University of Colorado
Department of Mathematics

2016/2017 Semester 2 | Math 8340 Functional Analysis 2 | Assignment 1

Due Friday Feb 3, 2017

1. Do the following problems in Conway: p. 7, Exercise 7; p. 11, Exercises 1, 5, 6.
2. Prove the *polarization identity*: if $\mathcal{H}$ is a complex Hilbert space and $f,\ g \in \mathcal{H}$, then

$$\langle f, g\rangle \;=\; \frac{1}{4}\sum_{j=0}^{3} i^j \|f + i^j g\|^2.$$

3. Suppose $\mathcal{X}$ is a Banach space whose norm satisfies the parallelogram law defined in lectures. Use Exercise 2 above to show that there is an inner product on X, $\langle\cdot,\cdot\rangle_{\mathcal{X}}$, satisfying

$$\langle x, x\rangle_{\mathcal{X}} = \|x\|^2.$$

(Thus $\mathcal{X}$ can be viewed as a Hilbert space.)

4. Give an example of a proper closed subspace $\mathcal{M}$ of an inner product space $\mathcal{X}$ for which $\mathcal{M}^\perp = \{0\}$.

5. Suppose that the bounded operator P is a projection in the Hilbert space $\mathcal{H}$, i.e. suppose $P^* = P$ and $P^2 = P$. Let $\mathcal{M} = \text{Range}(P)$. Prove that P is exactly the projection of $\mathcal{H}$ onto $\mathcal{M}$ as defined in lectures (Thus in a Hilbert space $\mathcal{H}$, the set of projections is in one-to-one correspondence with the set of closed subspaces of $\mathcal{H}$).