University of Colorado
Department of Mathematics

2018/19 Semester 1 | Math 6310 Real Analysis 1 | Assignment 1

Due Wednesday September 12, 2018

1. Let (X, ρ) be a metric space, let $a \in X$, and $\epsilon > 0$. Prove that $B(\epsilon, a) = \{x \in X : \rho(x, a) < \epsilon\}$ is open in X.

2. Let (X, d) be a metric space. Define $\rho : X \times X \to [0, \infty)$ by

$$\rho(x, y) = \frac{d(x, y)}{1 + d(x, y)}, \; x, y \in X.$$

Prove that ρ is a metric on X.

   (a) Prove that for any $x \in X$ and any $r > 0$,

$$B_d(r, x) = B_\rho(\frac{r}{1 + r}, x).$$

   (b) Use (a) to prove that if G is open in the metric space (X, d), then G is open in the metric space (X, ρ).

3. Let X be an uncountable set. Let

$$\mathcal{A} = \{U \subset X : U \text{ is finite}\} \cup \{V \subset X : X \backslash V \text{ is finite}\}.$$

   (a) Prove that $\mathcal{A}$ is an algebra of sets of X.

   (b) Is $\mathcal{A}$ a σ-algebra of subsets of X? If not, describe the smallest σ-algebra containing $\mathcal{A}$.

4. Do exercises 1, 2, 3 and 4 on p. 24 , and 7, 9 and 10 on p. 27 of the Folland textbook.