University of Colorado
Department of Mathematics

2017/2018 Semester 2 | Math 8370 Harmonic Analysis | Assignment 1

Due Wednesday January 31, 2018

1. Do the following problems in the Deitmar textbook: p. 19–21 Exercises no. 1.1, 1.3, 1.4, 1.5, 1.9, 1.13.
2. Let $f \in L^1(\mathbb{T})$, and suppose that $\sum_{n\in\mathbb{Z}} \hat{f}(n)e^{2\pi inx}$ converges almost everywhere to the function g defined on $\mathbb{T}$. Suppose there exists $h \in L^1(\mathbb{T})$ such that

$$|\sum_{n=-N}^{n=N} \hat{f}(n)e^{2\pi inx}| \ \leq \ h(x), \ \forall N \ \in \mathbb{N}.$$

[Here $\hat{f}(n) = \int_0^1 f(x)e^{-2\pi inx}dx$.]
Prove that $g \in L^1(\mathbb{T})$, and $f = g$ in $L^1(\mathbb{T})$. That is, f is equal to its Fourier expansion in $L^1(\mathbb{T})$.