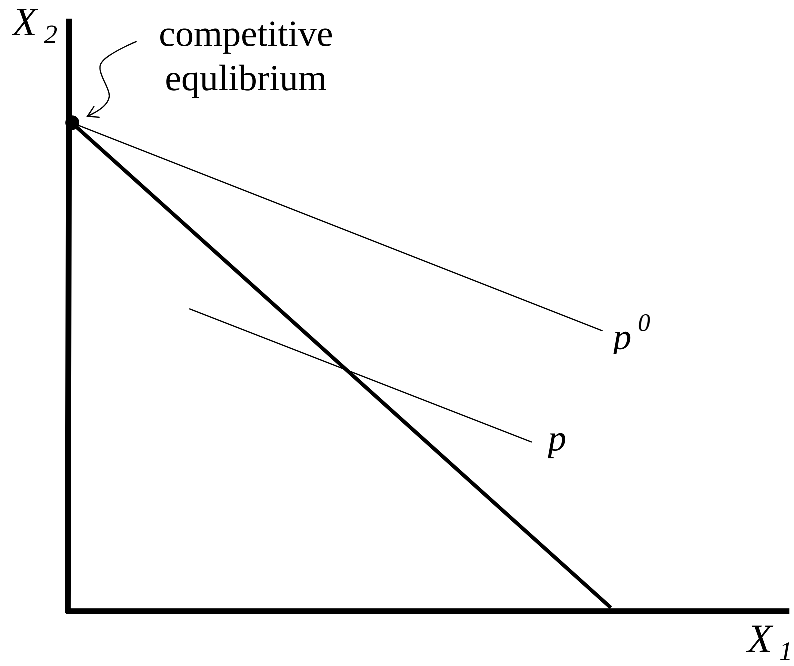


Figure 2.12



Suppose that we observed that the output vector X^0 , and the input matrix V^0 are chosen at commodity and factor prices p^0, $w^{0.}$

Suppose that X^1 , V^1 is any alternative *feasible* production plan.

For each industry i, profit maximization implies that

$$p_i^0 X_i^0 - \sum_j w_j^0 V_{ij}^0 \geq p_i^0 X_i^1 - \sum_j w_j^0 V_{ij}^1 \qquad (2.24)$$

If competitive producers are optimizing, the actual output and inputs chosen must yield profits which are greater than or equal to the profits obtained from any other feasible production plan. Sum over all i

$$\sum_i p_i^0 X_i^0 - \sum_i \sum_j w_j^0 V_{ij}^0 \geq \sum_i p_i^0 X_i^1 - \sum_i \sum_j w_j^0 V_{ij}^1 \qquad (2.25)$$

Suppose that endowments are fixed at $\bar{V}$. For each factor j

$$\sum_i w_j^0 V_{ij}^0 = \sum_i w_j^0 V_{ij}^1 = w_j^0 \bar{V}_j \tag{2.26}$$

Thus the summation terms on the two sides of (2.24) cancel out.

Therefore, the outputs X^0 chosen at prices p^0 maximize the value of production at those prices: (2.24) becomes

$$\sum_i p_i^0 X_i^0 \geq \sum_i p_i^0 X_i^1 \tag{2.27}$$

Geometrically, all feasible production points such as X^1 must lie on or below the price hyperplane p^0 through the actual production point X^0. The price plane is "supporting" to the production set.

(1) How the slope of c country's production frontier depends on its technology and its factor endowments

(2) How the curvature of the production frontier depends on

- (a) factor endowments
- (b) returns to scale

(3) At given prices, undistorted competitive economies produce efficiently.

this depends on:

- (a) competition in all markets
- (b) all firms face the same, true factor prices
- (c) all firms face the same true output prices
- (d) no externalities