

University of Colorado
Department of Mathematics

2017/18 Semester 2

Math 8370 Harmonic Analysis

Assignment 2

Due Friday February 16, 2018

1. Do problems 1.11, 1.12, 1.14, 1.15, and 1.17 pp. 21–23 of the Deitmar textbook.
2. Suppose that f is continuously differentiable on $[0, \frac{1}{2}]$ with $f(0) = f(\frac{1}{2}) = 0$. Prove that

$$\int_0^{\frac{1}{2}} |f(t)|^2 dt \leq \int_0^{\frac{1}{2}} |f'(t)|^2 dt.$$

[Hint: extend f to be an odd periodic function on $[-\frac{1}{2}, \frac{1}{2}]$, and then do some Fourier analysis.]

3. Let f be continuous on \mathbb{R} , with period 1, and suppose that α is an irrational real number. Prove that

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=0}^{N-1} f(t + k\alpha)$$

exists and is equal to $\int_0^1 f(t) dt$.

[Hint: first do the problem when f is a trigonometric polynomial, $f(x) = \sum_{j=-M}^M a_j e^{2\pi i j x}$.]