Homework 1
ECON 4818
Professor Martins-Filho
Due date: 2.3.2012

1. a) Let X be a random variable with density function given by

$$f(x) = \begin{cases} 2(1-x) & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}.$$

Obtain $E(X)$ and $V(X)$.
b) The joint density of random variables X_1 and X_2 is given by

$$f(x_1, x_2) = \begin{cases} 2 & \text{for } 0 < x_1 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}.$$

b1) Obtain the marginal densities of X_1, X_2. b2) Obtain the conditional density of X_1 given that $X_2 = x_2$ and $E(X_1|X_2 = x_2)$ and $V(X_1|X_2 = x_2)$.

2. Prove the following equalities for any two random variables X and Y.

a) $V(aX + bY) = a^2V(X) + b^2V(Y) + 2abCOV(X, Y)$.
b) Let $\{X_i\}_{i=1}^n$ be a sequence of independent and identically distributed random variables with $E(X_i) = \mu$ and $V(X_i) = \theta$, then $E\left(\sum_{i=1}^n X_i\right) = n\mu$ and $V\left(\sum_{i=1}^n X_i\right) = n\theta$.
c) $V(Y) = E\left(V(Y|X)\right) + V\left(E(Y|X)\right)$.
d) If X and Y are independent then $V(Y|X) = V(Y)$.
e) If $E(Y|X) = \beta_0 + \beta_1 X$ then $\beta_1 = \frac{Cov(X,Y)}{V(X)}$.

2. From your textbook do the following problems:

a) B.4, B.6, B.10 pages 745-746.